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Progress on pseudoscalar flavour-singlets in Sp(4) with mixed fermion representations

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We measure the masses of the pseudoscalar flavour-singlet meson states in the $Sp(4)$ gauge theory coupled to two Dirac fermions transforming in the fundamental representation and three Dirac fermions in the antisymmetric representation. This theory provides a compelling ultraviolet completion for the minimal composite Higgs model implementing also partial compositeness for the top quark. The spectrum contains two, comparatively light, pseudoscalar flavour-singlet states, which mix with one another. One of them is a Nambu-Goldstone boson (in the massless limit), whereas the other receives a mass from the $U(1)_A$ axial anomaly. We demonstrate how to measure the mixing between these two states. For moderately heavy fermion masses, we find that the two wave functions are dominated by one of the fermion representations, mixing effects being small.

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1. Introduction

Gauge theories with matter field content transforming as an admixture of representations of the gauge group have been proposed as short-distance completions of composite Higgs models that implement also partial compositeness for the top quark [1–4]. In these models, the Higgs boson is a composite state of fermions (hyperquarks) charged under a new, non-Abelian gauge symmetry mediated by additional gauge bosons (hypergluons). The global symmetries of these theories factorize into two (non-Abelian) cosets, one of which is related to the physics of the Higgs boson, and the other to the properties of the heavy quarks (the top).[1] The $Sp(4)$ gauge theory coupled to $N_{\mathrm{as}} = 3$ Dirac fermions transforming in the two-index antisymmetric representation and $N_{\mathrm{f}} = 2$ in the fundamental is the minimal such theory that meets all phenomenological requirements, and can be tested numerically on the lattice [6]—see also the reviews [7–9], and the summary tables in Refs. [10–12].

Interest in new gauge theories with $Sp(2N)$ gauge group has motivated the development of the research programme of *Theoretical Explorations on the Lattice with Orthogonal and Symplectic groups* (TELOS) [13–26]—see also Refs. [27–30] for related applications to dark matter models—and the work reported in this contribution, and the associated publication [26], are part of this programme. Other lattice studies of gauge theories with mixed representations include the $SU(4)$ gauge theory with fundamental and antisymmetric Dirac fermions [31–41], and $SU(2)$ gauge theory with fundamental and adjoint matter [42, 43].

A distinctive feature of gauge theories with matter in an admixture of multiple representations is the occurrence of an additional, flavour-singlet, Abelian pseudo-Nambu-Goldstone boson (pNGB), with peculiar phenomenological properties [44, 45]. This is due to the existence of an independent axial $U(1)$ current for each fermion representation. Only one combination of those currents is broken by the axial anomaly. The associated state (which in QCD can be identified with the η' meson) acquires a mass even in the limit in which the hyperquarks are massless [46–48], while the remaining flavour-singlet pseudoscalar state does not. This phenomenology of the additional pNGB has been studied in the context of composite Higgs models [12, 49, 50]. The relevant leading-order description within chiral perturbation theory has been developed in Ref. [45]. For the purposes of this work we refer to the light pNGB state as η'_l, while the state associated with the axial anomaly is denoted as η'_h. This contribution reports the highlights of non-perturbative lattice investigations into the masses of the η'_l and η'_h, as well as their mixing angle, ϕ, and we refer the reader to Ref. [26] for details and extensive discussions.

2. Lattice field theory observables

The (Minkowski space) Lagrangian density of the strongly interacting theory is given by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_{I=1}^{2} \bar{Q}^I \left(i\gamma^\mu D_\mu - m^{\mathrm{f}}\right) Q^I + \sum_{K=1}^{3} \bar{\Psi}^K \left(i\gamma^\mu D_\mu - m^{\mathrm{as}}\right) \Psi^K, \tag{1}$$

where Q denotes the fermions transforming in the fundamental representation and Ψ those in the antisymmetric one. We discretize the theory as described in Sect. 3. We perform lattice

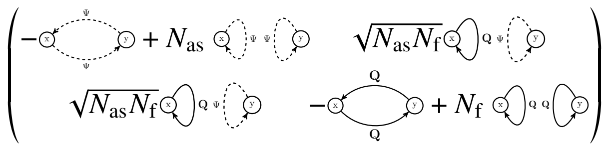
[1] See Ref. [5] for a model in which this can be achieved with one species of fermions.

measurements using gauge-invariant interpolating operators built with two fermions, transforming in a single representation. We define

$$O_{\eta^{\rm as}}(x) \equiv \frac{1}{\sqrt{N_{\rm as}}} \sum_{K=1}^{N_{\rm as}} \bar{\Psi}^K(x)\gamma_5\Psi^K(x), \qquad O_{\eta^{\rm f}}(x) \equiv \frac{1}{\sqrt{N_{\rm f}}} \sum_{I=1}^{N_{\rm f}} \bar{Q}^I(x)\gamma_5 Q^I(x)\,. \tag{2}$$

From these operators we build a correlation matrix for the pseudoscalar flavour-singlet sector. Diagrammatically, the correlation matrix has the following form

$$C(x,y) = \begin{pmatrix} \langle \bar{O}_{\eta^{\rm as}} O_{\eta^{\rm as}} \rangle & \langle \bar{O}_{\eta^{\rm f}} O_{\eta^{\rm as}} \rangle \\ \langle \bar{O}_{\eta^{\rm as}} O_{\eta^{\rm f}} \rangle & \langle \bar{O}_{\eta^{\rm f}} O_{\eta^{\rm f}} \rangle \end{pmatrix} = $$



(3)

We further enlarge the correlation matrix $C(x,y)$ by including several layers of Wuppertal smearing on the operators, as described in Ref. [25]. We extract the energy levels by performing a variational analysis, within the context of a generalized eigenvalue problem (GEVP) defined by projecting to the zero-momentum components, as $C_{ij}(t-t') = \langle \bar{O}_i(t, \mathbf{p}=0) O_j(t', \mathbf{p}=0) \rangle$. The eigenvalues, $\lambda_n(t,t_0)$, and eigenvectors, $v_n(t,t_0)$, of $C(t)$ are defined to obey the relations

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0) C(t_0) v_n(t,t_0). \tag{4}$$

At large Euclidean times, the $n^{\rm th}$ eigenvalue relates to the $n^{\rm th}$ energy level in this channel, E_n, as

$$\lambda_n(t \to \infty, t_0) = A_0 e^{-E_n(t-t_0)}\,. \tag{5}$$

We fit the eigenvalues to a lattice-periodic fit function. The fit-interval is chosen by a visual examination of the effective mass of the corresponding eigenvalue $\lambda_n(t,t_0)$, with t_0 fixed.

The determination of the mixing angle is based on Eq. (3), restricted to unsmeared operators [26]. The eigenvectors correspond to the matrix elements [51]:

$$\begin{pmatrix} \langle 0|O_{\eta^{\rm f}}|\eta'_l\rangle & \langle 0|O_{\eta^{\rm as}}|\eta'_l\rangle \\ \langle 0|O_{\eta^{\rm f}}|\eta'_h\rangle & \langle 0|O_{\eta^{\rm as}}|\eta'_h\rangle \end{pmatrix} = \begin{pmatrix} A_{\rm f}^{\eta'_l} & A_{\rm as}^{\eta'_l} \\ A_{\rm f}^{\eta'_h} & A_{\rm as}^{\eta'_h} \end{pmatrix} \equiv \begin{pmatrix} A_{\eta'_l}\cos\phi_{\eta'_l} & A_{\eta'_l}\sin\phi_{\eta'_l} \\ -A_{\eta'_h}\sin\phi_{\eta'_h} & A_{\eta'_h}\cos\phi_{\eta'_h} \end{pmatrix}. \tag{6}$$

In general, two mixing angles are needed to parameterize the matrix elements [52]. We test whether the mixing could be described by a single mixing angle, ϕ, by examining the following quantities

$$\tan\phi_{\eta'_l} \equiv \frac{A_{\rm as}^{\eta'_l}}{A_{\rm f}^{\eta'_l}}, \qquad -\tan\phi_{\eta'_h} \equiv \frac{A_{\rm f}^{\eta'_h}}{A_{\rm as}^{\eta'_h}}, \qquad -\tan^2\phi \equiv \frac{A_{\rm f}^{\eta'_h} A_{\rm as}^{\eta'_l}}{A_{\rm as}^{\eta'_h} A_{\rm f}^{\eta'_l}}. \tag{7}$$

If the system is parameterized by a single mixing angle, we should find $\phi_{\eta'_l} \approx \phi_{\eta'_h} \approx \phi$ [53]. For this study, we ignore other possible contributions, for instance due to mixing with pseudoscalar glueballs, or other excited states.

Table 1: For each available ensemble, lattice parameters and masses extracted for the pseudoscalar singlet states (η'_l and η'_h) as well as the pseudoscalar non-singlets (PS and ps) and vector non-singlets (V and v).

Label	β	N_t	N_l	am_0^{f}	am_0^{as}	$am_{\eta'_l}$	$am_{\eta'_h}$	am_{PS}	am_{ps}	am_{V}	am_{v}
M1	6.5	48	20	-0.71	-1.01	0.3769(96)	0.6334(59)	0.3639(14)	0.6001(11)	0.4030(33)	0.6452(18)
M2	6.5	64	20	-0.71	-1.01	0.3867(68)	0.619(13)	0.3648(13)	0.59856(82)	0.4038(17)	0.6421(15)
M3	6.5	96	20	-0.71	-1.01	0.3826(67)	0.588(12)	0.3652(16)	0.59940(79)	0.4040(18)	0.6467(21)
M4	6.5	64	20	-0.7	-1.01	0.4381(33)	0.6433(88)	0.4067(13)	0.62426(85)	0.4476(17)	0.6742(13)
M5	6.5	64	32	-0.72	-1.01	0.3591(53)	0.637(26)	0.31076(68)	0.57718(85)	0.3518(12)	0.6223(15)

3. Numerical Setup

Full details of the numerical simulation are provided in Refs. [25, 26]. We generate the gauge configurations on GPU-based machines, using the Grid software environment [54–56], extended to implement symplectic gauge theories [22]. We generate gauge configurations on a lattice volume $L^3 \times T = a^4 N_s^3 \times N_t$ using the Wilson plaquette action and the standard Wilson discretization for the fermions without the clover term. We restrict ourselves to a single lattice spacing ($\beta = 6.5$) and keep the bare mass of the three antisymmetric fermions fixed at $am_0^{\mathrm{as}} = -1.01$. We consider three different values of the bare fundamental fermion mass, $am_0^{\mathrm{f}} = -0.70, -0.71, -0.72$. For $am_0^{\mathrm{f}} = -0.71$, we study three different temporal extensions $N_t = 48, 64, 90$.

We measure correlation functions using the HiRep code [57–59], on CPU-based machines. We implement both APE [60] and Wuppertal [61] smearings. APE smearing is performed with smearing parameters $N_{\mathrm{APE}} = 50$ and $\epsilon_{\mathrm{APE}} = 0.4$. We use three different levels of Wuppertal smearing for each fermion representation, characterized by $N^{\mathrm{smear}} = 0, 40, 80$ smearing steps, respectively, with Wuppertal smearing parameters $\epsilon_f = 0.2$ and $\epsilon_{as} = 0.12$ for the fundamental and antisymmetric fermions. For the disconnected diagrams, we use $n_{\mathrm{src}} = 64$ stochastic sources.

The masses of all accessible flavour non-singlet states have been reported in Ref. [25]. We denote the masses of the pseudoscalar mesons made of fundamental and antisymmetric fermions as m_{PS} and m_{ps}, respectively. Similarly, for vector mesons, m_{V} and m_{v} stand for the flavoured, vector mesons. On a finite lattice, an additive constant contribution appears in the correlation function for the pseudoscalars [62], that we subtract by performing a numerical derivative at the level of the correlation matrix:

$$C_{ij}(t) \to \tilde{C}_{ij}(t) = \frac{C_{ij}(t-1) - C_{ij}(t+1)}{2} \,. \tag{8}$$

All statistical uncertainties are determined using the jackknife method.

4. Results

We find plateaux with modest time extent, both in the ground state and the first excited state in the pseudoscalar flavour-singlet sector. The signal is marginally better for the ground state. As a consequence, we expect the first excited state to be affected by systematic uncertainties due to the short plateaux. We perform fits over a minimum of four time slices. In Fig. 1, we show the effective masses and the effective mixing angle for each of the five ensembles. The extracted energies are

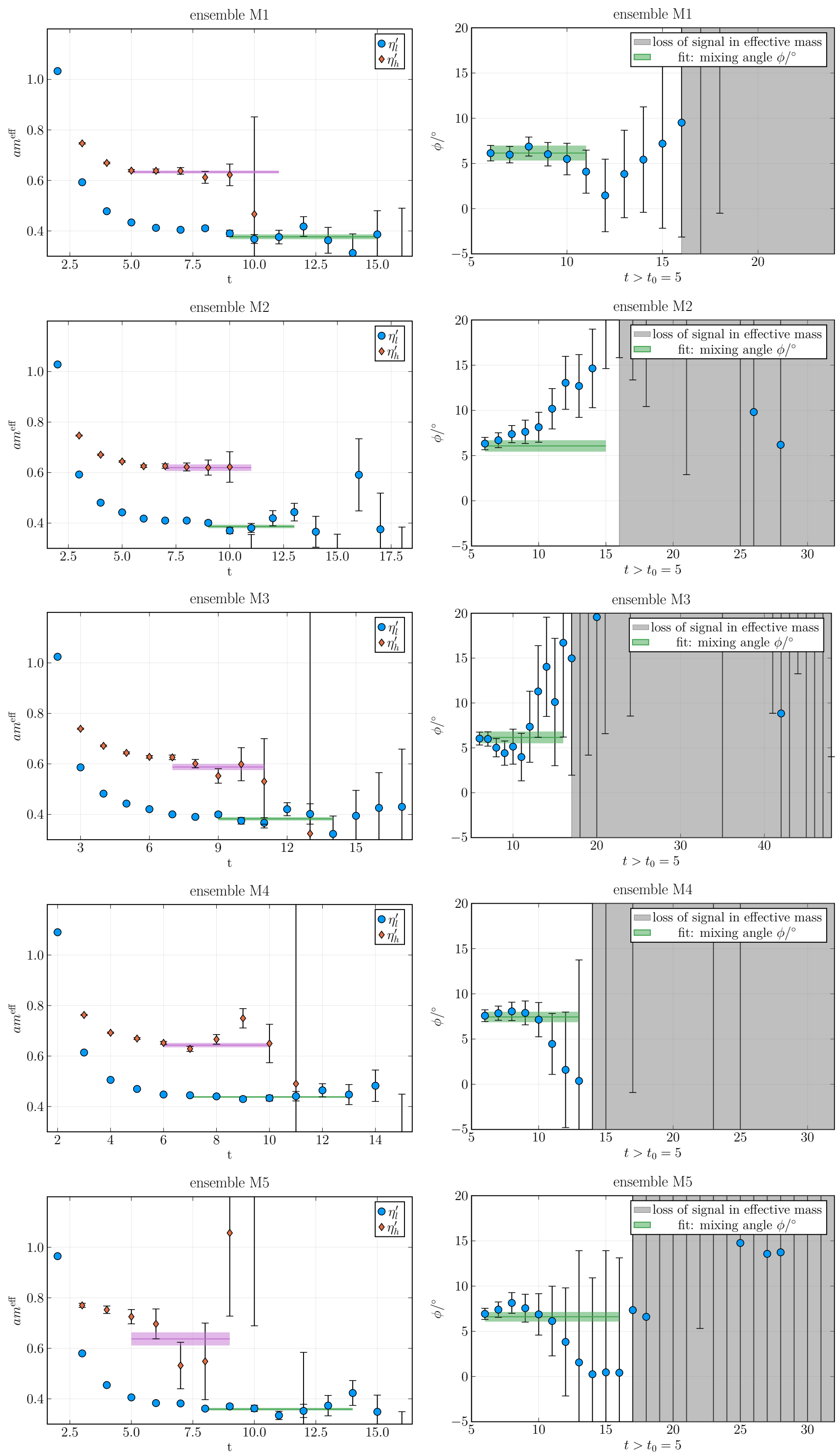


Figure 1: Effective mass (left) and effective average mixing angle (right) for each of the five ensembles.

Label	β	N_t	N_s	$\phi/^\circ$	$\phi_{\eta'_l}/^\circ$	$\phi_{\eta'_h}/^\circ$
M1	6.5	48	20	6.15(83)	3.83(57)	9.8(1.1)
M2	6.5	64	20	6.07(63)	3.74(43)	9.78(89)
M3	6.5	96	20	6.16(66)	3.76(44)	10.00(92)
M4	6.5	64	20	7.44(58)	4.77(42)	12.26(86)
M5	6.5	64	32	6.61(54)	5.87(52)	7.67(64)

Table 2: Mixing angles according to Eq. (7), for each available ensemble

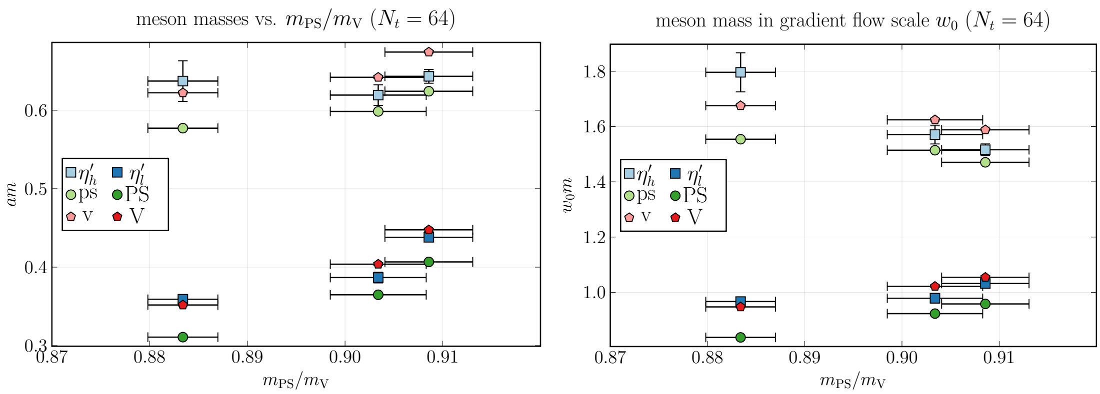


Figure 2: Mass spectrum of light pseudoscalar and vector mesons. In the left panel we report the masses in lattice units, am. In the right panel, the masses are expressed in units of the gradient flow scale, mw_0.

tabulated in Tab. 1. We include the lightest non-singlet masses as a reference value. We further report all mixing angles, extracted according to Eq. (7), in Tab. 2.

We find that the masses of the η'_l and η'_h are close to those of the pseudoscalar and vector mesons in the corresponding representations. This observation already suggests that these states are dominated by a single fermion representation. This hint is supported by the small mixing angles reported in Tab. 2. We also observe a large value of the ratio $m_{\mathrm{PS}}/m_{\mathrm{V}}$ close to unity, suggesting that the parameter space explored in this study corresponds to comparatively large values of the mass of the underlying hyperquarks (fermions). Indeed, we expect mixing effects to be dominated by the disconnected-diagram contributions to the off-diagonal terms of Eq. (3), which are suppressed in the presence of heavy fermions.

The results in Tab. 2 seem to suggest that the parametrization of Eq. (6) by a single mixing angle be insufficient. While the overall angle is small throughout all ensembles, the difference between mixing angles $\phi_{\eta'_l}$ and $\phi_{\eta'_h}$ is statistically significant. We empirically found that using a comparatively large value of t_0 in the GEVP method was advantageous in the determination of the mixing angles. We have chosen $t_0 = 5$ for this investigation.

We observe, in Fig. 2, that the masses of the mesons, expressed in lattice units, show some decrease when decreasing the mass of the fundamental fermions, even though only a modest change in $m_{\mathrm{PS}}/m_{\mathrm{V}}$ is observed. When plotting the fermion masses in units of the gradient flow scale, w_0, we observe that the masses of the η_l, PS, and V states decrease as a function of $m_{\mathrm{PS}}/m_{\mathrm{V}}$, whereas the masses of the η_h, ps, and v are increasing. A more extended discussion, including our operational choices for the definition of the Wilson flow scale, w_0, can be found in Refs. [25, 26].

5. Summary and outlook

We performed the first direct determination of flavour-singlet meson masses and their mixing angles in a gauge theory with multiple fermion representations. The fermion masses are comparatively large, and as a consequence the mixing angle is small. The two singlet meson masses are close to the pseudoscalar and vector flavoured meson masses, respectively. The composition of the mass eigenstates is dominated by fermions of a single species.

Systematic uncertainties are probably sizeable, arising from noisy signal, short plateaux and coarse lattices available. Yet, this investigation is a stepping stone towards further, more sophisticated measurements. These include exploring lighter fermion masses, while combining the scalar flavour-singlet meson channel with the glueball states.

In order to map out the physics of composite Higgs models, an obvious next step is to decrease both the fundamental and antisymmetric fermion masses. Furthermore, the system should be studied at different lattice spacings. This could be either achieved by increasing the inverse gauge coupling, β, or by switching to a fermion action which has $O(a)$ improvement.

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Research Data Access Statement—The data and analysis code generated for these proceedings and the extended publication in Ref. [26] can be downloaded from Ref. [63] and [64], respectively.

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