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Progress on the spectroscopy of lattice gauge theories using spectral densities

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Spectral densities encode non-perturbative information crucial in computing physical observables in strongly coupled field theories. Using lattice gauge theory data, we perform a systematic study to demonstrate the potential of recent technological advances in the reconstruction of spectral densities. We develop, maintain and make publicly available dedicated analysis code that can be used for broad classes of lattice theories. As a test case, we analyse the $Sp(4)$ gauge theory coupled to an admixture of fermions transforming in the fundamental and two-index antisymmetric representations. We measure the masses of mesons in energy-smeared spectral densities, after optimising the smearing parameters for available lattice ensembles. We present a summary of the mesons mass spectrum in all the twelve (flavored) channels available, including also several excited states.

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1. Lattice theory, ensembles, and observables

The analysis of spectral densities provides a novel tool to understand non-perturbative aspects of lattice gauge theories—see, e.g., Refs. [1–16]. This proceedings contribution discusses our approach to reconstructing spectral densities using smeared correlation functions, focusing on the implementation of numerical techniques and their application to meson spectroscopy. We exemplify the potential of such approach on the $Sp(4)$ gauge theory coupled to an admixture of fermions transforming in the fundamental and 2-index antisymmetric representations, which serves as a testbed for

exploring new physics scenarios, including composite Higgs models. We analyse new ensembles made available by the development of the research programme of *Theoretical Explorations on the Lattice with Orthogonal and Symplectic groups* (TELOS) [17–30]—see also Refs. [31–34].

The target theory of this study is the $Sp(4)$ gauge theory coupled to $N_{(\mathrm{f})} = 2$ fundamental and $N_{(\mathrm{as})} = 3$ antisymmetric fermions. We employ Wilson-Dirac fermions, with gauge configurations generated using an admixture of the Hybrid Monte Carlo (HMC) and Rational HMC (RHMC) algorithms. The action on the lattice is written as $S = S_g + S_f$, where S_g is the Wilson plaquette gauge action, with coupling $\beta = 8/g_0^2$, while S_f is the Wilson fermion action—for details, see Ref. [29]. We assume the presence of two diagonal mass matrices for the two species of fermions, denoted as am_f^0 and am_{as}^0. A summary of the parameters characterising the ensembles is provided in Table 1.

We focus our attention on the twelve gauge invariant operators built as fermion bilinears, $O(\vec{x},t)$, with all the admissible spin structures, and off-diagonal flavor structure—for the singlets, see Ref. [30]. One can extract the effective masses from correlation functions $C(t)$, defined as

$$C(t) = \sum_{\vec{x}} \langle 0|O(\vec{x},t)O^{\dagger}(\vec{0},0)|0\rangle \,. \tag{1}$$

In order to improve the signal, we introduce APE [35] and Wuppertal [36] smearings, and solve a Generalized Eigenvalue Problem (GEVP), to further optimise the numerical quality of the ground state signal, as well as to detect excited states. We construct the operator basis by varying the smearing parameters. Numerical results are listed in Tables V to XVII of Ref [29].

2. Spectral Density Reconstruction Method

The spectral density, $\rho(E)$, is the inverse Laplace transform of the correlation function, $C(\tau)$:

$$C(\tau) = \int_0^{\infty} dE\, \rho(E) e^{-E\tau}. \tag{2}$$

We reconstruct it using the Hansen-Lupo-Tantalo (HLT) method [2], a variation of the Backus-Gilbert method [37]. To regularise this inversion, we introduce a smearing kernel $\Delta_\sigma(E,\omega)$, which defines the smeared spectral density, $\rho_\sigma(E)$:

$$\rho_\sigma(\omega) = \int_0^{\infty} dE\, \Delta_\sigma(E,\omega)\rho(E). \tag{3}$$

Table 1: Summary table of the properties of the ensembles used in this study. The inverse coupling is denoted by β, and the bare masses of the two species of fermions as am_f^0 and am_{as}^0, respectively [29].

Label	β	am_f^0	am_{as}^0	Lattice Volume $(N_t \times N_s^3)$
M1	6.5	-1.01	-0.71	48×20^3
M2	6.5	-1.01	-0.71	64×20^3
M3	6.5	-1.01	-0.71	96×20^3
M4	6.5	-1.01	-0.70	64×20^3
M5	6.5	-1.01	-0.72	64×32^3

The parameter σ controls the smearing width, and therefore it controls the tradeoff between resolution and quality of the reconstruction. A larger σ broadens the kernel, reducing noise but blurring spectral features, while a smaller σ preserves fine details but increases statistical fluctuations. In order to balance the effects of statistical and systematic effects, we minimise, at fixed σ, a combined cost functional, $W[\vec{g}]$, defined as follows. We write the spectral density as $\rho_\sigma(E) = \sum_\tau g_\tau(E)\, C(\tau)$, the coefficients $\vec{g} = \{g_1, \cdots, g_{\tau_{\max}}\}$ corresponding to fixed-time lattice slices. We then define

$$W[\vec{g}] \equiv A[\vec{g}]/A[0] + \lambda\, B[\vec{g}]/B_{\text{norm}}, \tag{4}$$

where $A[\vec{g}] = \int_0^\infty d\omega\, e^{\alpha\omega} \left(\rho_\sigma(\omega) - \rho_{\text{target}}(\omega)\right)^2$, $B[\vec{g}] = \sum_{\tau\tau'} g_\tau \,\text{Cov}_{\tau\tau'}[C]\, g_{\tau'}$, $B_{\text{norm}}(E) = C^2(1)/E^2$, ρ_{target} is the target spectral density extracted from the data, λ is a trade-off parameter between systematic and statistical error and $\text{Cov}[C]$ is the covariance matrix of the correlators $C(\tau)$. In principle, a particular choice of λ introduces a source of bias that needs to be removed. Therefore, we perform a scan over λ values, and we search for plateaus in the reconstructed spectral density. Figure 1 illustrates how the spectral reconstruction depends on α and λ. Having identified optimal values of these parameters, details of which can be found in Ref. [29], the minimisation of $W[\vec{g}]$ yields the coefficients, $\vec{g}$, and hence the reconstructed spectral density, $\rho_\sigma(E)$. The process is repeated for each value of E in the range of interest.

The systematic errors associated with the spectral density reconstruction are evaluated by varying the parameters α and λ. The first component of the systematic error is estimated as $\sigma_{1,\text{sys}}(\rho(E)) = \left|\rho_{\lambda^*}(E) - \rho_{\lambda^*/10}(E)\right|$, and the second as $\sigma_{2,\text{sys}}(\rho(E)) = \left|\rho_{\lambda^*,\alpha_2}(E) - \rho_{\lambda^*,\alpha_1}(E)\right|$, where λ^* is defined by the optimisation procedure, and α_i are limiting values of α. While most of

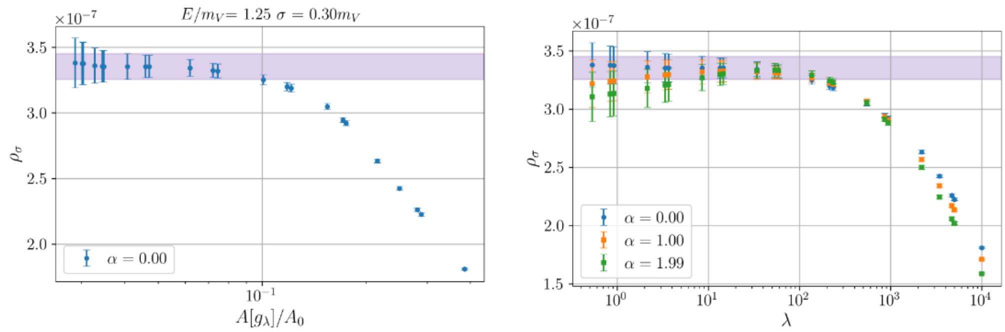

Figure 1: Examples of the optimisation of the spectral density reconstruction, for vector mesons (V) [29].

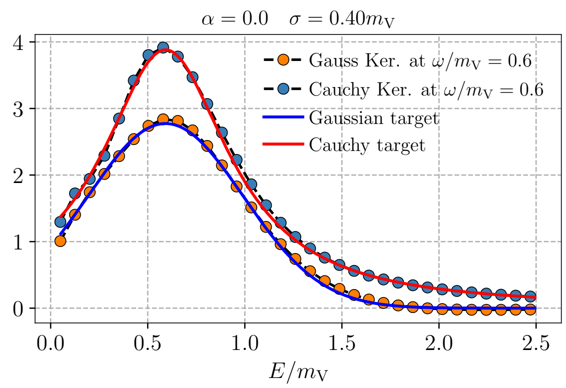

Figure 2: Spectral density kernels reconstructed with the HLT method, compared to target [29].

the bias-removal comes from the scan over λ, by absorbing the bias into the statistical noise, $\sigma_{1,\mathrm{sys}}$ takes care of possible residual effect.

Figure 2 shows a comparison of the reconstructed smearing kernel, $\bar{\Delta}_\sigma(E,\omega) = \sum_\tau g_\tau(E) e^{-(t+1)E}$, with the target one, for two choices of kernel. The Gaussian kernel is

$$\Delta_\sigma^{(\mathrm{Gaussian})}(E,\omega) = e^{-\frac{(E-\omega)^2}{2\sigma^2}} / Z(\omega), \quad Z(\omega) = \int_0^\infty dE\, e^{-\frac{(E-\omega)^2}{2\sigma^2}}, \tag{5}$$

while the Cauchy kernel reads

$$\Delta_\sigma^{(\mathrm{Cauchy})}(E,\omega) = \frac{\sigma}{(E-\omega)^2 + \sigma^2}. \tag{6}$$

3. Meson Spectroscopy

The spectral densities, $\rho_\sigma(E)$, associated with the meson correlation function, are fitted with both the Gaussian and Cauchy kernels, by minimising the functional:

$$\chi^2 \equiv \sum_{E,E'} \left(f_\sigma^{(k)}(E) - \rho_\sigma(E) \right) \mathrm{Cov}^{-1}_{E,E'}[\rho_\sigma] \left(f_\sigma^{(k)}(E') - \rho_\sigma(E') \right), \tag{7}$$

where the fitting functions are, respectively, $f_\sigma^{(k)}(E) = \sum_{n=1}^k A_n \Delta_\sigma^{(\mathrm{Gauss})}(E - E_n)$, and $f_\sigma^{(k)}(E) = \sum_{n=1}^k B_n \Delta_\sigma^{(\mathrm{Cauchy})}(E - E_n)$. The difference between energy levels determined with different kernels provides an estimate of systematic error, $\sigma_{1,\mathrm{sys}}(aE_n) = |aE_{n,\mathrm{Gauss}} - aE_{n,\mathrm{Cauchy}}|$. Figure 3 shows a numerical example demonstrating the level of consistency: both ground and first excited states measured with the two kernels are compatible, within statistical uncertainties.

The spectral density fitting method allows for a detailed exploration of excited states, which are often challenging. Figure 4 shows a comparison between energy levels obtained with the GEVP and HLT methods, demonstrating consistency. A comprehensive summary of numerical results for meson masses obtained with the HLT method are reported together with the GEVP results in Tables V to XVII of Ref. [29]. The mesonic spectrum for the case study theory is displayed in Fig. 5.

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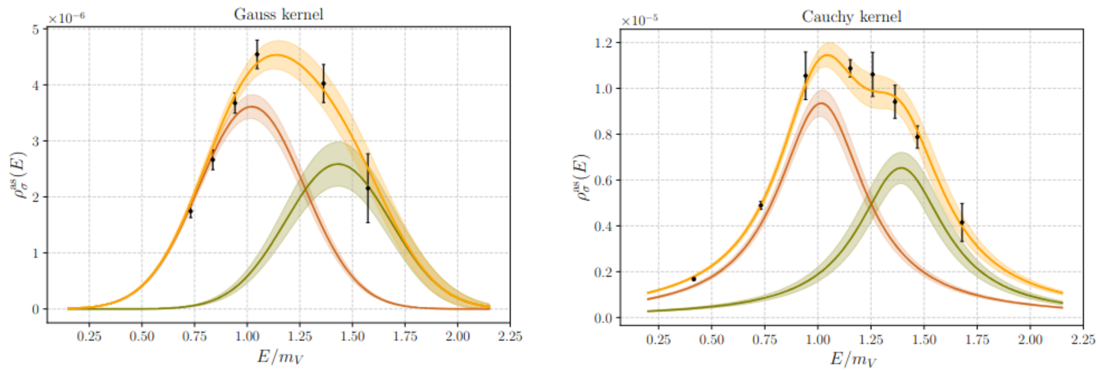

Figure 3: Reconstructed spectral density, using Gaussian (left) and Cauchy (right) smearing kernels [29].

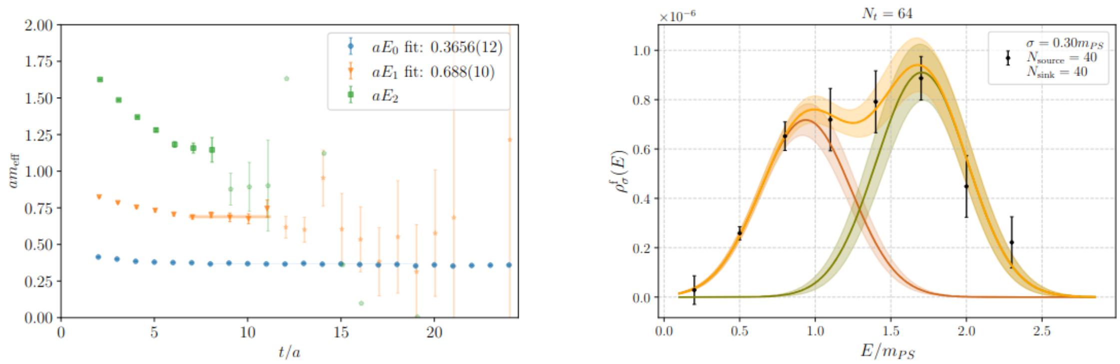

Figure 4: Comparison between GEVP (left) and HLT (right) analysis used to measure ground state and excited state energy levels, for the pseudoscalar (PS) mesons [29].

4. Outlook

This study demonstrates the effectiveness of using smeared spectral densities, by deploying the HLT method to the spectroscopy of flavored mesons in a special $Sp(4)$ gauge theory, used as a case study. Future development will seek to apply these techniques to other correlation functions, and to gain access to off-shell observables. These technical developments have the potential to impact of future studies of QCD as well as new physics scenarios, offering new insights into the non-perturbative dynamics of strongly coupled theories.

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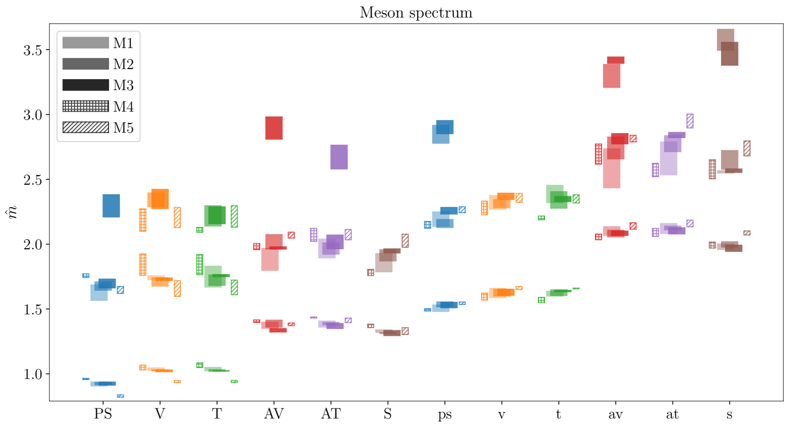

Figure 5: Mass spectrum of flavored mesons extracted with the HLT spectral density reconstruction method [29].

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Research Data Statement—The data generated and analysis code used to prepare for these proceedings and the full paper [29] can be downloaded from Refs. [38] and [39].

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