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The network origins of the gains from trade ☆

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ABSTRACT

This paper develops a network perspective on the gains from trade in today's international supply chains. In particular, we demonstrate that the comparative statics predictions of a standard general-equilibrium trade model with input-output linkages can be expressed as a network diffusion model. This model captures the relevant dimensions of the production network's structure by just two easily quantifiable statistics: A country's upstream exposure to supply shocks further up in the network and its downstream exposure to demand shocks further down. We then show how up- and downstream exposure crucially determine the welfare effects from various types of trade cost shocks. In some cases, they even capture the entire welfare effect.

1. Introduction

Global supply chains are a defining feature of the modern world economy. This paper emphasizes a key consequence of their emergence for our understanding of the welfare gains from trade. In particular, we show that in the presence of global supply chains, the welfare gains from trade are no longer primarily determined by a country's access to the technologies and markets of its direct trade partners. Instead, how much a country gains (or loses) depends on its precise position in the global production network in a way that can be measured by two easily quantifiable network statistics.

To develop our network perspective, we build on the simplest conceivable general-equilibrium framework: the Armington (1969) model where each country offers a unique product that is used by all other countries for both consumption and as an intermediate input in production.[1] We start by investigating very generally within this framework how an arbitrary but small trade cost shock along any number of trade routes affects each and every country's welfare. To do this, we perform classic comparative statics analysis

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[1] Despite the simplicity of our setup, the Armington framework fully serves our purposes because it encompasses a rich set of potential production sequences, ranging from a simple linear supply chain between countries to a complex production network with loops. Nevertheless, we encourage interested readers to also consult our Supplementary Online Material, where we demonstrate that our main results extend to richer general-equilibrium setups, incorporating multiple products and market power on the parts of the sellers.

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and decompose the total welfare effect in every country into several meaningful components. This decomposition yields our first main result: The comparative statics predictions of our framework can be written as a network diffusion model that describes how the local effects of a shock, that is, the well-known goods supply and factor demand effects in the countries directly involved in the affected trade routes, diffuse to all the other nations.

Two very different types of channels are responsible for this shock diffusion:

(i) the *general-equilibrium multipliers* that capture the interdependencies between the goods and factor markets. In particular, the *foreign trade multiplier* is responsible for the repercussions between different countries' factor markets, while the *terms of trade multiplier* governs the spillovers from the factor onto the goods markets.

(ii) the *supply chain diffusion* channels. They only emerge whenever the countries are connected through a production network, and they determine how the local effects propagate along the network's links. *Upstream exposure* thereby captures the extent to which each country's production costs are affected by shocks occurring to its intermediate goods suppliers, while *downstream exposure* measures how shocks occurring to a country's downstream customers affect its factor demand.

While all these channels are integral parts of any modern trade theory, the novelty of our approach is its ability to set them apart, both analytically as well as quantitatively based on readily available data. Most importantly, our approach allows us to show exactly how the emergence of global supply chains has changed the welfare consequences of any type of trade cost or technology shock. In the second part of the paper, we focus on three specific types of shocks:

First, a unilateral export cost reduction along a single trade route. We first demonstrate that, in a world without production linkages, the second Hicksian law of comparative statics would apply in this case so that the factor incomes in all nations but the exporter decline. Things are very different in a production network though, because parts of the exporter's gains spill over to other nations. We show that these spillovers can, in fact, be very sizable, and we develop two conditions on each country's supply chain exposure so that the Hicksian law is even overturned.

Second, we study a uniform cost reduction along all trade routes. As each country improves its market access to all other countries alike in this case, one may expect such an 'equal opportunity' cost reduction to also lead to equally sized welfare gains in every country. Yet, the logic only goes through in the absence of production linkages between countries. In their presence, the welfare gains crucially depend on a country's upstream exposure to every other nation. In some model specifications, upstream exposure is even all that matters.

Finally, we look at what could be regarded as the flipside of the classic gains from trade analysis. We isolate one nation after the other from the world economy and ask how much, and through which channels, the remaining nations are affected. Our findings put a group of countries in the spotlight that are important for others not so much because of their own value added and the size of their own markets, but because of their role as intermediaries of other nations' supply and demand. It is the access to these key intermediaries that explains the cross-country variation in up- and downstream exposure in our model.

Overall, our paper thus sheds new light on the classic question about the origins of the welfare gains from trade. We show that who benefits and how much in today's integrated supply chains depends on three central concepts in network theory: positive spillovers, network centrality, and trade intermediation. It is thus also not surprising that our measures of supply chain exposure and trade intermediation are closely related to measures of diffusion centrality (Bonacich, 1987; Banerjee et al., 2013) and bridging capital (Ballester et al., 2006) from that literature.[2] Our contribution to this work is that we derive our network statistics from a general-equilibrium setup where, unlike in the earlier theories, both diffusion directions (up- and downstream) matter.

Of course, our paper is also not the first to study the economics of global supply chains. Already the early theories of Ethier (1979) and Dixit and Grossman (1982) have made clear that their emergence had important implications for the location of production and for the sensitivity of factor incomes to changes in trade barriers or factor costs. Our paper is less ambitious than one group of extensions to this work, notably Costinot et al. (2013), Fally and Hillberry (2018), or Antràs and De Gortari (2020), in that we do not study the gradual emergence of a supply chain or the endogenous sorting of countries into its production steps. Instead, we investigate how various shocks to the links within an existing network affect each country's welfare, allowing, in contrast to these earlier studies, for a much richer network structure embedded in a general-equilibrium framework.

In this respect, our paper is closer to a second group of extensions with a similar ambition as ours, notably di Giovanni et al. (2014), Caliendo and Parro (2015), Ossa (2015), Blaum et al. (2018), or Huo et al. (2019). It is particularly close to the contemporaneous work by Baqaee and Farhi (2022), who also develop a general-equilibrium framework to identify the network determinants of the gains from trade. The main distinctive feature of our paper is probably the simpler economic setup. This allows us to derive comparative statics predictions for our framework where we can directly relate the welfare effects of any type of trade cost shock to the underlying network structure of production. Introducing a production network into a general-equilibrium framework adds a layer of complexity that generally makes it hard to derive simple analytic results, even in a first-order approximation. Nevertheless, we are able to derive several benchmark results where the network structure does not play a role, develop general conditions under which it matters, and even identify some model specifications where a country's network exposure alone predicts its entire welfare effect from a shock.

Our findings also speak to a parallel line of network studies in macroeconomics. Responding to the foundational works of Hulten (1978) and Lucas (1977), who argued that, under the efficient-market assumption, the microstructure of production does not matter for aggregate economic outcomes, earlier studies in this field have investigated different departures from this assumption under

[2] See also Jackson (2020) for a review of social network models and the centrality measures that follow from these models.

which the network structure does make a difference (e.g., Acemoglu et al., 2012; Grassi, 2017; Huneeus, 2018; Tintelnot et al., 2018; Liu, 2019; Baqaee and Farhi, 2019). Our paper highlights another such circumstance: imperfect mobility of goods and factors across space. As we show, the network structure matters in this case as it determines how the total welfare effect of a shock is distributed across the network's nodes (countries, sectors, etc.).

Finally, our paper contributes to a group of studies with the aim of developing some meaningful measures of a country's position in the global production network. Our findings provide a general-equilibrium foundation for some of the measures developed there. In particular, the up- and downstreamness measures of Fally (2012), Antràs et al. (2012), and Antràs and Chor (2013) turn out to be closely related to our two supply chain exposure statistics. Moreover, what Hummels et al. (2001) call vertical specialization trade captures in our framework a country's importance as a trade intermediary and, thus, its contribution to other nations' supply chain exposure. As such, our findings support the usefulness of these measures for ex-ante impact evaluations of trade cost or technology shocks.

We proceed from here as follows. Section 2 presents our basic framework that we use to bring our network perspective across. Section 3 sets out our comparative statics approach and introduces the measures of supply chain exposure. Their importance for understanding the welfare gains from trade is demonstrated in Sections 4–6. The Appendix contains the proofs of all our statements and the Supplementary Online Material several additional statements as well as two extensions of our basic framework.

2. The model

Consider a world economy consisting of n countries, indexed $i \in \mathcal{N} = \{1, 2, ..., n\}$. Each country produces a unique, horizontally differentiated product that is used in all other countries $i \in \mathcal{N}$ for final consumption and as an intermediate input in production. Consumption and production are specified as follows.

Consumption Country i hosts l_i consumers who each have CES preferences over the available products and who each supply one unit of labor against the wage $w_i > 0$. Specifically, when $p_{ji} > 0$ denotes the price paid in country i for the products from country j and $\gamma > 1$ denotes the elasticity of substitution, then i's consumers purchase $q^f_{ji} \geq 0$ units from every j so as to maximize

$$u_i = \left(\sum_{j \in \mathcal{N}} (q^f_{ji})^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}}$$

subject to $\sum_{j \in \mathcal{N}} q^f_{ji} p_{ji} \leq w_i$. It is straightforward to verify that utility maximization yields an indirect utility of $u_i = w_i / p^f_i$, where

$$p^f_i = (\sum_{j \in \mathcal{N}} p_{ji}^{1-\gamma})^{\frac{1}{1-\gamma}} \tag{1}$$

is the so-called consumer price index. Moreover, the value of all final goods shipped from j to i can be written as $x^f_{ji} = \pi_{ji} e^f_i$, where $e^f_i = w_i l_i$ denotes the final goods expenditures in country i and

$$\pi_{ji} = \frac{p_{ji}^{1-\gamma}}{(p^f_i)^{1-\gamma}} \tag{2}$$

i's expenditure share on the product made in country j.

Production Each country's output is produced by a homogeneous set of firms that operates under conditions of perfect competition employing a two-tier constant-returns CES technology. More concretely, the producers in country i substitute on the first stage between labor and a composite of the available intermediate products at the elasticity β, $\beta \geq 0$ and $\beta \neq 1$, and on the second stage between the available products at the same elasticity γ with which also consumers substitute between them (with $\gamma \geq \beta$).

Thus, in order to sell $q_{ij} \geq 0$ units to a $j \in \mathcal{N}$, the producers in i use a combination of labor $l^d_i > 0$ and intermediate inputs $q^i_{ki} \geq 0$ so as to minimize $c_i = l^d_i w_i + \sum_{k \in \mathcal{N}} q^i_{ki} p_{ki}$, subject to

$$\sum_{j \in \mathcal{N}} \tau_{ij} q_{ij} \equiv q_i \leq \mu_i \left(\kappa^l_i \left(l^d_i\right)^{\frac{\beta-1}{\beta}} + \kappa^i_i \left(\sum_{k \in \mathcal{N}} (q^i_{ki})^{\frac{\gamma-1}{\gamma}} \right)^{\frac{\gamma}{\gamma-1}\frac{\beta-1}{\beta}} \right)^{\frac{\beta}{\beta-1}}. \tag{3}$$

The function's parameters are the total factor productivity, $\mu_i > 0$, the relative factor productivity of labor and intermediate products, $\kappa^l_i > 0$ and $\kappa^i_i \geq 0$, and the productivity of each country's shipping technologies, τ_{ij}. In particular, we think of this parameter as an 'iceberg' trade cost parameter, $\tau_{ij} \in [1, \infty]$, measuring how many units need to be shipped from country i for one unit to arrive in j.

It is easily shown that for a given $q_i > 0$, the cost-minimizing input combination costs $c_i = p_i q_i / \mu_i$, where

$$p_i = \left((\kappa^l_i)^\beta w_i^{1-\beta} + (\kappa^i_i)^\beta (p^f_i)^{1-\beta} \right)^{\frac{1}{1-\beta}} \tag{4}$$

is the so called producer price index.

Market structure Perfect competition means marginal cost pricing. Hence, the sales price of i's product in j is given by

$$p_{ij} = \frac{p_i \tau_{ij}}{\mu_i} \,. \tag{5}$$

Market clearing, in turn, implies that the total value of all shipments from i to j, $x_{ij} \equiv p_{ij} q_{ij}$, is given by $x_{ij} = \pi_{ij} e^f_j + \pi_{ij} e^i_j$. Moreover, it implies that i's total wage income is

$$w_i l^d_i = \lambda_i \sum_{j \in \mathcal{N}} \pi_{ij} (e^f_j + e^i_j) \,, \tag{6}$$

where we refer to

$$\lambda_i = \frac{(\kappa^l_i)^\beta w_i^{1-\beta}}{p_i^{1-\beta}} \tag{7}$$

as the labor cost share of i's producers and to $1 - \lambda_i = (\kappa^i_i)^\beta (p^f_i)^{1-\beta} / p_i^{1-\beta}$ as the complementary intermediate input cost share. These costs share are endogenously determined in our model. In particular, labor and intermediate inputs might be complements ($\beta < 1$) or substitutes ($\beta > 1$) in production, with the Leontief specification ($\beta = 0$), the Cobb-Douglas case ($\beta \to 1$), and uniform elasticities ($\beta = \gamma$) as the limit cases.

Equilibrium An equilibrium of our economy is defined by the following two key equations. First, it follows from the identities in (1), (4), and (5) that, for a given vector of wages $\mathbf{w}$, the producer price indexes must satisfy

$$p_i = \left((\kappa^l_i)^\beta w_i^{1-\beta} + (\kappa^i_i)^\beta \left(\sum_{j \in \mathcal{N}} \left(\frac{p_j \tau_{ji}}{\mu_j} \right)^{1-\gamma} \right)^{\frac{1-\beta}{1-\gamma}} \right)^{\frac{1}{1-\beta}} \qquad \forall\, i \in \mathcal{N} \,. \tag{8}$$

Given a $\mathbf{p}$ that satisfies this equation, all other prices, p_{ij} and p^f_i, derive from here. When we then combine (6) with the expressions for λ_i and π_{ij} in (2) and (7), an equilibrium wage vector $\mathbf{w}$ must satisfy

$$\begin{aligned} w_i l_i &= \lambda_i \sum_{j \in \mathcal{N}} \pi_{ij} (e^f_j + e^i_j) \\ &= \frac{(\kappa^l_i)^\beta w_i^{1-\beta}}{p_i(\mathbf{w})^{1-\beta}} \sum_{j \in \mathcal{N}} \frac{(p_i(\mathbf{w}) \tau_{ij} / \mu_i)^{1-\gamma}}{\sum_{k \in \mathcal{N}} (p_k(\mathbf{w}) \tau_{kj} / \mu_k)^{1-\gamma}} (e^f_j + e^i_j) \qquad \forall\, i \in \mathcal{N} \,. \end{aligned} \tag{9}$$

As we show in Appendix A.1, such an equilibrium exists for our economy under the plausible assumption that all countries add some value to the global production network, that is, when $\lambda_i \in (0, 1] \; \forall\, i \in \mathcal{N}$.[3]

Proposition 1 *(Equilibrium existence). Suppose that $\lambda_i \in (0, 1] \; \forall\, i \in \mathcal{N}$. Then, there exists at least one profile of wages and producer prices* $(\mathbf{w}, \mathbf{p})$ *that satisfies the equations in* (8) *and* (9).

For our comparative statics approach, we require a bit more, however, because the approach is only valid when the labor demand system, $w_i l^d_i(\mathbf{w}, \mathbf{p}(\mathbf{w})) \; \forall\, i \in \mathcal{N}$, is locally invertible around an equilibrium point. To ensure this, we follow the literature and assume that, in an equilibrium point $(\mathbf{w}, \mathbf{p})$, the system's own- and cross-price derivatives lie in the unit interval (cf. Morishima, 1960; Alvarez and Lucas, 2007; Adao et al., 2017):

$$\frac{\partial (w_i l^d_i)}{\partial (w_j l_j)} (\mathbf{w}, \mathbf{p}(\mathbf{w})) \in (0, 1) \quad \text{for all } ij \in \mathcal{N} \times \mathcal{N} \,. \tag{10}$$

Intuitively, what we require here is a mild to moderate *positive* response of each country's labor income with respect to a foreign or domestic expenditure change.[4]

Making this additional assumption leads us to our next result (proven in Appendix A.1):

Proposition 2 *(Equilibrium uniqueness). Suppose that condition* (10) *is satisfied in an equilibrium point* $(\mathbf{w}, \mathbf{p})$*. The equilibrium is then locally unique (up to normalization), and comparative statics are admissible.*

[3] In Appendix A.1, we derive an equivalent condition on the parameters of our economy that guarantees $\lambda_i \in (0, 1] \; \forall\, i \in \mathcal{N}$ (see inequality (A.1)).

[4] In other words, we require that the interaction between different countries' labor demands is primarily be determined by the foreign income multiplier (Samuelson, 1943), and less so by either the labor demand complementarities that naturally arise when production processes are dispersed across countries (and which push the cross-price derivatives in (10) into the negative range) or by the competition for market shares (which increases the derivatives to values greater than one).

3. The network origins of the gains from trade

So far, we have described a fairly standard general-equilibrium trade model. In this section, we introduce our network perspective on this model. Towards this end, we study the comparative statics predictions of an arbitrary trade cost shock, and show that the model's predictions can be interpreted as a network diffusion model that describes how the well-known goods supply and factor demand effects in the countries immediately affected by the shock spill over to all other nations.

To formalize our diffusion model, we first need a bit of extra notation, however. In particular, we often summarize the country-specific variables or parameters of our trade model (e.g., the prices, productivities, etc.) in either a column vector $\mathbf{y} \in \mathbb{R}^{n\times 1}$ or in a diagonal matrix $\mathbf{Y} \in \mathbb{R}^{n\times n}$. For the intermediate input cost shares, for instance, we write $\mathbf{1} - \lambda$ or $\mathbf{I} - \mathbf{\Lambda}$, where $\mathbf{1}$ denotes a column vector of ones and $\mathbf{I}$ the identity matrix. The bilateral terms (e.g., trade costs, expenditure shares, etc.) are, in turn, summarized in a square matrix $\mathbf{Z} \in \mathbb{R}^{n\times n}$. Matrix $\mathbf{\Pi} = (\pi_{ij})$, for instance, collects the expenditure shares of the importers j on the products of the exporters i.

Furthermore, we make use of several matrix transformations. For instance, we sometimes need to compute the transpose of a matrix $\mathbf{Z}$, which we denote by $\mathbf{Z}^\top$, and the inverse of a matrix, which we denote by $\mathbf{Z}^{-1}$. Of particular importance to us, when $\mathbf{Z}$ has a row or column norm smaller than one (i.e., $\mathbf{1}^\top\mathbf{Z} < \mathbf{1}^\top$ or $\mathbf{Z1} < \mathbf{1}$), then the inverse of $\mathbf{I} - \mathbf{Z}$ can be expressed in terms of the following Neumann series

$$[\mathbf{I} - \mathbf{Z}]^{-1} = \mathbf{I} + \mathbf{Z} + \mathbf{Z}^2 + \ldots = \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \mathbf{Z}^{\mathbf{h}}.$$

Moreover, we sometimes dispense with one row and column of a matrix, which by Walras' Law are redundant. We, therefore, introduce the transformation $\mathbf{Z}^{-\mathbf{i}}$ which denotes the matrix that follows after removing row i and column i from $\mathbf{Z}$, while $\mathbf{Z}^{+\mathbf{i}}$ follows after insertion of a row vector of zeros above row i and a column vector of zeros before column i of $\mathbf{Z}$. Finally, we need a notation to describe the impact on one vector or matrix after a change to another vector or matrix. Towards this end, we introduce $\mathbf{d_Y Z}$ to denote the *direct* impact of a change to $\mathbf{Y}$ on $\mathbf{Z}$ (i.e., $\mathbf{d_Y Z} = \mathbf{Z}' - \mathbf{Z}$, where $\mathbf{Z}'$ is evaluated at the new value of $\mathbf{Y}$) and $\mathbf{dZ}$ to denote the *total* impact on $\mathbf{Z}$, also taking the adjustments to all the other variables in $\mathbf{Z}$ into account.

Now, consider an arbitrary shock to any number of elements in the trade cost matrix $\mathbf{T}$, i.e., $\mathbf{dT} = \left(d\tau_{ij}\right)_{ij\in\mathcal{N}\times\mathcal{N}}$. The shock has, in the first instance, a direct impact on the labor demands and the goods prices of all the exporters and importers involved in the affected trade routes. We call these the local effects of the shock, which are all well-known in the literature. In fact, any neoclassic trade theory considers the *local price effects* of a shock on the goods prices in the importer countries,

$$\delta^{\mathbf{p}} \equiv \Big(\sum_i d\ln(\tau_{i1})x_{i1}, \ldots, \sum_i d\ln(\tau_{ij})x_{ij}, \ldots, \sum_i d\ln(\tau_{in})x_{in} \Big)^\top, \tag{11}$$

also known as the *supplier access* effect (Redding and Venables, 2004).

Moreover, any neoclassic theory incorporates the direct impact on the labor demands of all the immediately involved exporters and importers (the *local demand effect* $\delta^{\mathbf{d}}$). Unlike the local price effects, the expression for $\delta^{\mathbf{d}}$ is, however, somewhat more cumbersome because there are several effects at play here (see (A.22) in Appendix A.2 for the full expression). Two of them are active regardless of whether or not there are production linkages between countries. First, the direct impact of a trade cost shock on the *exporters' market access* to the importers and second, the shocks' impact on the market access of all the other nations selling to the importers (the *import competition* channel). Two additional demand channels emerge from theories that take the effects from supply chain linkages into account: first, a *productivity* channel (e.g., Grossman and Rossi-Hansberg, 2008) which captures the idea that, with traded intermediate inputs, the importers' labor demands respond to a trade cost shock as well because of its impact on the importers' production costs. Second, as long as $\beta \neq 1$, an additional *input substitution* channel emerges (e.g., Dixit and Grossman, 1982; Costinot et al., 2013), capturing the idea that a trade cost shock also has an impact on the importer's optimal combination of labor versus intermediate inputs in production.

In our diffusion model, all these channels are subsumed under the vector of local demand effects $\delta^{\mathbf{d}}$. The main value of our model is to demonstrate how these local demand effects, together with the local price effects, propagate to all the other nations through, on the one hand, the general-equilibrium multipliers in the world economy and, on the other, the links within the production network. Our following main result elucidates the mechanics of this diffusion process.

Theorem 1 *(The diffusion model). The percentage real income effect, $d\ln(u_i) = d\ln(w_i) - d\ln(p_i^f)$, of an arbitrary trade cost shock, $\mathbf{dT}$, is in a first-order approximation given by the following linear mapping of the shock's local effects, $\delta^{\mathbf{d}}, \delta^{\mathbf{p}} \in \mathbb{R}^{n\times 1}$, on each country's wages and prices:*

$$\begin{aligned} \mathbf{d}\ln(\mathbf{w}) &= [\mathbf{LW}]^{-\mathbf{1}}\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}\,\mathbf{\Phi}^{down}\,\delta^{d} \\ \mathbf{d}\ln(\mathbf{p^f}) &= [\mathbf{E^f}]^{-\mathbf{1}}\mathbf{\Phi}^{up}\,\delta^{p} \;+\; [\mathbf{E^f}]^{-1}\mathbf{\Phi}^{tot}\,\mathbf{LW}\,\mathbf{d}\ln(\mathbf{w}), \end{aligned} \tag{12}$$

with the general-equilibrium multiplier matrices, $\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}$ and $\mathbf{\Phi}^{\mathbf{tot}}$ defined in (13) and (14), and the supply chain exposure matrices, $\mathbf{\Phi}^{\mathbf{down}}$ and $\mathbf{\Phi}^{\mathbf{up}}$ defined in (15), as coefficients.

The proof can be found in Appendix A.2, the matrices are defined in turns:

General-equilibrium multipliers Matrices $\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}$ and $\mathbf{\Phi}^{\mathbf{tot}}$ define the interdependencies between the goods and factor markets in our model. Their spillover channels are active regardless of whether supply chain linkages are present between countries or not.

In particular, the *terms of trade multiplier* matrix, $\mathbf{\Phi}^{\mathbf{tot}}$, collects the (rescaled) elasticities of each country's final goods prices with respect to a factor price change in every other nation. Formally,

$$\mathbf{\Phi}^{\mathbf{tot}} \equiv \mathbf{E}^{\mathbf{f}} \frac{\partial \ln(\mathbf{p}^{\mathbf{f}})}{\partial \ln(\mathbf{w})} [\mathbf{LW}]^{-\mathbf{1}} \quad \in \left[0,1\right)^{n\times n}, \tag{13}$$

where $\mathbf{E}^{\mathbf{f}}$ and $\mathbf{LW}$ denote the diagonal matrices of final goods expenditures and labor incomes, respectively, which satisfy $\mathbf{E}^{\mathbf{f}} = \mathbf{LW}$. In other words, $\mathbf{\Phi}^{\mathbf{tot}}$ captures the spillovers from the labor to the goods markets in our model.[5]

The *foreign trade multiplier*, $\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}$, by contrast, captures the interdependencies between different countries' labor markets. It is formally defined by the inverse of the excess labor supply functions', $w_i l_i - w_i l_i^d(\mathbf{w}, \tilde{\mathbf{p}}(\mathbf{w}, \boldsymbol{\omega}), \boldsymbol{\omega})$, partial derivatives with respect to a change in j's labor income, $w_j l_j$. Let i^* denote the reference country for which $d\ln(w_{i^*}) = 0$. Then, $\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}$ can—due to assumption (10)—also be written in terms of the following Neumann series:

$$\begin{aligned}\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*} &= \left\{ \mathbf{I}^{-\mathbf{i}^*} + \left\{\frac{\partial(\mathbf{Wl}^{\mathbf{d}})}{\partial(\mathbf{Wl})}\right\}^{-\mathbf{i}^*} + \left\{\frac{\partial(\mathbf{Wl}^{\mathbf{d}})}{\partial(\mathbf{Wl})}\right\}^{-\mathbf{i}^*}\left\{\frac{\partial(\mathbf{Wl}^{\mathbf{d}})}{\partial(\mathbf{Wl})}\right\}^{-\mathbf{i}^*} + \dots \right\}^{+\mathbf{i}^*} \\ &= \left\{ \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \left[\left\{\frac{\partial(\mathbf{Wl}^{\mathbf{d}})}{\partial(\mathbf{Wl})}\right\}^{-\mathbf{i}^*}\right]^{\mathbf{h}} \right\}^{+\mathbf{i}^*},\end{aligned} \tag{14}$$

where $\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*} \in \mathbb{R}^{n\times n}_{++}$. The series nicely illustrates the multiplier effect. Country i not only gains from an increase in income in country j because the latter buys more products from country i (the second summand with $\mathbf{h} = \mathbf{1}$), but also because j's higher income positively impacts the labor income of a shared trade partner, who subsequently buys more products from i, etc. (the summands $\mathbf{h} = \mathbf{2}, \mathbf{3}, \dots$).[6]

Supply chain diffusion channels The other two matrices in (12) capture the interdependencies between different countries due to their shared production linkages. Specifically, the *downstream exposure* matrix, $\mathbf{\Phi}^{\mathbf{down}}$, measures the extent to which each country's labor income is dependent on a local demand shock in every other country 'further down' in the global production network. The *upstream exposure* matrix $\mathbf{\Phi}^{\mathbf{up}}$, by contrast, measures by how much each country's goods prices respond to a local supply shock 'further up' in the network.

Both spillover types can thereby be direct, meaning i is exposed to j because i buys intermediate products from or sells them to j; or indirect, meaning that i is exposed to j via one or more countries on intermediate production steps. The elements in the $\mathbf{\Phi}^{\mathbf{down}}$- and $\mathbf{\Phi}^{\mathbf{up}}$-matrices capture all these direct and indirect spillovers. This becomes most apparent from their Neumann series representations:

$$\begin{aligned}\text{Upstream exposure: } \mathbf{\Phi}^{\mathbf{up}} &\equiv \mathbf{E}^{\mathbf{f}} \sum_{h=0}^{\infty} \left[\mathbf{\Pi}^{\top}(\mathbf{I} - \mathbf{\Lambda})\right]^{\mathbf{h}} \mathbf{E}^{-\mathbf{1}} \\ \text{Downstream exposure: } \mathbf{\Phi}^{\mathbf{down}} &\equiv \mathbf{\Lambda} \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \left[\mathbf{\Pi}(\mathbf{I} - \mathbf{\Lambda})\right]^{\mathbf{h}}.\end{aligned} \tag{15}$$

In the absence of production linkages (i.e., when $\boldsymbol{\kappa}^{\mathbf{i}} = \mathbf{0}$), there are no spillovers between countries because it holds in this case $\mathbf{\Lambda} = \mathbf{I}$ and $\mathbf{E}^{\mathbf{f}} = \mathbf{E}$, so that the exposure matrices reduce to $\mathbf{\Phi}^{\mathbf{down}} = \mathbf{\Phi}^{\mathbf{up}} = \mathbf{I}$. In their presence, the direct spillovers are contained in the second summands of the series, $\mathbf{E}^{\mathbf{f}}\mathbf{\Pi}^{\top}(\mathbf{I} - \mathbf{\Lambda})\mathbf{E}^{-\mathbf{1}}$ and $\mathbf{\Lambda}\mathbf{\Pi}(\mathbf{I} - \mathbf{\Lambda})$, and the indirect spillovers via $1, 2, 3\dots$ countries on intermediate steps in the higher-order summands with $\mathbf{h} = \mathbf{2}, \mathbf{3}, \mathbf{4}, \dots$.

Country i is, for instance, directly exposed to an input price shock to its trade partner j at a rate $(\phi^{up}_{ij})^{[1]} = (1/e_j)(1-\lambda_j)\pi_{ji} e^f_i$, because j's producers are themselves only affected in proportion to their intermediate input cost share, $(1-\lambda_j)$, and they only pass on a fraction π_{ji} of the shock's effects to country i. The country is, moreover, indirectly exposed to the same shock via one country on an intermediate step at the rate

$$(\phi^{up}_{ij})^{[2]} = (1/e_j)(1-\lambda_j) \sum_{k\in\mathcal{N}} \pi_{jk}(1-\lambda_k)\pi_{ki} e^f_i$$

because k's producers just pass on $(1-\lambda_k)\pi_{ki}$ of their input price reductions to i, etc.

Diffusion properties To gain more intuition about diffusion model (12), we first describe some of its general properties.

First, note that the sole function of the $\mathbf{\Phi}^{\mathbf{down}}$- and $\mathbf{\Phi}^{\mathbf{up}}$-matrices is to determine how the total welfare effect of a (trade cost) shock is distributed across the countries in our model. No more and no less. This makes intuitive sense from their very definition. The matrices only appear in (12) whenever production processes are fragmented across borders. Yet, production fragmentation means

[5] The full expression for the terms of trade and foreign income multiplier can be found in Appendix A.2, equations (A.17) and (A.25).

[6] Appendix A.2.2 summarizes some other useful properties of the Jacobian matrix $\partial(\mathbf{Wl}^{\mathbf{d}})/\partial(\mathbf{Wl})$.

no more than that the trade in value added is decoupled from the trade in products. Thus, while production fragmentation has consequences for where the effects of a trade cost shock are borne out, it does not have any bearing on the total effect size of a shock.

This intuition can, in fact, be formalized, as done in the following lemma (proven in Appendix A.2.2):

Lemma 1 *(Diffusion properties). The supply chain exposure matrices and the terms of trade matrix are norm-preserving transformations, that is,*

$$\mathbf{1}^\top \boldsymbol{\Phi}^{\mathbf{down}} = \mathbf{1}^\top \boldsymbol{\Phi}^{\mathbf{up}} = \mathbf{1}^\top \boldsymbol{\Phi}^{\mathbf{tot}} = \mathbf{1}^\top .$$

The foreign trade multiplier is, by contrast, norm-amplifying. That is, when $\mathbf{1}_{\mathbf{i}^*}^\top$ *denotes a row vector of ones with a zero in element* i^**, we then get* $\mathbf{1}^\top \boldsymbol{\Phi}_{\mathbf{i}^*}^{\mathbf{mult}} > \mathbf{1}_{\mathbf{i}^*}^\top$.

Hence, in contrast to the supply chain exposure matrices (and the terms of trade matrix), the foreign trade multiplier, $\boldsymbol{\Phi}_{\mathbf{i}^*}^{\mathbf{mult}}$, amplifies the effect sizes in our model. Nonetheless, as this matrix only amplifies the effects on wages and since wages merely distribute incomes in a Walrasian economy like ours, this type of effect amplification can be ignored from a total world welfare perspective. This leads us to our next observation:

Proposition 3 *(Worldwide total welfare effect). The worldwide total welfare effect of an arbitrary trade cost shock* **dT** *just depends, in a first-order approximation, on the local price effects:*

$$\sum_{i \in \mathcal{N}} e_i^f \, d\ln(u_i) = -\sum_{i \in \mathcal{N}} \delta_i^p ,$$

where e_i^f *denotes the Domar weights on the real income effects in each country.*

The result (proven in Appendix A.2.2) is essentially an application of Hulten (1978)'s theorem.[7] It says that all we need to know from a total world welfare perspective is how much a trade cost shock improves or deteriorates the supplier access of all the countries immediately involved in the affected trade routes. The effects on each country's wages and the downstream diffusion of the price effects are, by contrast, of no further relevance.

The wage and price effects are, however, key to understanding how the total effect of a shock is distributed across countries. In the remainder of the paper, we study three specific trade cost scenarios to investigate how the supply chain diffusion channels, in particular, shape their distributional consequences. Besides deriving several general propositions on the channels' importance, we also take our predictions to the data based on the following empirical approach.

Empirical implementation Diffusion model (12) can be easily quantified. As can be seen from the expressions for its components in (11), (15), and Appendix A.2, all we need to have is data on bilateral trade shares, π_{ij}, total outputs, x_i, national factor incomes, $w_i l_i$, and estimates for the two elasticity parameters β and γ. The remaining variables can be simply inferred from the equilibrium identities $x_i = e_i^f + e_i^i$, $w_i l_i = e_i^f$, and $\lambda_i = w_i l_i / x_i$.

For our own illustrations, we use data from the CEPII Trade and Production Database for the period 1980–2006 and from a self-collected data set based on UN Comtrade, the UN Industrial Statistics, and the World Development Indicators for the period 2000–2011. In both cases, we solely collect information on each country's manufacturing sectors defined by ISIC revision 3. Data availability leads to yearly variations in the numbers of countries included: the minimum, median, and maximum number is 64, 88, and 96, respectively. The missings are typically smaller developing economies.

As for the model's elasticity parameters, we keep things simple and fix the elasticity of product substitution at $\gamma = 5$ throughout all our illustrations. This number lies in the middle of the range of available estimates by Eaton and Kortum (2002), Romalis (2007), and Caliendo and Parro (2015). Because of the absence of any comparable estimate for the elasticity of factor substitution β, we treat it as a floating parameter and report results using different values taken from $\{.001, .5, 1.001, 1.5, \ldots, 4.5, 5\}$.[8]

4. The spillovers from local trade cost shocks

In our first exercise, we focus on the potential magnitude of the supply chain spillover channels. We study the simplest possible trade cost shock for this purpose: a unilateral export cost reduction (e.g., removing import or export restraints).[9]

[7] Proposition 3 has been developed independently in Baqaee and Farhi (2022).

[8] The existing evidence on the magnitude of β is mixed. While Barrot and Sauvagnat (2016), Atalay (2017), and Boehm et al. (2019) suggest a strong complementary relationship between labor and (foreign) intermediate inputs (β close to zero), there is also evidence in support of input substitutability ($\beta > 1$), for instance, Hummels et al. (2001) and Timmer et al. (2014).

[9] The arguments of this section can be easily extended to study trade cost shocks on multiple trade routes. The intuition is that, in a first-order approximation, the welfare effect of a shock to multiple cells of the trade cost matrix is simply the sum of the constituent cell-specific shocks' effects. Even more can be said, however, when we look at an inframarginal trade cost shock and the interaction between the constituent shocks on different trade routes. Supplementary Material S.2 presents an analysis in this direction. There, we investigate the conditions under which one trade cost reduction raises the incremental gains from another one so that each cost reduction becomes a 'building bloc' for a free trading zone.

We know from Proposition 3 that when this cost reduction is proportional to the initial trade costs between exporter i and importer j (i.e., $d\tau_{ij} = -y\tau_{ij}$ with $y > 0$), then the worldwide total welfare gain is simply given by $-\delta_j^p = yx_{ij}$. Naturally, parts of these gains accrue to the exporter and the importer, but our ambition here is to go beyond these obvious effects and to compare their gains with the spillovers to third countries.

No production linkages For comparison, let us first investigate a unilateral export cost reduction in a world where only final products are traded. This is our result:

Proposition 4 *(Export cost reduction without supply chains). Consider a unilateral export cost reduction, $d\tau_{ij} = -y\tau_{ij}$, $y > 0$, in a world where only final goods are traded ($\boldsymbol{\kappa}^{\mathbf{i}} = \mathbf{0}$). Then, (4a) wages in all $k \neq i$ decline relative to the exporter: $d\ln(w_k) < d\ln(w_i)$. Moreover, (4b) the exporter's real income increases, and when $(\gamma - 1)\sum_{k\neq i} \frac{x_{ki}}{e_k}\pi_{kj}x_{ij} > x_{ij}$, then the average income in all other countries declines: $e_i^f\, d\ln(u_i) > 0 > \frac{1}{n-1}\sum_{k\neq i} e_k^f\, d\ln(u_k)$.*

The result (proven in Appendix A.3) is essentially an application of the second Hicksian law of comparative statics (see Morishima, 1960). The underlying logic is simple. In response to the cost reduction, importer j buys more products from exporter i. This comes at the expense of other nations who also sold their products to j and who now see their sales shares decline (the *import competition channel:* $(1-\gamma)\pi_{kj}x_{ij}$). As a result, their wages decline relative to the wage paid by the exporter (Part 4a). Moreover, as the exporter's *terms of trade* unambiguously improve this way, the country also gains in real income terms. The average real income in all other nations $k \neq i$ declines, by contrast, in particular when both i and j are important markets to them (i.e., when both $\frac{x_{ki}}{e_k}$ and $\pi_{kj} = \frac{x_{kj}}{e_j}$ are large) so that their combined terms of trade losses outweigh the improved supplier access, x_{ij}, of importer j (Part 4b).

With production linkages Things can be very different in a production network. The reason is that parts of the exporter's gains spill over to other nations so that the tight connection between the country's product and labor markets is broken. In fact, nothing speaks against a scenario where a third country's workers benefit more from an export cost reduction than the exporter's own workers because that third country is either a major input supplier to the exporter or an important downstream customer of the importer who substantively benefits from the resultant input cost savings.

Our next result shows that, under certain conditions on each country's pre-shock supply chain exposure of to the affected trade link, such a scenario indeed becomes an inevitable fact (see Appendix A.3 for proof).

Proposition 5 *(Export cost reduction with supply chains). Consider a unilateral trade cost reduction, $d\tau_{ij} = -y\tau_{ij}$, $y > 0$, in a global production network. (5a) When*

$$(\gamma - 1)\sum_{k\neq i}\phi_{ki}^{down} + \sum_{k\neq i}(\gamma-\beta)\phi_{kj}^{up} >$$
$$(\gamma - 1)\sum_{k\neq i,\, l,m\in\mathcal{N}}\phi_{kl}^{down}\pi_{lm}\phi_{mj}^{up} + (\gamma-\beta)\sum_{k\neq i,\, l\in\mathcal{N}}\phi_{kl}^{down}\phi_{lj}^{up},$$

then there is at least one $k \neq i$ with $d\ln(w_k) > d\ln(w_i)$. (5b) Moreover, when

$$(\gamma-1)\phi_{ki}^{down} + (\gamma-\beta)\phi_{kj}^{up} >$$
$$(\gamma-1)\sum_{l,m\in\mathcal{N}}\phi_{kl}^{down}\pi_{lm}\phi_{mj}^{up} + (\gamma-\beta)\sum_{l\in\mathcal{N}}\phi_{kl}^{down}\phi_{lj}^{up} + \phi_{ij}^{up}$$

holds for every single $k \neq i$, then we even have $d\ln(w_k) > d\ln(w_i)$ for all $k \neq i$ and $e_i^f\, d\ln(u_i) < \frac{1}{n-1}\sum_{k\neq i} e_k^f\, d\ln(u_k)$.

Hence, according to Part (5a), the exporter and importer should ideally be some hub countries that pass on much of the additional demand generated by the export cost reduction to the exporter's upstream suppliers (as measured by a high value for $\sum_{k\neq i}\phi_{ki}^{down}$) or much of the input cost savings to the importers' downstream customers (measured by a high $\sum_{k\neq i}\phi_{kj}^{up}$).

Yet, both spillovers need to be compared to some critical values. In particular, the relevant benchmark for the upstream spillover is $\sum_{k\neq i,\, l,m\in\mathcal{N}}\phi_{kl}^{down}\pi_{lm}\phi_{mj}^{up}$ because this measures the extent to which i's suppliers are, at the same time, hurt by the more intense competition in all their other sales markets, where their competitors from countries m benefit from the input cost savings passed on to them from importer j (a *competitors' productivity effect*). The downstream spillover to j's customers must, in turn, be benchmarked against $\sum_{l\in\mathcal{N}}\phi_{kl}^{down}\phi_{lj}^{up}$ because this measures in how far their competitors can reduce their input costs as well (another *competitors' productivity effect*). When the sum of the up- and downstream spillover surpasses the sum of these critical values, at least one country's workers gain more from the cost reduction than the exporter's own workers.

In a production network, an export cost reduction may, however, also lead to a larger wage gain in every single other country. According to the condition in Part (5b), this is the case when the exporter and importer are important hub countries for every single other nation $k \neq i$. The cost reduction may then even backfire on the exporter in terms of a real income loss, in particular when the exporter's *terms of trade* losses are so strong that they even outweigh the costs savings to the country's own consumers (due to their own upstream exposure to importer j: ϕ_{ij}^{up}).

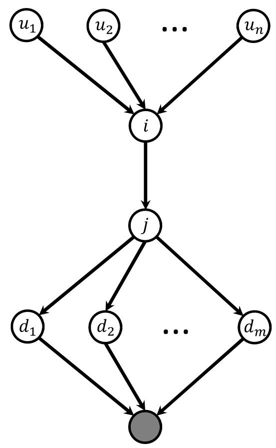

Fig. 1. A tree network.

To make the scenarios of Proposition 5 more concrete, consider the following two examples.

Example 1 Start from the network in Fig. 1. All countries, except for countries $d \in \mathcal{D}$, are intermediate goods suppliers. Countries $d \in \mathcal{D}$ are, by contrast, the final goods producers, and they sell their products to all other nations. Regarding the technologies of production, the upstream countries $u \in \mathcal{U}$ just use labor; all the other countries combine their own labor with the intermediate goods assembled at the previous stage using technology (3). How large are the spillovers from a trade cost reduction along the "bridging tie" between countries i and j?

To keep things simple, let us assume that all $u \in \mathcal{U}$ and all $d \in \mathcal{D}$ are symmetric with respect to their technologies, sizes, and trade costs. This allows us to focus on the pure positional impact of the link in the network. Now, when exporter i is the reference country, the wage effects of a cost reduction on ij become

$$d\ln(w_k) = \begin{cases} 0 & \text{for } k \in \mathcal{U} \cup \{i\} \\ y\frac{1-\beta}{\beta} & \text{otherwise} \end{cases} . \tag{16}$$

See Supplementary Material S.3 for the derivation. Clearly, the expression suggests that there is always at least one country gaining as much from the cost reduction as the exporter himself.[10] Moreover, who is gaining from the cost reduction depends on just a single parameter: the elasticity of input substitution, β. In particular, when labor and intermediate inputs are complements ($\beta < 1$) in production, it is the countries downstream of the link ij who gain the most; by contrast, the upstream countries gain the most when labor and intermediates are substitutes ($\beta < 1$).

The intuition lies in the unique position of the link ij in the network. As country i is the sole supplier of j (and j is the sole supplier of all $d \in \mathcal{D}$), the cost reduction between them does not improve i's market access to j. Moreover, for the same reason, all the downstream countries $k \in \mathcal{D}$ experience the same cost reduction on their imports of intermediate products so that no country gains a competitive edge in any of its sales markets (i.e., it is as if $\gamma = 1$ in the conditions of Proposition 5). The only effect of relevance is, thus, the *input substitution channel*, capturing the shock's impact on the optimal use of labor and intermediate inputs in production. Specifically, when labor and intermediate inputs are complements (substitutes), importer j and its downstream customers $d \in \mathcal{D}$ increase (decrease) their expenditure shares on labor. In other words, the sole effect of the cost reduction on link ij is a shift of the demand for value added to the up- or downstream stages of production, depending on the value of β.

Our next example highlights another supply chain channel that can backfire on the exporter:

Example 2 Consider the network of Fig. 1 again, but this time, we look at a trade cost reduction on the link between j and one of its downstream customers $d_i \in \mathcal{D}$. How does the resultant cost advantage to importer d_i play out for exporter j?

To keep things simple again, let us shut down the input substitution channel in addition, that is, set $\beta = 1$. With importer d_i as the reference country, the wage effect of the cost reduction can then be written as

$$d\ln(w_k) = \begin{cases} -(\gamma-1)\frac{|\mathcal{D}|-1}{|\mathcal{D}|}\frac{1-\lambda_d}{1+(\gamma-1)\lambda_d} & \text{for } k \in \mathcal{U} \cup \{i,j\} \\ 0 & \text{for } k = d_i \\ -(\gamma-1)\frac{1-\lambda_d}{1+(\gamma-1)\lambda_d} & \text{for } k \in \mathcal{D}\backslash\{d_i\} \end{cases} .$$

[10] As in this simple example of a tree network, consumer prices are the same in all $i \in \mathcal{N}$, the wage effects of a cost reduction on link ij are also the sole determinants of the cross-country variation in real income effects.

Table 1
Positive spillover links.

	% links passing Prop. 4a (1)	% 3rd countries benefiting (2)	% positive welfare spillovers (3)
All countries			
1980–2011	3.7	33.3	83.3
before 1996	2.2	32.7	81.9
after 1996	4.6	33.4	83.6
Top 7 - exporters			
Hong Kong	28.2	8.1	71.1
Macao	17.4	9.4	100.0
Singapore	17.1	16.5	43.0
Belgium/Luxembourg	14.4	39.3	69.5
Malta	12.2	37.5	100.0
Netherlands	10.9	43.8	46.4
Malaysia	9.6	20.2	44.6

NOTES: Column 1 reports averages across all active trade links and across all different values for $\beta \in \{.001, .5, 1.001, 1.5, ..., 4.5, 5\}$. Columns 2 and 3 report averages across the subset of links passing Proposition 5a. In Column 3, a link ij is said to produce positive welfare spillovers if $\frac{1}{n-1}\sum_{k\neq i} e_k^f\, d\ln(u_k) > e_i^f\, d\ln(u_i)$.

As becomes clear, a cost reduction on jd_i inflicts, in the first instance, a negative spillover on all countries $k \in \mathcal{D}\backslash\{d_i\}$ because these countries lose some market share to their competitor d_i (the *competitor's productivity channel*). Importantly, however, the negative competition spillover is also passed on to exporter j (and all its upstream suppliers) because they are also the sole suppliers of all $d \in \mathcal{D}$. As a result, exporter j unambiguously loses in wage income terms relative to importer d_i, and this loss is larger the more competitors country j supplies.

Network spillovers in the data So far, our findings suggested the possibility of sizable network spillovers from an export cost reduction in a production network. But how demanding are the conditions that we needed in the construction of our results?

To answer this question, we went to our data and checked whether we could actually find some links that satisfy the rather demanding requirements of Proposition 5. More concretely, we considered each one of the 235,655 active trade links (with $\pi_{ij} > 0$) in our two data sets and imposed, one by one, a 1% unilateral export cost reduction on them. Table 1 summarizes our findings on the resultant wage and real income effects on the exporter, the importer, and all the remaining nations.

We first take a look at Column 1 which shows in the top panel the shares of links passing the general condition of Proposition 5a, for both the first (1980-1995) and the second (1996-2011) half of our data. Clearly, the number of links qualifying as a 'positive spillover link' is substantive. Across all years and β-specifications looked at, $8,719$ of all active trade links (3.7%) qualify as such a link, and this share even doubles over time. Moreover, if a link satisfies the condition of Proposition 5a, there is typically more than one country benefiting (33.3 countries on average). In 7,263 cases (83.3% of all positive spillover links), the average third country benefits even more than the exporter in real income terms (see Column 3). Remember that, according to Proposition 4, all this would be impossible in a world where only final goods are traded.

What can we say about the location of these positive spillover links within the production network? One way to shed light on this is by looking at the exporters and importers involved. Table 1 lists in the bottom panel the seven countries that most often appear on the exporters' side. Probably not surprisingly, all of them are generally regarded as countries with a high ratio of trade over value added or, in our terminology, hub countries that heavily expose their upstream suppliers to shocks on their outgoing links. Hong Kong, Singapore, and Malaysia are the lead examples for East Asia, and Belgium and the Netherlands for Europe. These countries alone account for 39.5% of all positive spillover links, and they maintain their critical role over time. On the importers' side, we find, by contrast, a very diverse set of smaller countries which, not surprisingly, source a large fraction of their imports from one of the hub countries. The reason is that this creates the ideal breeding ground for positive spillovers to emerge because no third country is significantly hurt by the more intense competition with the exporter.

Finally, what can we say about the importance of the up- and downstream diffusion channel? Based on Example 1, we would expect that their relative importance depends crucially on the assumed value for β. For this reason, we re-calculated the number of positive spillover links separately for different βs and then asked, for each of link, which one of the two terms in Proposition 5a, $(\gamma - 1)\sum_{k\neq i}\phi_{ki}^{down}$ or $(\gamma - \beta)\sum_{k\neq i}\phi_{kj}^{up}$, is larger and, thus, primarily responsible for why a link has passed the condition in 5a. Our findings in Table 2 confirm the expected pattern: when labor and intermediate inputs are substitutes (i.e., when $\beta \geq 1$), virtually all beneficiaries of a cost reduction on a positive spillover link can be found on the upstream side of the link. When labor and intermediates are complements, by contrast, downstream diffusion plays a role as well.

Table 2
Up- and downstream spillovers.

Input elasticity	% links passing Proposition 5a (1)	% spillovers to upstream countries (2)
$\beta = 0.001$	3.7	23.7
$\beta = 0.5$	3.7	47.3
$\beta = 1.01$	3.7	97.7
$1.5 \leq \beta \leq 5$	3.6	100.0

NOTES: Column 1 (2) reports averages across all active links (links passing Proposition 5a). In Column 2, a link ij is said to primarily generate upstream spillovers if $(\gamma - 1)\sum_{k\neq i} \phi_{ki}^{down} > (\gamma - \beta)\sum_{k\neq i} \phi_{kj}^{up}$.

5. The gains from a global trade cost reduction

We saw before that trade cost shocks can trigger sizable network spillovers along the supply chain links connecting countries. Here, we show that a country's supply chain exposure may even be the single most important determinant of the welfare gains from trade.

We study a global trade cost shock for this purpose, more concretely a proportional decline of all the domestic and international trade costs (i.e., better transport technologies). Intuitively, and based on the idea of Hicks neutrality, one might expect this cost reduction to improve the economic prospects of all countries alike. As each country scales up on its initial access to suppliers and markets, it is tempting to conclude that also the welfare gains are just proportional to each country's initial level of welfare. Yet, it turns out that this logic only goes through in the absence of production linkages.

No production linkages The following result (proven in Appendix A.4) serves as our benchmark.

Proposition 6 *(Global trade cost reduction without supply chains). Consider a world where only final goods are traded ($\boldsymbol{\kappa}^{\mathbf{i}} = \mathbf{0}$). Then, the real income gains from a global trade cost reduction,* $\mathbf{dT} = -y\mathbf{T}$*, are just proportional to each country's initial level of welfare:* $d\ln(u_i) = y \;\forall\, i \in \mathcal{N}$.

The intuition is just as outlined above. Since all countries improve their market access to all other nations alike, the cost reduction gives no country a comparative edge in its sales markets. The cost reduction is, thus, 'demand neutral' in the sense that $d\ln(w_i) = 0 \;\forall\, i \in \mathcal{N}$. What remains is the pure consumer price effect resulting from their improved access to final goods suppliers. And, since trade costs fall proportionally everywhere, this leads to a worldwide uniform welfare increase.

With production linkages The above logic is no longer valid in the presence of production linkages between countries.[11] A first important difference is that the consumer price effects are no longer the same across countries. Instead, by how much consumers gain from a global trade cost reduction depends on their upstream exposure to the local price effects in every other nation. This is exactly what the direct consumer price effect in Theorem 1 says:

$$\text{Direct consumer price effect: } d_{\mathbf{T}}\ln(p_i^f) = \frac{1}{e_i^f}\sum_{j\in\mathcal{N}} \phi_{ij}^{up}\,\delta_j^p\,, \tag{17}$$

where in the case of a globally uniform cost reduction, $\delta_j^p = -y\,x_j$. Intuitively, the cost reduction not only enhances consumers' direct access to their final goods suppliers but also initiates cost savings across all upstream stages of production which are passed on to the consumers based on their home countries' entries in the $\boldsymbol{\Phi}^{\mathbf{up}}$-matrix.

The second major departure from a world without production linkages is that the logic of 'demand neutrality' no longer applies. Even though all countries improve their market access to their sales markets alike, the wage effects of the cost reduction are additionally shaped by the *productivity* and *input substitution* channels, which can vary significantly from one country to another. Clearly, by how much a country is affected by these channels depends on its exposure to the resultant input cost savings for its upstream suppliers. We thus get

$$\text{Productivity effect: } d_{\mathbf{T}}\ln(p_i) = \frac{1-\lambda_i}{e_i^f}\sum_{j\in\mathcal{N}} \phi_{ij}^{up}\,\delta_j^p\,, \tag{18}$$

$$\text{Input substitution effect: } d_{\mathbf{T}}\ln(\lambda_i) = (\beta - 1)\,d_{\mathbf{T}}\ln(p_i)\,.$$

[11] The logic of Proposition 6 still upholds in the case of some related counterfactual shocks. For instance, it fully carries over to the case of a global trade cost reduction on just the final outputs or to the case of a proportional increase in each country's labor productivity, $\boldsymbol{\kappa}^{\mathbf{l}}$. In both cases, the shock is, as in Proposition 6, demand neutral, leading to a uniform welfare increase in every country.

Yet, whether the exposure to these channels is a blessing or a curse is less clear: Each country's workers benefit, to a lesser or greater extent, from the productivity gains of their domestic producers. But, as a global trade cost reduction also improves the productivity of their foreign competitors and, at the same, triggers input substitution at home, it also puts the labor demands in each nation under pressure. These opposing forces of a global trade cost reduction on the wages in each nation are highlighted in the following expression (derived in Appendix A.4):

$$\begin{aligned} \mathbf{d}\ln(\mathbf{w}) &= [\mathbf{LW}]^{-1}\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}\,\mathbf{d_T}\ln(\mathbf{Wl^d}), \quad \text{with} \\ \mathbf{d_T}\ln(\mathbf{Wl^d}) &= \underbrace{\Big(\mathbf{E} - \boldsymbol{\Phi}^{\mathbf{down}}\boldsymbol{\Pi}\,\mathbf{E}\Big)\mathbf{d_T}(\lambda)}_{\text{Own and customers' input substitution}} \\ &\quad + (\gamma-1)\underbrace{\Big(\mathbf{E^f} + \boldsymbol{\Phi}^{\mathbf{down}}\boldsymbol{\Pi}\,\mathbf{E^i} - \boldsymbol{\Phi}^{\mathbf{down}}\boldsymbol{\Pi}\,\mathbf{E}\,\boldsymbol{\Pi}^\top\Big)\big(-\mathbf{d_T}\ln(\mathbf{p})\big)}_{\text{Own, customers, and competitors' productivity effects}}. \end{aligned} \tag{19}$$

Leaving the general-equilibrium multiplier, $\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}$, aside for a moment, the second line suggests that a country's labor demand is directly affected by, on the one hand, the extent of input substitution at home and, on the other hand, the input substitution in all the other nations buying intermediate products from this country. By how much a country is affected by this foreign input substitution channel depends on its downstream exposure to every other nation and, thus, on its row entries in matrix $\boldsymbol{\Phi}^{\mathbf{down}}\boldsymbol{\Pi}\,\mathbf{E}$.[12]

The third line of (19) then illustrates the productivity effects of a global trade cost reduction. Since $\gamma > 1$, all nations benefit from their domestic productivity gains (the first summand in line three), while they are hurt by the productivity gains of their foreign competitors (the final summand). Unlike the foreign input substitution channel, which may backfire on a country depending on the value of β (see (18)), the productivity channels mentioned above are clearly in the advantage of a country, given that this country holds a downstream position within the global production network. This is because downstream countries capture a greater share of the productivity gains in the upstream stages of production compared to their more upstream competitors. By contrast, the second summand on line three of (19) is in the advantage of countries in the upstream stages because these countries benefit more from the productivity gains of their downstream customers.

Thus, in total, expression (19) encompasses several conflicting channels that make it difficult to unambiguously sign the welfare effects in the general case of our model. Nevertheless, the above arguments do suggest that the advantage is on the side of downstream countries in the production network, that is, those with a high upstream exposure. For one thing, input substitution between labor and intermediate products is more difficult than substitution between different product varieties (i.e., $\beta \le \gamma$), so there is a larger weight on the productivity than on the input substitution channels in (19). For another, also consumers benefit from a higher upstream exposure of their home country because they experience larger price reductions this way (see (17)).

Indeed, we can find at least three plausible model specifications where the advantages of upstream exposure are unequivocally evident:

Homogeneous input cost shares The first specification is the classic one in international trade: a model with flexible expenditure shares but fixed and identical input cost shares in production, i.e., $\gamma > 1$, $\beta = 1$, and $\lambda_i = \lambda$ (e.g., Krugman and Venables, 1995; Eaton and Kortum, 2002; Arkolakis et al., 2012).

In this case, a global trade cost reduction improves each country's supplier access to its direct intermediate goods suppliers at a rate that is just proportional to the common intermediate input cost share $(1-\lambda)$. Furthermore, upon inspection of the definition of $\boldsymbol{\Phi}^{\mathbf{up}}$ in (15), it follows that, with a common cost share of $1-\lambda_i = 1-\lambda$, each country's exposure to these improvements in the upstream stages of production is identical as well and can be expressed as

$$\begin{aligned} d_{\mathbf{T}}\ln(p_i) &= \frac{1-\lambda}{e_i^f}\sum_{j\in\mathcal{N}}\phi_{ij}^{up}\,\delta_j^p \\ &= -y(1-\lambda)\Big(1 + \sum_{j\in\mathcal{N}}\pi_{ji}(1-\lambda) + \sum_{j\in\mathcal{N}}\pi_{ji}(1-\lambda)\sum_{k\in\mathcal{N}}\pi_{kj}(1-\lambda) + \ldots\Big) \\ &= -y\frac{1-\lambda}{\lambda}. \end{aligned} \tag{20}$$

Hence, when input cost shares are identical, a global trade cost reduction just lowers each country's producer prices at the same rate. As a result, no country gains from its relative up- or downstreamness in the production network because the conflicting productivity and substitution channels in (19) just cancel each other out. In other words, we retain the demand neutrality of Proposition 6 again

[12] Noteworthy, this modified downstream exposure measure, $\boldsymbol{\Phi}^{\mathbf{down}}\boldsymbol{\Pi}\,\mathbf{E}$, is closely related to the Antràs et al. (2012) measure of *upstreamness*, which intends to capture the position of each country in the global production network. The exact relationship with the column vector of *upstreamness* υ is $\upsilon = [\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{down}}\boldsymbol{\Pi}\,\mathbf{E}\,\mathbf{1}$. Thus, in a sense, upstreamness measures the downstream exposure to a vector of homogeneous local effects.

(i.e., $d\ln(w_i) = 0$), so that the real income effects of a global trade cost reduction are solely determined by the consumer price effects[13]:

Proposition 7 *(Global trade cost reduction with homogeneous input shares). Consider a model specification with flexible expenditure shares but fixed and identical input cost shares (i.e., $\gamma > 1$, $\beta = 1$, and $\lambda_i = \lambda$). The real income effects of a global trade cost reduction, $\mathbf{dT} = -y\mathbf{T}$, are then given, in a first-order approximation, by*

$$d\ln(u_i) = \frac{y}{e_i^f} \sum_{j\in\mathcal{N}} \phi_{ij}^{up} x_j \qquad \forall\, i \in \mathcal{N}\,. \tag{21}$$

Based on the same argument as in (20), this can be further simplified to $d\ln(u_i) = y/\lambda$. Hence, similar to a world without production linkages, all countries gain alike; however, distinct from this scenario, the welfare effect is amplified by a factor $1/\lambda$.

Fixed expenditure and input cost shares The same prediction as in (21) emerges from the classic model specification in macroeconomics: the Long and Plosser (1983) model with fixed, but potentially heterogeneous input and expenditure shares (i.e., $\beta = \gamma = 1$). By inspection of the formulas in (18) and (19), it becomes immediately evident that a global trade cost reduction does not have any impact on labor demands and wages in this case. What remains is the pure consumer price effect, which is solely determined by a country's upstream exposure.

Leontief Consider finally a Leontief specification, where labor and intermediate inputs are perfect complements in production (i.e., $\gamma > 1$ and $\beta = 0$). As full employment is assumed in our model and since labor and intermediate inputs are used in fixed proportion when $\beta = 0$, the demand for intermediate inputs must then remain unchanged following any type of trade cost shock, that is, $d(\sum_{j\in\mathcal{N}} q_{ji}^i) = 0\ \forall\, i \in \mathcal{N}$. Moreover, when the cost reduction is uniform, producer prices remain unaffected as well, i.e., $d\ln(p_i) = 0$, so that goods prices change in proportion to the size of the shock, i.e., $d\ln(p_i^f) = -y$, and wages according to $d\ln(w_i) = y(1-\lambda_i)/\lambda_i$. The real income effect is, therefore, given by

Proposition 8 *(Global trade cost reduction in the Leontief case). Consider a Leontief specification for our model (i.e., $\beta = 0$). The real income effect of a global trade cost reduction, $\mathbf{dT} = -y\mathbf{T}$, is then given by*

$$d\ln(u_i) = \frac{y}{\lambda_i} \qquad \forall\, i \in \mathcal{N}.$$

See Appendix A.4.3 for the proof. In other words, the real income effects of a global trade cost reduction are higher in countries with a higher intermediate input cost share, that is, those with a higher upstream exposure.

Global trade cost reduction in the data Throughout all our preceding predictions, the real income effects of a global trade cost reduction were both positive and more pronounced in countries with higher upstream exposure.[14] To see how far these predictions carry over to the empirical network structure of production, we now take them to our data and study the welfare effects of a 1% uniform trade cost reduction on all the active trade links in each year of our two data sets.

Our first set of findings are summarized in Fig. 2, which illustrates each country's predicted real income gain of such a cost reduction over time. Clearly, the figure suggests that the presence of production linkages works to the advantage of each and every country. Across all countries and years, the smallest predicted welfare gain is always strictly larger than the 1%-gain we would have expected in the absence of these linkages (see Proposition 6). In fact, the average predicted welfare gain is even 3.2%, and it slowly increases over time.[15]

More importantly, Fig. 2 suggests substantive cross-country variation in welfare effects. Fig. 3 explores this variation in more detail. There, we 'zoom in' onto the most recent year in our data and plot each country's predicted welfare gain against its consumers' upstream exposure, $(1/e_i^f)\sum_{j\in\mathcal{N}} \phi_{ij}^{up} e_j$, its workers' upstream exposure (separated out in expression (19)),

---

[13] For the proof of Proposition 7, just notice that labor market clearing implies $\mathbf{\Phi^{down}\,\Pi\,e^f} = \mathbf{e^f}$ (see equation (A.19) in Appendix A.2). In combination with the homogeneous price effects, this, in turn, suffices for the conflicting channels in (19) to cancel each other out.

[14] These results cannot be generalized easily to all the possible specifications for our model and to all network structures. In Supplementary Material S.3, for instance, we expand on our analysis of the tree network of Fig. 1 to study a uniform trade cost reduction on all its links. Our findings on this case suggest that the real income effects may increase in a country's downstream exposure, and some countries may even lose out.

[15] At first sight, an average welfare gain of 3.2% might seem at odds with Proposition 3, where we concluded that the presence of production linkages does not have any impact on the average gains from trade. The fact is, however, that the average welfare effect of a 1% global trade cost reduction is, according to this proposition, larger than 1% because

$$\frac{\sum_{i\in\mathcal{N}} e_i^f\, d\ln(u_i)}{\sum_{i\in\mathcal{N}} e_i^f} = \frac{0.01\sum_{i\in\mathcal{N}} e_i}{\sum_{i\in\mathcal{N}} e_i^f} > 0.01\,.$$

This is the well-known effect magnification that occurs when goods pass the same affected border multiple times before reaching final consumers (see, e.g., Yi, 2003; Bems et al., 2011).

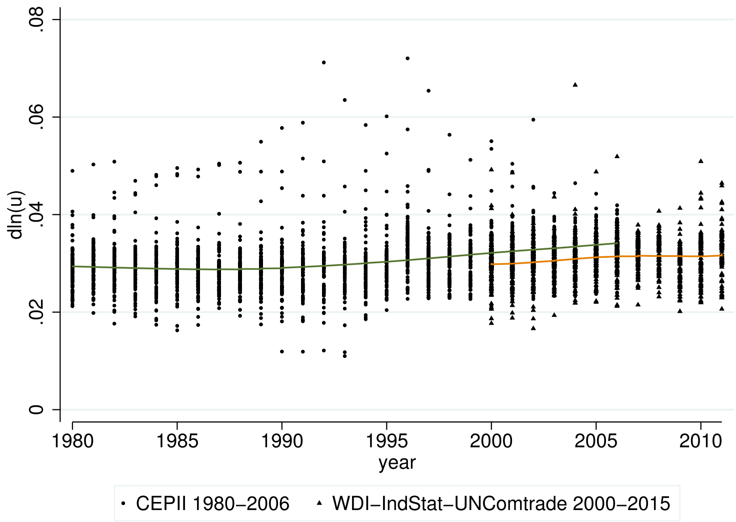

Fig. 2. Gains from a global trade cost reduction, 1980–2011. NOTES: Real income effect by country (in %) of a 1% global trade cost reduction in a specification with $\beta = \gamma = 5$. The picture looks very similar under any other specification with $\beta \in \{.001, .5, 1.001, 1.5, ..., 4.5\}$. In the years covered by both data sets (2000–2006), we always find larger income effects in the CEPII data, which is due to the fact that our self-collected data covers about 10–20 more countries that are not so well-connected in the production network.

$$\underbrace{e_i\, d_{\mathbf{T}} \lambda_i}_{\text{Own substitution}} + \underbrace{(1-\gamma)\, e_i^f\, d_{\mathbf{T}} \ln(p_i)}_{\text{Own productivity}} = y(\gamma - \beta)(1-\lambda_i) \sum_{j \in \mathcal{N}} \phi_{ij}^{up} x_j \,,$$

and its workers' downstream exposure (see again (19)),

$$\sum_{j,k \in \mathcal{N}} \phi_{ij}^{down} \pi_{jk} \Bigg(\underbrace{-e_k\, d_{\mathbf{T}} \lambda_k}_{\text{Customers' substitution}} + \underbrace{(1-\gamma) e_k^i\, d_{\mathbf{T}} \ln(p_k)}_{\text{Customers' productivity}} \underbrace{- (1-\gamma) e_k \sum_{l \in \mathcal{N}} \pi_{lk}\, d_{\mathbf{T}} \ln(p_l)}_{\text{Competitors' productivity}} \Bigg) \,,$$

for a model specification with input substitutability ($\beta = 2.5$). This allows us to evaluate whether, as predicted before, consumer upstream exposure alone (as in Proposition 7) or worker upstream exposure alone (as in Proposition 8) is able to predict each country's entire welfare effect. The figure, moreover, contrasts their predictive power with workers' downstream exposure and the general-equilibrium multipliers,

$$\mathbf{LW} \Bigg(\underbrace{\left(\mathbf{\Phi}_{\mathbf{i}*}^{\mathbf{mult}} - \mathbf{I}\right)}_{\text{Income multiplier}} - \underbrace{\mathbf{\Phi}^{\mathbf{tot}} \mathbf{\Phi}_{\mathbf{i}*}^{\mathbf{mult}}}_{\text{Terms of trade}} \Bigg) \mathbf{d}_{\mathbf{T}} \ln(\mathbf{Wl^d}) \,, \tag{22}$$

as the other two important channels in our diffusion model.

The most striking observation from Fig. 3 is the high goodness of fit of our two upstream exposure measures, consumers' upstream exposure (upper left panel) and workers' upstream exposure (upper right). Both measures on their own account for more than 52% of the cross-country variation in welfare effects. Workers' downstream exposure exhibits, by contrast, a much lower goodness of fit, and it is even negatively associated with the full welfare effect whenever $\beta < 4$ (lower left panel). Combined with our first finding, this is not at all surprising. Downstream exposure is high in the upstream stages of production where upstream exposure is low.[16]

Finally, the general-equilibrium effects always have the lowest predictive power of all determinants (lower right panel). Again, this is not surprising. As we said before, the primary role of the foreign income multiplier is to amplify the direct labor demand effects, $d_{\mathbf{T}} \ln(w_i l_i^d)$, in our model (see also Lemma 1). The terms of trade multiplier, by contrast, governs their spillovers on the goods prices, thereby compressing their real income effects, so that the general equilibrium effects as a whole have little predictive power. Overall, thus, it appears that also in the data, a country's upstream exposure alone is the most powerful predictor for a global trade cost reduction's full welfare effect.

[16] The illustrations in Fig. 3 are highly representative of all of the other β-specifications we examined. Across all specifications, the average R-squared is 0.65 for consumers' upstream exposure, 0.71 for workers' upstream exposure, 0.15 for workers' downstream exposure, and 0.07 for the general-equilibrium effects.

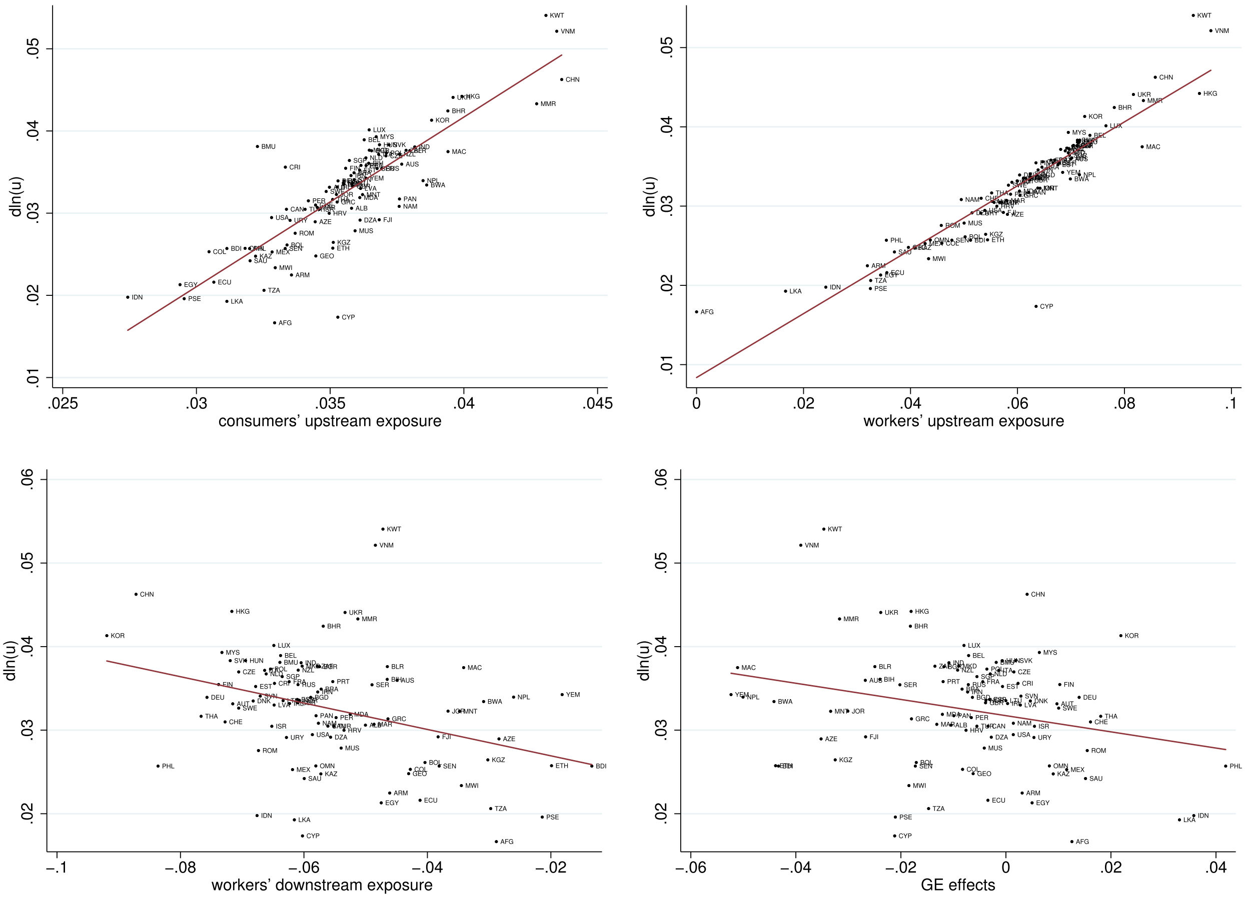

Fig. 3. Determinants of the gains from a global trade cost shock ($\beta = 2.5$). NOTES: Specification with $\beta = 2.5$ and $\gamma = 5$. Data from 2011. A linear regression on the full welfare effect gives an R-squared of 0.71 for consumers' upstream exposure, 0.85 for workers' upstream exposure, 0.12 for workers' downstream exposure, and 0.07 for the general-equilibrium effects.

6. Key trading partners

The preceding sections highlighted the importance of a country's up- and downstream exposure in shaping the welfare consequences of various types of trade cost shocks. In this section, we go a step further by asking where a country's supply chain exposure comes from. Leveraging existing network literature, we identify each country's key trade partners for this purpose——countries whose removal from the production has the most substantial impact on their welfare, and their up- and downstream exposure in particular.[17]

Our first result is as follows. Suppose one is merely interested in the importance of a country for the world as a whole. Then, it follows as a corollary of Proposition 3 that all one needs to know is the value of its total output:

Corollary 1 *(Worldwide effect of a country's isolation). The worldwide total welfare effect of country i's partial isolation, i.e.,* $\mathbf{dT} = \left(y\tau_{ji},\, y\tau_{ik} \text{ for } j,k \in \mathcal{N};\ 0 \text{ otherwise}\right)$ *with $y > 0$, is in a first-order approximation given by*

$$\sum_{j\in\mathcal{N}} e_j^f \, d\ln(u_j) = -2yx_i\,.$$

If one wants to know more, however, and understand which nations are most affected by a country's isolation, and through which channels, knowing a country's total output is not enough. For one thing, different countries might be differently dependent on a given trade partner, leading to potentially very unequal divisions of the total welfare loss. For another, the output measure does

[17] The literature on key players in networks is huge. Applications can be found in such diverse fields as computer network (Goyal and Vigier, 2014), disaster impact analysis (Foti et al., 2013), optimal policies for R&D networks (König et al., 2019), or multi-party armed conflicts (König et al., 2017). We contribute to this literature by breaking down the distinct effects of a key player's removal from a network. Moreover, there is a close relationship between our analysis here and the classic 'gains from trade' analysis, which looks at the flipside of what we are interested in, namely at the loss in the isolated country itself.

not say much about how other nations are affected. How much do they lose because of their foregone access to the isolated country's own value added and final demand, and how much because of their lowered up- and downstream exposure through this country?

These two pieces of information are provided by the following 'key trade partner' formula. It calculates the welfare loss in every other nation and distinguishes between several meaningful channels. For expositional clarity, we thereby focus on a uniform elasticity specification ($\beta = \gamma$) and consider the effects of a country's entire isolation.[18]

Key trade partner formula Consider the total isolation of any country $i \in \mathcal{N}$. The real income effects in every other nation $j \neq i$ are in a first-order approximation given by

$$d\ln(u_j) = \frac{1}{z_i}\left(d\ln(w_j) - d_{\mathbf{T}}\ln(p_j^f) - \frac{1}{e_j^f}\sum_{k\neq i}\phi_{jk}^{tot}\, l_k w_k\, d\ln(w_k)\right),$$

where $z_i > 0$ is a scale factor defined in Appendix A.5.1,

$$d_{\mathbf{T}}\ln(p_j^f) = \frac{1}{e_j^f}\sum_{k\in\mathcal{N}}\phi_{jk}^{up}\Big(\underbrace{\lambda_i x_{ik}}_{\substack{\text{(i) local}\\ \text{value added}}} + \underbrace{(1-\lambda_i)x_{ik}}_{\substack{\text{(ii) intermediated}\\ \text{value added}}}\Big), \text{ and} \tag{23}$$

$$d\ln(w_j) = \frac{1-\gamma}{l_j w_j}\sum_{l\in\mathcal{N}}\phi_{jl,i*}^{mult}\Bigg(\sum_{m\in\mathcal{N}}\phi_{lm}^{down}\cdot \underbrace{x_{mi}^f z_i}_{\substack{\text{(iii) local}\\ \text{demand}}} + \underbrace{\phi_{li}^{down}\cdot\frac{1}{\lambda_i}\sum_{m\in\mathcal{N}}\phi_{im}^{down}\sum_{n\neq i}x_{mn}^f}_{\substack{\text{(iv) intermediated}\\ \text{demand}}}$$
$$-\sum_{m\in\mathcal{N}}\phi_{lm}^{down}\cdot \underbrace{\sum_{n\neq i}x_{mn}^f d_{\mathbf{T}}\ln(p_n^f)}_{\substack{\text{softer competition}\\ \text{due to foregone (v) local}\\ \text{and (vi) int. value added}}}\Bigg).$$

The formula (developed in Appendix A.5.1) distinguishes a total of six different channels through which other nations' welfare is affected: Three channels are active regardless of whether or not countries share production linkages. First, workers from all nations lose access to the isolated country's final demand, putting their wages under pressure (channel iii). Second, consumers forego access to the isolated country's local value added, putting their prices under pressure (channel i). Finally, all nations lose a competitor in their sales markets, which, unlike the previous two channels, leads to a positive effect on their wages (channel v).

Three additional channels appear when production linkages are present. First, workers from all nations need to accept further rounds of wage cuts because they also lose access to the final demands of third countries, which they have accessed indirectly through their intermediate products sold to the isolated country i (channel iv). The expression for this *intermediated demand* is

$$\frac{1}{\lambda_i}\sum_{m\in\mathcal{N}}\phi_{im}^{down}\sum_{n\neq i}x_{mn}^f,$$

which is related to several other established measures for an actor's centrality in a network. It is, for instance, closely related to Hummels et al. (2001)'s measure of vertical specialization trade, which captures a country's engagement in global supply chains based on the proportion of imported intermediated inputs used for exports. Furthermore, when pre-multiplied by $\sum_{l\in\mathcal{N}}\phi_{jl,i*}^{mult}\phi_{li}^{down}$—as in Formula (23)—, we receive a measure of bridging capital, that is, a measure for how much country i is a vital connector between the final demands in one country $n \neq i$ and the factors used in the productions of these goods from another country $j \neq i$ (Jackson, 2020).

The second channel emerging in a production network is the remaining nations' foregone access to the foreign value added content incorporated in the isolated country's products (channel ii). This foregone *intermediated value added* can simply be measured by a country's intermediate input cost share

$$(1-\lambda_i).$$

Premultiplied by $\sum_{k\in\mathcal{N}}\phi_{jk}^{up}x_{ik}$, this again turns into a measure of bridging capital; this time, however, between the value added in country $n \neq i$ and the demand for it in any other country $j \neq i$. Third, and finally, all other nations experience an additional positive

[18] A formula very similar to (23) emerges from a partial-isolation counterfactual and a model specification with $\beta < \gamma$. The advantage of the uniform-elasticity case is the ability to study the first-order impact of an infra-marginal trade cost shock, such as a country's *entire* isolation. This makes Formula (23) in several respects easier because a number of channels that are present in the partial-isolation case cancel each other out. Nevertheless, do note that also in the case of $\beta < \gamma$, the same local and intermediation channels are active that are also key to Formula (23).

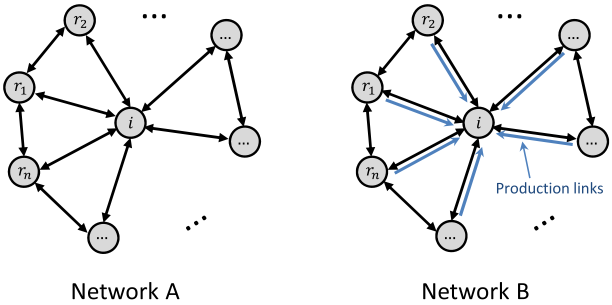

Fig. 4. Two key player networks.

impact on their wages because they do not need to compete anymore with the isolated country's intermediated value added (channel vi).

The different channels and their relative importance are illustrated in the following example:

Example 3 Consider the two networks in Fig. 4. All countries sell final goods on the links of these networks, next to the final goods they sell at home. In Network B, the ring countries additionally supply intermediate products to country i, which the latter uses in the production of its final goods. What is the welfare loss incurred by the ring countries when their connections to the center i are broken, and how does the size of this loss depend on their production links to country i?

Isolating country i from Network A results in a welfare loss that can be broken into the same three channels (i), (iii), (v) as in the general formula (23). On top of this, in Network B, the ring countries lose indirect access to the value added and markets of all other ring countries via country i, while the latter foregoes access to their intermediate inputs, rendering it a weaker competitor. Which of these opposing effects prevail?

To quantify them, assume as before that $\beta = \gamma > 1$. Suppose, moreover, that all ring countries are symmetric with respect to their sizes, technologies, and trade costs and that $\mu_r = \mu_i = \kappa_i^l = 1$. Using their wages as our reference price (i.e., $w_r = 1$) and making use of equations (1) and (5), we can write for the ring countries' real incomes $u_r = 1/p_r^f$, with a consumer price after i's isolation of $p_r^f = (3(w_r\tau_{rr})^{1-\gamma})^{\frac{1}{1-\gamma}} = (3\tau_{rr}^{1-\gamma})^{\frac{1}{1-\gamma}}$ and a consumer price before i's isolation of

$$p_r^f = \left(3\tau_{rr}^{1-\gamma} + (w_i\tau_{ir})^{1-\gamma} + n\tau_{ri}^{1-\gamma}(\kappa_i^i)^\gamma \tau_{ir}^{1-\gamma}\right)^{\frac{1}{1-\gamma}}.$$

Therefore, as p_r^f increases for all $\kappa_i^i \geq 0$, the ring countries lose out starting from either Network A or B. The magnitude of their welfare loss tends, however, to be larger in Network B because they also forego access to country i's intermediated value added (as captured by $n\tau_{ri}^{1-\gamma}(\kappa_i^i)^\gamma\tau_{ir}^{1-\gamma}$) and intermediated demand. The latter effect leads to a net welfare loss if and only if, before the shock, country i was primarily intermediating the value added from one ring country to another—rather than using it to become a fiercer competitor—and, hence, if and only if τ_{ir} is small compared to τ_{ii}.[19]

The value of trade intermediation Now that we understand how the effect decomposition in Formula (23) works, what is it useful for? Here, we explore one application, directly addressing our earlier question about the origins of a country's up- and downstream exposure.

The earlier arguments are already suggestive of a simple fact: a country's supply chain exposure is determined by the intermediation capacities of its trading partners (channels ii and iv in Formula (23)). The fundamental idea here is that these are the two channels in (23) through which a trade partner exposes a country to the value added and demand generated elsewhere in the world.

This insight can, in fact, be formalized. For this purpose, suppose we partially isolate each of country j's trade partners from the rest of the world. Summing up j's upstream exposure to the foregone intermediated value added (channel ii), we reach a total loss of

[19] Formally, whether country i's isolation leads to a larger or smaller welfare loss in Network B depends on the initial level of w_i in the two networks. The loss is larger in Network B if and only if $w_i^{netA} > w_i^{netB}$ because the ring countries' relative wage is larger in Network B in this case. Country i's initial wage is, however, in both networks defined by the unique solution to

$$\begin{aligned} w_i l_i &= n\frac{(w_i\tau_{ir})^{1-\gamma}}{(p_r^f)^{1-\gamma}} l_r + \frac{(w_i\tau_{ii})^{1-\gamma}}{(p_i^f)^{1-\gamma}} w_i l_i \\ &= \frac{(w_i\tau_{ir})^{1-\gamma}\left((w_i\tau_{ii})^{1-\gamma} + n\tau_{ri}^{1-\gamma} + n\tau_{ri}^{1-\gamma}(\kappa_i^i)^\gamma\tau_{ii}^{1-\gamma}\right)}{\left(3\tau_{rr}^{1-\gamma} + (w_i\tau_{ir})^{1-\gamma} + n\tau_{ri}^{1-\gamma}(\kappa_i^i)^\gamma\tau_{ir}^{1-\gamma}\right)\left(\tau_{ri}^{1-\gamma} + \tau_{ri}^{1-\gamma}(\kappa_i^i)^\gamma\tau_{ii}^{1-\gamma}\right)} l_r . \end{aligned} \quad (24)$$

Hence, the production links within Network B lead to two conflicting forces on w_i: firstly, the intermediated demand channel (the term $n\tau_{ri}^{1-\gamma}(\kappa_i^i)^\gamma\tau_{ir}^{1-\gamma}$ in the denominator of (24)) exerting downward pressure on w_i; secondly, the intermediated competition channel (the term $n\tau_{ri}^{1-\gamma}(\kappa_i^i)^\gamma\tau_{ii}^{1-\gamma}$ in the numerator) exerting upward pressure on w_i. When τ_{ir} is small compared to τ_{ii}, then the former effect dominates so that $w_i^{netA} > w_i^{netB}$.

$$(1/e_j^f)\sum_{i\in\mathcal{N}}\phi_{ji}^{up}x_i - 1$$

and, thus, a loss for country j that is proportional to its initial upstream exposure to every other country's total output (similar to effect (21) in Section 5).[20] Similarly, on the demand side, when we sum up j's downstream exposure to the foregone intermediated demand from its trade partners, we arrive at[21]

$$\sum_{i\in\mathcal{N}}\phi_{ji}^{down}e_i\,.$$

This is nothing but j's downstream exposure to the total income in every other nation. In other words, Formula (23) allows one to reconstruct a country's up- and downstream exposure from the intermediation capacities of its trade partners.

This, in turn, establishes the following connection to Section 5 (proven in Appendix A.5):

Proposition 9 *(Trade intermediation). Consider a global trade cost reduction,* $\mathbf{dT} = y\mathbf{T}$*, and suppose that* $\beta = \gamma$*. The relative welfare gains of any two countries* j *and* k *are solely dependent on the intermediation capacities of their trading partners. That is, let* $d_{\mathbf{T}_\mathbf{i}^{\mathbf{int}}}\ln(u_j)$ *and* $d_{\mathbf{T}_\mathbf{i}^{\mathbf{int}}}\ln(u_k)$ *denote the sum of channels (ii), (iv), and (vi) in Formula* (23)*. We then get*

$$d_{\mathbf{T}}\ln(u_j) - d_{\mathbf{T}}\ln(u_k) = -\frac{1}{2}\sum_{i\in\mathcal{N}}\left(d_{\mathbf{T}_\mathbf{i}^{\mathbf{int}}}\ln(u_j) - d_{\mathbf{T}_\mathbf{i}^{\mathbf{int}}}\ln(u_k)\right).$$

Key intermediaries in the data Fig. 5 depicts the intermediation capacities (sum of channels (ii), (iv), and (vi) in Formula (23)) of each country in the latest year of our data. The size of a node in this figure indicates the importance of a country as a trade intermediary for the average other nation; the size of an arrow ij the specific importance of country i for another country j.

Two basic observations explain the overall picture in Fig. 5. Firstly, trade intermediation is a geographically very confined phenomenon.[22] Secondly, it is typically the larger nation that acts as an intermediary for its smaller neighbors. Not surprisingly, then, each country serves as its own most important trade intermediary.[23] Furthermore, most of the important *international* intermediation ties originate from the same few countries. China, as one of the key intermediaries in Asia, stands out in this regard. It has the strongest intermediation ties of all nations with its direct neighbors in Southeast Asia. Yet, for the world as a whole, China is only of minor importance as an intermediary because many other countries are also significantly hurt by the foreign value-added content in Chinese exports (the competition channel (vi) in Formula (23)). A similar, albeit less pronounced, pattern emerges for the other key intermediaries in Asia (India and Russia) and Europe (Germany, France, and Italy) as well. The U.S., in contrast, stands out as a key intermediary that is important not only for countries in the Americas but also for several other nations beyond its immediate geographic neighborhood.

In the light of Proposition 9, it is therefore not surprising that many of the largest beneficiaries of a global trade cost reduction are, according to our quantitative predictions in Section 5, located in Europe and East Asia. As can be seen from Fig. 5, many countries in these regions benefit from their proximity to, sometimes multiple, key intermediaries. At the same time, the low density of intermediation ties in the Americas explains why the top gainer on this continent, Brazil, is only ranked 31st in the world.

7. Conclusion

In this paper, we develop a novel network perspective on the welfare gains from trade. We show that the comparative statics predictions of a standard general-equilibrium trade model can be expressed in terms of a network diffusion model that allows one to isolate the classic general-equilibrium multipliers of a (trade cost) shock from the resultant spillovers through the production network. Our key insight derived from this model is that the important dimensions of the production network's structure are fully captured by two simple statistics: a country's up- and downstream exposure in the network relative to the location of the shock.

Applying the model's predictions to three specific types of shocks delivers several additional insights. First, in all our exercises, up- and downstream exposure are the two key determinants of the shocks' welfare effects, and they typically supersede the general-equilibrium effects. Their relative importance depends, however, on the specific type of shock. While upstream exposure is the single most important welfare determinant under a global trade cost shock, downstream exposure tends to be important under a local shock,

[20] The expression can be most easily derived using matrix notation: Country j's upstream exposure to channel (ii) is $(1/e_j^f)\sum_{k\in\mathcal{N}}\phi_{jk}^{up}x_{ik}(1-\lambda_i)$ with $x_{ik} = \pi_{ik}e_k$. Summing up over all i and presenting the resultant expression in vector notation gives $[\mathbf{E^f}]^{-1}\mathbf{\Phi^{up}}\mathbf{E}\,\mathbf{\Pi}^\top(\mathbf{I}-\mathbf{\Lambda})\mathbf{1}$. By the definition of upstream exposure in (15) and $\mathbf{e} = \mathbf{x}$, this is, however, nothing but $[\mathbf{E^f}]^{-1}\mathbf{\Phi^{up}}\mathbf{x} - \mathbf{1}$.

[21] To arrive at this expression, note that j's exposure to i's intermediated demand can, in the partial-isolation case, be written as $\phi_{ji}^{down}\frac{1}{\lambda_i}\sum_{l\in\mathcal{N}}\phi_{il}^{down}\sum_{m\in\mathcal{N}}x_{lm}^f$. Using the labor market identity (A.19) in Appendix A.2, this immediately simplifies to $\phi_{ji}^{down}e_i$.

[22] This is quite different from the network of key trade partners in 'local capacities' shown in Supplementary Material S.4. The difference with Fig. 5 is particularly striking for the top two trade partners in local capacities, the U.S. and Germany, that sell their own value added and source their final goods from a number of locations significantly further away. Nevertheless, as the total welfare losses from the average country's isolation can for 67% be attributed to their foregone intermediation capacities, the network of overall key trade partners in the Supplementary Material looks quite the same as the network of key intermediaries in Fig. 5.

[23] This rather obvious pattern is omitted from Fig. 5, which only displays the *international* intermediation ties. However, note that these international ties already account for 52% of the typical country's supply chain exposure.

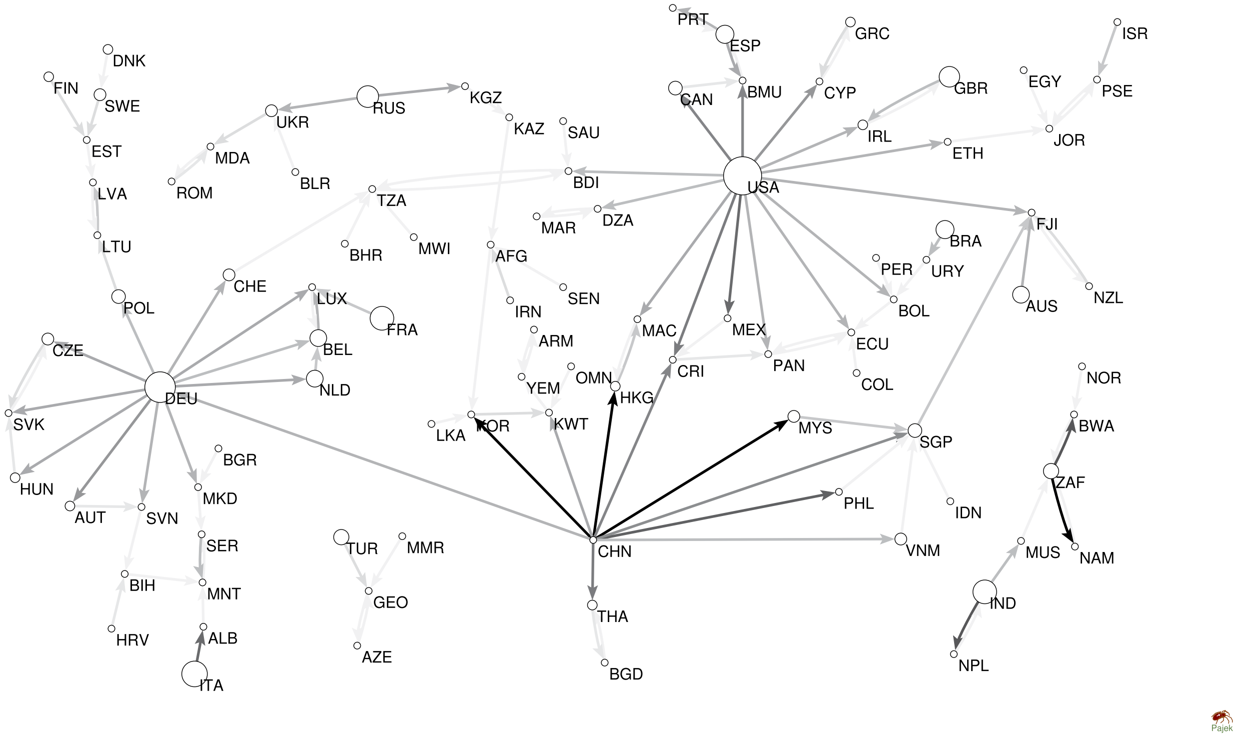

Fig. 5. Key Intermediaries in 2011. NOTES: For each country in 2011, the figure shows its most important trade intermediary in terms of channels (ii), (iv), and (vi) in Formula (23) for a model specification with $\beta = \gamma = 5$ and a 1% trade cost increase on all the in- and outgoing links of the trade partner. Node sizes indicate country i's overall importance as a trade intermediary for the average other nation. Arrows indicate that either (a) trade intermediary i contributes most to the welfare in j among all $l \neq i$ or (b) country pair ij belongs to the top 50 of intermediation ties.

especially during the transition to a long-run equilibrium, where producers can substitute between labor and intermediate products in their production processes. Finally, our paper makes clear how up- and downstream exposure are connected to two characteristics of a country's trade partners: their capacities in intermediating the supply and demand of other nations.

Beyond the immediate value of these insights, our paper also makes two methodological contributions with potential value for future research. Firstly, our counterfactual approach to distinguish the different effect channels may prove useful in future studies aiming to formulate new trade policies for global supply chains (see, e.g., Antràs and Staiger, 2012; Ornelas and Turner, 2012; Erbahar and Zi, 2017). Its particular relevance to this literature lies in the fact that it is only one step from the system of total derivatives that lies at its heart to the first-order conditions for an optimal tariff regime. Secondly, our approach may be valuable for future studies seeking to understand the determinants of the welfare gains from trade. The more so, as the different effect channels can be easily quantified using readily available macroeconomic data and estimates of the model's elasticity parameters. Prior studies have shown that the emergence of global supply chains has opened up new transmission channels for foreign shocks (e.g., Caselli et al., 2020; Huo et al., 2019). As our approach helps to single out and quantify these different channels, particularly the foreign trade multipliers versus the up- and downstream diffusion channels, it may provide further insights into the factors that dampen or exacerbate shocks.

CRediT authorship contribution statement

Maarten Bosker: Conceptualization, Formal analysis, Investigation, Methodology, Software, Writing – original draft, Data curation. **Bastian Westbrock:** Conceptualization, Formal analysis, Investigation, Methodology, Writing – original draft, Writing – review & editing.

Declaration of competing interest

None

Data availability

Data will be made available on request.

Appendix A

A.1. Equilibrium

Here, we verify Propositions 1 and 2 stating that the economy from Section 2 has a locally unique equilibrium point with well-behaved comparative statics properties.

A.1.1. Equilibrium existence

In a first step, we will show that, for given wages $\mathbf{w}$, the producer price indexes, $\mathbf{p}$, are uniquely defined. To get there, note that by expression (8), the price indexes can be written in terms of the differentiable function

$$f_i(\mathbf{p},\mathbf{w}) = \left((\kappa_i^l)^\beta w_i^{1-\beta} + (\kappa_i^i)^\beta \left(\sum_{j\in\mathcal{N}} \left(\frac{p_j\tau_{ji}}{\mu_j}\right)^{1-\gamma} \right)^{\frac{1-\beta}{1-\gamma}} \right)^{\frac{1}{1-\beta}}$$

which has partial derivatives given by

$$\begin{aligned}\frac{\partial \ln(f_i)}{\partial \ln(p_j)} &= \frac{(\kappa_i^i)^\beta (p_i^f)^{1-\beta}}{p_i^{1-\beta}} \frac{(p_j\tau_{ji})^{1-\gamma}}{(\mu_j(p_i^f))^{1-\gamma}} = \pi_{ji}(1-\lambda_i)\\ \frac{\partial \ln(f_i)}{\partial \ln(w_i)} &= \frac{(\kappa_i^l)^\beta w_i^{1-\beta}}{p_i^{1-\beta}} = \lambda_i\,.\end{aligned}$$

Based on this expression, we can prove the following claim:

Lemma 2 *(Unique price equilibrium). Let $\boldsymbol{\omega} = (\boldsymbol{\mu}, \boldsymbol{\kappa}^{\mathbf{l}}\,\boldsymbol{\kappa}^{\mathbf{i}}, \mathbf{l}, \mathbf{T}, \boldsymbol{\beta}, \boldsymbol{\gamma}) \in \boldsymbol{\Omega}$ denote the parameters of our economy and let $\bar{\omega}$ and $\underline{\omega}$ denote the largest, respectively, smallest element of a parameter vector or matrix. When*[24]

$$\begin{cases} n^{\frac{1-\beta}{1-\gamma}} (\bar{\kappa}^i)^\beta \left(\underline{\tau}/\bar{\mu}\right)^{1-\beta} < 1 & \text{if } 1 < \beta \le \gamma \\ n^{\frac{1-\beta}{1-\gamma}} (\bar{\kappa}^i)^\beta \left(\bar{\tau}/\underline{\mu}\right)^{1-\beta} < 1 & \text{if } 0 \le \beta < 1 \end{cases}, \tag{A.1}$$

then there exists an implicit function $\tilde{\mathbf{p}}(\mathbf{w},\boldsymbol{\omega}) : \mathbb{R}^{\mathbf{n}}_{++} \times \boldsymbol{\Omega} \to \mathbb{R}^{\mathbf{n}}_{++}$ such that (i) $\mathbf{p} = \tilde{\mathbf{p}}(\mathbf{w},\boldsymbol{\omega})$ is the unique solution to $\mathbf{p} = \mathbf{f}(\mathbf{p},\mathbf{w},\boldsymbol{\omega})$ and (ii) $\tilde{\mathbf{p}}(\cdot)$ is continuously differentiable in $\mathbf{w}$ and $\boldsymbol{\omega}$ with partial derivatives given by

$$\begin{aligned}\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})} &= \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \left[(\mathbf{I}-\boldsymbol{\Lambda})\boldsymbol{\Pi}^\top\right]^{\mathbf{h}} \frac{\partial \ln(\mathbf{f})}{\partial \ln(\mathbf{w})}\\ \frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \boldsymbol{\omega}} &= \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \left[(\mathbf{I}-\boldsymbol{\Lambda})\boldsymbol{\Pi}^\top\right]^{\mathbf{h}} \frac{\partial \ln(\mathbf{f})}{\partial \boldsymbol{\omega}}\,.\end{aligned} \tag{A.2}$$

Proof. We verify that $\mathbf{f}(\mathbf{p},\cdot)$ has the following properties:

1.) it is an *endomorphic* function on a compact and complete space $\mathcal{P} \subset \mathbb{R}^n_{++}$ (i.e., $\mathbf{f} : \mathcal{P} \to \mathcal{P}$),
2.) it is a *contraction* mapping, and
3.) the Jacobian matrix of $\ln(\mathbf{p}) - \ln \mathbf{f}\left(\mathbf{ln}(\mathbf{p})\right)$ is invertible.

Existence of the implicit function $\tilde{\mathbf{p}}(\mathbf{w},\boldsymbol{\omega})$ then follows from the Banach Fixed Point Theorem.

1.) To verify existence of an endomorphic function, note first that $f_i(\mathbf{p},\mathbf{w_i},\boldsymbol{\omega})$ is monotonically increasing in w_i and $p_j\tau_{ji}/\mu_j$. Moreover, the function is monotonically increasing (decreasing) in κ_i^l and κ_i^i whenever $\beta < 1$ ($\beta > 1$).

This means that a conservative upper bound for p_i is given by

$$p_i \le \bar{p} \equiv \begin{cases} (\underline{\kappa}^l)^{\frac{\beta}{1-\beta}}\,\bar{w} & \text{if } 1 < \beta \le \gamma \\ \left((\bar{\kappa}^l)^\beta \bar{w}^{1-\beta} + (\bar{\kappa}^i)^\beta \left((\bar{\tau}/\underline{\mu})^{1-\gamma} n \bar{p}^{1-\gamma} \right)^{\frac{1-\beta}{1-\gamma}} \right)^{\frac{1}{1-\beta}} & \text{if } 0 \le \beta < 1 \end{cases}. \tag{A.3}$$

A conservative lower bound for p_i is, on the other hand, given by

[24] The assumption mirrors the familiar constraint on κ_i^i from models with a Cobb-Douglas technology ($\beta = 1$). In that case, the condition simplifies to $\kappa_i^i = 1 - \lambda_i < 1\ \forall\, i \in \mathcal{N}$. In the more general case of a CES technology, the condition is combined with an additional constraint on the number of nodes, n, and the link intensity, $\tau_{ij}^{1-\beta}$, that is common to social network models (e.g., Ballester et al., 2006).

$$p_i \geq \underline{p} = \begin{cases} \left((\bar{\kappa}^l)^\beta \underline{w}^{1-\beta} + (\bar{\kappa}^i)^\beta \left((\underline{\tau}/\bar{\mu})^{1-\gamma} n\underline{p}^{1-\gamma} \right)^{\frac{1-\beta}{1-\gamma}} \right)^{\frac{1}{1-\beta}} & \text{if } 1 < \beta \leq \gamma \\ (\underline{\kappa}^l)^{\frac{\beta}{1-\beta}} \underline{w} & \text{if } 0 \leq \beta < 1 \end{cases}. \tag{A.4}$$

When $0 \leq \beta < 1$, then $n\bar{p}^{1-\gamma}$ satisfies by (A.3),

$$n\bar{p}^{1-\gamma} = n\left((\bar{\kappa}^l)^\beta \bar{w}^{1-\beta} + (\bar{\kappa}^i)^\beta \left((\bar{\tau}/\underline{\mu})^{1-\gamma} n\bar{p}^{1-\gamma} \right)^{\frac{1-\beta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\beta}}.$$

When the parameter restrictions of Assumption (A.1) are satisfied in addition, this yields a unique, positive solution for $n\bar{p}^{1-\gamma}$. Hence, the upper bound in (A.3) can be rewritten as

$$\bar{p} = \begin{cases} (\underline{\kappa}^l)^{\frac{\beta}{1-\beta}} \bar{w} & \text{if } 1 < \beta \leq \gamma \\ \left(\frac{(\bar{\kappa}^l)^\beta}{1 - n^{\frac{1-\beta}{1-\gamma}} (\bar{\kappa}^i)^\beta \left(\bar{\tau}/\underline{\mu} \right)^{1-\beta}} \right)^{\frac{1}{1-\beta}} \bar{w} & \text{if } 0 \leq \beta < 1 \end{cases}. \tag{A.5}$$

On the other hand, when $1 < \beta \leq \gamma$, then $n\underline{p}^{1-\gamma}$ must, by (A.4), solve

$$n\underline{p}^{1-\gamma} = n\left((\bar{\kappa}^l)^\beta \underline{w}^{1-\beta} + (\bar{\kappa}^i)^\beta \left((\underline{\tau}/\bar{\mu})^{1-\gamma} n\underline{p}^{1-\gamma} \right)^{\frac{1-\beta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\beta}}.$$

Again, this has a unique solution when Assumption (A.1) is satisfied, so that the lower bound in (A.4) becomes

$$\underline{p} = \begin{cases} \left(\frac{1 - n^{\frac{1-\beta}{1-\gamma}} (\bar{\kappa}^i)^\beta \left(\bar{\mu}/\underline{\tau} \right)^{\beta-1}}{(\bar{\kappa}^l)^\beta} \right)^{\frac{1}{\beta-1}} \underline{w} & \text{if } 1 < \beta \leq \gamma \\ (\underline{\kappa}^l)^{\frac{\beta}{1-\beta}} \underline{w} & \text{if } 0 \leq \beta < 1 \end{cases}. \tag{A.6}$$

Hence, in total, regardless of the value for β, there always exists a compact and complete space $\mathcal{P} = [\underline{p}, \bar{p}]^n$ such that $\mathbf{f} : \mathcal{P} \to \mathcal{P}$.

2.) To verify that $\mathbf{f}(\cdot)$ is also a contraction mapping, note that for any two (log-linearized) vectors $\ln(\mathbf{p}), \ln(\mathbf{p}') \in \ln \mathcal{P}$ it holds that

$$\left(\ln(\bar{p}) - \ln(\underline{p}) \right) \mathbf{1} \geq \ln(\mathbf{p}') - \ln(\mathbf{p}). \tag{A.7}$$

Now, let $s \equiv (\ln(\bar{p}) - \ln(\underline{p}))$ denote the sup norm of $\ln(\mathbf{f}(\cdot))$. We get

$$\begin{aligned} \ln \mathbf{f}\left(\ln(\mathbf{p}') \right) - \ln \mathbf{f}\left(\ln(\mathbf{p}) \right) &\leq \ln \mathbf{f}\left(\ln(\mathbf{p}) + s\mathbf{1} \right) - \ln \mathbf{f}\left(\ln(\mathbf{p}) \right) \\ &= \frac{\partial \ln \mathbf{f}\left(\ln(\mathbf{p}) + s\mathbf{z} \right)}{\partial \ln(\mathbf{p})} s\mathbf{1} \\ &= (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^\top s\mathbf{1} \\ &= s(\mathbf{I} - \mathbf{\Lambda})\mathbf{1}. \end{aligned}$$

To get there, note that the inequality in line one follows directly from (A.7). The equality in line two is, in turn, the consequence of the Mean Value Theorem applied to an interior point $s\mathbf{z} = (sz_1, sz_2, ..., sz_n)^\top$, $\mathbf{z} \in (0,1)^{n\times 1}$, between $\ln(\mathbf{p})$ and $\ln(\mathbf{p}) + s\mathbf{1}$. For line three, note that $\partial \ln(\mathbf{f})/\partial \ln(\mathbf{p})$ is nothing but the Jacobian matrix of the producer price indexes evaluated at $\ln(\mathbf{p}) + s\mathbf{z}$, with matrix entries as shown in (A.2). Finally, for line four, we use $\mathbf{\Pi}^\top \mathbf{1} = \mathbf{1}$.

The contraction property follows, now, from the fact that $\mathbf{I} - \mathbf{\Lambda} \leq (1 - \underline{\lambda})\mathbf{I} < \mathbf{I}$ so that

$$\ln \mathbf{f}\left(\ln(\mathbf{p}') \right) - \ln \mathbf{f}\left(\ln(\mathbf{p}) \right) \leq s(1 - \underline{\lambda})\mathbf{1} < s\mathbf{1}.$$

Here, $1 - \underline{\lambda}$ denotes the *modulus* of $\ln(\mathbf{f}(\cdot))$, given by

$$\underline{\lambda} = \begin{cases} \frac{\bar{w}^{1-\beta}(\underline{\kappa}^l)^\beta}{\underline{p}^{1-\beta}} = \frac{\underline{w}^{\beta-1}(\underline{\kappa}^l)^\beta}{\bar{w}^{\beta-1}(\bar{\kappa}^l)^\beta} \left(1 - n^{\frac{1-\beta}{1-\gamma}} (\bar{\kappa}^i)^\beta \left(\bar{\mu}/\underline{\tau} \right)^{\beta-1} \right) & \text{if } 1 < \beta \leq \gamma \\ \frac{\underline{w}^{1-\beta}(\underline{\kappa}^l)^\beta}{\bar{p}^{1-\beta}} = \frac{\underline{w}^{1-\beta}(\underline{\kappa}^l)^\beta}{\bar{w}^{1-\beta}(\bar{\kappa}^l)^\beta} \left(1 - n^{\frac{1-\beta}{1-\gamma}} (\bar{\kappa}^i)^\beta \left(\bar{\tau}/\underline{\mu} \right)^{1-\beta} \right) & \text{if } 0 \leq \beta < 1 \end{cases}. \tag{A.8}$$

Therefore, by Banach's Fixed Point Theorem, there exists a unique $\mathbf{p} \in \mathcal{P}$ with $\mathbf{p} = \mathbf{f}(\mathbf{p}, \mathbf{w}, \boldsymbol{\omega})$.

3.) Existence of an implicit, continuously differentiable function $\tilde{\mathbf{p}}(\mathbf{w}, \boldsymbol{\omega})$ follows from the fact that the Jacobian matrix of $\ln(\mathbf{p}) - \ln \mathbf{f}\left(\mathbf{ln}(\mathbf{p}) \right)$,

$$\mathbf{I} - \frac{\partial \ln(\mathbf{f})}{\partial \ln(\mathbf{p})} = \mathbf{I} - (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^\top$$

is indeed invertible, as the row sum norm of $\partial \ln(\mathbf{f})/\partial \ln(\mathbf{p})$ is smaller than one. For the same reason, the inverse of this matrix, $[\mathbf{I}-(\mathbf{I}-\boldsymbol{\Lambda})\boldsymbol{\Pi}^\top]^{-1}$, can also be expressed in terms of the Neumann series in (A.2). □

As a consequence of Lemma 2, the proof of equilibrium existence reduces to finding a vector of wages that satisfies the labor market clearing condition in (9). The proof of this claim is presented in the following.

Proof of Proposition 1. The proof proceeds in two steps. In the first step, we derive an alternative expression for the labor demand vector, $\mathbf{Wl^d}$, that is just a function of $\mathbf{w}$ and $\boldsymbol{\varpi}$. To get there, make repeated use of the product market clearing condition $\mathbf{e}=\mathbf{x}=\boldsymbol{\Pi}\mathbf{e}=\boldsymbol{\Pi}\mathbf{e^f}+\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\mathbf{e}$ to arrive at:

$$\begin{aligned}\mathbf{Wl^d} &= \boldsymbol{\Lambda}\boldsymbol{\Pi}\,(\mathbf{e^f}+\mathbf{e^i})\\ &= \boldsymbol{\Lambda}\Big(\mathbf{I}+\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})+\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})+\dots\Big)\boldsymbol{\Pi}\,\mathbf{e^f}\\ &= \boldsymbol{\Lambda}\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\big[\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big]^{\mathbf{h}}\boldsymbol{\Pi}\,\mathbf{Wl}\,. \end{aligned}\tag{A.9}$$

The desired function, now, emerges from the fact that $\boldsymbol{\Pi}$ and $\boldsymbol{\Lambda}$ are, by Lemma 2, functions of $(\mathbf{w},\tilde{\mathbf{p}}(\mathbf{w},\boldsymbol{\varpi}),\boldsymbol{\varpi})$.

In the next step, we verify that the system of excess demand functions,

$$\mathbf{Wz}(\mathbf{w}) \equiv \boldsymbol{\Lambda}\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\big[\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big]^{\mathbf{h}}\boldsymbol{\Pi}\,\mathbf{Wl}\;-\;\mathbf{Wl}\,,\tag{A.10}$$

has the following properties: For all rows i of $\mathbf{Wz}(\mathbf{w})$

1. $w_i z_i(\mathbf{w})$ is continuous on the domain $\mathbf{w}\in\mathbb{R}^n_{++}$,
2. $w_i z_i(\mathbf{w})$ is homothetic (i.e., homogeneous of degree one),
3. $\sum_{i\in\mathcal{N}} w_i z_i(\mathbf{w})=0$ (Walras' Law),
4. for all $\mathbf{w}\in\mathbb{R}^n_{++}$, there is a $y\in\mathbb{R}_{++}$ such that $z_i(\mathbf{w})>-y$,
5. if $\mathbf{w}\to\mathbf{w}^0$, where $w^0_{-i}\neq 0$ and $w^0_i=0$ for some i, then $z_i(\mathbf{w})\to\infty$.

Equilibrium existence then follows from Proposition 17.C.1 of Mas-Collel et al. (1995, p.585).

1.) As already said before, all entries in $\boldsymbol{\Pi}$ and $\boldsymbol{\Lambda}$ are the products of wages and the implicit functions $\tilde{\mathbf{p}}(\mathbf{w},\boldsymbol{\varpi})$, both of which are continuous on $\mathbb{R}^n_{++}$. The continuity of $z_i(\mathbf{w})$ thus depends on the continuity of the Neumann series in (A.10). Note, however, that for every $\mathbf{w}\in\mathbb{R}_{++}$ and every $\boldsymbol{\varpi}$ satisfying condition (A.1), the series $\sum_{\mathbf{h}=\mathbf{0}}^{\bar{\mathbf{h}}}[\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})]^{\mathbf{h}}$ is uniformly converging when $\bar{h}\to\infty$. Thus, by the Uniform Limit Theorem, the Neumann series is continuous as well.

2.) Note, first, that $\tilde{\mathbf{p}}(\mathbf{w},\boldsymbol{\varpi})$ is homothetic with respect to $\mathbf{w}$. To see this, start from the partial derivatives of $\tilde{\mathbf{p}}(\mathbf{w},\boldsymbol{\varpi})$ in (A.2),

$$\frac{\partial \ln(\tilde{\mathbf{p}})}{\partial \ln(\mathbf{w})}=\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\big[(\mathbf{I}-\boldsymbol{\Lambda})\boldsymbol{\Pi}^\top\big]^{\mathbf{h}}\frac{\partial \ln(\mathbf{f})}{\partial \ln(\mathbf{w})}\;=\;\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\big[(\mathbf{I}-\boldsymbol{\Lambda})\boldsymbol{\Pi}^\top\big]^{\mathbf{h}}\boldsymbol{\Lambda}\,.$$

Next, make use of the following elementary identity[25]

$$\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\big[\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big]^{\mathbf{h}}\lambda=\mathbf{1}\,.\tag{A.11}$$

For a proportional wage change, $\mathbf{d}\ln(\mathbf{w})=\mathbf{1}$, this immediately implies $(\partial\ln(\tilde{\mathbf{p}})/\partial\ln(\mathbf{w}))\,\mathbf{d}\ln(\mathbf{w})=\mathbf{1}$. Hence, by the Wicksteed-Euler Theorem, $\tilde{\mathbf{p}}(\mathbf{w})$ is homothetic. It remains to be seen that, as a result of this, $\boldsymbol{\Pi}$ and $\boldsymbol{\Lambda}$ are both homogeneous of degree zero.

3.) To verify Walras' Law, simply note that from the elementary identity in (A.11) and $\mathbf{1}^\top\boldsymbol{\Pi}=\mathbf{1}^\top$, we get

$$\mathbf{1}^\top\mathbf{Wz}=\mathbf{1}^\top\boldsymbol{\Lambda}\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\big[\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big]^{\mathbf{h}}\boldsymbol{\Pi}\,\mathbf{Wl}\;-\;\mathbf{1}^\top\mathbf{Wl}=0.$$

4.) Because the total output satisfies $x_i>0$ in each nation i, it must be $z(\mathbf{w})>-l_i$. Hence, $z_i(\mathbf{w})>-\max_{k\in\mathcal{N}}\{l_k\}\equiv -y$.

5.) Suppose that $\mathbf{w}\to\mathbf{w}^0$, where $w^0_{-i}>0$, and $w^0_i=0$. Let $\underline{z}$ and $\bar{z}$ denote the smallest, respectively largest, element of a vector or matrix. It holds for any i that

[25] Identity (A.11) can be derived from the following series of simple identities

$$\lambda=\mathbf{1}-\mathbf{1}+\lambda=\mathbf{1}-\boldsymbol{\Pi}^\top\mathbf{1}+\boldsymbol{\Lambda}\boldsymbol{\Pi}^\top\mathbf{1}=\big[\mathbf{I}-(\mathbf{I}-\boldsymbol{\Lambda})\boldsymbol{\Pi}^\top\big]\mathbf{1}\,.$$

Finally, take the inverse of the matrix on the right-hand side.

$$z_i(\mathbf{w}) \geq \frac{1}{w_i}\lambda_i \min_{j\neq i}\{\pi_{ij}w_jl_j\} - \max_{k\in\mathcal{N}}\{l_k\}$$
$$\geq \begin{cases} \frac{1}{w_i}\frac{(\kappa_i^l)^\beta w_i^{1-\beta}}{\underline{p}^{1-\beta}}\frac{p_i^{1-\gamma}(\bar{\tau}/\underline{\mu})^{1-\gamma}}{\underline{p}^{1-\gamma}n(\underline{\tau}/\bar{\mu})^{1-\gamma}}\min_{j\neq i}\{l_jw_j\} - \max_{k\in\mathcal{N}}\{l_k\} & \text{if } 1<\beta \\ \frac{1}{w_i}\frac{(\kappa_i^l)^\beta w_i^{1-\beta}}{\bar{p}^{1-\beta}}\frac{p_i^{1-\gamma}(\bar{\tau}/\underline{\mu})^{1-\gamma}}{\underline{p}^{1-\gamma}n(\underline{\tau}/\bar{\mu})^{1-\gamma}}\min_{j\neq i}\{l_jw_j\} - \max_{k\in\mathcal{N}}\{l_k\} & \text{if } \beta<1 \end{cases}.$$

The first inequality follows from $x_i = \sum_{j\in\mathcal{N}}\pi_{ij}w_jl_j > \min_{j\neq i}\{\pi_{ij}w_jl_j\}$, and the second inequality from

$$\pi_{ij} \geq \frac{p_i^{1-\gamma}(\bar{\tau}/\underline{\mu})^{1-\gamma}}{\underline{p}^{1-\gamma}n(\underline{\tau}/\bar{\mu})^{1-\gamma}}.$$

Applying the expressions for $\underline{p}$ and $\bar{p}$ in (A.5) and (A.6), we finally get

$$z_i(\mathbf{w}) \geq \begin{cases} \frac{1}{w_i}\left(\frac{w_i}{\underline{w}}\right)^{2-\gamma-\beta}\sigma_i(\boldsymbol{\omega})\min_{j\neq i}\{l_jw_j\} - \max_{k\in\mathcal{N}}\{l_k\} & \text{if } 1<\beta\leq\gamma \\ \frac{1}{w_i^\beta\bar{p}^{1-\beta}}\left(\frac{w_i}{\underline{w}}\right)^{1-\gamma}\rho_i(\boldsymbol{\omega})\min_{j\neq i}\{l_jw_j\} - \max_{k\in\mathcal{N}}\{l_k\} & \text{if } 0\leq\beta<1 \end{cases}, \tag{A.12}$$

where $\sigma_i(\boldsymbol{\omega}) > 0$ and $\rho_i(\boldsymbol{\omega}) > 0$ are two compound parameters. As w_i converges to $w_i^0 = 0$ and w_{-i} to $w_{-i}^0 > 0$, we get $\lim(\underline{w}) = \lim(w_i)$ on the right hand side of (A.12). Therefore, $z_i(\mathbf{w})$ is growing unboundedly and, by Proposition 17.C.1 of Mas-Collel et al. (1995, p.585), we have established existence of an equilibrium $\mathbf{w}$. □

A.1.2. Equilibrium uniqueness

So far, we have established existence of at least one equilibrium $(\mathbf{w}, \mathbf{p})$ for any parameter constellation satisfying (A.1). The following lemma shows that, under the additional condition stated in (10), the equilibrium points are also locally unique and stable:

Lemma 3 *(Locally unique wage equilibrium). Let $\hat{\mathbf{w}} \equiv \frac{1}{|\mathbf{w}|}\mathbf{w}$, where $|\mathbf{w}| \equiv \sum_{i\in\mathcal{N}} w_i$. Suppose that conditions (A.1) and (10) are satisfied for an equilibrium point $\mathbf{w}$. Then, (i) there exists a* locally *unique $\hat{\mathbf{w}}$, i.e., an implicit function $\mathbf{g} : \boldsymbol{\Omega} \to \mathbb{R}^n_{++}$ such that for $\hat{\mathbf{w}} = \mathbf{g}(\boldsymbol{\omega})$ it holds $\hat{\mathbf{W}}\mathbf{z}(\hat{\mathbf{w}}) = \mathbf{0}$ for the $\mathbf{Wz}(\mathbf{w})$-function defined in (A.10). Moreover, (ii) there exists an open subset $\boldsymbol{\Omega}' \subset \boldsymbol{\Omega}$, such that for any $\boldsymbol{\omega} \in \boldsymbol{\Omega}'$ it is $\mathbf{g}(\boldsymbol{\omega})$ continuously differentiable with partial derivatives given by*

$$\frac{\partial \ln(\mathbf{g})}{\partial\boldsymbol{\omega}}(\boldsymbol{\omega}) = \left\{\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\left[\left\{\frac{\partial\ln(\mathbf{Wl^d})}{\partial\ln(\boldsymbol{Wl})}(\hat{\mathbf{w}},\boldsymbol{\omega})\right\}^{-i^*}\right]^h\right\}^{+i^*}\frac{\partial\ln(\mathbf{Wl^d})}{\partial\boldsymbol{\omega}}(\hat{\mathbf{w}},\boldsymbol{\omega}). \tag{A.13}$$

Proof. Following up on the properties of $\hat{\mathbf{W}}\mathbf{z}(\hat{\mathbf{w}})$ in Proposition 1, the excess demand system is homogeneous of degree one and satisfies Walras' Law. Hence, we are free to fix $\hat{w}_{i^*} = 1$ and to remove row i^* and column i^* from $\hat{\mathbf{W}}\mathbf{z}(\hat{\mathbf{w}})$, which are redundant.

It thus remains to show that the reduced system of excess supply functions, $\{-\hat{\mathbf{W}}\mathbf{z}(\hat{\mathbf{w}})\}^{-\mathbf{i}^*} = \mathbf{0}^{-\mathbf{i}^*}$, satisfies the conditions of the Implicit Function Theorem, i.e.,

1. $\left\{-\hat{\mathbf{W}}\mathbf{z}(\hat{\mathbf{w}})\right\}^{-\mathbf{i}^*} : \mathbb{R}^{n-1}_{++}\times\boldsymbol{\Omega} \to \mathbb{R}^{n-1}_{++}$ is continuously differentiable in $\hat{\mathbf{w}}^{-\mathbf{i}^*}$ and $\boldsymbol{\omega}$, and
2. in an equilibrium point $(\hat{\mathbf{w}}, \boldsymbol{\omega})$, the (log-linearized) Jacobian matrix of the excess supply function,
$$\frac{\partial\ln\{-\hat{\mathbf{W}}\mathbf{z}(\hat{\mathbf{w}})\}^{-\mathbf{i}^*}}{\partial\ln\{\mathbf{w}\}^{-\mathbf{i}^*}} = \left\{\mathbf{I} - \frac{\partial\ln(\mathbf{Wl^d})}{\partial\ln(\mathbf{Wl})}\right\}^{-\mathbf{i}^*}, \tag{A.14}$$
is invertible.

1.) Simply note that, by Proposition 1, the excess demand in (A.10) is continuously differentiable in $\mathbf{w}$ and $\boldsymbol{\omega}$. The same thus holds for the reduced system of excess supply functions.

2.) The Jacobian matrix (A.14) is invertible when its row sum norm is not equal to zero at an equilibrium point, that is, when at $(\hat{\mathbf{w}}, \boldsymbol{\omega})$:

$$\left\{\mathbf{I} - \frac{\partial\ln(\mathbf{Wl^d})}{\partial\ln(\mathbf{Wl})}\right\}^{-\mathbf{i}^*}\mathbf{1}^{-\mathbf{i}^*} \neq \mathbf{0}^{-\mathbf{i}^*}.$$

This follows immediately from the facts that (a) $\mathbf{Wz}(\mathbf{w})$ is homogeneous of degree one and (b) that, by condition (10), $\mathbf{Wl^d}(\mathbf{w}, \boldsymbol{\omega})$ satisfies the gross substitutes property, that is,

$$\frac{\partial\ln(w_il_i^d)}{\partial\ln(w_jl_j)}(\hat{\mathbf{w}},\boldsymbol{\omega}) > 0 \qquad \text{for all } i \text{ and } j\neq i.$$

As this also implies that the row sums of $\{\partial\ln(\mathbf{Wl^d})/\partial\ln(\mathbf{Wl})\}^{-\mathbf{i}^*}$ satisfy

$$\{\frac{\partial \ln(\mathbf{Wl^d})}{\partial \ln(\mathbf{Wl})}\mathbf{1}\}^{-i^*} < \mathbf{1}^{-i^*},$$

we can alternatively express the inverse of $\{\mathbf{I} - \partial \ln(\mathbf{Wl^d})/\partial \ln(\mathbf{Wl})\}^{-i^*}$ by the Neumann series in (A.13). □

Lemma 3 immediately establishes Proposition 2.

A.2. The network diffusion model

Here, we derive diffusion model (12) and describe some of its properties.

A.2.1. Proof of Theorem 1

The first-order effects of an arbitrary trade cost shock $\mathbf{dT}$ on an equilibrium point $(\mathbf{p}, \mathbf{w})$ can be written in the following way.

The price effects:

$$\begin{aligned}
\mathbf{d}\ln(\mathbf{p}) &= \underbrace{(\mathbf{I}-\mathbf{\Lambda})[\mathbf{E^f}]^{-1}\mathbf{\Phi^{up}}\,\delta^{\mathbf{p}}}_{\mathbf{d_T}\ln(\mathbf{p})} + \underbrace{\left(\mathbf{\Lambda} + (\mathbf{I}-\mathbf{\Lambda})[\mathbf{E^f}]^{-1}\mathbf{\Phi^{tot}}\mathbf{LW}\right)\mathbf{d}\ln(\mathbf{w})}_{\mathbf{d_w}\ln(\mathbf{p})} \\
\mathbf{d}\ln(\mathbf{p^f}) &= \underbrace{[\mathbf{E^f}]^{-1}\mathbf{\Phi^{up}}\,\delta^{\mathbf{p}}}_{\mathbf{d_T}\ln(\mathbf{p^f})} + \underbrace{[\mathbf{E^f}]^{-1}\mathbf{\Phi^{tot}}\mathbf{LW}\,\mathbf{d}\ln(\mathbf{w})}_{\mathbf{d_w}\ln(\mathbf{p^f})}.
\end{aligned} \tag{A.15}$$

These terms result immediately from the expressions (A.2) in Lemma 2, the definition of $\mathbf{\Phi^{up}}$ in (15), and the following vector expression for the local price effect (11):

$$\delta^{\mathbf{p}} = \mathbf{E}\,[\mathbf{\Pi}^\top \circ (\mathbf{d}\ln(\mathbf{T}))^\top]\,\mathbf{1}, \tag{A.16}$$

where $\circ$ denotes the Hadamard (pointwise) product, which we give priority in the order of operations. Finally, $\mathbf{\Phi^{tot}}$ in (A.15) denotes the

$$\text{Terms of trade multiplier: } \mathbf{\Phi^{tot}} \equiv \mathbf{E^f}\mathbf{\Pi}^\top \sum_{\mathbf{h=0}}^{\infty}[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^\top]^{\mathbf{h}}\,[\mathbf{E}]^{-1}. \tag{A.17}$$

The wage effects: Following up on expression (A.13) of Lemma 3, the wage effect of a trade cost shock $\mathbf{dT}$ can, to a first order, be approximated by

$$\mathbf{d}\ln(\mathbf{w}) = \underbrace{\mathbf{\Phi^{mult}_{i^*}}}_{\text{Trade multiplier}} \cdot \underbrace{\mathbf{d_T}(\mathbf{Wl^d})}_{\text{Direct demand effect}}, \tag{A.18}$$

with $\mathbf{\Phi^{mult}_{i^*}}$ and $\mathbf{d_T}(\mathbf{Wl^d})$ given as follows. Regarding the direct demand effect, $\mathbf{d_T}(\mathbf{Wl^d})$, start from the vector expression for the labor demands which we already developed in (A.9):

$$\mathbf{Wl^d} = \mathbf{\Lambda}\sum_{\mathbf{h=0}}^{\infty}\left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}}\mathbf{\Pi}\,\mathbf{e^f}. \tag{A.19}$$

A trade cost shock $\mathbf{dT}$ (or likewise a wage shock $\mathbf{dw}$) has, in the first instance, a direct impact on $\mathbf{\Lambda}$ and $\mathbf{\Pi}$. In the second instance, the shock propagates through the endogenous entries of the Leontief inverse matrix, $\sum_{\mathbf{h=0}}^{\infty}\left[\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{\mathbf{h}}$. Nevertheless, for a small shock $\mathbf{dT}$ (or $\mathbf{dw}$), the full impact on this matrix can be determined by means of the derivative rule for inverse matrices:

$$\mathbf{d_T}\left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1} = \left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}\mathbf{d_T}\left(\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right)\left[I-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}.$$

The full impact of $\mathbf{dT}$ on $\mathbf{Wl^d}$ is, therefore, given by

$$\begin{aligned}
\mathbf{d_T}(\mathbf{Wl^d}) = {} & \mathbf{d_T}(\mathbf{\Lambda})\left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}\mathbf{\Pi}\,\mathbf{e^f} \\
& + \mathbf{\Lambda}\left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}\mathbf{d_T}\left(\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right)\left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}\mathbf{\Pi}\,\mathbf{e^f} \\
& + \mathbf{\Lambda}\left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}\mathbf{d_T}(\mathbf{\Pi})\,\mathbf{e^f}.
\end{aligned} \tag{A.20}$$

To simplify this expression, note that labor market clearing, $\mathbf{Wl} = \mathbf{Wl^d}$, and product market clearing, $\mathbf{x} = \mathbf{e}$, imply

$$\left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}\mathbf{\Pi}\,\mathbf{e^f} = [\mathbf{\Lambda}]^{-1}\mathbf{Wl} = \mathbf{e}.$$

Moreover, when we pre-multiply the first summand in line one of (A.20) by $\mathbf{\Lambda}[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})]^{-1}$ and its inverse $[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})][\mathbf{\Lambda}]^{-1}$, and solve $\mathbf{d_T}(\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda}))$ in line two by means of the product rule for matrices and the identity $\mathbf{d_T}(\mathbf{I}-\mathbf{\Lambda}) = -\mathbf{d_T}\mathbf{\Lambda}$, we arrive at

$$\mathbf{d_T}(\mathbf{Wl^d}) = \underbrace{\mathbf{\Lambda}\big[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\big]^{-1}}_{\text{Downstream exposure } \mathbf{\Phi^{down}}} \cdot \underbrace{\left(\left(\mathbf{I}-\mathbf{\Pi}\right)[\mathbf{\Lambda}]^{-1}\,\mathbf{d_T\Lambda} + \mathbf{d_T\Pi}\right)\mathbf{e}}_{\text{Local demand effects } \delta^{\mathbf{d}}}, \tag{A.21}$$

where the full expression for $\delta^{\mathbf{d}}$ is

$$\begin{aligned}\delta^{\mathbf{d}} \equiv{} & (1-\gamma)\Big(\underbrace{\big[\mathbf{\Pi}\circ\mathbf{d}\ln(\mathbf{T})\big]\,\mathbf{e}}_{\text{Market access}} - \underbrace{\mathbf{\Pi}\,\mathbf{E}\,[\mathbf{\Pi}^{\top}\circ(\mathbf{d}\ln(\mathbf{T}))^{\top}]\,\mathbf{1}}_{\text{Import competition}}\Big)\\ & + (1-\gamma)\Big(\underbrace{\mathbf{E}\,\mathbf{d_T}\ln(\mathbf{p})}_{\text{Exporter's productivity}} - \underbrace{\mathbf{\Pi}\,\mathbf{E}\,\mathbf{\Pi}^{\top}\,\mathbf{d_T}\ln(\mathbf{p})}_{\text{Competitors' productivity}}\Big)\\ & + \Big(\underbrace{\mathbf{E}\,\mathbf{d_T}\ln(\lambda)}_{\text{Exporter's substitution}} - \underbrace{\mathbf{\Pi}\,\mathbf{E}\,\mathbf{d_T}\ln(\lambda)}_{\text{Importer's substitution}}\Big).\end{aligned} \tag{A.22}$$

The terms $\mathbf{d_T}\ln(\mathbf{p})$ and $\mathbf{d_T}\ln(\lambda)$ in (A.22) are, finally, given by

$$\text{Productivity channel: } \mathbf{d_T}\ln(\mathbf{p}) = (\mathbf{I}-\mathbf{\Lambda})[\mathbf{E^f}]^{-1}\mathbf{\Phi^{up}}\,\delta^{\mathbf{p}}, \tag{A.23}$$

$$\text{Input substitution channel: } \mathbf{d_T}\ln(\lambda) = (\beta-1)\,\mathbf{d_T}\ln(\mathbf{p}).$$

We, now, turn to the foreign trade multiplier in (A.18). By Lemma 3 in Appendix A.1, we have

$$\mathbf{\Phi^{mult}_{i^*}} \equiv \mathbf{E^f}\left\{\sum_{h=0}^{\infty}\left[\left\{\frac{\partial\ln(\mathbf{Wl^d})}{\partial\ln(\mathbf{Wl})}\right\}^{-i^*}\right]^h\right\}^{+i^*}[\mathbf{E^f}]^{-1}. \tag{A.24}$$

Key here is the $n\times n$ Jacobian matrix $\partial\ln(\mathbf{Wl^d})/\partial\ln(\mathbf{Wl})$. It can be expressed in terms of our model's primitives in the following way: Start from the same steps that we used in our derivation of (A.21) from (A.9). This gives

$$\begin{aligned}\mathbf{d}_{\ln(\mathbf{w})}(\mathbf{Wl^d}) &= \mathbf{E^f}\,\frac{\partial\ln(\mathbf{Wl^d})}{\partial\ln(\mathbf{Wl})}\,\mathbf{d}\ln(\mathbf{w})\\ &= \mathbf{d}_{\ln(\mathbf{w})}(\mathbf{\Lambda})\,\mathbf{e} + \mathbf{\Phi^{down}}\Big(\mathbf{d}_{\ln(\mathbf{w})}(\mathbf{\Pi})\,\mathbf{e} + \mathbf{\Pi}\,\mathbf{d}_{\ln(\mathbf{w})}(\mathbf{I}-\mathbf{\Lambda})\,\mathbf{e} + \mathbf{\Pi}\,\mathbf{d}_{\ln(\mathbf{w})}\mathbf{e^f}\Big).\end{aligned}$$

The expressions in line two can most easily be derived element by element[26]:

$$\begin{aligned}\mathbf{d}_{\ln(\mathbf{w})}(\mathbf{\Lambda})\,\mathbf{e} &= (1-\beta)\mathbf{E^f}\left(\mathbf{I}-\frac{\partial\ln(\mathbf{p})}{\partial\ln(\mathbf{w})}\right)\mathbf{d}\ln(\mathbf{w})\\ \mathbf{d}_{\ln(\mathbf{w})}(\mathbf{I}-\mathbf{\Lambda})\,\mathbf{e} &= (1-\beta)\mathbf{E^i}\left(\frac{\partial\ln(\mathbf{p^f})}{\partial\ln(\mathbf{w})}-\frac{\partial\ln(\mathbf{p})}{\partial\ln(\mathbf{w})}\right)\mathbf{d}\ln(\mathbf{w})\\ \mathbf{d}_{\ln(\mathbf{w})}(\mathbf{\Pi})\,\mathbf{e} &= (1-\gamma)\left(\mathbf{E}\frac{\partial\ln(\mathbf{p})}{\partial\ln(\mathbf{w})}-\mathbf{\Pi}\,\mathbf{E}\frac{\partial\ln(\mathbf{p^f})}{\partial\ln(\mathbf{w})}\right)\mathbf{d}\ln(\mathbf{w})\\ \mathbf{d}_{\ln(\mathbf{w})}(\mathbf{e^f}) &= \mathbf{E^f}\,\mathbf{d}\ln(\mathbf{w}).\end{aligned}$$

Together, this gives

$$\begin{aligned}\frac{\partial\ln(\mathbf{Wl^d})}{\partial\ln(\mathbf{Wl})} = (1-\beta)\,\mathbf{I} + \big[\mathbf{E^f}\big]^{-1}\mathbf{\Phi^{down}}\Big(&(\beta-\gamma)\,\mathbf{E}\,\frac{\partial\ln(\mathbf{p})}{\partial\ln(\mathbf{w})} + (\gamma-1)\mathbf{\Pi}\,\mathbf{E}\,\frac{\partial\ln(\mathbf{p^f})}{\partial\ln(\mathbf{w})}\\ &+(1-\beta)\mathbf{\Pi}\,\mathbf{E^i}\,\frac{\partial\ln(\mathbf{p^f})}{\partial\ln(\mathbf{w})} + \mathbf{\Pi}\,\mathbf{E^f}\Big),\end{aligned} \tag{A.25}$$

with

$$\frac{\partial\ln(\mathbf{p})}{\partial\ln(\mathbf{w})} = \mathbf{\Lambda} + (\mathbf{I}-\mathbf{\Lambda})[\mathbf{E^f}]^{-1}\mathbf{\Phi^{tot}}\mathbf{LW} \quad\text{and}\quad \frac{\partial\ln(\mathbf{p^f})}{\partial\ln(\mathbf{w})} = [\mathbf{E^f}]^{-1}\mathbf{\Phi^{tot}}\mathbf{LW}. \quad \square \tag{A.26}$$

[26] The element in row i of vector $\mathbf{d}_{\ln(\mathbf{w})}(\mathbf{\Pi})\,\mathbf{e}$ can, for instance, be written as

$$\sum_{j\in\mathcal{N}}\mathbf{d}_{\ln(\mathbf{w})}(\pi_{ij})\,e_j = (1-\gamma)\left(\sum_{j\in\mathcal{N}}\pi_{ij}e_j\sum_{k\in\mathcal{N}}\frac{\partial\ln(p_i)}{\partial\ln(w_k)}d\ln(w_k) - \sum_{j\in\mathcal{N}}\pi_{ij}e_j\sum_{k\in\mathcal{N}}\frac{\partial\ln(p^f_j)}{\partial\ln(w_k)}d\ln(w_k)\right),$$

where market clearing implies that $\sum_{j\in\mathcal{N}}\pi_{ij}e_j = x_i = e_i$.

A.2.2. Diffusion properties

Diffusion model (12) has several useful properties. We begin with the proof of Lemma 1:

Proof of Lemma 1. Concerning the property $\mathbf{1}^\top \mathbf{\Phi}^{\mathbf{down}} = \mathbf{1}^\top$, start from the elementary identity in (A.11):

$$[\mathbf{I} - (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^\top]^{-1} \lambda = \mathbf{1}\,.$$

Combined with the rule for transpose matrices, $\left([\mathbf{I} - (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^\top]^{-1}\mathbf{\Lambda}\right)^\top = \mathbf{\Lambda}[\mathbf{I} - \mathbf{\Pi}(\mathbf{I} - \mathbf{\Lambda})]^{-1}$, we immediately arrive at $\mathbf{1}^\top \mathbf{\Phi}^{\mathbf{down}} = \mathbf{1}^\top \mathbf{\Lambda}[\mathbf{I} - \mathbf{\Pi}(\mathbf{I} - \mathbf{\Lambda})]^{-1} = \mathbf{1}^\top$.

Concerning $\mathbf{1}^\top \mathbf{\Phi}^{\mathbf{tot}} = \mathbf{1}^\top$, all we need to note is that $\mathbf{1}^\top \mathbf{\Phi}^{\mathbf{tot}}$ is the transpose of the column vector of product market equations in (A.9). Hence, we get

$$\mathbf{1}^\top \mathbf{\Phi}^{\mathbf{tot}} = \left([\mathbf{E}]^{-1} \sum_{\mathbf{h}=\mathbf{0}}^{\infty} \left[\mathbf{\Pi}(\mathbf{I} - \mathbf{\Lambda})\right]^{\mathbf{h}} \mathbf{\Pi}\, \mathbf{e}^{\mathbf{f}}\right)^\top = \mathbf{1}^\top\,. \tag{A.27}$$

Concerning $\mathbf{1}^\top \mathbf{\Phi}^{\mathbf{up}} = \mathbf{1}^\top$, note that

$$\begin{aligned}\mathbf{1}^\top \mathbf{\Phi}^{\mathbf{up}} &= \mathbf{1}^\top \mathbf{E}^{\mathbf{f}} [\mathbf{E}]^{-\mathbf{1}} \;+\; \mathbf{1}^\top \mathbf{E}^{\mathbf{f}} \mathbf{\Pi}^\top [\mathbf{I} - (\mathbf{I} - \mathbf{\Lambda})\mathbf{\Pi}^\top)]^{-\mathbf{1}} (\mathbf{I} - \mathbf{\Lambda})[\mathbf{E}]^{-\mathbf{1}} \\ &= \lambda^{\mathbf{T}} \;+\; \mathbf{1}^\top \mathbf{\Phi}^{\mathbf{tot}} (\mathbf{I} - \mathbf{\Lambda})\,.\end{aligned}$$

The claim follows now from the fact that, by (A.27), the second summand simplifies to $(\mathbf{1} - \lambda)^\top$.

Concerning $\mathbf{1}_{\mathbf{i^*}}^\top \mathbf{\Phi}_{\mathbf{i^*}}^{\mathbf{mult}} > \mathbf{1}_{\mathbf{i^*}}^\top$, note that by Lemma 4 (presented below), it holds

$$\mathbf{1}^\top \frac{\partial(\mathbf{Wl^d})}{\partial(\mathbf{Wl})} = \mathbf{1}^\top\, \mathbf{WL} \frac{\partial \ln(\mathbf{Wl^d})}{\partial \ln(\mathbf{Wl})} [\mathbf{WL}]^{-\mathbf{1}} = \mathbf{1}^\top\,.$$

Hence, consider an arbitrary reference country i^* and define $y \equiv \max\{\partial(w_{i^*} l^d_{i^*})/\partial(l_j w_j) \mid j \in \mathcal{N} \backslash \{i^*\}\}$, which by condition (10) satisfies $0 < y < 1$. It follows that

$$\begin{aligned}\mathbf{1}^\top \mathbf{\Phi}_{\mathbf{i^*}}^{\mathbf{mult}} &= \mathbf{1}_{\mathbf{i^*}}^\top \;+\; \mathbf{1}^\top \left\{ \left\{ \frac{\partial(\mathbf{Wl^d})}{\partial(\mathbf{Wl})} \right\}^{-\mathbf{i^*}} \right\}^{+\mathbf{i^*}} \;+\; \mathbf{1}^\top \left\{ \left\{ \frac{\partial(\mathbf{Wl^d})}{\partial(\mathbf{Wl})} \right\}^{-\mathbf{i^*}} \left\{ \frac{\partial(\mathbf{Wl^d})}{\partial(\mathbf{Wl})} \right\}^{-\mathbf{i^*}} \right\}^{+\mathbf{i^*}} \;+\; \ldots \\ &\geq \mathbf{1}_{\mathbf{i^*}}^\top \;+\; (1-y)\mathbf{1}_{\mathbf{i^*}}^\top \;+\; (1-y)^2 \mathbf{1}_{\mathbf{i^*}}^\top \;+\; \ldots \;=\; \frac{1}{y} \mathbf{1}_{\mathbf{i^*}}^\top \;>\; \mathbf{1}_{\mathbf{i^*}}^\top\,. \quad \square\end{aligned}$$

We next characterize the Jacobian matrix inside the foreign trade multiplier in (A.24):

Lemma 4 *(Properties of the Jacobian). The Jacobian matrix of the labor demand system satisfies:*

1. $\left(\partial \ln(\mathbf{Wl^d})/\partial \ln(\mathbf{Wl})\right) \mathbf{1} \;=\; \mathbf{1}$,
2. $\mathbf{1}^\top \left(\partial(\mathbf{Wl^d})/\partial(\mathbf{Wl})\right) \;=\; \mathbf{1}^\top$.

Proof. Because the labor demand function $\mathbf{Wl^d}$ is homothetic in $\mathbf{Wl}$, Property 1 follows immediately from the Wicksteed-Euler Theorem. Regarding Property 2, as the labor demand function satisfies Walras' Law in addition, i.e., $\mathbf{1}^\top \mathbf{Wl^d} - \mathbf{1}^\top \mathbf{Wl} \;=\; \mathbf{0}$, we must have for all $\mathbf{d(Wl)}$ that

$$\mathbf{1}^\top \frac{\partial(\mathbf{Wl^d})}{\partial(\mathbf{Wl})} \mathbf{d(Wl)} - \mathbf{1}^\top \mathbf{d(Wl)} \;=\; \mathbf{0}\,.$$

This requires $\mathbf{1}^\top(\partial \mathbf{Wl^d})/(\partial \mathbf{Wl}) = \mathbf{1}^\top$. $\square$

Our next result presents a useful property of the local demand effects in (A.22):

Lemma 5 *(Average demand neutrality). The total worldwide local demand effect of an arbitrary trade cost shock is zero. That is, for any* $\mathbf{dT}$*, it holds that* $\mathbf{1}^\top \delta^{\mathbf{d}} = 0$.

Proof. By inspection of equation (A.22), it should become clear that the cross-country sum of *market access* effects cancels against the cross-country sum of *import competition* effects because $\mathbf{1}^\top \mathbf{\Pi} = \mathbf{1}^\top$ and

$$\mathbf{1}^\top \left[\mathbf{\Pi} \circ \mathbf{d} \ln(\mathbf{T})\right] \mathbf{e} = \mathbf{e}^\top \left[\mathbf{\Pi}^\top \circ (\mathbf{d} \ln(\mathbf{T}))^\top\right] \mathbf{1}.$$

Regarding the *productivity effects*, note that by the definitions in (A.26) and (A.23) and since $\mathbf{1}^\top \mathbf{\Pi} = \mathbf{1}^\top$ and $\mathbf{1}^\top \mathbf{E}\, \mathbf{\Pi}^\top = \mathbf{1}^\top \mathbf{E}$ (product market clearing), their cross-country sum can be written as

$$
\begin{aligned}
&(1-\gamma)\mathbf{1}^{\top}\left(\mathbf{E} - \mathbf{\Pi}\mathbf{E}\mathbf{\Pi}^{\top}\right)(\mathbf{I}-\mathbf{\Lambda})[\mathbf{E}^{\mathbf{f}}]^{-1}\mathbf{\Phi}^{\mathbf{up}}\,\delta^{\mathbf{p}} \\
&=(1-\gamma)\mathbf{1}^{\top}\left(\mathbf{E} - \mathbf{E}\right)(\mathbf{I}-\mathbf{\Lambda})[\mathbf{E}^{\mathbf{f}}]^{-1}\mathbf{\Phi}^{\mathbf{up}}\,\delta^{\mathbf{p}} \\
&=0\,.
\end{aligned}
$$

For the same reason, also the input substitution effects cancel each other out. □

Based on these properties, we can finally prove Proposition 3:

Proof of Proposition 3. The worldwide total welfare effect of an arbitrary trade cost shock can be written as

$$
\begin{aligned}
\mathbf{1}^{\top}\mathbf{E}^{\mathbf{f}}\,\mathbf{d}\ln(\mathbf{u}) &= \mathbf{1}^{\top}\mathbf{L}\mathbf{W}\,\mathbf{d}\ln(\mathbf{w}) - \mathbf{1}^{\top}\left(\mathbf{\Phi}^{\mathbf{up}}\,\delta^{\mathbf{p}} + \mathbf{\Phi}^{\mathbf{tot}}\mathbf{L}\mathbf{W}\,\mathbf{d}\ln(\mathbf{w})\right) \\
&= -\mathbf{1}^{\top}\,\delta^{\mathbf{p}}
\end{aligned}
$$

In the first line, we made use of the expression for $\mathbf{d}\ln(\mathbf{u})$ in (12) and, in the second line, the properties summarized in Lemma 1. □

A.3. A local trade cost shock

A.3.1. Proof of Proposition 4

Part 4a Suppose that $\mathbf{\Lambda}=\mathbf{I}$ (i.e., $\mathbf{\Phi}^{\mathbf{down}}=\mathbf{\Phi}^{\mathbf{up}}=\mathbf{I}$ in diffusion model (12)). The wage effects of a unilateral export cost reduction, $d\tau_{ij}=-y\tau_{ij}$, can then be written as

$$
\mathbf{d}\ln(\mathbf{w}) = y(\gamma-1)[\mathbf{L}\mathbf{W}]^{-1}\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}\left(\mathbf{x}^{\mathbf{f}}_{\mathbf{ij}} - \pi_{ij}\mathbf{x}^{\mathbf{f}}_{\mathbf{j}}\right),
$$

where $\mathbf{x}^{\mathbf{f}}_{\mathbf{ij}}=(0,0,\dots,x^{f}_{ij},0,\dots,0)^{\top}$ and $\mathbf{x}^{\mathbf{f}}_{\mathbf{j}}=(x^{f}_{1j},x^{f}_{2j},\dots,x^{f}_{ij},x^{f}_{i+1j},\dots,x^{f}_{nj})^{\top}$. Take i as the reference country (i.e., set $d\ln(w_i)=0$) and note that $\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}}$ has all its entries positive (except for the zeros in row i and column i). Then, the positive vector $\mathbf{x}^{\mathbf{f}}_{\mathbf{ij}}$ has no bearing for the wage effects in $k\neq i$ and, hence, we get $d\ln(w_k)<0$ for all $k\neq i$.

Part 4b From $d\ln(w_i)=0$ and $d\ln(w_k)<0\;\forall\,k\neq i$, it immediately follows that $d\ln(u_i)>0$. Regarding the real income effect in $k\neq i$, note that by Lemma 1 (see Appendix A.2), it is $\mathbf{1}^{\top}\mathbf{\Phi}^{\mathbf{tot}}=\mathbf{1}^{\top}$ and, thus, $\sum_{k\in\mathcal{N}}e^{f}_{k}\left(d\ln(w_k)-(1/e^{f}_{k})\sum_{l\in\mathcal{N}}\phi^{tot}_{kl}\,w_l l_l\,d\ln(w_l)\right)=0$.

Combined with j's improved supplier access to i's products (which is simply x_{ij}) and $e^{f}_{k}=w_k l_k$, the total real income effect in $k\neq i$ can thus be written as

$$
\sum_{k\neq i} e^{f}_{k}\,d\ln(u_k) = -\,d(w_i l_i) + \sum_{k\neq i^*}\phi^{tot}_{ik}\sum_{l\neq i^*}\phi^{mult}_{kl,i^*}\delta^{ld}_{l} + y x_{ij}\,.
$$

With (i) $i^*=i$ and thus $d(w_i l_i)=0$, (ii) $e^{f}_{i}=e_i$, (iii) $\phi^{tot}_{ik}=(1/e_k)x_{ki}$, (iv) $\delta^{ld}_{l}=y(1-\gamma)x_{ij}\pi_{lj}$ for all $l\neq i$, (iv) $\pi_{ki}e_i=x_{ki}$, and (v) the amplification property of $\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i}}$ in Lemma 1, this gives

$$
\sum_{k\neq i} e^{f}_{k}\,d\ln(u_k) < y(1-\gamma)\sum_{k\neq i} x_{ki}\frac{\pi_{kj}}{e_k}x_{ij} + y x_{ij}\,.
$$

When the condition in Proposition 4 is met, the total real income effect in $k\neq i$ is thus negative. □

A.3.2. Proof of Proposition 5

Part 5a Consider a unilateral export cost reduction, $d\tau_{ij}=-y\tau_{ij}$, with $\mathbf{0}<\mathbf{\Lambda}<\mathbf{I}$ and take exporter i as the reference country (i.e., $d\ln(w_i)=0$). We build up our proof that $d\ln(w_k)>d\ln(w_i)=0$ for at least one $k\neq i$ in two steps.

Step 1: $d\ln(w_k)>0$ for some $k\neq i$ if the cross-country sum of direct demand effects in $k\neq i$ is positive, i.e., if $\sum_{k\neq i}d_{\mathbf{T}}(w_k l^{d}_{k})\equiv\sum_{k\neq i}\sum_{l\in\mathcal{N}}\phi^{down}_{kl}\delta^{ld}_{l}>0$.

The total derivative of the labor market equation for a $k\neq i$ with respect to changes in $w_l l_l\;\forall\,l\in\mathcal{N}$ and τ_{ij} is

$$
d(w_k l_k) = \sum_{l\neq i}\frac{\partial(w_k l^{d}_{k})}{\partial(w_l l_l)}\,d(w_l l_l) + d_{\mathbf{T}}(w_k l^{d}_{k})\,.
$$

Summing up over all $k\neq i$ gives

$$
\begin{aligned}
\sum_{k\neq i} d(w_k l_k) &= \sum_{l\neq i}\sum_{k\neq i}\frac{\partial(w_k l^{d}_{k})}{\partial(w_l l_l)}\,d(w_l l_l) + \sum_{k\neq i} d_{\mathbf{T}}(w_k l^{d}_{k}) \\
&= \sum_{k\neq i}\left(1-\frac{\partial(w_i l^{d}_{i})}{\partial(w_k l_k)}\right)d(w_k l_k) + \sum_{k\neq i} d_{\mathbf{T}}(w_k l^{d}_{k}),
\end{aligned}
$$

where in line two we made use of Lemma 4 Part 2 (Appendix A.2.2). We thus get

$$\sum_{k\neq i} \frac{\partial(w_i l_i^d)}{\partial(w_k l_k)}\, d(w_k l_k) = \sum_{k\neq i} d_{\mathbf{T}}(w_k l_k^d)\,. \tag{A.28}$$

Contrary to the claim, suppose now that $d(w_k l_k) < 0$ for all $k \neq i$. Combined with (10) this means that the left hand side of (A.28) must be strictly negative. A contradiction to the right hand side being positive. Hence, there must be at least one k with $d(w_k l_k) > 0$.

Step 2: $\sum_{k\neq i} d_{\mathbf{T}}(w_k l_k^d) \equiv \sum_{k\neq i}\sum_{l\in\mathcal{N}} \phi_{kl}^{down}\delta_l^{l^d} > 0$ *when the condition in Proposition 5a is met.*

Start from the general expression for the direct labor demand effect, $d_{\mathbf{T}}(w_k l_k^d)$, in equation (A.21). Because the sum of direct effects is zero when summing over all $k \in \mathcal{N}$ (Lemmas 1 and 5 in Appendix A.2.2), $\sum_{k\neq i} d_{\mathbf{T}}(w_k l_k^d)$ is identical to $-d_{\mathbf{T}}(w_i l_i^d)$ and, so, we can write

$$\begin{aligned}\sum_{k\neq i} d_{\mathbf{T}}(w_k l_k^d) = {} & y(1-\gamma)\phi_{ii}^{down}x_{ij} \;-\; y(1-\gamma)\sum_{l\in\mathcal{N}}\phi_{il}^{down}\pi_{lj}x_{ij}\\ & + y(1-\gamma)\sum_{l\in\mathcal{N}}\phi_{il}^{down}\frac{e_l(1-\lambda_l)}{e_l^f}\phi_{lj}^{up}x_{ij}\\ & - y(1-\gamma)\sum_{l,m,n\in\mathcal{N}}\phi_{il}^{down}x_{lm}\frac{(1-\lambda_n)\pi_{nm}}{e_n^f}\phi_{nj}^{up}x_{ij}\\ & + y(\beta-1)\sum_{l\in\mathcal{N}}\phi_{il}^{down}\frac{e_l(1-\lambda_l)}{e_l^f}\phi_{lj}^{up}x_{ij}\\ & - y(\beta-1)\sum_{l,m\in\mathcal{N}}\phi_{il}^{down}\frac{x_{lm}(1-\lambda_m)}{e_m^f}\phi_{mj}^{up}x_{ij}\,.\end{aligned}$$

Simplifying the summand in line three by means of the following properties of $\mathbf{\Phi}^{\mathbf{up}}$:

$$(i)\quad \sum_{n\in\mathcal{N}}\frac{(1-\lambda_n)\pi_{nm}}{e_n^f}\phi_{nj}^{up} = \frac{1}{e_m^f}\phi_{mj}^{up}\quad \text{for } m\neq j\,,$$

$$(ii)\quad \sum_{n\in\mathcal{N}}\frac{(1-\lambda_n)\pi_{nj}}{e_n^f}\phi_{nj}^{up} = \frac{1}{e_j^f}\phi_{jj}^{up} - 1/e_j\,,$$

we get

$$\begin{aligned}\sum_{k\neq i} d_{\mathbf{T}}(w_k l_k^d) = {} & y(1-\gamma)\phi_{ii}^{down}x_{ij}\\ & + y(\beta-\gamma)\sum_{l\in\mathcal{N}}\phi_{il}^{down}\frac{e_l(1-\lambda_l)}{e_l^f}\phi_{lj}^{up}x_{ij}\\ & + y(\gamma-\beta)\sum_{l,m\in\mathcal{N}}\phi_{il}^{down}\frac{x_{lm}(1-\lambda_m)}{e_m^f}\phi_{mj}^{up}x_{ij}\\ & + y(\gamma-1)\sum_{l,m\in\mathcal{N}}\phi_{il}^{down}\pi_{lm}\phi_{mj}^{up}x_{ij}\,.\end{aligned}$$

The summand in line three can be further simplified by means of the following property of $\mathbf{\Phi}^{\mathbf{down}}$:

$$\sum_{l,m\in\mathcal{N}}\phi_{il}^{down}x_{lm}(1-\lambda_m) = \sum_{m\in\mathcal{N}}\phi_{im}^{down}e_m - e_i^f\,.$$

Hence, we get

$$\begin{aligned}\sum_{k\neq i} d_{\mathbf{T}}(w_k l_k^d) = {} & y(1-\gamma)\phi_{ii}^{down}x_{ij} \;+\; y(\gamma-\beta)\sum_{l\in\mathcal{N}}\phi_{il}^{down}\phi_{lj}^{up}x_{ij}\\ & + y(\beta-\gamma)\phi_{ij}^{up}x_{ij} \;+\; y(\gamma-1)\sum_{l,m\in\mathcal{N}}\phi_{il}^{down}\pi_{lm}\phi_{mj}^{up}x_{ij}\,,\end{aligned}$$

or, once we make use of the properties in Lemma 1 again, we get

$$\begin{aligned}\sum_{k\neq i} d_{\mathbf{T}}(w_k l_k^d) = {} & y(\gamma-1)\sum_{k\neq i}\phi_{ki}^{down}x_{ij} \;-\; y(\gamma-\beta)\sum_{l\in\mathcal{N},\,k\neq i}\phi_{kl}^{down}\phi_{lj}^{up}x_{ij}\\ & + y(\gamma-\beta)\sum_{k\neq i}\phi_{kj}^{up}x_{ij} \;-\; y(\gamma-1)\sum_{l,m\in\mathcal{N},\,k\neq i}\phi_{kl}^{down}\pi_{lm}\phi_{mj}^{up}x_{ij}\,.\end{aligned} \tag{A.29}$$

Expression (A.29) contains two negative and two positive effects. Nevertheless, the latter prevail when the condition in Proposition 5a is met. Combined with **Step 1**, this means that there is at least one $k \neq i$ with $d\ln(w_k) > d\ln(w_i)$.

Part 5b: $d\ln(w_k) > d\ln(w_i)\ \forall\, k \neq i$ *and* $e_i^f\, d\ln(u_i) < \frac{1}{n-1}\sum_{k\neq i} e_k^f\, d\ln(u_k)$ *when the condition in Proposition 5b is met.*

Following the same steps that lead to expression (A.29) in Part 5a gives a direct labor demand effect in any single country $k \neq i$ of

$$\begin{aligned} d_{\mathbf{T}}(w_k l_k^d) = {} & y(\gamma-1)\phi_{ki}^{down} x_{ij} \;+\; y(\beta-\gamma)\sum_{l\in\mathcal{N}} \phi_{kl}^{down}\phi_{lj}^{up} x_{ij} \\ & + y(\gamma-\beta)\phi_{kj}^{up} x_{ij} \;+\; y(1-\gamma)\sum_{l,m\in\mathcal{N}} \phi_{kl}^{down}\pi_{lm}\phi_{mj}^{up} x_{ij}\,. \end{aligned} \tag{A.30}$$

There are two negative and two positive effects. However, the latter prevail when the condition in Proposition 5b is met. We thus have $d_{\mathbf{T}}(w_k l_k^d) > 0\ \forall\, k \neq i$. Combined with the fact $\mathbf{\Phi_i^{mult}}$ has all its entries strictly positive (except for the entries in row i and column i), it immediately follows from here that also $d\ln(w_k) > d\ln(w_i) = 0$ for all $k \neq i$.

Moreover, the real income effect in country i, $d\ln(u_i)$, is bounded from above by $y\phi_{ij}^{up}x_{ij}/e_i^f - d_{\mathbf{T}}(w_j l_j^d)/e_i^f$ because

$$\begin{aligned} d\ln(u_i) &= \frac{y}{e_i^f}\phi_{ij}^{up}x_{ij} \;-\; \frac{1}{e_i^f}\sum_{k\neq i}\phi_{ik}^{tot}\sum_{l\neq i}\phi_{kl}^{mult} d_{\mathbf{T}}(w_l l_l^d) \\ &< \frac{y}{e_i^f}\phi_{ij}^{up}x_{ij} \;-\; \frac{1}{e_i^f}\sum_{k\neq i}\phi_{ik}^{tot}\frac{d_{\mathbf{T}}(w_k l_k^d)}{e_k^f} \\ &= \frac{y}{e_i^f}\phi_{ij}^{up}x_{ij} \;-\; \frac{1}{e_i^f}\sum_{k\neq i}\frac{\phi_{ik}^{up}}{1-\lambda_k} d_{\mathbf{T}}(w_k l_k^d) \\ &\leq \frac{y}{e_i^f}\phi_{ij}^{up}x_{ij} \;-\; \frac{\phi_{ij}^{up}}{e_i^f(1-\lambda_j)} d_{\mathbf{T}}(w_j l_j^d) \\ &\leq \frac{y}{e_i^f}\phi_{ij}^{up}x_{ij} \;-\; \frac{\phi_{ij}^{up}}{e_i^f\sum_{k\neq j}\phi_{kj}^{up}} d_{\mathbf{T}}(w_j l_j^d) \\ &\leq \frac{y}{e_i^f}\phi_{ij}^{up}x_{ij} \;-\; \frac{\phi_{ij}^{up}}{e_i^f\phi_{ij}^{up}} d_{\mathbf{T}}(w_j l_j^d)\,. \end{aligned} \tag{A.31}$$

Line one displays the expression for $d\ln(u_i)$. The inequality in line two makes use of the effect amplification through $\mathbf{\Phi^{mult}}$ (stated in Lemma 1). For the identity in line three, we then use the definitions of ϕ_{ij}^{up} and ϕ_{ik}^{tot} in (15) and (A.17), and for the inequality in line four, the fact that $d_{\mathbf{T}}(w_k l_k^d) > 0$ for all $k \neq i$. Finally, in lines five and six, we utilize the inequality $\phi_{jj}^{up} \geq \lambda_j$, leading to the chain of inequalities $1-\lambda_j \geq \sum_{k\neq j}\phi_{kj}^{up} \geq \phi_{ij}^{up}$.

When we, now, combine the final expression in (A.31) with the expression for $d_{\mathbf{T}}(w_j l_j^d)$ in (A.30) and the condition in Proposition 5b, we immediately reach at the insight that this final expression is strictly negative. Hence, we have $d\ln(u_i) < 0$. By contrast, we have $\sum_{k\neq i} e_k^f\, d\ln(u_k) > 0$ because the worldwide total sum of real income effects of $d\tau_{ij}$ must, by Proposition 3, be strictly positive. □

A.4. A global trade cost shock

A.4.1. Proof of Proposition 6

Consider a global trade cost reduction, $\mathbf{dT} = -y\mathbf{T}$, in a model specification with $\boldsymbol{\kappa}^{\mathbf{i}} = \mathbf{0}$. Then, $\mathbf{\Lambda} = \mathbf{\Phi^{down}} = \mathbf{I}$. Moreover, the local effects of our diffusion model (12) are given by

$$\begin{aligned} \boldsymbol{\delta}^{\mathbf{p}} &= \mathbf{E}\big[\mathbf{\Pi}^\top \circ (\mathbf{d}\ln(\mathbf{T}))^\top\big]\mathbf{1} \;=\; -y\,\mathbf{e} \\ \boldsymbol{\delta}^{\mathbf{d}} &= (1-\gamma)\Big(\big[\mathbf{\Pi}\circ\mathbf{d}\ln(\mathbf{T})\big]\,\mathbf{e^f} \;-\; \mathbf{\Pi}\,\mathbf{E^f}\,\big[\mathbf{\Pi}^\top\circ(\mathbf{d}\ln(\mathbf{T}))^\top\big]\mathbf{1}\Big) \\ &= y(\gamma-1)\Big(\mathbf{\Pi}\,\mathbf{e^f} \;-\; \mathbf{\Pi}\,\mathbf{E^f}\,\mathbf{\Pi}^\top\,\mathbf{1}\Big) \;=\; \mathbf{0}\,. \end{aligned}$$

Thus, by (A.18), we get $\mathbf{d}\ln(\mathbf{w}) = \mathbf{0}$. Moreover, because $\mathbf{\Phi^{up}} = \mathbf{I}$, we get $\mathbf{d}\ln(\mathbf{u}) = -[\mathbf{E^f}]^{-1}\mathbf{\Phi^{up}}\boldsymbol{\delta}^{\mathbf{p}} = y\,\mathbf{1}$. □

A.4.2. Derivation of expression (19)

Expression (19) is a special case of the more general expression for the wage effect in (A.18). To see this, note first that the *market access* and *import competition* effects in $\delta^{\mathbf{d}}$ cancel each other out when $\mathbf{dT} = -y\mathbf{T}$ (see (A.22) for the expressions). The remaining effects on wages can then be written as

$$\begin{aligned}
\mathbf{d}\ln(\mathbf{w}) &= y\,[\mathbf{LW}]^{-1}\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}\,\boldsymbol{\Phi}^{\mathbf{down}}\Big[(\gamma-1)\Big(\mathbf{E} - \boldsymbol{\Pi}\,\mathbf{E}\,\boldsymbol{\Pi}^{\top}\Big) \\
&\qquad + (1-\beta)\Big(\mathbf{E} - \boldsymbol{\Pi}\,\mathbf{E}\Big)\Big](\mathbf{I}-\boldsymbol{\Lambda})[\mathbf{E}^{\mathbf{f}}]^{-1}\boldsymbol{\Phi}^{\mathbf{u}p}\,\mathbf{e} \\
&= y\,[\mathbf{LW}]^{-1}\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}\,\boldsymbol{\Phi}^{\mathbf{down}}\Big[(\gamma-1)\Big(\big[\mathbf{I}-\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big]\mathbf{E} + \boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\mathbf{E} - \boldsymbol{\Pi}\,\mathbf{E}\,\boldsymbol{\Pi}^{\top}\Big) \\
&\qquad + (1-\beta)\Big(\big[\mathbf{I}-\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big]\mathbf{E} - \boldsymbol{\Pi}\,\mathbf{E}^{\mathbf{f}}\Big)\Big](\mathbf{I}-\boldsymbol{\Lambda})[\mathbf{E}^{\mathbf{f}}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\,\mathbf{e} \\
&= y\,[\mathbf{LW}]^{-1}\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i}^*}\Big[(\gamma-1)\Big(\mathbf{E}^{\mathbf{f}} + \boldsymbol{\Phi}^{\mathbf{down}}\big(\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\mathbf{E} - \boldsymbol{\Pi}\,\mathbf{E}\,\boldsymbol{\Pi}^{\top}\big)\Big) \\
&\qquad + (1-\beta)\Big(\mathbf{E} - \boldsymbol{\Phi}^{\mathbf{down}}\boldsymbol{\Pi}\,\mathbf{E}\Big)\boldsymbol{\Lambda}\Big](\mathbf{I}-\boldsymbol{\Lambda})[\mathbf{E}^{\mathbf{f}}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\,\mathbf{e}\,.
\end{aligned}$$

Lines two and three follows from expansion of the expression in line one; and lines four and five from the fact that $\boldsymbol{\Phi}^{\mathbf{down}}\big(\mathbf{I}-\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big)\mathbf{E} = \mathbf{E}^{\mathbf{f}}$. That final expression is identical to (19) after considering the definitions in (18). □

A.4.3. Proof of Proposition 8

Consider a global trade cost reduction, $\mathbf{dT} = -y\mathbf{T}$, under a Leontief specification for our model ($\beta = 0$). A first thing to note is that the producer price effects, $d\ln(p_i)$, must be identical in this case, that is,

$$d\ln(p_i) = z \qquad \forall\, i \in \mathcal{N} \text{ and any } z \in \mathbb{R}\,.$$

To see this, note that labor market clearing, $w_i l_i = \lambda_i x_i$, implies

$$\frac{w_i l_i}{\lambda_i} = \sum_{j\in\mathcal{N}} \pi_{ij} e_j \;=\; \sum_{j\in\mathcal{N}} \pi_{ij}\frac{w_j l_j}{\lambda_j} \qquad \forall\, i \in \mathcal{N}\,. \tag{A.32}$$

When $\beta = 0$, it is $w_i l_i/\lambda_i = l_i \kappa_i^l p_i$ and $\pi_{ij} = p_{ij}^{1-\gamma}/(\sum_{k\in\mathcal{N}} p_{kj}^{1-\gamma})$. Hence, the system of equations in (A.32) has $\mathbf{p}$ as its sole variable. In fact, as $\sum_{j\in\mathcal{N}} \pi_{ij} l_j \kappa_j^l p_j$ satisfies the gross substitutes property, there is a unique solution $\mathbf{p}$ to this system.

Now, let us determine the total derivative of (A.32). For $\mathbf{dT} = -y\mathbf{T}$, this derivative is

$$d\ln(p_i) = \sum_{j\in\mathcal{N}} \frac{x_{ij}}{x_i}\left[(1-\gamma)\Big(d\ln(p_i) - y - \sum_{k\in\mathcal{N}} \pi_{ki}\big(d\ln(p_k) - y\big)\Big) + d\ln(p_j)\right] \qquad \forall\, i \in \mathcal{N}\,.$$

Hence, we arrive at our claim because the unique solution to the above equation is $d\ln(p_i) = z\;\forall\, i \in \mathcal{N}$.

As a result, the wage effects $d\ln(w_i)$ of $\mathbf{dT} = -y\mathbf{T}$ are determined by the shock's impacts on the wedges between the homogeneous producer price effects and the country-specific changes in intermediate input cost shares. These impacts can be backed out from the total derivative of the producer price index:

$$\begin{aligned}
d\ln(p_i) &= \lambda_i d\ln(w_i) + (1-\lambda_i) d\ln(p_i^f) \\
&= \lambda_i d\ln(w_i) + (1-\lambda_i)\sum_{j\in\mathcal{N}} \big(d\ln(p_j) - y\big)\pi_{ji}\,.
\end{aligned}$$

The expression for $d\ln(u_i)$ follows immediately from here by setting $d\ln(p_i) = 0\;\forall\, i \in \mathcal{N}$. □

A.5. Key trading partners

A.5.1. Deriving the key player formula

To obtain Formula (23), we first need to spell out the impact of country i's isolation on the Leontief inverse matrix in (A.9). When $\beta = \gamma$, this matrix can be written as

$$\big[\mathbf{I}-\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\big]^{-1}\boldsymbol{\Pi} = \mathbf{P}^{\mathbf{1}-\gamma}\big[\mathbf{I}-\mathbf{M}^{\gamma-\mathbf{1}}\,\mathbf{T}^{\mathbf{1}-\gamma}(\mathbf{K}^{\mathbf{i}})^{\gamma}\big]^{-1}\mathbf{M}^{\gamma-\mathbf{1}}\,\mathbf{T}^{\mathbf{1}-\gamma}(\mathbf{P}^{\mathbf{f}})^{\gamma-\mathbf{1}}\,,$$

where $\mathbf{P}^{\mathbf{1}-\gamma} = (p_i^{1-\gamma})$ and $(\mathbf{P}^{\mathbf{f}})^{\gamma-\mathbf{1}} = ((p_i^f)^{\gamma-1})$ denote the diagonal matrices of augmented producer and consumer price indexes respectively, $\mathbf{M}^{\gamma-\mathbf{1}} = (\mu_i^{\gamma-1})$ the diagonal matrix of augmented total factor productivities, $\mathbf{T}^{\mathbf{1}-\gamma} = (\tau_{ij}^{1-\gamma})$ the full matrix of augmented trade costs, and $(\mathbf{K}^{\mathbf{i}})^{\gamma} = ((\kappa_i^i)^{\gamma})$ the diagonal matrix of augmented intermediate goods productivities. For convenience, we write

$$\left[\mathbf{I}-\mathbf{\Pi}(\mathbf{I}-\mathbf{\Lambda})\right]^{-1}\mathbf{\Pi}=\mathbf{P}^{1-\gamma}\left[\mathbf{I}-\mathbf{Z}\right]^{-1}\mathbf{Z}\,(\mathbf{K}^{\mathbf{i}})^{-\gamma}(\mathbf{P}^{\mathbf{f}})^{\gamma-1}, \tag{A.33}$$

where $\mathbf{Z}\equiv\mathbf{M}^{\gamma-1}\mathbf{T}^{1-\gamma}(\mathbf{K}^{\mathbf{i}})^{\gamma}$ consists of exogenous parameters only.

Making use of Lemma S.1 Property (3.) in Supplementary Online Material S.1 (in particular, the expression in the proof), the impact of isolating country i on the exogenous matrix $[\mathbf{I}-\mathbf{Z}]^{-1}\mathbf{Z}$ is given by[27]

$$\begin{aligned}\mathbf{d}\big([\mathbf{I}-\mathbf{Z}]^{-1}\mathbf{Z}\big)&=\left[\mathbf{I}-\mathbf{I}_{-\mathbf{i}}\mathbf{Z}\mathbf{I}_{-\mathbf{i}}\right]^{-1}\mathbf{I}_{-\mathbf{i}}\mathbf{Z}\mathbf{I}_{-\mathbf{i}}-\left[\mathbf{I}-\mathbf{Z}\right]^{-1}\mathbf{Z}\\&=\left[\mathbf{I}-\mathbf{Z}\right]^{-1}\frac{1}{\sum_{h=0}^{\infty}z_{ii}^{[h]}}\mathbf{I}_{\mathbf{i}}\left[\mathbf{I}-\mathbf{Z}\right]^{-1}\mathbf{Z}\mathbf{I}_{-\mathbf{i}}\;+\;\left[\mathbf{I}-\mathbf{Z}\right]^{-1}\mathbf{Z}\mathbf{I}_{\mathbf{i}}\end{aligned} \tag{A.34}$$

where $\mathbf{I}_{\mathbf{i}}$ is a square matrix with a one in element ii and zero everywhere else, $\mathbf{I}_{-\mathbf{i}}=\mathbf{I}-\mathbf{I}_{\mathbf{i}}$, and $\sum_{h=0}^{\infty}z_{ii}^{[h]}$ denotes entry ii of matrix $[\mathbf{I}-\mathbf{Z}]^{-1}$.

Going from here to the price and wage effects of a country's isolation, note that for $\beta=\gamma$, the vectors of producer and consumer price indexes can be explicitly solved for by

$$\mathbf{p}^{1-\gamma}=(\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{w}^{1-\gamma}\;+\;\mathbf{Z}^{\top}\mathbf{p}^{1-\gamma}\;=\;[\mathbf{I}-\mathbf{Z}^{\top}]^{-1}(\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{w}^{1-\gamma}$$

and

$$(\mathbf{p}^{\mathbf{f}})^{1-\gamma}=(\mathbf{K}^{\mathbf{i}})^{-\gamma}\mathbf{Z}^{\top}\mathbf{p}^{1-\gamma},$$

where $(\mathbf{K}^{\mathbf{l}})^{\gamma}((\kappa_i^{l})^{\gamma})$ denotes the diagonal matrix of augmented labor productivities, and $\mathbf{w}^{1-\gamma}=(w_i^{1-\gamma})$ the column vector of augmented wages. Hence, the first-order impact of country i's isolation becomes

$$\begin{aligned}\mathbf{d}(\mathbf{p}^{\mathbf{f}})^{1-\gamma}&=(\mathbf{K}^{\mathbf{i}})^{-\gamma}\Big(\mathbf{d}\big([\mathbf{I}-\mathbf{Z}]^{-1}\mathbf{Z}\big)\Big)^{\top}(\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{w}^{1-\gamma}\\&\quad+(1-\gamma)\,(\mathbf{K}^{\mathbf{i}})^{-\gamma}\mathbf{Z}^{\top}[\mathbf{I}-\mathbf{Z}^{\top}]^{-1}(\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{W}^{1-\gamma}\,\mathbf{d}\ln(\mathbf{w}),\end{aligned}$$

where $\mathbf{W}^{1-\gamma}$ denotes the diagonal matrix corresponding to $\mathbf{w}^{1-\gamma}$ and

$$(\mathbf{K}^{\mathbf{i}})^{-\gamma}\mathbf{Z}^{\top}[\mathbf{I}-\mathbf{Z}^{\top}]^{-1}(\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{W}^{1-\gamma}=(\mathbf{P}^{\mathbf{f}})^{1-\gamma}[\mathbf{E}^{\mathbf{f}}]^{-1}\mathbf{\Phi}^{\mathbf{tot}}\mathbf{L}\mathbf{W}.$$

Application of (A.34) thus gives

$$\begin{aligned}\mathbf{d}\ln(\mathbf{p}^{\mathbf{f}})&=\bigg(\mathbf{I}_{-\mathbf{i}}\mathbf{\Pi}^{\top}\left[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\top}\right]^{-1}\frac{1}{\sum_{h=0}^{\infty}z_{ii}^{[h]}}\mathbf{I}_{\mathbf{i}}\left[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\top}\right]^{-1}\\&\quad+\mathbf{I}_{\mathbf{i}}\mathbf{\Pi}^{\top}\left[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\top}\right]^{-1}\bigg)\lambda\;+\;[\mathbf{E}^{\mathbf{f}}]^{-1}\mathbf{\Phi}^{\mathbf{tot}}\mathbf{L}\mathbf{W}\,\mathbf{d}\ln(\mathbf{w})\\&=\mathbf{I}_{-\mathbf{i}}\mathbf{\Pi}^{\top}\sum_{\mathbf{h}=\mathbf{0}}^{\infty}\left[(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\top}\right]^{\mathbf{h}}\frac{1}{\sum_{h=0}^{\infty}z_{ii}^{[h]}}\Big((1-\lambda)_{\mathbf{i}}\;+\;\lambda_{\mathbf{i}}\Big)\\&\quad+\mathbf{1}_{\mathbf{i}}\;+\;[\mathbf{E}^{\mathbf{f}}]^{-1}\mathbf{\Phi}^{\mathbf{tot}}\mathbf{L}\mathbf{W}\,\mathbf{d}\ln(\mathbf{w})\\&=\mathbf{I}_{-\mathbf{i}}[\mathbf{E}^{\mathbf{f}}]^{-1}\mathbf{\Phi}^{\mathbf{up}}\bigg(\underbrace{\mathbf{E}\,\mathbf{\Pi}^{\top}\frac{1}{\sum_{h=0}^{\infty}z_{ii}^{[h]}}(\mathbf{1}-\lambda)_{i}}_{\substack{\text{intermediated}\\\text{value added}}}\;+\;\underbrace{\mathbf{E}\,\mathbf{\Pi}^{\top}\frac{1}{\sum_{h=0}^{\infty}z_{ii}^{[h]}}\lambda_{\mathbf{i}}}_{\substack{\text{ctr. }i\text{'s}\\\text{value added}}}\bigg)\\&\quad+[\mathbf{E}^{\mathbf{f}}]^{-1}\mathbf{\Phi}^{\mathbf{tot}}\mathbf{L}\mathbf{W}\,\mathbf{d}\ln(\mathbf{w}),\end{aligned} \tag{A.35}$$

where, for lines one and two, we took advantage of the identities

$$\begin{aligned}&(i)\;\;(\mathbf{P}^{\mathbf{f}})^{\gamma-1}(\mathbf{K}^{\mathbf{i}})^{-\gamma}\mathbf{Z}^{\top}\mathbf{P}^{1-\gamma}=\mathbf{\Pi}^{\top}\\&(ii)\;\mathbf{P}^{\gamma-1}[\mathbf{I}-\mathbf{Z}^{\top}]^{-1}\mathbf{P}^{1-\gamma}=\left[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\top}\right]^{-1}\\&(iii)\,\mathbf{P}^{\gamma-1}(\mathbf{K}^{\mathbf{l}})^{\gamma}\mathbf{w}^{1-\gamma}=\lambda.\end{aligned} \tag{A.36}$$

In lines three and four of (A.35), we then decomposed the terms in lines one and two into the channels (i) and (ii) of Formula (23). Here, we additionally made use of the elementary identity in (A.11), implying that $\mathbf{\Pi}^{\top}\left[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\top}\right]^{-1}\lambda=\mathbf{1}$. The resultant expression

[27] Lemma S.1 in Supplementary Online Material S.1 expands on a collection of results from the regional science and social networks literature to perform comparative statics for an inverse matrix $[\mathbf{I}-\mathbf{Z}]^{-1}$.

in line four of (A.35), $\mathbf{I_i 1} \equiv \mathbf{1_i}$, is eventually omitted in line six because (a) we ignore the welfare effects in the isolated country i itself and (b) we ignore the relaxed import competition in country i, since no other country $j \neq i$ is going to sell in i anyhow.

Regarding the wage effects of a country's isolation, labor demand equation (A.9) can be written as

$$\begin{aligned}\mathbf{Wl} &= \boldsymbol{\Lambda}\left[\mathbf{I} - \boldsymbol{\Pi}(\mathbf{I} - \boldsymbol{\Lambda})\right]^{-1}\boldsymbol{\Pi}\mathbf{Wl} \\ &= (\mathbf{K^l})^{\gamma}\,\mathbf{W}^{1-\gamma}\left[\mathbf{I} - \mathbf{Z}\right]^{-1}\mathbf{Z}\,(\mathbf{K^i})^{-\gamma}\,(\mathbf{P^f})^{\gamma-1}\mathbf{Wl}.\end{aligned}$$

Application of (A.34) and the identities in (A.36) onto the exogenous matrix $[\mathbf{I} - \mathbf{Z}]^{-1}\mathbf{Z}$, and recalling the definition of $\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}$ in (A.24), gives a first-order effect of

$$\begin{aligned}\mathbf{d}\ln(\mathbf{w}) = (1-\gamma)\,[\mathbf{LW}]^{-1}\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}\boldsymbol{\Phi}^{\mathbf{down}}\Bigg[&\underbrace{\frac{1}{\sum_{h=0}^{\infty} z_{ii}^{[h]}}\mathbf{I_i}\sum_{\mathbf{h=0}}^{\infty}\left[\boldsymbol{\Pi}(\mathbf{I}-\boldsymbol{\Lambda})\right]^{\mathbf{h}}\boldsymbol{\Pi}\,\mathbf{I_{-i}}\,\mathbf{e^f}}_{\text{intermediated demand}} + \underbrace{\boldsymbol{\Pi}\,\mathbf{I_i}\,\mathbf{e^f}}_{\text{ctr } i\text{'s demand}} \\ &\underbrace{-\,\boldsymbol{\Pi}\,\mathbf{E^f}\,\mathbf{I_{-i}}\left(\mathbf{d}\ln(\mathbf{p^f}) - [\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{tot}}\mathbf{LW}\,\mathbf{d}\ln(\mathbf{w})\right)\Bigg]}_{\text{softer import competition}}.\end{aligned} \quad \text{(A.37)}$$

Combining expressions (A.35) and (A.37) results in Formula (23). □

A.5.2. Proof of Proposition 9

Starting from our diffusion model defined in (12) and assuming $\beta = \gamma$, the wage and price effects of country i's partial isolation (in vector notation: $\mathbf{d_i T} = y(\mathbf{I_i T} + \mathbf{T\,I_i})$ with $y > 0$) can be written as

$$\begin{aligned}\mathbf{d}\ln(\mathbf{w}) = (1-\gamma)\,[\mathbf{LW}]^{-1}\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}\,\boldsymbol{\Phi}^{\mathbf{down}}\Bigg(&\underbrace{\left[\boldsymbol{\Pi}\circ\mathbf{d_i}\ln(\mathbf{T})\right](\mathbf{e^f} + \mathbf{e^i})}_{\substack{\text{Foregone local (iii) and}\\ \text{intermediated demand (iv)}}} - \underbrace{\boldsymbol{\Pi}\,\delta^{\mathbf{p}}}_{\substack{\text{Softer import}\\ \text{competition (v+vi)}}} \\ &+ \underbrace{\boldsymbol{\Pi}\,\mathbf{E}\,(\mathbf{I}-\boldsymbol{\Lambda})[\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\,\delta^{\mathbf{p}}}_{\substack{\text{Key player's productivity}\\ \text{losses (i+ii)}}} - \underbrace{\boldsymbol{\Pi}\,\mathbf{E}\,\boldsymbol{\Pi}^{\top}(\mathbf{I}-\boldsymbol{\Lambda})[\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\delta^{\mathbf{p}}}_{\substack{\text{Rivals' productivity}\\ \text{losses (v+vi)}}}\Bigg)\\ \mathbf{d}\ln(\mathbf{p^f}) = \underbrace{[\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\delta^{\mathbf{p}}}_{\substack{\text{Foregone value}\\ \text{added (i+ii)}}} &+ [\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{tot}}\mathbf{LW}\,\mathbf{d}\ln(\mathbf{w}),\end{aligned}$$

where

$$\delta^{\mathbf{p}} = \underbrace{\mathbf{E}\left[\boldsymbol{\Pi}\circ(\mathbf{d_i}\ln(\mathbf{T}))\right]^{\top}\lambda}_{\text{local value added (i)}} + \underbrace{\mathbf{E}\left[\boldsymbol{\Pi}\circ(\mathbf{d_i}\ln(\mathbf{T}))\right]^{\top}(\mathbf{1}-\lambda)}_{\text{intermediated value added (ii)}}$$

and where the different channels are enumerated in accordance with Formula (23).

Isolating one country after the other from the rest and summing up the *total* (local and intermediated) effects gives

$$\sum_{i\in\mathcal{N}}\left[\boldsymbol{\Pi}\circ\mathbf{d_i}\ln(\mathbf{T})\right] = 2y\boldsymbol{\Pi} \qquad \text{and} \qquad \sum_{i\in\mathcal{N}}\left[\boldsymbol{\Pi}\circ(\mathbf{d_i}\ln(\mathbf{T}))\right]^{\top} = 2y\boldsymbol{\Pi}^{\top},$$

that is, we emulate the welfare effects of a global trade cost increase. By contrast, when we just sum up the *local* effects (i), (iii), and (v), the real income effects are given by

$$\sum_{i\in\mathcal{N}}\mathbf{d_i^{loc}}\ln(\mathbf{u}) = \mathbf{d^{loc}}\ln(\mathbf{w}) - \mathbf{d^{loc}}\ln(\mathbf{p^f}),$$

where

$$\begin{aligned}\mathbf{d^{loc}}\ln(\mathbf{w}) = (1-\gamma)\,[\mathbf{LW}]^{-1}\boldsymbol{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}\,\boldsymbol{\Phi}^{\mathbf{down}}\Bigg(&\left[\boldsymbol{\Pi}\circ\mathbf{d}\ln(\mathbf{T})\right]\mathbf{e^f} + \boldsymbol{\Pi}\,\mathbf{E}\,(\mathbf{I}-\boldsymbol{\Lambda})[\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\,\delta^{\mathbf{loc}} \\ &-\boldsymbol{\Pi}\,\mathbf{E}\left[[\mathbf{E}]^{-1}\delta^{\mathbf{loc}} + \boldsymbol{\Pi}^{\top}(\mathbf{I}-\boldsymbol{\Lambda})[\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\,\delta^{\mathbf{loc}}\right]\Bigg),\\ \mathbf{d^{loc}}\ln(\mathbf{p^f}) = [\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{up}}\delta^{\mathbf{loc}} &+ [\mathbf{E^f}]^{-1}\boldsymbol{\Phi}^{\mathbf{tot}}\mathbf{LW}\,\mathbf{d^{loc}}\ln(\mathbf{w}),\end{aligned}$$

$\mathbf{\Pi}\circ\mathbf{d}\ln(\mathbf{T}) = 2y\,\mathbf{\Pi}$, and $\delta^{\mathbf{loc}} = 2y\,\mathbf{E}\,\mathbf{\Pi}^{\top}\lambda$. To arrive at our claim, it remains to be seen that

$$\begin{aligned}[\mathbf{E^f}]^{-1}\mathbf{\Phi}^{up}\,\delta^{\mathbf{loc}} &= [\mathbf{E}]^{-1}\delta^{\mathbf{loc}} + \mathbf{\Pi}^{\top}(\mathbf{I}-\mathbf{\Lambda})[\mathbf{E^f}]^{-1}\mathbf{\Phi}^{\mathbf{up}}\,\delta^{\mathbf{loc}} \\ &= 2y\mathbf{\Pi}^{\top}\left[\mathbf{I}-(\mathbf{I}-\mathbf{\Lambda})\mathbf{\Pi}^{\top}\right]^{-1}\lambda \\ &= 2y\,\mathbf{1}\,.\end{aligned}$$

Line one is nothing but a decomposition, line two a rearrangement of the terms, and line an immediate consequence of the elementary identity (A.11) in Appendix A.2. We thus get

$$\mathbf{d^{loc}}\ln(\mathbf{w}) = 2y(1-\gamma)\,[\mathbf{LW}]^{-1}\mathbf{\Phi}^{\mathbf{mult}}_{\mathbf{i*}}\,\mathbf{\Phi}^{\mathbf{down}}\left[\mathbf{\Pi}\,\mathbf{e} - \mathbf{\Pi}\,\mathbf{e}\right] = \mathbf{0} \qquad \text{and}$$

$$\mathbf{d^{loc}}\ln(\mathbf{p^f}) = 2y\,\mathbf{1},$$

which means that the cross-country variation in welfare effects of $\mathbf{dT} = -y\mathbf{T}$ is solely determined by the emanating intermediation effects (ii), (iv), and (vi) of Formula (23). □

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jet.2024.105800.

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