Diffusion Prediction of Competitive Information with Time-varying Attractiveness in Social Networks

Abstract: The ubiquity of social media has facilitated the simultaneous dissemination of large-scale information within online social networks. By assuming that information attractiveness is static, numerous studies have been devoted to the analysis of multiple information dissemination. However, real-world information attractiveness often exhibits variations throughout the dissemination process. This paper delves into the study of how time-varying information attractiveness influences the diffusion process, particularly in the context of competitive information dissemination regarding multiple products. First, we propose a Markov multi-information diffusion model, incorporating three critical parameters: the attractiveness degree, the information-boom time, and the information prosperity index, to address the dynamic nature of attractiveness. The basic reproduction number is derived and the accuracy of the proposed model is verified. Furthermore, our numerical simulation results illustrate that both the maximal attractiveness value and the information prosperity duration significantly enhance information competitiveness, while delayed information boom time may undermine this competitiveness. In addition, it is indicated that improving the peak attractiveness is the key for time-varying information to achieve a better spreading effect. Moreover, we find that the growth of information attractiveness can even mitigate the impact of blocking the spread caused by information discarding behavior, highlighting intrinsic quality as the paramount determinant of information promotion outcomes.

Keywords: Competitive information dissemination; Time-varying, Markov model; Social networks; Information attractiveness

1. Introduction

The rise of social media platforms like Facebook, Twitter, and WeChat has marked the advent of an unprecedented era of user-generated content, spanning from content sharing to Time-sync comment (TSC) [1], which permeates online social networks. Enterprises that enhance their marketing management through various means [2-4], actively utilize social media interactions for marketing activities [5,6]. Businesses seek to cultivate deeper consumer engagement for their products by disseminating product information through these platforms. There exist numerous instances of highly successful marketing campaigns, exemplified by Wren's "First Kiss" advertisement, which garnered over 42 million views on YouTube within three days and led to a substantial surge in sales by nearly 140 times [7]. Decisions related to factors such as selecting the appropriate advertising channels and determining the optimal timing for advertisement release can significantly impact the popularity of advertisements, thereby influencing the number of consumers who make purchases. Thus, predicting advertisement popularity is paramount in guiding decision-making for companies [8].

In practice, the abundance of coexisting information surpasses an individual user's processing capacity, leading to competition for the limited attention of users [9]. This competition for attention highlights the importance of metrics such as cumulative retweets or shares on social networks, which are crucial indicators of marketing effectiveness. The persuasive cues embedded within brand information, termed information attractiveness, encompass both rational and emotional dimensions [10,11], which determine the level of user attention to the information [12]. Previous research has highlighted the pivotal role of information attractiveness in influencing users' retweet decisions, thereby affecting the success of viral marketing advertising [13,14]. Existing research has explored the impact of information attractiveness on user decision-making [15-18].

Of noteworthy significance is the temporal variability observed in information attractiveness in real-world scenarios, which underscores the dynamic nature of this phenomenon [19]. For instance, in industries like mobile phones, where well-established models like the iPhone sustain unwavering popularity, while new smartphones experience a transient surge in appeal followed by a decline as novelty fades. Numerous studies validate the pattern of rise and fall in information attractiveness associated with popular products [20]. Temporal dynamics is a critical factor in predicting information cascades [8], with the rise-and-fall pattern predominating the popularity of hot topics in online social networks [21,22]. Variations in information content attractiveness lead to significant differences in retweeting probabilities, highlighting the dynamic nature of information infectiousness [23]. Consequently, this work focuses on understanding the temporal dynamics of attractiveness, which is critical to elucidating the mechanisms driving information propagation.

We employ the information diffusion modeling approach to simulate the dissemination process of information within social networks. Existing research on information diffusion modeling can be categorized into several branches [24]. The first branch, exemplified by the linear threshold model [25] and the independent cascade model [26], operates under the assumption of static social network structures. The second branch, represented by models like CODA [27], analyzes user opinion evolution based on neighboring node relationships, while the third branch, game theory models [28], describes individual behavior strategies in information diffusion. However, these branches often overlook overall dissemination outcomes [29]. To address this, we adopt the fourth branch, comprising non-graph-based methods, which focus on the dynamics of information diffusion and overcome limitations associated with unclear network structures, emphasizing variations in propagation stages and diffusion outcomes. The most common model in this field is the epidemic model, with the SIS and SIR models being prominent examples [30,31].

While numerous studies have examined information dissemination in social networks, some models primarily focused on single pieces of information [32-34]. Some research has explored competitive information propagation, elucidating how users' behaviors and network characteristics influence the transmission process [35-40]. However, existing studies have largely overlooked time-dependent attractiveness dynamics and individual information discarding behaviors.

To address these gaps, this paper evaluates competitive information propagation within a Markov model framework, incorporating time-varying attractiveness. We assume that while static attractiveness conforms to a uniform distribution, time-varying attractiveness follows a bell-shaped distribution with short durations. In addition, we evaluated the effectiveness of incorporating the features considered in the improved models. We also consider scenarios where individuals discard information based on its intrinsic quality.

Our study has the following main contributions. First, we analyzed the information competitive propagation processes of two types of information with time-varying attractiveness and proposed an attractiveness function that integrates attractiveness degree, information-boom time, and information prosperity index. This function unveils the dynamic nature of information attractiveness in online social networks, unlike the static attractiveness assumed by traditional models. Second, we highlight the pivotal role of the initial promotion stage and peak attractiveness in information performance. Investing more effort in the launch phase can significantly enhance information performance while focusing on maximizing attractiveness during product promotion periods can optimize impact while potentially reducing overall promotional efforts. These findings offer valuable insights for further research on product information popularity. Third, we proposed an improved Markov process model suitable for product information popularity prediction, expanding the model to a time-dependent framework. Our study delves into the intricate dynamics of information propagation in social networks, unraveling the complexities of competing information from various sources.

The remainder of this paper is organized as follows: Section 2 outlines the model structure with competitive information, presenting its notions and assumptions. Section 3 delineates the simulations, alongside analytic and numerical findings. Finally, Section 4 offers a summary of the paper and furnishes prescriptive insights derived from the results.

2. Theoretical Framework

2.1. Information transmission mechanism

We consider the interactions among users within online social networks, where connections are formed through interactive engagements, and engage nodes are considered neighbors. The relationships formed are undirected, treating the network as an undirected graph. These connections are dynamic, and we assume the network forms a dynamic random network, where the likelihood of any two nodes forming a connection is finite. We assume that the probability of establishing a connection between two users obeys a Poisson process. Therefore, the event that whether two users exchange information is an independent event and within any given equal-length time interval, the communication inter-arrival time between two users follows an exponential distribution with an arrival rate of λ. To gain a deeper insight into information competition performance, we assume that users follow the same communication contact model.

Our primary focus is on scenarios where multiple messages compete with each other during the process of dissemination in a social network. For instance, in the context of online social networks, various similar product advertisements may be spreading, and users might choose one based on their appeal. These messages may have mutual effects on the propagation performance. To address this, we consider a social network comprising N users and M different messages. Initially, M source users are chosen randomly as seeds to spread the M different messages that involve competitive relationships but belong to the same product category.

Our emphasis is on optimizing the promotion strategy, so we will consider a promotion phase. Considering the information overload in online social networks and users facing constraints of limited attention and budget, the typical behavior is for users to select information about a single product within a specified time frame. For instance, in product categories like smartphones, users tend to adopt information from a singular product during a specific period. As a result, each user in the network adopts only one of these messages throughout the propagation process. In addition, we classified users in the network into three types: *interested*, *satisfied,* and *discarded*. *Interested* users have an interest in this product category but have not received the relevant information yet. S*atisfied* users have received information from others and are actively involved in propagating the product details. Moreover, the *discarded* users have already received the information but are not interested in it and will not propagate it any further.

As described in Section 1, the attractiveness of information varies depending on the product type, and individuals accept information based on its attractiveness. We consider the promotion of two different products in the same category, such as mobile phones, each with different attractions. In the first scenario, consider a classic product, such as the iPhone, which has maintained enduring popularity, establishing a consistent customer base. The attractiveness of marketing information related to such classic products tends to remain relatively stable throughout the information dissemination process. Here, we assume that the information attractiveness of classic products follows a uniform distribution with parameter denoted by C. In the second scenario, we focus on emerging smartphones with innovative features or unique designs, such as the trend of foldable-screen phones that gained popularity for a period. Here, information attractiveness is time-varying, and the product's popularity is short-lived. Since smartphones may initially lack widespread recognition and exposure, making them less appealing. As their publicity and information exposure increase, their attractiveness gradually ascends. However, as their novelty diminishes over time, their appeal may also decline. Therefore, we assume that the attractiveness distribution for this scenario follows a bell-shaped function. We define the attractiveness distribution $p(t)$ as follows:

$$p(t)=\begin{cases} C, & t\geq 0,\ \ \textit{attractiveness static}, \\ L\cdot e^{-\frac{(t-\mu)^2}{2\sigma^2}}, & t\geq 0, \mu>0, \sigma>0, 0\leq L\leq 1,\ \ \textit{attractiveness time-varying}. \end{cases} \tag{1}$$

- L refers to the attractiveness degree of the message, signifying the highest attractiveness of the message throughout its transmission.

- μ stands for the information-boom point-in-time, representing the mean time at which the message reaches its peak attractiveness. It represents the moment when the message's attractiveness reaches its zenith.
- σ denotes the message's prosperity index, reflecting the fluctuation in message attractiveness. A high value of σ implies that the message sustains its competitiveness for an extended duration. It might commence with low attractiveness, increase rapidly during subsequent phases, and then slowly decline over a prolonged period. Conversely, a low value of σ corresponds to initial low attractiveness, gradual rise in later stages, and rapid decline over a short period.

The information disseminates in an epidemic–routing-like manner, as depicted in Figure 1. When interested users have an opportunity to communicate with any satisfied user, the latter will forward the message to interested users with a certain probability. *Interested* users accept the information i depending on its attractiveness, and subsequently transition to a *satisfied* state while carrying the message i. On the other hand, owing to the intrinsic value of information and individual preferences, *satisfied* users may cease to transmit information with a certain probability. Different information may possess varying levels of intrinsic product quality. Messages with higher attractiveness are likely to have a large share of interest and, consequently, a lower probability of being discarded. In certain instances, despite some messages boasting high attractiveness owing to dramatic promotions, the underlying product may possess inferior intrinsic quality. Similar to the work in [41], we assume that M types of *satisfied* users cease to share the information, each following an exponential distribution with parameter $\rho_i (i \in [1, M])$, respectively. A snapshot of the information dissemination process is illustrated in Figure 1(a).

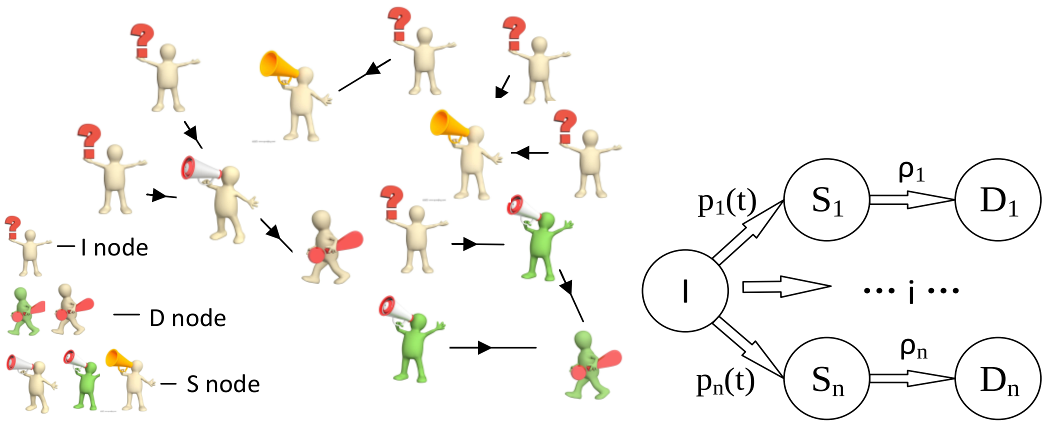

Figure 1. (a) Illustration of information dissemination in social networks. (b) A typical information dissemination process.

2.2. Information dissemination model

Based on the transmission mechanism described in Subsection 2.1, we are ready to investigate the model of the information dissemination process. $I(t)$ stands for the quantity of *interested* users at time t, and $S_i(t)$ refers to the quantity of *satisfied* users with message i $(i \in [1, M])$ at time t, respectively. Additionally, let $D_i(t)$ represents the quantity of *discarded* users with message i $(i \in [1, M])$ at time t. As described in Subsection 2.1, due to the limited nature of consumer attention, we assume that each user selects only one of the M product messages during the transmission process. Based on the previous description, *interested* users may receive message i $(i \in [1, M])$ from *satisfied* users carrying message i. Given a small-time interval Δt, we have:

$$I(t+\Delta t) = I(t) - \sum_{i \in [1,M]} \sum_{j \in \{I(t)\}} \phi i_j (t, t+\Delta t), \tag{2}$$

Here, $\phi i_j (t, t+\Delta t)$ refers to whether the *interested* user j transitions to *satisfied* user carrying message i $(i \in [1, M])$ during the time interval $[t, t+\Delta t]$. If $\phi i_j (t, t+\Delta t) = 1$, the *interested* user

j chooses the message i. Otherwise, $\phi i_j(t,t+\Delta t)=0$ indicates that the user does not elect message i. On the other hand, $\{I(t)\}$ denotes the set of *interested* users at time t. Additionally, user j adopts the message depending on the attractiveness of the message at time t. According to the definition of attractiveness described in Subsection 2.1, we assume that the attractiveness of message i $(i \in [1, M])$ is $p_i(t)$:

$$p_i(t)=\begin{cases} C_i, & t \geq 0, \ \ attractiveness \ \ static, \\ L_i \cdot e^{-\frac{(t-\mu_i)^2}{2\sigma_i^2}}, & t \geq 0, \mu_i > 0, \sigma_i > 0, 0 \leq L_i \leq 1, \ \ attractiveness \ \ time\text{-}varying. \end{cases} \tag{3}$$

In the scenario where information attractiveness remains static, the attractiveness degree of message i remains C_i throughout the transmission process. However, when attractiveness is time-varying, the attractiveness degree of the message i as L_i, the information-boom time as μ_i, and the information prosperity index as σ_i.

When Δt is sufficiently small, the message attractiveness remains unchanged during the time interval $[t,t+\Delta t]$. We suppose that the attractiveness of message i is $p_i(t)$ in this time interval $[t,t+\Delta t]$. As the communication interarrival time between two users obeys to an exponential distribution with an arrival rate λ, user j contacts a *satisfied* user carrying message i with a probability of $1-e^{-\lambda\Delta t}$ during the time interval $[t,t+\Delta t]$. Thus, user j receives the message from a *satisfied* user carrying message i with probability $(1-e^{-\lambda\Delta t})p_i(t)$ during $[t,t+\Delta t]$. Furthermore, the number of *satisfied* users carrying message i is $S_i(t)$ at time t, and user j may receive message i from any of them. Hence, we obtain the following expression:

$$p(\phi i_j(t,t+\Delta t)=1)=1-(1-(1-e^{-\lambda\Delta t})p_i(t))^{S_i(t)}. \tag{4}$$

By combining Equations (2)-(4), we can obtain the expected number of *interested* users as follows:

$$E(I(t+\Delta t))=E(I(t))-E(I(t))\sum_{i=1}^{M}E(\phi i_j(t,t+\Delta t)). \tag{5}$$

Furthermore, based on the mean-field limit, we derive the following ordinary differential equation (ODE):

$$\begin{aligned} E(\dot{I}(t)) &= \lim_{\Delta t\to 0}\frac{E(I(t+\Delta t))-E(I(t))}{\Delta t} \\ &= -\sum_{i=1}^{M}\lim_{\Delta t\to 0}\frac{E(I(t))E(1-(1-(1-e^{-\lambda\Delta t})p_i(t))^{S_i(t)})}{\Delta t} \\ &= -\lambda E(I(t))\sum_{i=1}^{M}E(S_i(t))E(p_i(t)). \end{aligned} \tag{6}$$

It's essential to consider that *interested* users may switch to the *satisfied* users by adopting message i, and *satisfied* users may cease to propagate messages. Then, we can calculate the number of *satisfied* users carrying message i at time $t+\Delta t$

$$S_i(t+\Delta t)=S_i(t)+\sum_{j\in\{I(t)\}}\phi i_j(t,t+\Delta t)-\sum_{k\in\{S_i(t)\}}\tau i_k(t,t+\Delta t), \tag{7}$$

where $\tau i_k(t,t+\Delta t)$ refers to the event that the *satisfied* users discard the carried message i in time interval $[t,t+\Delta t]$. They discard messages probability according to the exponential distribution with a parameter ρ_i as described in Subsection 2.1. Therefore, we have:

$$p(\tau i_k(t,t+\Delta t)=1)=1-e^{-\rho_i\Delta t}. \tag{8}$$

Similar to Equations (6), we obtain the ODE equation as follows:

$$\begin{aligned} E(\dot{S}_i(t)) &= \lim_{\Delta t\to 0}\frac{E(S_i(t+\Delta t))-E(S_i(t))}{\Delta t} \\ &= \lim_{\Delta t\to 0}\frac{E(I(t))E((1-e^{-S_i(t)\lambda\Delta t})p_i(t))}{\Delta t}-\lim_{\Delta t\to 0}\frac{E(S_i(t))E(1-e^{-\rho_i\Delta t})}{\Delta t} \\ &= \lambda E(I(t))E(S_i(t))E(p_i(t))-\rho_i E(S_i(t)). \end{aligned} \tag{9}$$

Furthermore, for the *discarded* users with message i, their number can be increased by switching from *satisfied* users, we get ODE:

$$D_i(t+\Delta t) = D_i(t) + \sum_{k \in \{S_i(t)\}} \tau i_k(t, t+\Delta t). \tag{10}$$

Therefore, we can derive another ODE using a similar method:

$$E(\dot{D}_i(t)) = \rho_i E(S_i(t)). \tag{11}$$

Combining Equations (6), (9) and (11), we arrive at a system of $2M+1$ unknown variables and $2M+1$ equations. The solution to this ODE-based system can be easily derived from the Matlab ODE suite.

Notably, the purpose of the work is to predict the number of users who have adopted the message and investigate the speed of message transmission. Predicting the popularity of advertisements is very important for businesses. Therefore, the performance of message transmission is estimated as follows:

$$R_i(t) = E(S_i(T)) + E(D_i(T)). \tag{12}$$

Here, the symbol $R_i(t)$ indicates the number of users who have adopted message i at time t, comprising both *satisfied* users and *discarded* users. Apparently, the larger $R_i(t)$ is, the more individuals will adopt the message.

2.3. Basic reproduction number

The basic reproduction number, denoted as R^0, signifies the expected number of secondary cases infected by a infectious user [42]. It serves as a pivotal parameter governing information transmission. In essence, the larger the value of R^0, the more challenging it becomes to control information transmission. Our analysis centers on the impact of information attractiveness on information outbreaks. According to the method in [42], the metric R^0 can be expressed by the spectral radius of the regeneration matrix. For clarity, we employ the notations s_i, d_i, p_i, z, $i=1,2,\cdots,M$ to denote $E(S_i(t))$, $E(D_i(t))$, $E(p_i(t))$, $E(I(t))$, $i=1,2,\cdots,M$, respectively. Consequently, we define $X=(s_1,\cdots,s_M,d_1,\cdots,d_M,z)^T$. In this context, $F(x)$ represents the change rate of satisfied users, while $V(x)$ is the rate of change for other users. Thus, our propagation model can be expressed as follows:

$$\frac{dX}{dt} = F(x) - V(x). \tag{13}$$

where $F(x) = \begin{bmatrix} \lambda x_1 p_1 z \\ \vdots \\ \lambda x_M p_M z \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(2M+1)\times 1}$, $V(x) = \begin{bmatrix} \rho_1 x_1 \\ \vdots \\ \rho_M x_M \\ \rho_1 x_1 \\ \vdots \\ \rho_M x_M \\ z\sum_{i=1}^{i=M} x_i p_i \end{bmatrix}_{(2M+1)\times 1}$.

For the system defined by Equations (13), we can derive the disease-free equilibrium as $X_0 = (1,\cdots,1,0,\cdots,0,N-M)^T$, which represents the initial number of users. We can then derive:

$$DF(x_0) = \begin{bmatrix} F & 0 \\ 0 & 0 \end{bmatrix}_{(2M+1)\times(2M+1)}, \quad DV(x_0) = \begin{bmatrix} V & 0 \\ J_3 & J_4 \end{bmatrix}_{(2M+1)\times(2M+1)}, \tag{14}$$

Here,

$$F = \left[\frac{\partial F_i}{\partial x_j}(X_0)\right] = \begin{bmatrix} \lambda z p_1 & 0 & \cdots & 0 \\ 0 & \lambda z p_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \lambda z p_M \end{bmatrix}_{M\times M}, V = \left[\frac{\partial V_i}{\partial x_j}(X_0)\right] = \begin{bmatrix} \rho_1 & 0 & \cdots & 0 \\ 0 & \rho_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \rho_M \end{bmatrix}_{M\times M}.$$

The regeneration matrix is derived as follows:

$$FV^{-1}=\begin{bmatrix} \frac{\lambda \zeta p_1}{\rho_1} & 0 & \cdots & 0 \\ 0 & \frac{\lambda \zeta p_2}{\rho_2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \frac{\lambda \zeta p_M}{\rho_M} \end{bmatrix}_{M\times M}.$$

Notably, the spectral radius is represented as $\rho(FV^{-1})=\max_{1\le i\le M}|\xi_i|$, where ξ_i denotes the eigenvalues of the regeneration matrix [42], and $R^i=\xi_i=\lambda\zeta_0 p_i/\rho_i$. Thus, we have:

$$R^0=\max_{1\le i\le M}\lambda\zeta_0\frac{p_i}{\rho_i}=\max_{1\le i\le M}\lambda(N-M)\frac{p_i}{\rho_i}. \tag{15}$$

3. Simulation and Numerical Results

With the theoretical model of information dissemination established in the previous section, we are ready to present our numerical analysis. To provide a comprehensive evaluation, we will evaluate the performance of two competitive messages transmission in social networks. Moreover, the impact of the information attractiveness and the intrinsic quality of information on competitive information transmission will also be analyzed.

3.1. Simulation results

In this section, we will validate the accuracy of the proposed theoretical model in a Matlab simulation environment. Due to the theoretical nature of the analysis and the discrete nature of network sizes within integer ranges, we selected a network of an appropriately sized population for experimental analysis, setting $N=2000$ users, with a subset of 1998 *interested* users. At time 0, one *satisfied* user carries message 1 and another *satisfied* user carries message 2. The attractiveness of a message ranges from [0,1]. A higher value indicates that the message is more easily adopted by users. We assume that the attractiveness of message 1 is static with a degree C_1 as 0.2. While the attractiveness of message 2 is time-varying, and its attractiveness is $L_2=0.4$. We assume that the information-boom time is $\mu_2=6000$, and the information prosperity index is $\sigma_2=2000$. This paper focuses more on the impact of parameter variations rather than the effects of the parameters themselves. Therefore, this section uses these specific values as an example to analyze the model. Additionally, according to the previous study on the process of information spreading on a mobile social network [43], we set the inter-meeting time to follow an exponential distribution with parameter $\lambda=3.71\times10^{-6}$. In reality, the parameter λ determines the probability of information propagation between two users, representing the speed of online information dissemination. Its numerical value and magnitude are contingent on the propagation platform and the characteristics of the information, and are not fixed. The discarding behaviors related to the user's interest transfer behavior. According to literature [41], setting the two messages conform to exponential distribution with parameter $\rho_1=0.0002$, $\rho_2=0.0001$, respectively. In this configuration, it ensures that the termination of information propagation does not occur too rapidly due to user losing interest. Simultaneously, it reflects the changing dynamics of newly added, interested users in information propagation. By setting the maximal lifetime T from $1s$ to $10,000s$, we conduct Monte Carlo simulation for 100 times. The theoretical results are obtained based on our theoretical model proposed in Section 3 and the Matlab ODE toolbox. The spreading results are shown in Figure 2.

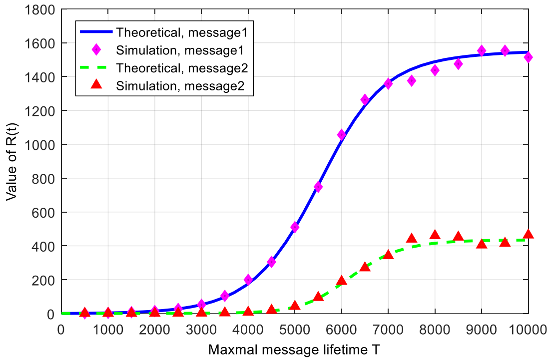

Figure 2. Comparison of the simulation and the theoretical results for spreading a static-attractive message 1 and a time-varying-attractiveness message 2.

As depicted in Figure 2, the simulation results closely align with the theoretical results, with the average deviation between them being remarkably small, typically less than 5.02%. This close match verifies the accuracy of our theoretical model. Then, we can conduct further performance analysis using the results obtained from the theoretical model. In addition, the results reveal that the performance of the two messages changes significantly as message attractiveness varies. For example, when $T = 10,000$ s, the value of $R_1(t)$ is around 1600 and value of $R_2(t)$ is roughly 400. Although the attractiveness of message 1 is 0.2, which is less than the maximum attractiveness of message 2, the outbreak time of message 1 is earlier than that of message 2. As a result, the final spread of message 1 is larger in scope than that of Information 2. Therefore, why do such results occur, and are there any other phenomena to consider? The competitive dynamics of information propagation between information with constant attractiveness and information with time-varying attractiveness warrant further analysis. In the subsequent sections, we will delve deeper into this phenomenon through numerical results.

3.2. Impacts of information attractiveness

As mentioned earlier, we have obtained a preliminary understanding of how attractiveness influences information competition. To gain further insight, we proceed to analyze the impact of various parameters on competitive information propagation. In this subsection, we explore scenarios where both messages exhibit time-varying attractiveness. Specifically, we set the total number of users as $N = 5000$, comprising one *satisfied* user carrying message 1 and another *satisfied* user carrying message 2, with the remaining users classified as *interested* users. We consider three groups of messages with different attractiveness degree for message 2, while maintaining a constant attractiveness degree for message 1. The dual information propagation processes, spanning a maximal message lifetime of T from $0s$ to $10,000s$, are shown in Figure 3(a). Furthermore, Figure 3(b) illustrates the temporal variation of attractiveness value for different attractiveness degrees L.

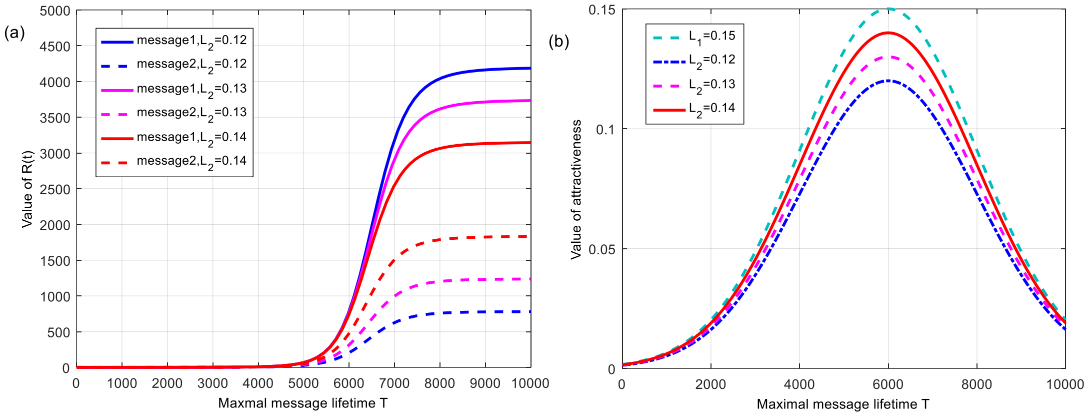

Figure 3. (a) The propagation process of dual information with varying attractiveness degrees for three groups. (b) The attractiveness value of different attractiveness degrees with time. Results are obtained based on our theoretical model with $N = 5000$, $T = 10,000s$, $L_1 = 0.15$, $\mu_1 = \mu_2 = 6000$, $\sigma_1 = \sigma_2 = 2000$, $\rho_1 = \rho_2 = 0.00001$. $L_2 = 0.12,\ 0.13,\ 0.14$ in the three groups, respectively.

By analyzing the dissemination patterns of three groups of messages, we can conclude from Figure 3(a) that the final propagation extent of message 2 expands with its increasing attractiveness degree. For example, when $L_2 = 0.12$, the final propagation range of message 1 reaches its peak, while the range of message 2 is minimized. In contrast, when $L_2 = 0.14$, the final propagation range of message 1 is minimized and that of message 2 is maximized. On the other hand, the information initially undergoes constrained diffusion, with a rapid increase in its spread rate at $T = 5000s$. Subsequently, the spreading range stabilizes around $T = 8000s$ for all three groups. The final outbreak size of the two competitive messages is significantly influenced by the attractiveness degree of the message. This observation is elucidated by examining Figure 3(b), where initial attractiveness of information remains relatively low during the early propagation phase. Consequently, only a limited number of individuals initially receive the message across all experimental scenarios. However, as the information booms, the more attractive message effectively captivates a larger audience. As a result, with more individuals acquiring the information, there are more *satisfied* users to spread it compared to competitive information. This increases the likelihood of *interested* individuals receiving the information, leading to the remarkable difference in the final outbreak size. Therefore, businesses can increase the maximum attractiveness during the promotion phase, amplifying the intensity of promotions to achieve greater impact while potentially saving overall promotional efforts and costs.

Now, we investigate the impact of information-boom time by examining three groups of messages with distinct information-boom time values for message 2 while maintaining a fixed information-boom time for message 1. The results of these propagation processes are displayed in Figure 4(a), while Figure 4(b) demonstrates the temporal evolution of attractiveness values for different information boom times μ.

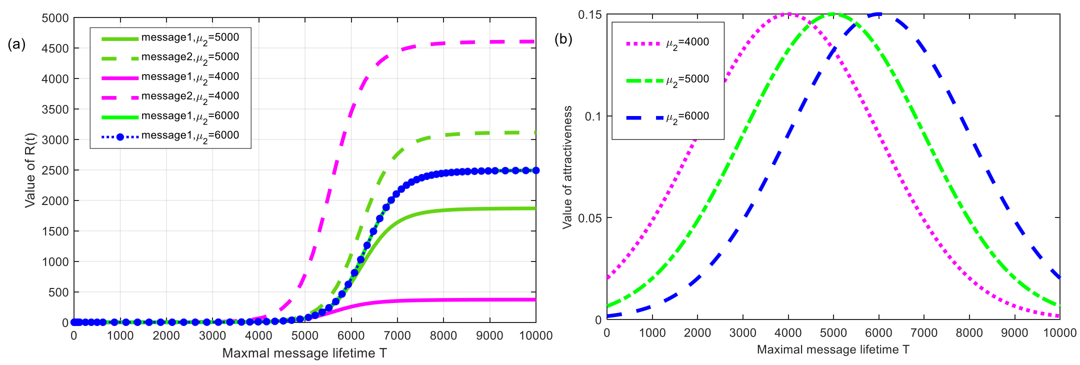

Figure 4. (a) The propagation process of dual information with varying information boom times with three groups. (b) The attractiveness value of different information-boom time. Results are obtained based on our theoretical model with $N = 5000$, $T = 10,000s$, $L_1 = L_2 = 0.15$, $\mu_1 = 6000$, $\sigma_1 = \sigma_2 = 2000$, $\rho_1 = \rho_2 = 0.00001$. $\mu_2 = 4000, 5000, 6000$ in the three groups respectively.

Analyzing the results shown in Figure 4(a), we observe that the dissemination of information initiates with constrained reach, then amplifies at different times, and eventually attains peak outbreak size at varying times. Notably, the dissemination scope of message 1 exhibits a propensity for augmentation, contrasting the diminishing trend observed for message 2 as information boom times μ_2 ascend. For example, when $\mu_2 = 4000$, message 1 manifests minimal spread, while massage 2 achieves the largest final propagation range. In contrast, when $\mu_2 = 6000$, the final propagation range of both message 1 and message 2 exhibiting a same range close to 2500. Furthermore, we can see that the spreading speed of message 2 increases rapidly from $T = 3000s$ when $\mu_2 = 4000$, while it begins from $T = 5000s$ otherwise when $\mu_2 = 6000$. The spreading rise time of message 1 is from $T = 5000s$ across all three groups. In conclusion, it is apparent that the temporal dynamics of information boom time wield significant influence over dual information propagation. From Figure 4(b), the attractiveness gradually increases at different times, with the highest value being consistent across the groups, indicating the same length of boom duration. When message 2 experiences an early information-boom time, it garners higher initial attractiveness compared to other competing information. Consequently, it attracts more *satisfied* users to spread the message 2 during the initial stage. However, if message 1 commences broadcasting after message 2 has already established substantial transmission, it becomes challenging for message 1 to attract more *interested* users, resulting in fewer *satisfied* users spreading it. This highlights the intricacies involved in competing with pre-existing messages. Therefore, businesses can achieve better results by investing more in promoting their products early on, attracting more propagators and enhancing their promotional effectiveness.

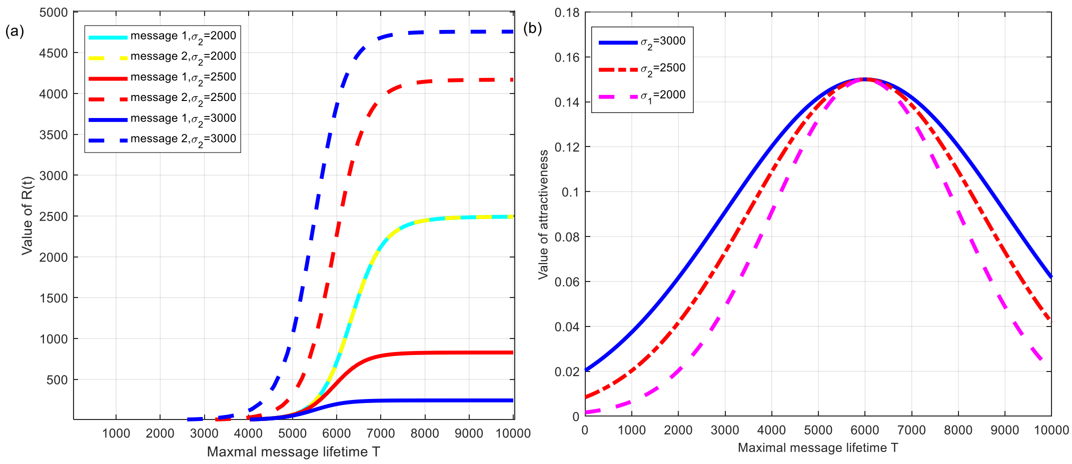

Figure 5. (a) The propagation process of dual information for different information prosperity index with time varying of three groups. (b) The attractiveness value varies with time. Results are obtained based on our theoretical model with $N = 5000$, $T = 10,000s$, $L_1 = L_2 = 0.15$, $\mu_1 = \mu_2 = 6000$, $\sigma_1 = 2000$, $\rho_1 = \rho_2 = 0.00001$. $\sigma_2 = 3000,\ 2500,\ 2000$ in the three groups respectively.

Figure 5(a) shows the propagation process of dual information concerning different values of the information prosperity index parameter σ_2 of message 2 and a fixed value σ_1 of message 1 across three groups. The results show that the outbreak size of message 2 increases while message 1 decreases with an increase in σ_2. For example, under the scenario when $\sigma_2 = 3000$, the outbreak size of message 1 approximates 200, whereas the final propagation range of message 2 surpasses considerably. In contrast, when $\sigma_2 = 2000$, both message 1 and message 2 exhibit final propagation range around 2500. Thus, it becomes evident that the effect of the information prosperity index on dual information propagation is great. As shown in Figure 5(b), when σ_2 is larger, the information booms for a longer time. The highest attractiveness value remains consistent across the groups, maintaining the same boom duration. The results in Figure 5(a) can be explained as follows: initially, the likelihood of individuals receiving this message is higher, not diminishing during the boom time compared to other messages. Consequently, it attracts more individuals during the early stages, resulting in more expansive propagation and higher spreading speed. The more extended boom duration leads to striking differences in the final outbreak size. In summary, these experiments emphasize the importance of attracting more individuals during the initial stages of the transmission process to enhance information competitiveness. Thus, businesses can improve attractiveness during the initial stage to enhance the information competitiveness, and enhance their promotional effectiveness.

3.3. Performance analysis of different types of information attractiveness

In this subsection, we delve into the impact of various types of information attractiveness on the propagation of multiple competitive information. We uphold identical initial user settings as in the preceding subsection. Specifically, we presume that the attractiveness of message 1 remains static at a degree of $C_1 = 0.3$, whereas message 2 exhibits time-varying attractiveness following a bell-sharped function. Based on the analysis from the preceding subsections, we investigate the impacts of attractiveness degree, information-boom time, and information prosperity index on information competition. Figure 6 visually represents the combined effects of parameters L, μ, and σ on information competition.

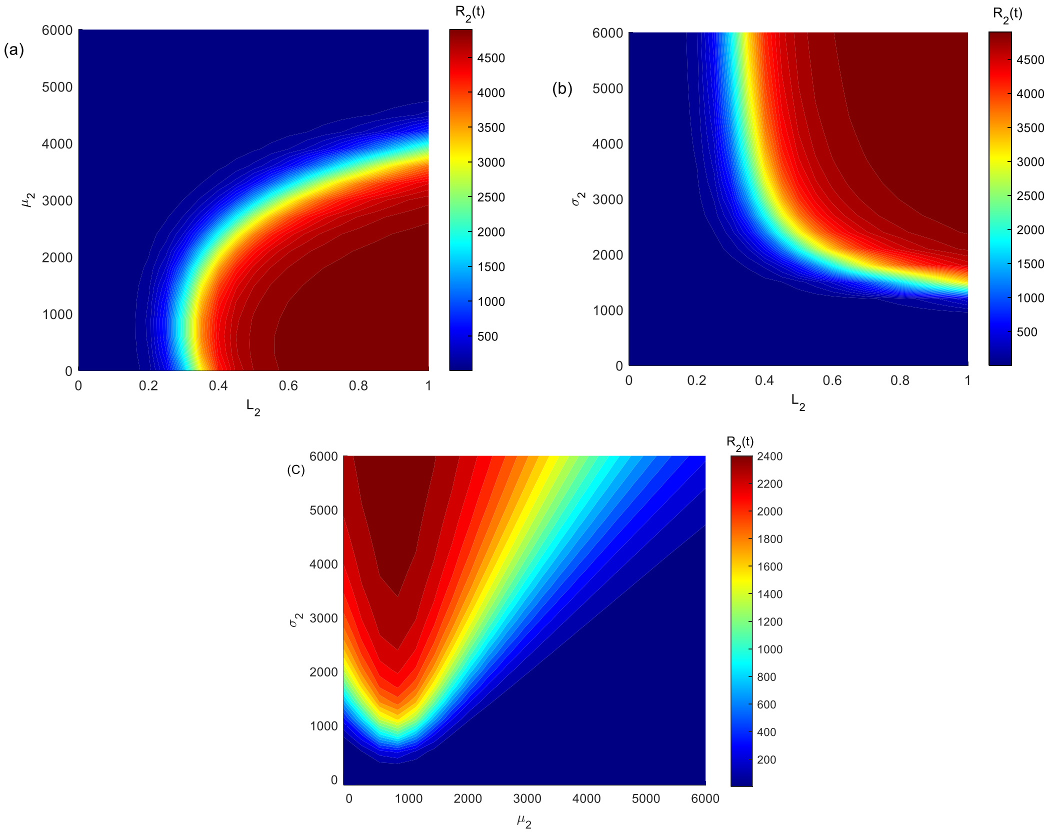

Figure 6. The impacts of different types of information attractiveness on the dual information propagation. The contours plot the final outbreak size of message 2. (a) Parameter L_2 varies from 0 to 1 and μ_2 varies from 0 to 6000 with σ_2=2000. (b) L_2 varies from 0 to 1 and σ_2 varies from 0 to 6000 with μ_2=3000. (c) Parameter μ_2 varies from 0 to 6000 and σ_2 varies from 0 to 6000 with $L_2 = 0.3$. Other parameters are set as $N = 5000$, $T = 10,000s$, $C_1 = 0.3$ and $\rho_1 = \rho_2 = 0.0001$.

From Figure 6 (a) and (b), a discernible trend emerges wherein the performance of message 2 with increasing attractiveness degree L_2. This underscores a fiercely competitive scenario in the propagation process. When L_2 is smaller than 0.2, the outbreak size of message 2 is near 0, implying that aside from the source user, regardless of the value of μ_2, scarcely any individual received message 2. When L_2 is smaller than 0.3, the outbreak size of message 2 fails to surpass that of message 1. However, it becomes evident that when L_2 surpasses a certain threshold 0.3, message 2 exhibits superior performance by adjusting parameter μ_2 and σ_2. These results can be explained as follows: Throughout the propagation phase, the information attractiveness of message 2 consistently lower than that of message 1 when L_2 does not exceed C_1. Consequently, it fails to vie effectively with message 1. Yet, as L_2 increases, and parameter μ_2 and σ_2 are adjusted, message 2 manages to attract more individuals to adopt the information during the information prosperity phase, despite message 1 maintaining a certain quantity holders. Therefore, the attractiveness degree plays a decisive role in the competition whether it is time-varying information or static-attractiveness information. Therefore, we emphasize the crucial role of peak attractiveness in time-varying information. Businesses can utilize this knowledge to tailor their promotional strategies, focusing on amplifying the maximum promotional intensity during product promotion periods. This approach enables them to achieve greater impact while potentially reducing overall promotional efforts and costs.

Figure 6 (c) visually depicts the joint effects of parameters μ and σ on the information competition. When the information prosperity duration σ is less than a specific threshold (Close to 800), regardless of whether it occurs earlier or later, the information propagation is almost negligible. However, upon surpassing this threshold, and with the information-boom time μ exceeding a certain value (approximately 800), an earlier occurrence of the information-boom time correlates with a broader dissemination of the information. Conversely, when μ falls below a particular threshold (approximately 800), an earlier information-boom time leads to a diminished eventual spread of the information. This phenomenon stems from the relatively brief period during which the information maintains a substantial level of attractiveness before commencing its decline, as depicted in Figure 4 (b). Notably, even with substantial value of σ, achieving a competitive advantage becomes challenging when μ is sizable. Therefore, in the realm of business promotion, it's imperative to avoid excessively short promotional durations. Understanding the critical threshold for information prosperity duration is highly beneficial for achieving maximum effectiveness in promotions with constrained budget. Extended promotional duration yield more favorable outcomes for businesses. Moreover, maintaining a high promotional intensity during the initial stages of product promotion markedly augments promotional efficacy.

3.4. Impact of the discarding probability

In this subsection, we investigate the impact of discarding probability on the propagation of multiple competitive information. As described in Section 3, *satisfied* users may opt to discard information, with the exponential distribution $1-e^{-\rho\Delta t}$, while considering information quality. Figure 7 depicts the final transmission range of two messages, with varying discarding value of ρ_1 and keeping a fixed value of ρ_2.

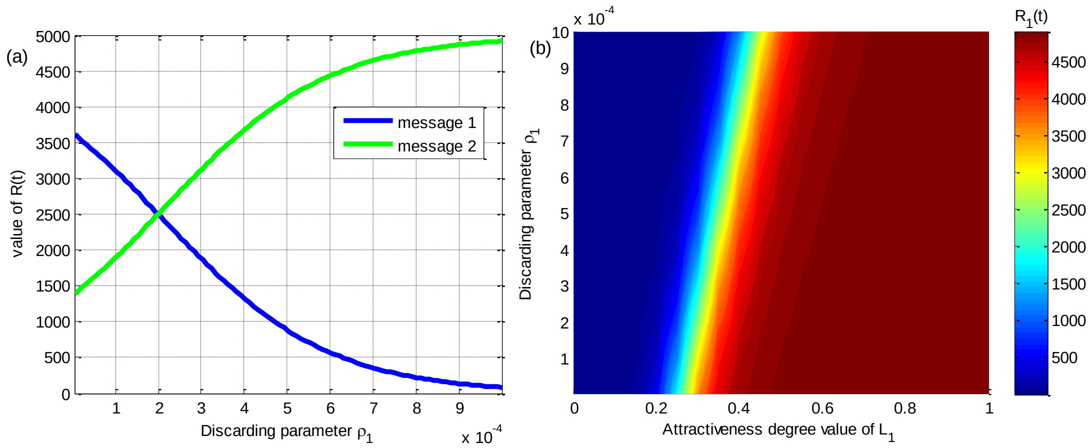

Figure 7. The impact of discarding probability on competitive information with varying parameter of ρ_1 from 0 to 1×10^{-3}. (a) The curve shows the outbreak size of the two messages. (b) The contour plots the final outbreak size of message 1 with parameter plane (L_1 , ρ_1), L_1 varying from 0 to 1. Other parameters are set as follows: $N = 5000$, $T = 10,000s$, $\rho_2 = 0.0002$, $L_1 = L_2 = 0.3$, $\mu_1 = \mu_2$=6000 and $\sigma_1 = \sigma_2$=2000 .

The results shown in Figure 7(a) demonstrate a decrease in the propagation efficacy of message 1 as the discarding parameter ρ_1 increases. Notably, as ρ_1 approaches approximately 1×10^{-3}, message 1 exhibits minimal spread throughout the network. Conversely, Figure 7 (b) illuminates a significant enhancement in the performance of message 1 with increasing values of L_1. Furthermore, when ρ_1 is below 1×10^{-3} and L_1 exceeds a certain threshold 0.7, message 1 emerges as the dominant force in the network. In contrast, as L_1 diminishes below 0.2, message 1 fades into obscurity within the network. In fact, when individuals carrying message 1 exhibit a higher tendency to discard information, augmenting the number of *satisfied* users spreading message 1 becomes challenging, especially during the initial stage. Furthermore, there is a risk of information vanishing from the network when the discarding parameter attains a sufficiently elevated level. Meanwhile, improving the attractiveness degree can precipitate an upsurge in the *satisfied* users base, mitigating the detrimental effects of discarding. Thus, enhancing the intrinsic quality of information proves to be of paramount importance in enhancing information competitiveness. Therefore, the importance of intrinsic quality serves as a practical lesson for businesses. While attracting customers through appealing promotions is essential, maintaining the quality of products or services is equally crucial for long-term success.

4. Discussion

Theoretical contributions:

Our study delved into the realm of online social networks, specifically focusing on the widespread adoption of competing product information. We introduced the attractiveness function, which encompasses both static and time-varying information attractiveness. This function incorporates critical factors, including the attractiveness degree, information-boom time, and information prosperity index, enabling a more comprehensive model that can accommodate the dynamic nature of information propagation. By introducing this time-dependent model, we extended the conventional understanding of information dissemination, providing new possibilities for information prediction and marketing strategies.

Furthermore, our study examined the impact of attractiveness on information competition highlighting the significance of both maximal attractiveness values and information prosperity durations. These findings contribute to the development of a deeper theoretical framework for information diffusion modeling.

Practical Implications:

Our findings offered valuable insights and implications for businesses and advertisers. Understanding the dynamics of information spread in online social networks is crucial for developing effective promotion strategies.

Based on our analytical results, it is shown that the maximal attractiveness value and the information prosperity duration play crucial roles in enhancing information competitiveness, while delayed information-boom time can undermine information competitiveness. Moreover, the importance of enhancing information attractiveness during the initial promotion stage has been underscored. This insight is particularly valuable for businesses and advertisers aiming to maximize their impact. By dedicating more effort to promoting their products or messages early on, they can effectively achieve better results, attracting more spreaders and improving their information's overall performance. This aligns with real-world advertising strategies where the launch phase is critical for capturing consumer attention. For instance, consider the launch of a new smartphone model. By enhancing the attractiveness of product information during the initial promotion, a company can create a buzz and attract early adopters, thereby achieving a robust market presence.

Additionally, we highlighted the critical role of peak attractiveness in time-varying information. Businesses can use this knowledge to tailor their promotion strategies, focusing on amplifying the maximum intensity of promotion during product promotion periods. This approach allows them to achieve a more significant impact while potentially reducing overall promotional efforts and costs. This finding holds practical applications in various industries. Take the example of new beverages, concentrating marketing efforts during promotional activities can create a sense of urgency and demand, ultimately boosting sales and brand visibility.

Furthermore, our analysis of the impact of discarding behavior on competitive information diffusion underscores the paramount role of intrinsic quality. The importance of intrinsic quality is a practical lesson for businesses. While attracting customers through appealing promotions is essential, maintaining the quality of products or services is equally vital for long-term success. Customer satisfaction and loyalty are often determined by the actual value delivered. Consider a restaurant chain's marketing campaign. While enticing promotions may attract customers, the quality of the food and service is ultimately what determines customer satisfaction and repeat business.

5. Conclusion

Considering the situations where the adoption of competing products and analogous information are widespread in online social networks, this research advances the understanding of competitive information dissemination in online social networks. By considering the time-varying nature of information attractiveness, we have expanded existing theoretical models and provided new insights into the dynamics of information spread. Our study emphasizes the importance of enhancing information attractiveness during the initial stages of promotion, offering practical implications for businesses and marketers seeking to optimize their strategies.

The findings highlight the pivotal role of peak attractiveness in information performance, underscoring the need for businesses to concentrate marketing efforts during product promotion periods to create a sense of urgency and demand. Moreover, the intrinsic quality of the information is identified as a paramount determinant of information promotion outcomes, emphasizing the importance of delivering value to customers.

In a rapidly evolving digital landscape, our study provides a foundational framework for businesses to refine their information promotion strategies and adapt to the dynamic nature of online social networks. By understanding the interplay of time-varying information attractiveness, peak attractiveness, and intrinsic quality, businesses can achieve better results in terms of audience engagement and product promotion. Overall, our contributions and findings offer a foundation for the development of more effective information promotion and marketing strategies within the realm of online social networks.

Our study also has its limitations. Although our research focuses on theoretical modeling and numerical simulations, future studies could benefit from empirical research that validates our findings using real-world data. Our work assumes a network structure of random networks, but future directions could involve integrating network structures with social graphs to better understand information competition in online social networks. For example, investigating the impact of network node degrees and analyzing the effects of selecting influential nodes that many users pay attention to as initial nodes on the final propagation could provide valuable insights. Exploring how community dynamics and social ties influence competitive information dissemination would be an interesting avenue for further research. Furthermore, understanding the role of external factors such as user behavior and platform-specific features in information propagation could offer a more comprehensive perspective for businesses and advertisers. In the future, we will assess and consider the influence of user sentiment on the popularity of product information, and incorporate user sentiment into predictive models.

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