Robust Tracking Control of Heterogeneous Robots With Uncertainty: A Super-Exponential Convergence Neurodynamic Approach

Dechao Chen, *Member, IEEE*, Lin Zhuo, Yifan Shao, Shuai Li, *Senior Member, IEEE*, Christian Griffiths, and Ashraf A. Fahmy

***Abstract*— The immediate feedback tracking control system design of heterogeneous robots with uncertainty is considered to be a significant i ssue i n r obotic r esearch. N ote t hat when the robot information is uncertain, the scale of computation would become increasingly large and the accuracy of tracking control would become exceptionally low. The realization of the immediate feedback control system of heterogeneous robots with uncertainty remains to be a challenging problem. Many conventional zeroing neural network (CZNN) models have been developed accordingly. However, most of them are supported by the hypothesis that the robot parameters are complete and accurate, and the associated models possess the exponential convergence property. To handle the robot uncertainty as well as to improve the convergence performance, a new zeroing neural network (ZNN) with super-exponential convergence (SEC) rate is put forward in this paper termed SEC-ZNN, to resolve the robust control issue of uncertain heterogeneous robots. The proposed SEC-ZNN takes full advantage of effector real-time information, with robust controlling and super-exponential convergence performance so far as to the robot information is uncertain. Theoretically, the super-exponential convergence properties including lower error bound and faster convergence rate are rigorously proved. Moreover, circular path-tracking example, comparisons and tests via MATLAB, Coppeliasim and experiment via robot INNFOS substantiate the efficaciousness and preponderance of the SEC-ZNN for the immediate feedback control system for heterogeneous robots with uncertainty.**

***Note to Practitioners*—This paper is motivated by the problem that most robots which need real-time tracking control in real applications come with uncertainty. It is important to note that traditional robot tracking control algorithms mostly require complete robot information or assume information complete, which does not correspond to the actual situation of robot control. Moreover, for practical applications in robotics, the real-time tracking control problem is very attractive. Therefore, an accurate, efficient and stable solution is of great significance to practitioners in this area. In this paper, the SEC-ZNN algorithm is proposed to solve the problem of real-time control of heterogeneous robots with uncertainty in real applications for practitioners. The proposed methos makes full use of the real-time feedback infromation to solve the real-time tracking control problem of heterogeneous robots with uncertainty at the velocity level. The algorithmic steps and principle explanation of the SEC-ZNN scheme are also presented for better understanding. Simulation studies and comparisons are performed on a Stewart robot to confirm the effectiveness and superiority of the proposed scheme. Furthermore, the simulation experiment in Coppeliasim platform is performed to confirm the possibility of portability of the SEC-ZNN to real robot operations. Finally, applications on a real-world robot INNFOS verify the physical relizability of the proposed SEC-ZNN for the engineering practice via heterogeneous robots.**

***Index Terms*— Zeroing neural network, super-exponential convergence, heterogeneous robots, uncertainty, robust tracking control.**

This work was supported in part by the National Natural Science Foundation of China under Grant 62276085 and Grant 61906054 and in part by the Natural Science Foundation of Zhejiang Provincial under Grant LY21F030006. (Corresponding author: Dechao Chen.)

Dechao Chen and Yifan Shao are with the School of Computer Science and Technology, Hangzhou Dianzi University, Hangzhou 310018, China (e-mail: chdchao@hdu.edu.cn; shaoyf001@hdu.edu.cn).

Lin Zhuo is with the HDU-ITMO Joint Institute, Hangzhou Dianzi Univer-sity, Hangzhou 310018, China (e-mail: lynn_zhuolin@163.com).
Shuai Li is with the Faculty of Information Technology and Electrical Engineering, University of Oulu, 90570 Oulu, Finland (e-mail: shuai.li@ oulu.fi).

Christian Griffiths and Ashraf A. Fahmy are both with the Faculty of Science and Engineering, Swansea University, SA1 8EN Swansea, U.K. (e-mail: c.a.griffiths@swansea.ac.uk; and e-mail: A.A.Fahmy@Swansea.ac.uk).

I. Introduction

ADVANCED robots, including heterogeneous robot manipulators, are expected to perform complicated tasks in changing environments in real-time [1], [2], [3]. Since the urgent requirement of the online solutions to advanced robots, immediate feedback control system design of robots is considered to be a fundamental and significant problem in robotic research. Specifically, immediate feedback tracking control system design focuses on the online computation to calculate the control signal and steer the motion of the end-effector equipped along with a user-preset path inside the robot workspace [4]. The real-time control system of heterogeneous robots involves broad robot applications to complete high-burden and precision-requirements operations, such as handling, packing, and sorting of materials [5]. On account of the fundamentality and universality, researchers and engineers have raised various strategies and finished a number of works in this field [4], [5]. For example, Zhang et al. [4] proposed a novel virtual plane tactic to analyze the head motion and generate the solution. The illustrative experiments in [4] showed that the robot could use head motion to effectively track itself and exterior targets in real-time. In addition, Mohammed and Li [5] developed and investigated a dynamic neural network to figure out the kinematic tracking control obstacle of Stewart platforms. The proposed dynamic neural network in [5] considered the issue in the dual space, can effectively disentangle the optimization problem recursively in real-time.

Recently, thanks to the benifits touching on parallelization, multithreaded architecture, adaptive learning and training ability, and feasible development via software and hardware, neural networks, especially adaptive neural networks [6], [7], have served as powerful solutions for solving various automation problems including the immediate feedback tracking control system design of robotic problems [8], [9]. For example, to the solve nonholonomic mobile robots (NMRs) trajectory tracking problem, the model predictive control (MPC) proposed by Li et al. [10] is schemed by amalgamating neural-dynamic optimization. For underactuated wheeled inverted pendulum (WIP) models control problem, by utilizing indirect control trajectory, Yang et al. [11] investigated a new neural network based motion control. Although the studies on neural networks for the immediate feedback tracking control system design of robots are feasible for both research and industry, there still exist some fundamental problems that remain unsolved. Specifically, some existing neural network based works for robot control are supported by the hypothesis that the robot parameters are complete and accurate [4], [5]. However, uncertain robot systems due to unexpected impacts, e.g., external disturbances and parameter perturbations, usually occur during the robot tracking process, and may make the related neural network models suffer from an intensive computational burden and low tracking accuracy [12].

Recurrent neural network (RNN), as a representative neural networks, has been raised and investigated as an effective alternative to a majority of real-time research problems [13]. The first RNN is proposed by Hopfield and Tank [14] to slove quadratic optimization. After such a seminal effort, numerous RNNs have been came up with. Notably, zeroing neural network [or termed Zhang neural network (ZNN)], as a novel RNN, can handle multidimension problems [15], [16], [17]. Specifically, this kind of RNN zeroizes factors in the error function one by one using a neural-dynamic method [18]. So that it is considered a systematic and scientific strategy to solve diverse time-varying problems (especially time-varying tracking control problems) [19]. For example, Benchabane et al. [20] used the discrete Zhang neural network (DZNN) to derive out an application of an online algorithm to inverse the covariance matrix. To make the ZNN model converges in finite time, Xiao [21] designed a novel formula to make progress on the research of convergence performance of the ZNN. Based on discre the ZNN, Guo et al. [22] recently proposed a discrete-time ZNN (DTZNN) model to inverse the time-varying matrix and applicated it to the kinematic control of a two-link planar robot manipulator. Moreover, as a seminal research in the robustness research of ZNN, researchers proposed a new integration-enhanced zeroing neural network (IEZNN) model in [23] to inverse time-varying matrix under diffirent kinds of noises of the neural network model.

Althouth researchers have made great process on the real-time control problems by using ZNNs, existing works based on ZNN based are designed and developed on the basis of the full and accurate system information of the involved robots [21], [22]. The system uncertainty remains to be a difficult and strenuous subject in the field of using the ZNN to control the robots. Because the system with uncertainty is universal, this kind of robust ZNN (RZNN) which can control the uncertain heterogeneous robots in real-time is in urgently need. At the same time, when the robot uncertainty issue is taken into account in complex applications, the scale of computation would become increasingly large and the accuracy of tracking control would become exceptionally low. Also, considering the shortcomings of traditional control algorithms, such as PID algorithms, although they have a simple structure and can get good control results, but they rely heavily on the actual engineering experience of engineers. For engineers without relevant engineering experience, the selection of parameters would become a big problem (see Remarks). To solve the problems discussed above, in this paper, we make a more in-depth study on the real-time tracking control problem of uncertain heterogeneous robot along the direction of ZNN research. A new neural network model is designed with the following characteristics: 1) handling the robot uncertainty to improve the system robustness; 2) converging with super-exponential rate; 3) tracking with a high accuracy. For better illustration, Table I lists a comprehensive comparison between the proposed neural network model and existing strategies [24], [25], [26]. Combined with the results observed in the table and our knowledge, it can be concluded that there is no control algorithm for the real-time tracking of uncertain robots with the excellent characteristics as the proposed one.

The rest of this paper is described in the following five chapters. The whole problem about tracking control heterogeneous robots in real-time with immediate feedback is described via Section II. Section III introduces the process of designing the SEC-ZNN model design, proposal, discretization and theoretical analysis. To further verify the proposed model, the simulation studies including tests and comparisons are carried out in Section IV via MATLAB. Section V introduces the result of the simulation test via Coopeliasiom by controlling a real Stewart platform using the SEC-ZNN. To further demonstrate the effectiveness of the proposed algorithm, Section VI shows the experiment based on a real serial robot INNFOS with 6 degree-of-freedom (6-DOF). In Section VII, the research content of this paper is summarized. This paper makes contributions in the following points:

- This is the first work that simultaneously improves both the robustness and convergence performance of the ZNN model to control heterogeneous robots in real-time with immediate feedback from the case with full and accurate robot information to the case with uncertain information. The proposed SEC-ZNN model in this paper fully utilized the feedback information of effector showing robust tracking and super-exponential convergence performance simultaneously.
- Theoretical analysis prove the super-exponential convergence properties including lower error bound and faster

TABLE I

ANALYSIS OF VARIOUS NEURAL NETWORK MODELS ABOUT CONTROL RESULTS OF ROBOTS

Model	Convergence property	Variable restriction◇	Error bound	Immediate feedback	Robot involved	Error
[24]	Exponential	Yes	Big	Yes	Planar robot manipulator	Ignorable
[25]	Asymptotic	Yes	Big	Yes	Planar robot manipulator	Ignorable
[26]	Asymptotic	Yes	Big	Yes	PUMA 560 robot manipulator	Ignorable
[27]	Exponential	No	Big	Yes	PUMA 560 robot manipulator	Ignorable
[28]	Asymptotic	Yes	Big	Yes	Planar robot manipulator	Ignorable
[29]	Asymptotic	Yes	Big	Yes	PUMA 560 robot manipulator	Ignorable
[30]	Asymptotic	Yes	Big	No	Humanoid robot	Inignorable
[31]	Asymptotic	No	Big	Yes	Delta robot manipulator	Inignorable
[32]	Asymptotic	Yes	Big	Yes	3PRS robot	Ignorable
This paper	**Super-exponential**	**No**	**Small**	**Yes**	**Serial and parallel robots**	**Ignorable**

Note: ◇The variable restriction means that the parameter details of robot should be provided in advance.

convergence rate of the proposed SEC-ZNN model when tracking control the heterogeneous robots in real-time with the absence of certain robot information. Meanwhile, a Lyapunov function is designed to prove the stability of the SEC-ZNN system.

- Circular path-tracking example, comparisons, and extensive tests via both MATLAB and Coppeliasim, in addition to the applications on serial robot INNFOS sufficiently illuminate the efficaciousness and preponderance of the proposed SEC-ZNN for the immediate feedback control system for heterogeneous robots with uncertainty.

II. PRELIMINARIES AND PROBLEM DESCRIPTIONS

In this section, we firstly introduce the kinematic modeling and equation about the parallel robot manipulator as the research preliminaries. Then, the problem about real-time tracking control together with the convergence issue for the uncertain parallel robots investigated is described.

A. Preliminaries

Due to the special pyhsical properties, parallel robot manipulators always have the characteristics of high stiffness and high load, which makes them are widely applied in many engineering applications. Stewart robot is a typical six degree of freedom parallel robot, which is driven by six independent motion joints set in parallel connected the mobile platform and the fixed platform. The motion of the mobile platform in any degree of freedom will cause different motions of six motion joints. Besides, on the top and center of the mobile platform, there is an effector set to complete the task. That means that we can make the end-effector fixed on the mobile platform move along the specific trajectory by controlling the changes of each moving joint respectively. In this paper, we consider the tracking control problem of the end-effector in three dimension about the position information to complete the specific task without generality. For more details about the mechanical structure of Stewart platform, the model of such kind of parallel robot manipulator can be referred to [5]. In addition, the corresponding specific meanings of the symbols appearing in this paper are as follows:

$\mathbf{f}_i$: coordinates of the connection point of the ith motion leg on the fixed platform in the global coordinate {O}.

$\mathbf{m}_i$: coordinates of the connection point of the ith motion leg on the mobile platform in the platform coordinate {O_2}.

$\mathbf{p}$: coordinates of the origin of the platform coordinate in the global coordinate, that is, the translational transformation.

Q: rotation matrix of the initial position of the platform coordinate defined by the Euler angle relative to the global coordinate.

l_i: length of the ith motion leg of the parallel robot manipulator with $\mathbf{l} = [l_1, l_2, \cdots, l_6]^{\mathrm{T}}$.

$\dot{l}_i$; velocity of the ith motion leg of the parallel robot manipulator with $\dot{\mathbf{l}} = [\dot{l}_1, \dot{l}_2, \cdots, \dot{l}_6]^{\mathrm{T}}$.

$\mathbf{r}_{\mathrm{a}}$: actual coordinates of the effector on the mobile platform.

$\dot{\mathbf{r}}_{\mathrm{a}}$: actual velocity of the effector on the mobile platform.

The kinematic equation of the Stewart platform at velocity level can be depicted in the following compact matrix form on the basis of the robot kinematic modeling [5]:

$$\dot{\mathbf{l}} = C(\mathbf{l}, \mathbf{d})\dot{\mathbf{r}}_{\mathrm{a}}, \tag{1}$$

where $C(\mathbf{l}, \mathbf{d}) \in \mathbb{R}^{n\times m}$ (with $n = 6$ for the 6 legs parallel robot manipulator and $m = 3$ for the 3D position tracking) represents the coefficient matrix depended on the position infromation of the effector and the length of legs that are relative to the robot system. It can be defined as

$$C(\mathbf{l}, \mathbf{d}) = \begin{bmatrix} \frac{1}{l_1}\mathbf{d}_1 & \frac{1}{l_2}\mathbf{d}_2 & \frac{1}{l_3}\mathbf{d}_3 & \frac{1}{l_4}\mathbf{d}_4 & \frac{1}{l_5}\mathbf{d}_5 & \frac{1}{l_6}\mathbf{d}_6 \end{bmatrix}^{\mathrm{T}}$$

with $\mathbf{d}_i \in \mathbb{R}^m$ denoting the ith leg parameter vector which is expressed as

$$\mathbf{d}_i = \mathbf{p} + Q\mathbf{m}_i - \mathbf{f}_i,$$

where $i = 1, 2, \cdots, 6$ refers to the index of the leg.

Considering a specified task for the effector fixed on the parallel robot manipulator to complete [that is, a preset path $\mathbf{r}_{\mathrm{d}}(t) \in \mathbb{R}^m$ for the effector fixed on the parallel robot manipulator to track] in real time t, we have

$$\Upsilon(\mathbf{l}(t), \mathbf{d}(t), t) = \mathbf{r}_{\mathrm{a}}(t) \to \mathbf{r}_{\mathrm{d}}(t), \tag{2}$$

where $\mathbf{r}_{\mathrm{d}}(t)$ is known, bounded, and piecewise continuous and $\Upsilon(\cdot,\cdot,\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ describes a real-time continuous nonlinear forward-kinematics mapping according to the parallel robot system with complete physical structure information of the model. By analyzing (2) with taking time t into account, we have

$$P(\mathbf{l}(t), \mathbf{d}(t), t)\dot{\mathbf{l}}(t) = \dot{\mathbf{r}}_{\mathrm{a}}(t) \to \dot{\mathbf{r}}_{\mathrm{d}}(t), \tag{3}$$

where matrix $P(\mathbf{l}(t), \mathbf{d}(t), t) \in \mathbb{R}^{m\times n}$ equals $\partial\Upsilon/\partial\mathbf{l}$, which is exactly the pseudo-inverse of parallel robot coefficient matrix $C(\mathbf{l}(t), \mathbf{d}(t), t)$, and $\dot{\mathbf{r}}_{\mathrm{d}}(t) \in \mathbb{R}^m$ is the derivative of time t of the preset path $\mathbf{r}_{\mathrm{d}}(t)$.

B. Problem Descriptions

The general real-time tracking control issue of a parallel robot is defined as given the preset path $\mathbf{r}_{\mathrm{d}}(t)$ and velocity $\dot{\mathbf{r}}_{\mathrm{d}}(t)$ of the parallel robot's effector, we should calculate the correct leg length $\mathbf{l}(t)$ and appropriate leg velocity $\dot{\mathbf{l}}(t)$ in real-time t. The following formula express the robot kinematics equation (1) at the speed level in real-time:

$$\dot{\mathbf{l}}(t) = C(\mathbf{l}(t), \mathbf{d}(t), t)\dot{\mathbf{r}}_{\mathrm{a}}(t). \tag{4}$$

By observing and analyzing the real-time kinematics equation (4), we can draw the conclusion that before the process of tracking control, we need to know the precise physical parameters (e.g., the vector of leg length $\mathbf{l}$) and the robot infromation (e.g., the position vector of leg-platform connection points $\mathbf{f}$ and $\mathbf{m}$) for the calculation of coefficient matrix $C(\mathbf{l}(t), \mathbf{d}(t), t)$. The traditional strategy of real-time tracking control issue mostly depends on the complete prior information of robot system to calculate the coefficient matrix $C(\mathbf{l}(t), \mathbf{d}(t), t)$. However, such complete prior information of robot system may not be accessibly and accurately attainable in real time with various unexpected influences. For better understanding, the concept of the immediate feedback (real-time tracking) control of uncertain parallel manipulators is presented as follows.

The uncertain robot systems due to unexpected impacts, e.g., external disturbances and parameter perturbations, usually appear for the duration of tracking the robot motion. In practical application, with the passage of time, the parallel manipulator will appear fatigue and wear influenced by some factors, which makes the difference between the information of the real manipulator and the marked information. Such cases can be all deemed as the robot with uncertain information. In this work, the complete robot information is denoted by $C(\mathbf{l}(t), \mathbf{d}(t), t)$, where $\mathbf{l}(t)$ represents the length of the legs and $\mathbf{d}(t)$ denotes the leg parameter vector. Due to the presence of uncertainty metioned above within the robot, the information $\mathbf{l}(t)$ and $\mathbf{d}(t)$ related to the outriggers is not accurate. Therefore, the robot parameters with uncertainty are noted as $\tilde{C}(\tilde{l}(t), \tilde{d}(t), t)$, where $\tilde{l}(t)$ and $\tilde{d}(t)$ are approximate outrigger information. That is, in this work, we use an inaccurate coefficient matrix $\tilde{C}(\mathbf{l}(t), \mathbf{d}(t), t)$ with assumptions and disturbances are considered in the real-time kinematics resolution for (4) of robot system to simulate the problem of parallel manipulator with uncertainty, which is termed the immediate feedback control of uncertain heterogeneous robot manipulators.

The computational scale would become intensively large when the robot uncertainty issue needs to be handled in complex applications. This means that it will cost additional time to obtain desirable result with sufficient high accuracy or not at all when the computational scale becomes larger. This may lead to troubles in time-critical (or to say, real-time) applications in practice. Therefore, higher convergence speed together with ignorable error bound for real-time control of uncertain robot in complex environment is needed urgently. To achieve such a request, unlike the existing CZNN with exponential convergence rate, a novel type of RNN model with super-exponential convergence rate, termed SEC-ZNN model is designed in this paper. Such a time-efficient convergence property will make the proposed SEC-ZNN model much more applicable and wide spread. For better understanding, the definition of the super-exponential convergence [33] is presented as follows.

Definition 1: Suppose that a given vector-valued function $\mathbf{f}(t)$ converges to zero with respect to time t starting from a random initial state. We say that this function converges to zero super-exponentially with respective to time t if it satisfies

$$\|\mathbf{f}(t)\|_{\mathrm{E}} \leq \alpha \|\mathbf{f}(0)\|_{\mathrm{E}} \exp(-\beta \exp(t)), \ \forall t > 0,$$

where constants α and β exist and $\beta \exp(t)/t$ is termed as the super-exponential convergence rate of function $\|\mathbf{f}(t)\|_{\mathrm{E}}$.

That means with the same coefficients, the super-exponential convergent neural network has a better convergence performance compared to the exponential convergent neural network. Note that solving a tracking control problem directly relying on the uncertain knowledge of a robot system would be time-consuming and of low accuracy. Even severely, because of the absence of prior knowledge of the robot system in some situations, the results of tracking control may be influenced deeply by the uncertain robot systems. Therefore, it is needed for an effective solution to work without knowing the exact physical parameters. At the same time, the requirements of faster convergence speed and lower error bound occur during the immediate feedback tracking control of uncertain parallel robots. These facts motivate us to investigate the immediate feedback tracking control of uncertain parallel robot manipulators by exploiting a novel SEC-ZNN in this paper.

III. Model Design and Theoretical Analyses

In this section, we firstly present the design process of the SEC-ZNN model. Moreover, theoretical analyses rigorously prove convergence properties including lower error bound and faster convergence rate of such an SEC-ZNN compared with CZNN for the immediate feedback tracking control of parallel robots with uncertainty.

A. SEC-ZNN Model

Firstly, along with the neural dynamic design framework [34], [35], the vector-valued error function (for the measurement of the gap between the time-varying preset path and the time-varying actual trajectory of the end-effector) is described by following formula:

$$\mathbf{e}(t) = \mathbf{r}_{\mathrm{d}}(t) - \mathbf{r}_{\mathrm{a}}(t). \tag{5}$$

Traditionally, utilize the zeroing neurodynamic design formula [34], [35] to zeroize each factor $e_i(t)$ (with $i = 1, 2, \cdots, m$) in the vector-valued error function (5):

$$\dot{\mathbf{e}}(t) = -\gamma \Psi(\mathbf{e}(t)), \tag{6}$$

where $\gamma \in \mathbb{R}^+$ is a parameter designed for scaling the convergence rate of the tracking process, and operator $\Psi(\cdot)$ [13] is the activation-function for array mapping, where each element is a monotonically increasing odd function. Note that design parameter γ is a fixed parameter and thus the neural network model designed by the above formula (6) would have the exponential convergence rate during the problem solving. Inspired by (6), to obtain a superior convergence property, i.e., the super-exponential convergence rate, during the tracking process, the following novel design formula can be applied:

$$\dot{\mathbf{e}}(t) = -\gamma \exp(t)\Psi(\mathbf{e}(t)). \tag{7}$$

To achieve the super-exponential convergence rate during the tracking control process, we thus employ (7), which yields

$$\dot{\mathbf{r}}_{\mathrm{d}}(t) - \dot{\mathbf{r}}_{\mathrm{a}}(t) = -\gamma \exp(t)\Psi(\mathbf{r}_{\mathrm{d}}(t) - \mathbf{r}_{\mathrm{a}}(t)). \tag{8}$$

The specific form control equation of the SEC-ZNN model can be obtained as following by replacing real-time kinematics equation (4) with above dynamical equation (8):

$$\dot{\mathbf{l}}(t) = C(\mathbf{l}(t), \mathbf{d}(t), t)\big(\dot{\mathbf{r}}_{\mathrm{d}}(t) + \gamma \exp(t)\Psi(\mathbf{r}_{\mathrm{d}}(t) - \mathbf{r}_{\mathrm{a}}(t))\big). \tag{9}$$

Equation (9) is for the immediate feedback tracking control of uncertain parallel robots of the SEC-ZNN model with super-exponential convergence property. It can be observed from equation (9) that the real-time information $\mathbf{r}_{\mathrm{a}}(t)$ feedback from the primary task execution collected by the sensors fixed on the effector makes the control system a closed-loop.

According to formula (9), in order to accurately calculate the real-time coefficient matrix $C(\mathbf{l}(t), \mathbf{d}(t), t)$, we need to know the complete physical parameters and model details of the robot system in advance. However, due to the existence of external disturbances, we can only obtain the approximate matrix $\tilde{C}(\mathbf{l}(t), \mathbf{d}(t), t)$ with the exact coefficient matrix $C(\mathbf{l}(t), \mathbf{d}(t), t)$ at time t. However, in the case of incomplete system information (e.g. inaccurate leg length information due to wear and tear, measurement errors, etc.), modeling different robots with traditional methods will consume additional time. Even in some practical application scenarios, this motion control method using uncertain system information directly may cause serious damage to the tracking control process. Taking the above facts into account, we proposed a novel strategy, i.e. the zeroing neurodynamic approach, to adapt the coefficient matrix in real-time t in this paper to solve the robot uncertainty problem to improve the control system robostness. Thus, equation (9) without knowing the system information is written as

$$\dot{\hat{\mathbf{l}}}(t) = \hat{C}(t)\big(\dot{\mathbf{r}}_{\mathrm{d}}(t) + \gamma \exp(t)\Psi(\mathbf{r}_{\mathrm{d}}(t) - \mathbf{r}_{\mathrm{a}}(t))\big), \tag{10}$$

where $\dot{\hat{\mathbf{l}}}(t)$ is the approximate velocity vector of the leg and $\hat{C}(t) \in \mathbb{R}^{6\times m}$ is the approximate coefficient matrix which is set to adapt the SEC-ZNN model as follows.

A vector-valued error function is given as

$$\varepsilon(t) = \dot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t) \tag{11}$$

with vector $\varepsilon(t) \in \mathbb{R}^m$. After that, for simplicity, we can applying the following zeroing neurodynamic design formula [34], [35] for the real-time adaption of approximate matrix $\hat{C}(t)$:

$$\dot{\varepsilon}(t) = -\nu\Psi(\varepsilon(t)), \tag{12}$$

where $\nu \in \mathbb{R}^+$ is a design parameter to scale the convergence rate of the adaption process, we have the dynamical equation

Algorithm 1 SEC-ZNN Model Design

Initialize: The initial leg length $\hat{\mathbf{l}}(0)$ and initial coefficient matrix $\hat{C}(0)$ of the uncertain parallel robot;
Set: The motion task duration T_{d}, the design parameters γ and ν;
while $t \leq T_{\mathrm{d}}$ **do**
  Input: The preset $\mathbf{r}_{\mathrm{d}}(t)$ and the velocity $\dot{\mathbf{r}}_{\mathrm{d}}(t)$;
  Read: The real-time actual coordinate, velocity of the effector $\mathbf{r}_{\mathrm{a}}(t)$, $\dot{\mathbf{r}}_{\mathrm{a}}(t)$;
  Calculate: The real-time actual acceleration via $\ddot{\mathbf{r}}_{\mathrm{a}}(t) = (\dot{\mathbf{r}}_{\mathrm{a}}(t) - \dot{\mathbf{r}}_{\mathrm{a}}(t-\Delta t))/\Delta t$ and the real-time actual leg acceleration vector via $\ddot{\hat{\mathbf{l}}}(t) = (\dot{\hat{\mathbf{l}}}(t) - \dot{\mathbf{l}}(t-\Delta t))/\Delta t$ with Δt being sufficiently
  Calculate: The real-time control signal via dynamical equation $\dot{\hat{\mathbf{l}}}(t) = \hat{C}(t)(\dot{\mathbf{r}}_{\mathrm{d}}(t) + \gamma \exp(t)\Psi(\mathbf{r}_{\mathrm{d}}(t) - \mathbf{r}_{\mathrm{a}}(t)))$;
  Calculate: The time derivative of real-time approximate matrix via $\dot{\hat{C}}(t) = \big(\ddot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\ddot{\mathbf{r}}_{\mathrm{a}}(t) + \nu(\dot{\mathbf{l}}(t) - \hat{C}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t))\big)\dot{\mathbf{r}}_{\mathrm{a}}^{\dagger}(t)$
  Update: The real-time coefficient matrix in the next moment via $\hat{C}(t+\Delta t) = \hat{C}(t) + \Delta t\dot{\hat{C}}(t)$ small;
  Update: The real-time leg length in the next moment via $\hat{\mathbf{l}}(t+\Delta t) = \hat{\mathbf{l}}(t) + \Delta t\dot{\hat{\mathbf{l}}}(t)$;
  Output: The robot leg length $\hat{\mathbf{l}}(t+\Delta t)$ in next moment;
end while

as follow for the adaption of the approximate matrix of the robot system with uncertainty in time t:

$$\ddot{\hat{\mathbf{l}}}(t) - \dot{\hat{C}}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t) - \hat{C}(t)\ddot{\mathbf{r}}_{\mathrm{a}}(t) = -\nu\Psi\big(\dot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t)\big). \tag{13}$$

Dynamical equation (13) can be clearly reformed as

$$\dot{\hat{C}}(t) = \big(\ddot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\ddot{\mathbf{r}}_{\mathrm{a}}(t) + \nu\Psi\big(\dot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t)\big)\big)\dot{\mathbf{r}}_{\mathrm{a}}^{\dagger}(t), \tag{14}$$

where superscript † represents the pseudo-inverse of a vector or matrix. Equation (14) describes the real-time adaption of coefficient matrix $\hat{C}(t)$ of the SEC-ZNN model. Consequently, the whole SEC-ZNN model is accomplished as follows:

$$\begin{cases} \dot{\hat{\mathbf{l}}}(t) = \hat{C}(t)\big(\dot{\mathbf{r}}_{\mathrm{d}}(t) + \gamma \exp(t)\Psi(\mathbf{r}_{\mathrm{d}}(t) - \mathbf{r}_{\mathrm{a}}(t))\big), \\ \dot{\hat{C}}(t) = \big(\ddot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\ddot{\mathbf{r}}_{\mathrm{a}}(t) + \nu\Psi\big(\dot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t)\big)\big)\dot{\mathbf{r}}_{\mathrm{a}}^{\dagger}(t). \end{cases} \tag{15}$$

where $\dot{\hat{\mathbf{l}}}(t)$ is the control signal of the SEC-ZNN model. For better comparison and investigation the convergence properties of SEC-ZNN model (15), the whole CZNN model [15], [16], [17] reads as follows:

$$\begin{cases} \dot{\hat{\mathbf{l}}}(t) = \hat{C}(t)\big(\dot{\mathbf{r}}_{\mathrm{d}}(t) + \gamma\Psi(\mathbf{r}_{\mathrm{d}}(t) - \mathbf{r}_{\mathrm{a}}(t))\big), \\ \dot{\hat{C}}(t) = \big(\ddot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\ddot{\mathbf{r}}_{\mathrm{a}}(t) + \nu\Psi\big(\dot{\hat{\mathbf{l}}}(t) - \hat{C}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t)\big)\big)\dot{\mathbf{r}}_{\mathrm{a}}^{\dagger}(t). \end{cases} \tag{16}$$

For clearer apprehension, the correlative algorithm description about the SEC-ZNN model is presented in Algorithm 1. It is deserved to be mentioned here that the immediate feedback control problem solving with super-exponential convergence rate can be achieve by utilizing the formula (7) for the control equation (9). For purpose of simplicity, we can still apply the CZNN design formula to adapt the approximate matrix (14) in real-time of the SEC-ZNN model.

B. Theoretical Analyses

To verify the convergence performance of the proposed SEC-ZNN model (15) for the immediate feedback tracking control of uncertain parallel robot manipulators, theoretical analyses are presented accordingly.

Definition 2 (Convergence of Dynamic Neural Network): For handling a immediate feedback tracking control problem (4) of a parallel robot, if the effector trajectory $\mathbf{r}_{\mathrm{a}}(t)$ in Cartesian space synthesized by a neural network model starting from any initial position $\mathbf{r}_{\mathrm{a}}(0)$ satisfies

$$\lim_{t\to\infty}(\mathbf{r}_{\mathrm{d}}(t)-\mathbf{r}_{\mathrm{a}}(t))=0,$$

it could said to be globally convergent to the desired path $\mathbf{r}_{\mathrm{d}}(t)$

Theorem 1 (Global Convergence of SEC-ZNN): Consider the real-time tracking control problem (4) of an uncertain parallel robot manipulator. If a linear activation-function processing-array $\Psi(\cdot)$ is utilized, starting from any initial position $\mathbf{r}_{\mathrm{a}}(0)$, the trajectory $\mathbf{r}_{\mathrm{a}}(t)$ of the end-effector of closed-loop SEC-ZNN model (15) globally converges to the preset path $\mathbf{r}_{\mathrm{d}}(t)$ in the sense of Lyapunov.

Proof: To solve the immediate feedback tracking control problem (4) of an uncertain parallel robot, the dynamical equations of the closed-loop SEC-ZNN model (15) can be expanded into the following two equations:

$$\dot{\mathbf{e}}(t)=-\gamma\exp(t)\Psi(\mathbf{e}(t)),\tag{17}$$

$$\dot{\varepsilon}(t)=-\nu\Psi(\varepsilon(t)),\tag{18}$$

where $\mathbf{e}(t)=\mathbf{r}_{\mathrm{d}}(t)-\mathbf{r}_{\mathrm{a}}(t)$ and $\varepsilon(t)=\dot{\mathbf{l}}(t)-\hat{C}(t)\dot{\mathbf{r}}_{\mathrm{a}}(t)$. Considering the linear activation-function $\Psi(\cdot)$ into account (that is $\Psi(\mathbf{e}(t))=k\mathbf{e}(t)$ and in this paper k is set to be 1), equations (17) and (18) can be updated as

$$\dot{\mathbf{e}}(t)=-\gamma\exp(t)\mathbf{e}(t),$$
$$\dot{\varepsilon}(t)=-\nu\varepsilon(t).$$

A Lyapunov function candidate is defined by the following equation:

$$L(t)=\frac{\|\mathbf{e}(t)\|_{\mathrm{E}}^2}{2}+\frac{\|\varepsilon(t)\|_{\mathrm{E}}^2}{2}=\frac{\mathbf{e}^{\mathrm{T}}(t)\mathbf{e}(t)}{2}+\frac{\varepsilon^{\mathrm{T}}(t)\varepsilon(t)}{2}.$$

We can draw the conclusion that $L(t)$ is positive-definite in view of $L(t)>0$ for $\mathbf{e}(t)\neq 0$ or $\varepsilon(t)\neq 0$, and $L(t)=0$ for both $\mathbf{e}(t)=0$ and $\varepsilon(t)=0$ only. Again, we can get the time-derivative of $L(t)$ as follows:

$$\begin{aligned}\dot{L}(t)&=\frac{\mathrm{d}L(t)}{\mathrm{d}t}=\mathbf{e}^{\mathrm{T}}(t)\frac{\mathrm{d}\mathbf{e}(t)}{\mathrm{d}t}+\varepsilon^{\mathrm{T}}(t)\frac{\mathrm{d}\varepsilon(t)}{\mathrm{d}t}\\&=-\gamma\exp(t)\mathbf{e}^{\mathrm{T}}(t)\mathbf{e}(t)-\nu\varepsilon^{\mathrm{T}}(t)\varepsilon(t).\end{aligned}$$

Therefore, we judge the conclusion that $\dot{L}(t)$ is negative-definite for time $t\in[0,+\infty)$ with design parameters $\gamma>0$ and $\nu>0$. Based on the Lyapunov theory [36], according to Definition 2, the trajectory $\mathbf{r}_{\mathrm{a}}(t)$ of the end-effector of the closed-loop SEC-ZNN model (15) globally converges to the desired path $\mathbf{r}_{\mathrm{d}}(t)$, i.e., $\mathbf{r}_{\mathrm{d}}(t)-\mathbf{r}_{\mathrm{a}}(t)$ globally converges to 0. The above is the whole content of proof. □

Theorem 2 (Convergence Bound of the SEC-ZNN): Consider the immediate feedback control problem (4) of an uncertain parallel robot manipulator. Supposed that under the condition of using a linear activation-function processing-array $\Psi(\cdot)$ and starting from any initial states $\mathbf{e}(t)$ at time instance $t=0$, the upper bound of residual error $\|\mathbf{e}_{\text{SEC-ZNN}}(t)\|_{\mathrm{E}}$ of the SEC-ZNN model (15) is lower than residual error $\|\mathbf{e}_{\text{CZNN}}(t)\|_{\mathrm{E}}$ of CZNN model (16) at the same instance $t^*\in(0,+\infty)$, i.e., $\|\mathbf{e}_{\text{SEC-ZNN}}(t^*)\|_{\mathrm{E}}<\|\mathbf{e}_{\text{CZNN}}(t^*)\|_{\mathrm{E}}$, with the same parameters.

Proof: According to residual errors of SEC-ZNN model (15) and CZNN model (16), define two function candidates to measure the residual errors respectively as follows:

$$\mathcal{L}_{\text{SEC-ZNN}}(t)=\frac{\|\mathbf{e}_{\text{SEC-ZNN}}(t)\|_{\mathrm{E}}^2}{2},$$
$$\mathcal{L}_{\text{CZNN}}(t)=\frac{\|\mathbf{e}_{\text{CZNN}}(t)\|_{\mathrm{E}}^2}{2}.$$

Noticed that both $\mathcal{L}_{\text{SEC-ZNN}}(t)$ and $\mathcal{L}_{\text{CZNN}}(t)$ are positive-definite because of $\mathcal{L}_{\text{SEC-ZNN}}(t)>0$ and $\mathcal{L}_{\text{CZNN}}(t)>0$ for $\mathbf{e}_{\text{SEC-ZNN}}(t)\neq 0$ and $\mathbf{e}_{\text{CZNN}}(t)\neq 0$, and $\mathcal{L}_{\text{SEC-ZNN}}(t)=0$ and $\mathcal{L}_{\text{CZNN}}(t)=0$ for both $\mathbf{e}_{\text{SEC-ZNN}}(t)=0$ and $\mathbf{e}_{\text{CZNN}}(t)=0$ only. Again, we can get the time-derivatives of $\mathcal{L}_{\text{SEC-ZNN}}(t)$ and $\mathcal{L}_{\text{CZNN}}(t)$ respectively as follows:

$$\dot{\mathcal{L}}_{\text{SEC-ZNN}}(t)=-\gamma\exp(t)\mathbf{e}_{\text{SEC-ZNN}}^{\mathrm{T}}(t)\mathbf{e}_{\text{SEC-ZNN}}(t),\tag{19}$$

$$\dot{\mathcal{L}}_{\text{CZNN}}(t)=-\gamma\mathbf{e}_{\text{CZNN}}^{\mathrm{T}}(t)\mathbf{e}_{\text{CZNN}}(t).\tag{20}$$

Compared (19) with (20), with the same parameter γ and the same initial states $\mathbf{e}_{\text{SEC-ZNN}}(t_{\text{ini}})=\mathbf{e}_{\text{CZNN}}(t_{\text{ini}})$ at time $t_{\text{ini}}\in[0,+\infty)$, we have

$$\dot{\mathcal{L}}_{\text{SEC-ZNN}}(t_{\text{ini}})<\dot{\mathcal{L}}_{\text{CZNN}}(t_{\text{ini}}).\tag{21}$$

For the next moment, i.e., $t^*=t_{\text{ini}}+\Delta t$ with $\Delta t\to 0$, for $t^*\in(0,+\infty)$, we can obtain

$$\mathcal{L}_{\text{SEC-ZNN}}(t_{\text{ini}}+\Delta t)=\mathcal{L}_{\text{SEC-ZNN}}(t_{\text{ini}})+\Delta t\dot{\mathcal{L}}_{\text{SEC-ZNN}}(t_{\text{ini}}),\tag{22}$$

and

$$\mathcal{L}_{\text{CZNN}}(t_{\text{ini}}+\Delta t)=\mathcal{L}_{\text{CZNN}}(t_{\text{ini}})+\Delta t\dot{\mathcal{L}}_{\text{CZNN}}(t_{\text{ini}}).\tag{23}$$

In view of (21), with the same initial states, i.e., $\mathbf{e}_{\text{SEC-ZNN}}(t_{\text{ini}})=\mathbf{e}_{\text{CZNN}}(t_{\text{ini}})\neq 0$, there is

$$\mathcal{L}_{\text{SEC-ZNN}}(t_{\text{ini}}+\Delta t)<\mathcal{L}_{\text{CZNN}}(t_{\text{ini}}+\Delta t).$$

Therefore, we have the conclusion that

$$\|\mathbf{e}_{\text{SEC-ZNN}}(t_{\text{ini}}+\Delta t)\|_{\mathrm{E}}<\|\mathbf{e}_{\text{SEC-ZNN}}(t_{\text{ini}}+\Delta t)\|_{\mathrm{E}},\tag{24}$$

where $t^*=t_{\text{ini}}+\Delta t\in(0,+\infty)$. Equation (24) indicates that the upper bound of residual error of the SEC-ZNN model (15) is lower than that of CZNN model (16) at instance $t^*\in(0,+\infty)$ with the same parameters. The above is the whole content of the proof. □

Theorem 3 (Convergence Rate of SEC-ZNN): Consider the immediate feedback tracking control problem (4) of an uncertain parallel robot. Supposed that under the condition of using a linear activation-function processing-array $\Psi(\cdot)$ and starting from any initial position $\mathbf{r}_{\mathrm{a}}(0)$, the trajectory $\mathbf{r}_{\mathrm{a}}(t)$ of the end-effector of the SEC-ZNN model (15) converges to the preset path $\mathbf{r}_{\mathrm{d}}(t)$ with super-exponential convergence rate.

Proof: Supposed using a linear activation-function processing-array $\Psi(\cdot)$, the ith dynamical subsystem relative to error function $\mathbf{e}(t)$ in SEC-ZNN model (15) can be presented as

$$\dot{\mathbf{e}}_i(t) = -\gamma \exp(t)\mathbf{e}_i(t), \text{ with } i = 1, 2, \cdots m. \tag{25}$$

According to the differential equation theory [37], the solution of (25) is

$$e_i(t) = \frac{e_i(0)}{\exp(-\gamma)} \exp(-\gamma \exp(t)). \tag{26}$$

The vectorization of equation (26) is expressed as follow

$$\mathbf{e}(t) = \frac{\mathbf{e}(0)}{\exp(-\gamma)} \exp(-\gamma \exp(t)). \tag{27}$$

Consequently, we have the residue error as

$$\|\mathbf{e}(t)\|_{\mathrm{E}} = \sqrt{\sum_{i=1}^{m} \frac{e_i^2(0)}{\exp(-2\gamma)}} \exp(-\gamma \exp(t)). \tag{28}$$

Equation (28) shows that the residue error of SEC-ZNN model (15) globally converges to 0 with a super-exponential convergence rate as $\gamma \exp(t)/t$. Thus, the trajectory $\mathbf{r}_{\mathrm{a}}(t)$ of the end-effector of the SEC-ZNN model (15) converges to the preset path $\mathbf{r}_{\mathrm{d}}(t)$ super-exponential convergence rate $\gamma \exp(t)/t$. The above is the whole content of the proof. □

IV. Simulations, Comparisons and Tests

In this section, we conduct simulations embed in the 3D position tracking control of the Stewart platform, a popular parallel manipulators, in real time t. Thus, in this paper, the Stewart platform is used in the form of redundant manipulator. Note that more details of establishment of kinematics model of Stewart platform can be referred to [5]. Simulations studies on the real-time tracking control problem via circular path-tracking example is conducted to show the effectiveness of the proposed SEC-ZNN model (15). Then, comparisons with two conventional RNN models, i.e., the CZNN model as well as the gradient neural network (GNN) model, are presented to illustrate that the proposed SEC-ZNN model with super-exponential convergence rate is superior for the immediate feedback control problem solving under the condition of complex environments and uncertainty of robot. Moreover, to further study the convergence property of the proposed SEC-ZNN model, we carry out several extensive tests. Without losing generality, we set the duration of the task as $T_{\mathrm{d}} = 4$ s. In addition, when initialize the model setting, the length vector and the velocity vector of the leg are set as $\mathbf{l}(t) = [1.184, 1.184, 1.184, 1.184, 1.184, 1.184]^{\mathrm{T}}$ m and $\dot{\mathbf{l}}(0) = [0, 0, 0, 0, 0, 0]^{\mathrm{T}}$ m/s respectively. According to engineering experience, the initial approximate coefficient matrix for the real-time adaption is set as $\hat{C}(0) = [0.307, -0.438, 0.844; 0.307, 0.438, 0.844; 0.226, 0.485, 0.844; -0.533, 0.046, 0.844; -0.533, -0.046, 0.844; 0.226, -0.485, 0.844]$. In the tracking examples, the design parameters for the control equation and the coefficient matrix adaption is predesigned to be $\gamma = 10$ and $\nu = 10$ respectively.

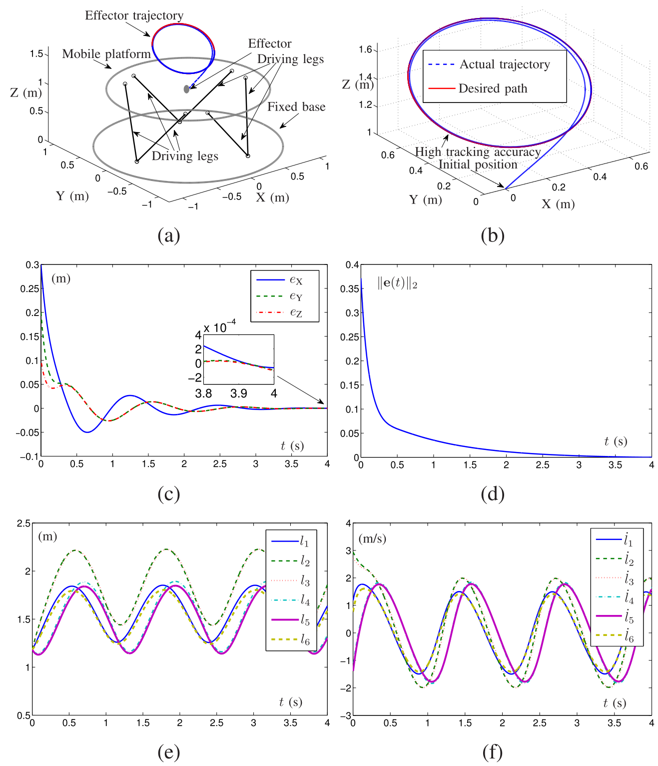

Fig. 1. The integrated the results when the effector fixed on the Stewart platform tracking the circular path by the proposed SEC-ZNN model (15). (a) Robot configuration and its trajectory. (b) Result curves of actual trajectory and preset path. (c) Result curves of position error. (d) Result curves of residual error. (e) Result curves of leg length. (f) Result curves of leg velocity.

Remarks: We provide the guidelines of parameter selection as follows. Firstly, we can set the initial value of approximation coefficient matrix $\hat{C}(t)$ for the adaption according to engineering experience and background knowledge. Note that the real-time adaptive updating approximate matrix $\hat{C}(t)$ can make the actual effector trajectory converge to the desired path.

Secondly, there are two important parameters for the proposed SEC-ZNN model. Theoretically, any value satisfying $\gamma > 0$ and $\nu > 0$ can be predefined as the parameters γ and ν for the SEC-ZNN model. In practice, the parameters γ and ν can be set to an appropriate large value allowed by the hardware to achieve rapid convergence without the need of engineering experience [38]. This is more practical value of the proposed SEC-ZNN for engineers without relevant engineering experience, with wide applicability and high generality.

A. Illustrative Example

Simulation verifications via circular path-tracking example is conducted to show the effectiveness of the proposed SEC-ZNN model (15) for the immediate feedback control problem of uncertain parallel robots.

In this example, the experiment is to make the end-effector of the Stewart platform track a circular path. Figs. 1 shows the corresponding simulation results of the Stewart platfrom following the circular path generated by the proposed SEC-ZNN model (15). Specifically, Fig. 1(a) depicts the motion of the entire parallel robot manipulator in a 3D plane during the tracking procedure. The effector's actual trajectory and the targeted path are both exact circles. Starting from an initial location, the actual trajectory of the parallel robot manipulator's effector quickly tracks the planned circular path, as shown in Fig. 1(b). After that, the effector's real trajectory convergences to the desired circular path in a short amount of time. The position error of the end-effector $\mathbf{e} = [e_X, e_Y, e_Z]^T$ (i.e., the gap between the anticipated path and the actual trajectory in the X-, Y-, and Z-axes) possesses an obvious convergence tendency with its steady-state absolute value being less than 3×10^{-4} m, as shown in Fig. 1(c) (in the workspace with a diameter of 0.8 m). Furthermore, during the real-time tracking process, the residual error presented in Fig. 1(d) exhibits the super-exponential convergence property with high speed and precision. Such results indicate that path-tracking task of the end-effector has been successfully completed. In addition, Fig. 1(e) and Fig. 1(f) show the profiles of the parallel robot manipulator's corresponding length $\mathbf{l}$ and velocity $\dot{\mathbf{l}}$ of the motion leg. Throughout the tracking process, all states are smooth and stable.

B. Comparisons

To further investigate the tracking performance of the proposed SEC-ZNN model (15), robustness tests and comprehensive comparisons influenced by uncertain system information are carried out in this subsection. The robustness tests and comparisons are designed and conducted by using the proposed SEC-ZNN model (15) and two conventional models, i.e., the CZNN model and the GNN model. We undertake the tracking process without sacrificing generality by using an approximate coefficient matrix $\tilde{C}(\mathbf{l}(t), \mathbf{d}(t), t)$ having unexpected impacts with assumptions and disturbances, which can be regarded as the deviation of physical parameters of the robot system in practical applications. In addition, in robustness testing, the step size is adjusted to $h = 0.8$ (with the sampling period ς set as 0.001 s and the design parameter γ set as 800).

1) Comparison With CZNN Model [15], [16], [17]: According to Zhang et al., the CZNN model is a common type of RNN that can tackle time-varying issues (including robotic challenges) efficiently by exploiting time-derivative information. It's worth noting that the corresponding CZNN models created using the fixed-parameter dynamical formula frequently have exponential convergence. The corresponding CZNN model is illustrated as (16) for handling the identical immediate feedback control problem of uncertain parallel robots, where the parameter γ is designed the same as in the SEC-ZNN model (15). Other simulation conditions are also identical to those in Section IV-A. Fig. 2 shows the comparing findings of CZNN model (16) for the uncertain parallel manipulator tracking the circular path mentioned above. In contrast, Fig. 2(a) illustrates that the parallel robot manipulator's effector is unable to correctly track the desired routes (or to say, with a relatively big error) without knowing the robot information. The corresponding residual errors illustrated in Fig. 2(b) shows that they converge with the exponential property and then with a relatively big (or termed non-ignorable) error bound within the motion task duration $T_d = 4$ s. That is, in terms of convergence speed and error bound, the suggested SEC-ZNN model (15) outperforms the CZNN model (16).

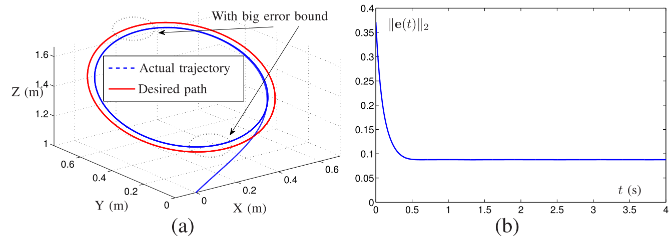

Fig. 2. The integrated the results when the effector fixed on the Stewart platform tracking the circular path by CZNN model (16).Synthesized motion results by CZNN model (16) when the effector of a parallel robot manipulator tracks circular path. (a) Result curves of actual trajectory and preset path. (b) Result curves of residual error.

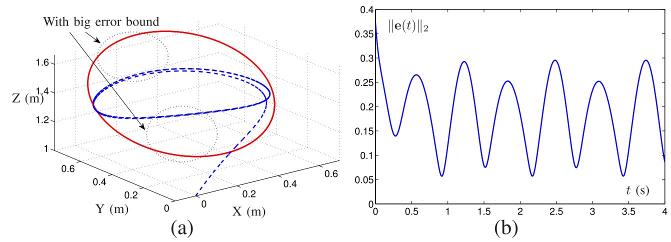

Fig. 3. The integrated the results when the effector fixed on the Stewart platform tracking the circular path by GNN model (29). (a) Result curves of actual trajectory and preset path. (b) Result curves of residual error.

2) Comparison With GNN Model [39], [40]: The GNN model, which is a type of traditional RNN model, has demonstrated its efficacy in engineering problem solving and optimization [39], [40]. It is known that the energy function of GNN model decreases monotonically when the dynamic system approaches the equilibrium point. The scalar-valued norm-based energy function according to GNN model for the immediate feedback tracking control problem of uncertain parallel robots is defined as follows:

$$\epsilon(t) = \frac{\|\mathbf{r}_d(t) - \mathbf{r}_a(t)\|_2^2}{2}.$$

Than, we can establish the following connection using the kinematics equation (4) of the robot system without the certain robot system information:

$$\frac{\partial \epsilon}{\partial \mathbf{l}} = -\left(\tilde{C}^{\dagger}(\mathbf{l}(t), \mathbf{d}(t), t)\right)^T (\mathbf{r}_d(t) - \mathbf{r}_a(t)).$$

By exploiting the GNN design formula $\dot{\mathbf{l}}(t) = -\rho \circ (\partial\epsilon/\partial\mathbf{l})$ where $\circ$ refers to the Hadamard product operator and $\rho = [\rho_1, \rho_2, \cdots, \rho_n]^T \in \mathbb{R}^n$ with ρ_i denoting the user-defined GNN parameter, the following continuous-time GNN model can be described as:

$$\dot{\mathbf{l}}(t) = \rho \circ \left(\tilde{C}^{\dagger}(\mathbf{l}(t), \mathbf{d}(t), t)\right)^T (\mathbf{r}_d(t) - \mathbf{r}_a(t)). \quad (29)$$

Comparatively, the corresponding test results synthesized by the conventional GNN model (29) for the parallel robot to track the intended path is shown in Fig. 3. We can draw the same conclusion that the effector of the parallel robot manipulator will not be able to trace the desired path accurately without the concrete robot system information [41], [42], [43], [44], [45]. The corresponding residual errors illustrate that they converge with the exponential property and then with a relatively big error bound within the motion task duration $T_d = 4$ s, which are also comparatively

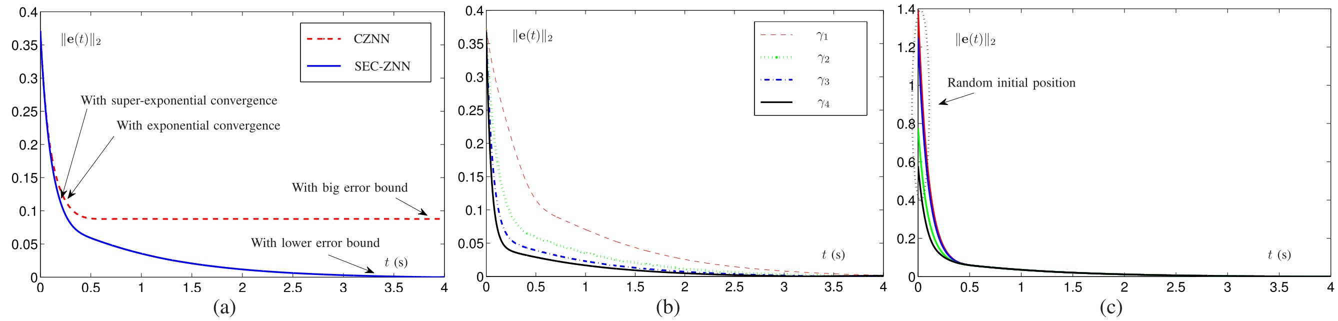

Fig. 4. Synthesized residual errors when the effector fixed on the Stewart platform tracking the circular path. (a) Result curves of residual errors by the proposed SEC-ZNN model (15) and CZNN model (16). (b) Result curves of residual errors by the proposed SEC-ZNN model (15) with different design parameter γ. (c) Result curves of residual errors by the proposed SEC-ZNN model (15) with randomly generated initial positions.

shown in Fig. 3(b). To put it another way, the proposed SEC-ZNN model (15) outperforms the traditional GNN model (29) on the convergence property.

C. Extensive Tests

To learn more about the convergency of the proposed SEC-ZND model (15), we conducted the extensive tests in respect of residual error utilizing different synthesized models, i.e., SEC-ZNN model (15) and CZNN model (16) by setting different design of parameter γ and starting from different position $\mathbf{r}_a(0)$ respectively. Fig. 4 depicts the test findings. In particular, Fig. 4(a) illustrates that residual errors generated by distinct neural network models, i.e., the SEC-ZNN model (15) and the CZNN model (16), preforms differently on the convergency, i.e., super-exponential and exponential convergence. In comparison to the residual error synthesized by the CZNN model (16), the residual error synthesized by the SEC-ZNN model (15) has a quicker convergence velocity in the transient state and a smaller error bound in the steady state, as demonstrated in this figure.

Then, the influence of the design parameter γ selection on convergence performance is investigated. When illustrated in Fig. 4(b), the residual errors exhibit a quicker tendency as the design parameter *gamma* increases from 5 to 20 by the step of 5 starting from the same position. That is, the convergence performance of residual errors of the proposed SEC-ZNN model (15) can be further improved by raising the design parameter properly. These graphical conclusions are also compatible with theoretical results in Theorem 3. In other words, the proposed SEC-ZNN (15) has a super-exponential convergence feature, with a $\gamma \exp(t)/t$ convergence rate. Thus, the suggested SEC-ZNN model (15) achieves achieves optimal computing speed and tracking accuracy by selecting suitable values for the design parameter γ.

Besides, we look at how the beginning position $\mathbf{r}_a(0)$ affects convergence performance. As shown in Fig. 4(c), the residual errors of the proposed SEC-ZNN (15) have cosmparable convergence properties, i.e., similar convergence perfromance throughout the transient state and similar error bound throughout the steady state, based on randomly generated beginning locations. These graphical conclusions are also compatible with Theorem 1's theoretical results. The residual errors synthesized of the proposed SEC-ZNN (15), in other words, converge to zero globally.

Another kind of important uncertainties, i.e., the external disturbance which are caused by varying load and unexpected impacts, is meaningfully to be investigated [46], [47], [48], [49], [50]. The white noise $\eta(t)$ existing in the process of leg control signal $\mathrm{l}(t)$ transmission of the robot tracing control as the external uncertainty is tested. The external uncertainty $\eta(t)$ is a kind of additive white noise with mean amplitude and covariance as $\eta(t) \sim \eta(\mu, \sigma^2)$ being $\mu = 2 \times 10^{-4}$ and $\sigma = 1$. A preliminary exploration of this external uncertainty problem is considered and investigated in the proposed SEC-ZNN model. We added white noise to the robot kinematic model and observed the control results. From the experimental results in Fig. 8, it can be seen that the additive white noise $\eta(t)$ does not bring a large impact during the process of leg control signal $\mathrm{l}(t)$ transmission of the robot tracing control as the external uncertainty. This shows that the proposed algorithm has some suppression effect on external noise as well.

V. APPLICATIONS ON COPPELIASIM

In this section, we use Coppeliasim to carry out the simulation experiment of Stewart platform, while the comtrol part using SEC-ZNN model (15) will be completed in MATLAB. Coppeliasim is a new open source robot simulation platform, which can well simulate the robot movement environment with rich details. By connecting with MATLAB, it can provide the most realistic robot simulation process with complex control. So that we use MATLAB to carry out SEC-ZNN model (15) control algorithm and use Coppeliasim to do the simulation. We pick the cicular route to test the performance of the proposed model on the robot without sacrificing generality. For better understanding, the Figs. 5 describes the whole workflow during the simulation. Firstly, we establish the communication between MATLAB and Coppeliasim. Secondly, MATLAB will read the data about the Stewart platfrom based on real drawings in Coppeliasim and generate a simplified model for further calculation. The real model is presented on the left and the simplified is on the right. Thirdly, MATLAB will use the SEC-ZNN model (15) to calculate the control signal using the data reading from Coppeliasim and return the results of calculation back. Finally, the Stewart platform in Coppeliasim will move according to control signals and visualize the movement process.

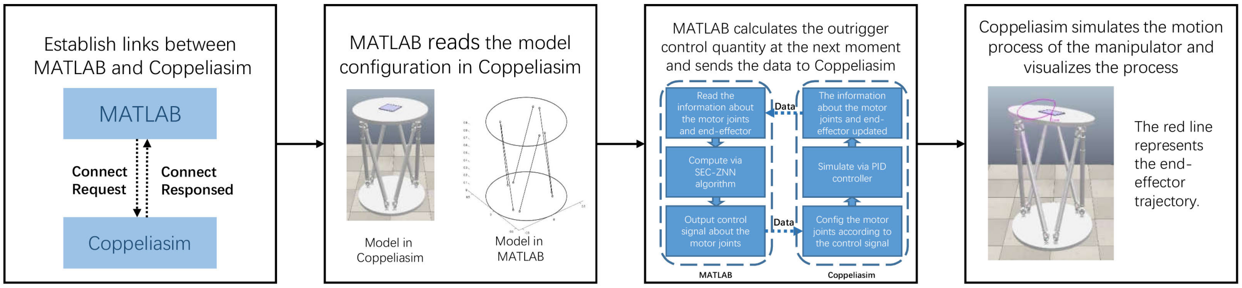

Fig. 5. Described the whole process of application tests via MATLAB and Coppeliasim.

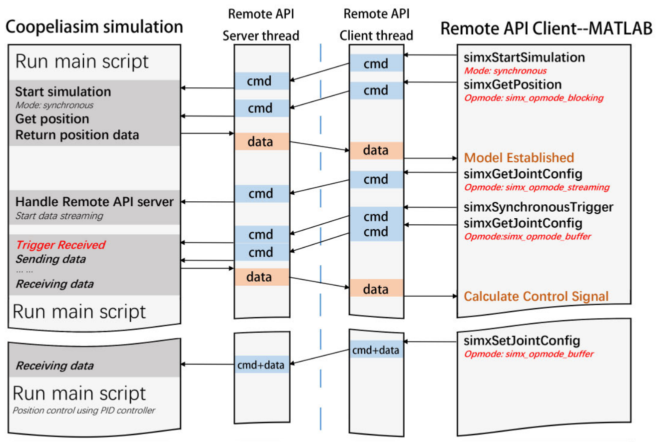

Fig. 6. Described the main communicate process of application tests between Coppeliasim and MATLAB.

A. Coppeliasim Communicate

CoppeliaSim is a robot simulator that uses a distributed control architecture and a common development environment. Embedded scripts, remote API clients, and other methods may be used to manipulate each object or model independently. This makes CoppeliaSim very versatile and ideal for multi-robot applications. MATLAB is a widely used advanced technical computing language and interactive environment for algorithm creation and execution. According to their respective advantages, the real robot automation control process is simulated by combining them together. The details of the communication between MATLAB and Coppeliasim is presented in the Figs. 6. There are four modes of communication we can choose from in Coppeliasim. In this experiment, the main mode we used is synchronous operation with blocking function calls and data streaming as an aid. That means, the data transfer between MATLAB and Coppeliasim is real-time so that the SEC-ZNN model (15) can use these data to further computation. As descriped in the Figs. 6, after established the connection between MATLAB and Coppeliasim and start the simulation, we use blocking function calls to get the initial parameters about the Stewart platform. Then a simplified model is established according to the data return from Coppeliasim. After that, MATLAB will give the first GetJointConfig signal to start the data streaming. At this time, the main data communicate and the computation mode in MATLAB is activated. Every time MATLAB give the SynchronousTrigger to Coppeliasim, the latter will perform

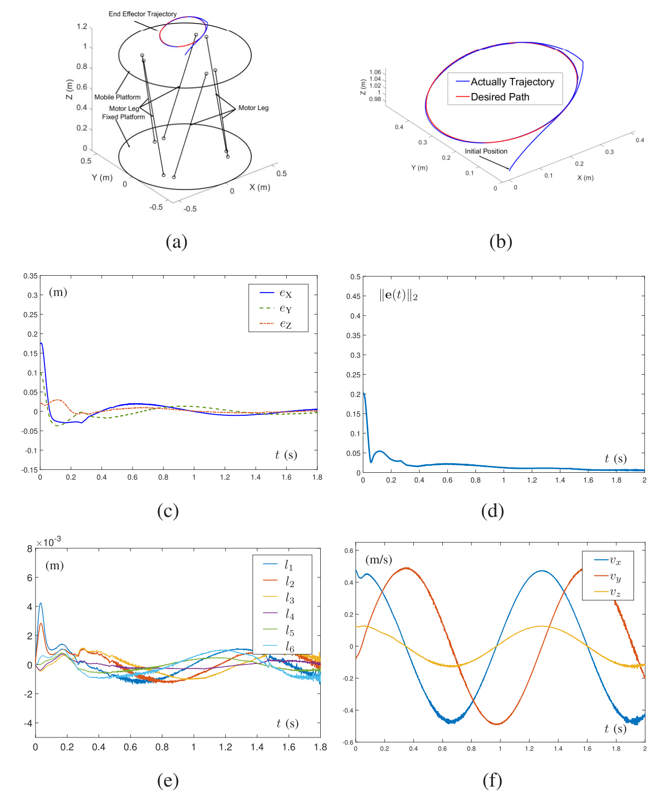

Fig. 7. The integrated the results when the effector fixed on the Stewart platform tracking the circular path by the proposed SEC-ZNN model (15). (a) Robot configuration and its trajectory. (b) Result curves of actual trajectory and preset path. (c) Result curves of position error. (d) Result curves of residual error. (e) Result curves of leg control error. (f) Result curves of leg velocity.

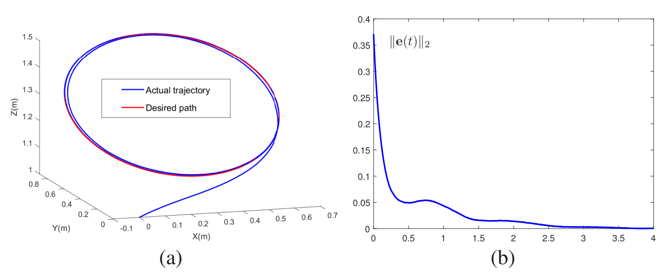

Fig. 8. The integrated the results when the effector fixed on the Stewart platform tracking the circular path by SEC-ZNN model with additive white noise. (a) Result curves of actual trajectory and preset path. (b) Result curves of residual error.

a simulation and send real-time data to the former and the former will use the SEC-ZNN (15) to compute the control signal for the next simulation and transmit the results back.

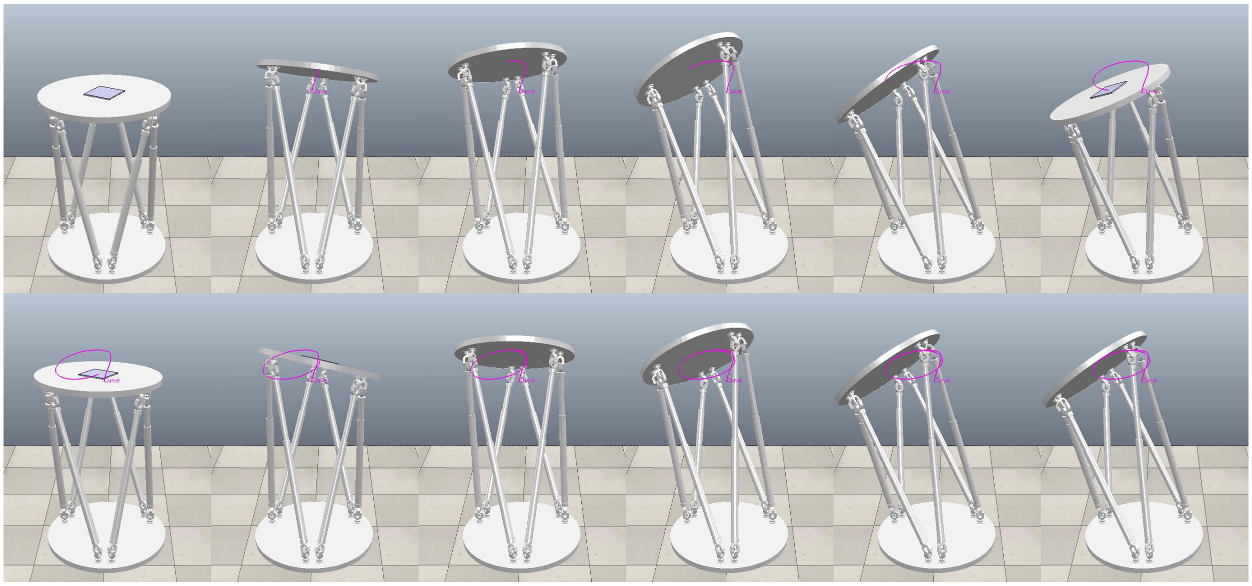

Fig. 9. Described the whole process of application tests on Coppeliasim.

B. Results and Discussions

To test the practicability and applicability of the algorithm, we establish a robot based on a drawing of an actual Stewart platform and use MATLAB to apply the SEC-ZNN model (15) to control it. Considering the workspace of the robot, we set the specified path to a circle with a radius of 0.5m. Without losing generality, we set the duration of the task as $T_\mathrm{d} = 2$ s. In addition, the motion control of the Stewart platfrom begins at rest, so that the initial velocity of the motion leg is set as $\dot{\mathbf{l}}(0) = [0, 0, 0, 0, 0, 0]^\mathrm{T}$ m/s. According to engineering experience, a rough initial value of the approximate coefficient matrix of the real-time adaption can be predefined as $\hat{C}(0) = [0.52099760, 0.01772071, 0.85337417; 0.11722871, 0.49529754, 0.86077742; -0.35778270, 0.43259082, 0.82756070; -0.20123115, 0.01002559, 0.97949247; 0.03785805, -0.40073872, 0.91540987; 0.16261162, -0.61514496, 0.77146233]$. In the tracking examples, the design parameters for the control equation and the coefficient matrix adaption is predesigned to be $\gamma = 8$ and $\nu = 10$ respectively.

Figs. 7 shows the corresponding simulation results of the Stewart platfrom following the circular path generated by the proposed SEC-ZNN model (15). Specifically, Fig. 7(a) depicts the motion of the entire parallel robot manipulator in a 3D plane during the tracking procedure. The effector's actual trajectory and the targeted path are both exact circles. Starting from an initial location, the actual trajectory of the parallel robot manipulator's effector quickly tracks the planned circular path, as detailedly shown in Fig. 7(b). After that, the effector's real trajectory convergences to the desired circular path in a short amount of time. The position error of the end-effector $\mathbf{e} = [e_\mathrm{X}, e_\mathrm{Y}, e_\mathrm{Z}]^\mathrm{T}$ (i.e., the gap between the anticipated path and the actual trajectory in the X-, Y-, and Z-axes) floating around zero, as shown in Fig. 7(c) (in the workspace with a diameter of 1.0 m). Furthermore, during the real-time tracking process, the residual error presented in Fig. 7(d) exhibits the super-exponential convergence property with high speed and precision. Such results indicate that path-tracking task of the end-effector has been successfully completed. In addition, Fig. 7(e) and Fig. 1(f) show the profiles of the parallel robot manipulator's corresponding leg's control error and velocity $\dot{\mathbf{l}}$. Throughout the tracking process, although there are some shocks due to the PID controller, all states are smooth and stableon the whole.

To show the result obviously, in Fig. 9 we captured some sence to illustrate the process of the experiment. The figure shows the applicability of SEC-ZNN (15) in real robot, and it can be seen that the whole motion trajectory is smooth and convergent. Then, we can conclude that the proposed SEC-ZNN (15) is effective and robust on the immediate feedback control problem of the real parallel robots.

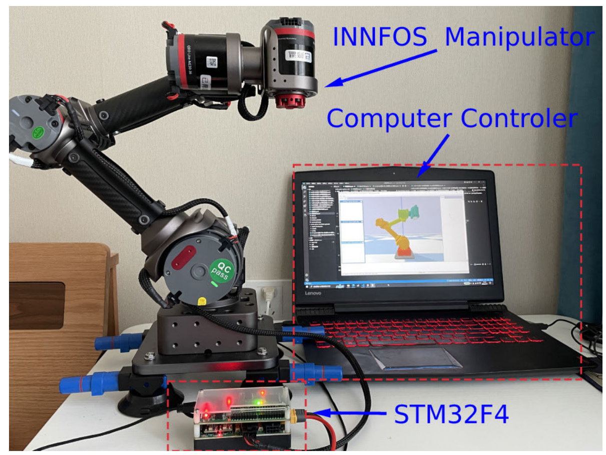

Fig. 10. Experiment environment with serial robot manipulator INNFOS.

VI. Applications on Robot Manipulator INNFOS

In this section, we use a serial robot manipulator INNFOS with 6 degree-of-freedom (6)-DOF) in forms of both real robot and computer virtual 3D module to carry out the proposed SEC-ZNN.

A. Experiment Environment

INNFOS Gluon is a lightweight and low cost robot manipulator that can perform more complex movements. INNFOS has created a new actuator by highly integrating servo motors, drivers, encoders and reducers, and built the INNFOS Gluon series robotic arm with it. The serial robot manipulator INNFOS has a wide range of work, and it can be equipped with

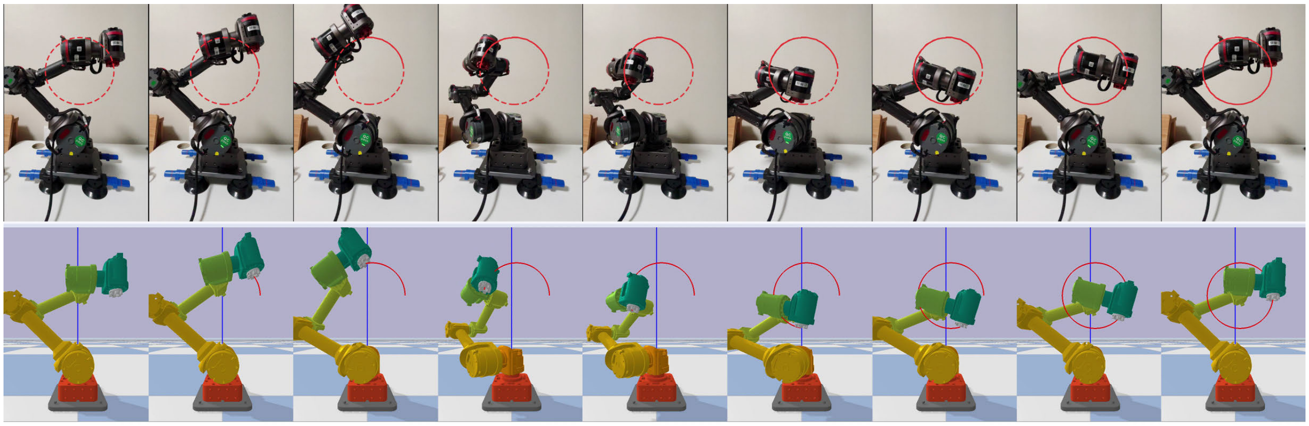

Fig. 11. Whole tracking process of applications in forms of both real robot and computer virtual 3D module with 6-DOF serial robot manipulator INNFOS by employing the proposed SEC-ZNN.

different auxiliary tools at the end, such as 3D printing heads, laser engraving heads, suction cups, clamps, jaws, welding heads, etc. By changing the auxiliary tools at the end, the INNFOS can perform a variety of functions. INNFOS is a multi-functional arm, which can be equipped with different auxiliary tools, which also has the problem of uncertainty appearing in the Jacobian matrix calculation [51], [52], [53]. In this work, we use Innfos Gluon to further validate the applicability of the proposed SEC-ZNN algorithm for physical robots. Fig. 10 shows our experimental environment, which mainly consists of a Windows laptop, a controller and a serial robot Innfos Gluon.

B. Results and Discussions

To test the practicability and applicability of the proposed SEC-ZNN, a serial robot manipulator Innfos with 6 degree-of-freedom (6)-DOF) is used and applied the SEC-ZNN model (15) to control it. Considering the workspace of the robot, we set the specified path to a circle with a radius of 0.08m. In the tracking examples, the design parameters for the control equation and the coefficient matrix adaption is predesigned to be $\gamma = 100$ and $\nu = 100$ respectively.

Figs. 11 shows the whole process of experiment (top row) and relative simulation (bottom row) 3D circle tracking control. The diagram in the first row shows the motion process of the physical robot manipulator INNFOS, and the diagram in the second row shows the motion process of the corresponding robot manipulator INNFOS 3D module via virtual simulation in computer. The two correspond one to another, and the results of the control of path tracking are described completely. It can be seen from the plots that the proposed SEC-ZNN algorithm is accurate and effective in controlling serial robot Innfos with 6 degree-of-freedom (6)-DOF), which proves the applicability of the proposed algorithm to the physical robot.

VII. Conclusion

Through further research and exploration in the direction of ZNN, a new SEC-ZNN (15) has been proposed for real-time tracking control problems solving of uncertain parallel robots to handle the robot uncertainty as well as to improve the convergence performance. The proposed SEC-ZNN (15) has emphasized the full exploitation of the feedback information from the end-effector, and has shown a robust tracking and super-exponential convergence performance even with uncertain robot information. The super-exponential convergence properties including error bound and convergence rate have been rigorously proved by theoretical analyses. Moreover, circular path-tracking example, comparison and tests via MATLAB and Coppeliasim have illuminated the efficaciousness and preponderance of the SEC-ZNN model (15) for the immediate feedback control problem for uncertain parallel robots. Finally, the applications on the real-world serial robot manipulator INNFOS have further verified the physical realizability for heterogeneous robots in engineering practice. As a final note, this is the first work in the field of ZNN that is able to address the immediate feedback tracking control problem of uncertain heterogeneous robots with theoretically guaranteed super-exponential convergence performance.

References

[1] Y. Zhang, S. Li, J. Gui, and X. Luo, "Velocity-level control with compliance to acceleration-level constraints: A novel scheme for manipulator redundancy resolution," *IEEE Trans. Ind. Informat.*, vol. 14, no. 3, pp. 921–930, Mar. 2018.

[2] B. Qiu and Y. Zhang, "Two new discrete-time neurodynamic algorithms applied to online future matrix inversion with nonsingular or sometimes-singular coefficient," *IEEE Trans. Cybern.*, vol. 49, no. 6, pp. 2032–2045, Jun. 2019.

[3] Z. Li, Z. Huang, W. He, and C.-Y. Su, "Adaptive impedance control for an upper limb robotic exoskeleton using biological signals," *IEEE Trans. Ind. Electron.*, vol. 64, no. 2, pp. 1664–1674, Feb. 2017.

[4] Z. Zhang, A. Beck, and N. Magnenat-Thalmann, "Human-like behavior generation based on head-arms model for robot tracking external targets and body parts," *IEEE Trans. Cybern.*, vol. 45, no. 8, pp. 1390–1400, Aug. 2015.

[5] A. M. Mohammed and S. Li, "Dynamic neural networks for kinematic redundancy resolution of parallel Stewart platforms," *IEEE Trans. Cybern.*, vol. 46, no. 7, pp. 1538–1550, Jul. 2016.

[6] H. Wang, X. Liu, and K. Liu, "Robust adaptive neural tracking control for a class of stochastic nonlinear interconnected systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 3, pp. 510–523, Mar. 2016.

[7] Z. Zhang, Z. Li, Y. Zhang, Y. Luo, and Y. Li, "Neural-dynamic-method-based dual-arm CMG scheme with time-varying constraints applied to humanoid robots," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 26, no. 12, pp. 3251–3262, Dec. 2015.

[8] C. Yang, Z. Li, and J. Li, "Trajectory planning and optimized adaptive control for a class of wheeled inverted pendulum vehicle models," *IEEE Trans. Cybern.*, vol. 43, no. 1, pp. 24–36, Feb. 2013.

[9] J. Na, Q. Chen, X. Ren, and Y. Guo, “Adaptive prescribed performance motion control of servo mechanisms with friction compensation,” *IEEE Trans. Ind. Electron.*, vol. 61, no. 1, pp. 486–494, Jan. 2014.

[10] Z. Li, J. Deng, R. Lu, Y. Xu, J. Bai, and C.-Y. Su, “Trajectory-tracking control of mobile robot systems incorporating neural-dynamic optimized model predictive approach,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 6, pp. 740–749, Jun. 2016.

[11] C. Yang, Z. Li, R. Cui, and B. Xu, “Neural network-based motion control of an underactuated wheeled inverted pendulum model,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 11, pp. 2004–2016, Nov. 2014.

[12] Y.-J. Liu, S. Tong, C. L. P. Chen, and D.-J. Li, “Neural controller design-based adaptive control for nonlinear MIMO systems with unknown hysteresis inputs,” *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 9–19, Jan. 2016.

[13] K. Chen, “Recurrent implicit dynamics for online matrix inversion,” *Appl. Math. Comput.*, vol. 219, no. 20, pp. 10218–10224, Jun. 2013.

[14] J. J. Hopfield and D. W. Tank, “‘Neural’ computation of decisions in optimization problems,” *Biol. Cybern.*, vol. 52, no. 3, pp. 141–152, Jul. 1985.

[15] Y. Zhang, B. Mu, and H. Zheng, “Link between and comparison and combination of Zhang neural network and quasi-Newton BFGS method for time-varying quadratic minimization,” *IEEE Trans. Cybern.*, vol. 43, no. 2, pp. 490–503, Apr. 2013.

[16] L. Jin, S. Li, B. Liao, and Z. Zhang, “Zeroing neural networks: A survey,” *Neurocomputing*, vol. 267, pp. 597–604, Dec. 2017.

[17] B. Liao and Y. Zhang, “Different complex ZFs leading to different complex ZNN models for time-varying complex generalized inverse matrices,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 25, no. 9, pp. 1621–1631, Sep. 2014.

[18] Y. Zhang, D. Jiang, and J. Wang, “A recurrent neural network for solving Sylvester equation with time-varying coefficients,” *IEEE Trans. Neural Netw.*, vol. 13, no. 5, pp. 1053–1063, Sep. 2002.

[19] S. Li, Y. Zhang, and L. Jin, “Kinematic control of redundant manipulators using neural networks,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 10, pp. 2243–2254, Oct. 2017.

[20] A. Benchabane, A. Bennia, F. Charif, and A. Taleb-Ahmed, “Multi-dimensional Capon spectral estimation using discrete Zhang neural networks,” *Multidimensional Syst. Signal Process.*, vol. 24, no. 3, pp. 583–598, Sep. 2013.

[21] L. Xiao, “A new design formula exploited for accelerating Zhang neural network and its application to time-varying matrix inversion,” *Theor. Comput. Sci.*, vol. 647, pp. 50–58, Sep. 2016.

[22] D. Guo, Z. Nie, and L. Yan, “Novel discrete-time Zhang neural network for time-varying matrix inversion,” *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 8, pp. 2301–2310, Aug. 2017.

[23] L. Jin, Y. Zhang, and S. Li, “Integration-enhanced Zhang neural network for real-time-varying matrix inversion in the presence of various kinds of noises,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 12, pp. 2615–2627, Dec. 2016.

[24] B. Liao and W. Liu, “Pseudoinverse-type bi-criteria minimization scheme for redundancy resolution of robot manipulators,” *Robotica*, vol. 33, no. 10, pp. 2100–2113, Dec. 2015.

[25] D. Chen and Y. Zhang, “Minimum jerk norm scheme applied to obstacle avoidance of redundant robot arm with jerk bounded and feedback control,” *IET Control Theory Appl.*, vol. 10, no. 15, pp. 1896–1903, Oct. 2016.

[26] D. Guo, K. Li, and B. Liao, “Bi-criteria minimization with MWVN–INAM type for motion planning and control of redundant robot manipulators,” *Robotica*, vol. 36, no. 5, pp. 655–675, May 2018.

[27] D. Chen, Y. Zhang, and S. Li, “Tracking control of robot manipulators with unknown models: A Jacobian-matrix-adaption method,” *IEEE Trans. Ind. Informat.*, vol. 14, no. 7, pp. 3044–3053, Jul. 2018.

[28] Y. Wang, X. Yan, L. He, H. Tan, and Y. Zhang, “Inverse-free solution of Z1G1 type to acceleration-level inverse kinematics of redundant robot manipulators,” in *Proc. 7th Int. Conf. Adv. Comput. Intell. (ICACI)*, Wuyi, China, Mar. 2015, pp. 57–62.

[29] Y. Zhang, S. S. Ge, and T. H. Lee, “A unified quadratic-programming-based dynamical system approach to joint torque optimization of physically constrained redundant manipulators,” *IEEE Trans. Syst., Man Cybern. B, Cybern.*, vol. 34, no. 5, pp. 2126–2132, Oct. 2004.

[30] B. Aghbali, A. Yousefi-Koma, A. G. Toudeshki, and A. Shahrokhshahi, “ZMP trajectory control of a humanoid robot using different controllers based on an offline trajectory generation,” in *Proc. 1st RSI/ISM Int. Conf. Robot. Mechatronics (ICRoM)*, Tehran, Iran, Feb. 2013, pp. 530–534.

[31] A. Gholami, T. Homayouni, R. Ehsani, and J.-Q. Sun, “Inverse kinematic control of a delta robot using neural networks in real-time,” *Robotics*, vol. 10, no. 4, p. 115, Oct. 2021.

[32] A. Zubizarreta, M. Larrea, E. Irigoyen, I. Cabanes, and E. Portillo, “Real time direct kinematic problem computation of the 3PRS robot using neural networks,” *Neurocomputing*, vol. 271, pp. 104–114, Jan. 2018.

[33] J. M. Mendel and F. Liu, “Super-exponential convergence of the Karnik–Mendel algorithms used for type-reduction in interval type-2 fuzzy logic systems,” in *Proc. Int. Conf. Fuzzy Syst.*, Vancouver, BC, Canada, 2006, pp. 1253–1260.

[34] Y. Zhang and D. Guo, *Zhang Functions and Various Models*. Berlin, Germany: Springer-Verlag, 2015.

[35] Y. Zhang, L. Xiao, Z. Xiao, and M. Mao, *Zeroing Dynamics, Gradient Dynamics, and Newton Iterations* Boca Raton, FL, USA: CRC Press, 2015.

[36] W. He, Z. Yin, and C. Sun, “Adaptive neural network control of a marine vessel with constraints using the asymmetric barrier Lyapunov function,” *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1641–1651, Jul. 2017.

[37] P.-F. Hsieh and Y. Sibuya, *Basic Theory of Ordinary Differential Equations*. Berlin, Germany: Springer, 2012.

[38] C. Mead, *Analog VLSI and Neural Systems*. Reading, MA, USA: Addison-Wesley, 1989.

[39] K. Chen and C. Yi, “Robustness analysis of a hybrid of recursive neural dynamics for online matrix inversion,” *Appl. Math. Comput.*, vol. 273, pp. 969–975, Jan. 2016.

[40] C. Yi, Y. Chen, and Z. Lu, “Improved gradient-based neural networks for online solution of Lyapunov matrix equation,” *Inf. Process. Lett.*, vol. 111, no. 16, pp. 780–786, Aug. 2011.

[41] D. Chen, X. Li, and S. Li, “A novel convolutional neural network model based on beetle antennae search optimization algorithm for computerized tomography diagnosis,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 34, no. 3, pp. 1418–1429, Mar. 2023.

[42] D. Chen, S. Li, and Q. Wu, “A novel supertwisting zeroing neural network with application to mobile robot manipulators,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 32, no. 4, pp. 1776–1787, Apr. 2021.

[43] Y. Shi, L. Jin, S. Li, J. Li, J. Qiang, and D. K. Gerontitis, “Novel discrete-time recurrent neural networks handling discrete-form time-variant multi-augmented Sylvester matrix problems and manipulator application,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 33, no. 2, pp. 587–599, Feb. 2022.

[44] D. Chen, S. Li, W. Li, and Q. Wu, “A multi-level simultaneous minimization scheme applied to jerk-bounded redundant robot manipulators,” *IEEE Trans. Autom. Sci. Eng.*, vol. 17, no. 1, pp. 463–474, Jan. 2020.

[45] L. Xiao, L. Jia, Y. Wang, J. Dai, Q. Liao, and Q. Zhu, “Performance analysis and applications of finite-time ZNN models with constant/fuzzy parameters for TVQPEI,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 33, no. 11, pp. 6665–6676, Nov. 2022.

[46] W. Li, P. W. Y. Chiu, and Z. Li, “A novel neural approach to infinity-norm joint-velocity minimization of kinematically redundant robots under joint limits,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 34, no. 1, pp. 409–420, Jan. 2023.

[47] L. Xiao, J. Tao, and W. Li, “An arctan-type varying-parameter ZNN for solving time-varying complex Sylvester equations in finite time,” *IEEE Trans. Ind. Informat.*, vol. 18, no. 6, pp. 3651–3660, Jun. 2022.

[48] D. Chen and Y. Zhang, “A hybrid multi-objective scheme applied to redundant robot manipulators,” *IEEE Trans. Autom. Sci. Eng.*, vol. 14, no. 3, pp. 1337–1350, Jul. 2017.

[49] D. Chen, S. Li, Q. Wu, and X. Luo, “New disturbance rejection constraint for redundant robot manipulators: An optimization perspective,” *IEEE Trans. Ind. Informat.*, vol. 16, no. 4, pp. 2221–2232, Apr. 2020.

[50] D. Chen, S. Li, F.-J. Lin, and Q. Wu, “New super-twisting zeroing neural-dynamics model for tracking control of parallel robots: A finite-time and robust solution,” *IEEE Trans. Cybern.*, vol. 50, no. 6, pp. 2651–2660, Jun. 2020.

[51] J. Darbon and T. Meng, “On some neural network architectures that can represent viscosity solutions of certain high dimensional Hamilton–Jacobi partial differential equations,” *J. Comput. Phys.*, vol. 425, Jan. 2021, Art. no. 109907.

[52] J. Darbon, G. P. Langlois, and T. Meng, “Overcoming the curse of dimensionality for some Hamilton–Jacobi partial differential equations via neural network architectures,” *Res. Math. Sci.*, vol. 7, no. 3, pp. 1–50, Jul. 2020.

[53] J. Darbon and S. Osher, “Algorithms for overcoming the curse of dimensionality for certain Hamilton–Jacobi equations arising in control theory and elsewhere,” *Res. Math. Sci.*, vol. 3, no. 1, pp. 1–26, Sep. 2016.