Tunable interface states in locally resonant acoustic chains with inerters

Milan Cajić[*,†], Danilo Karličić[†], Stepa Paunović[†] and Sondipon Adhikari [††]
[*]*Faculty of Science and Engineering, Swansea University, Swansea, United Kingdom*
[†]*Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia*
[††]*James Watt School of Engineering,University of Glasgow, Glasgow, UK*

Summary. In this work, we study locally resonant acoustic chains with inerters. Topological properties are investigated on the example of one-dimensional chain with diatomic mass-in-mass unit cells based on the band structure and eigenstate analysis. The existence of interface modes in finite chains is confirmed through natural frequencies and frequency response function analysis. Tuning of interface modes due to introduced inertia amplification effect is observed and investigated in details.

Introduction

The interest for investigation of topologically protected interface states in acoustic and mechanical metamaterials has significantly grown over the past years. Inspired by the phenomena from solid states physics such as the valley and quantum Hall effects, researchers discovered a number of equivalent phenomena in mechanical systems based on acoustic and elastic wave propagation analysis [1]. Since the beginning there was a need to control wave propagation properties in such periodic systems based on approaches that can be divided into two main groups. The first group encompasses active approaches where different external fields such as magnetic or electric field are employed for wave propagation control purposes [2]. The second group encompasses passive or semi-passive strategies that are often based on application of external passive damping devices [3]. Since their discovery, inerters have been widely accepted as efficient vibration attenuation devices [4]. Based on the inertia amplification effect, they are able to reduce the frequency and change properties of periodic structures and other mechanical systems [5]. Here, we apply ideal inerter elements to change the band structure of the locally resonant acoustic chain and at the same time keep the topological properties of the original lattice without inerters.

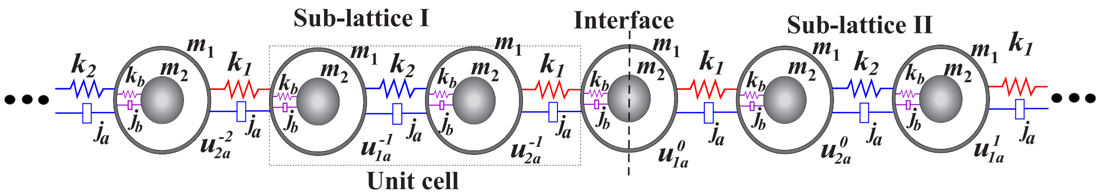


Figure 1: Illustration of one-dimensional locally resonant acoustic chain with interface.

Mathematical model

Let us consider a one-dimensional locally resonant acoustic chain with an interface, for which the equation for the unit cell in the general case can be expressed as

$$\begin{aligned} m_a \ddot{u}^p_{(\zeta)a} + (k_\zeta + k_{\zeta+1})\, u^p_{(\zeta)a} - k_\zeta u^p_{(\zeta-1)a} - k_{\zeta+1} u^p_{(\zeta+1)a} + k_b \left(u^p_{(\zeta)a} - u^p_{(\zeta)b}\right) + \\ j_a \left(2\ddot{u}^p_{(\zeta)a} - \ddot{u}^p_{(\zeta-1)a} - \ddot{u}^p_{(\zeta+1)a}\right) + j_b \left(\ddot{u}^p_{(\zeta)a} - \ddot{u}^p_{(\zeta)b}\right) = 0, \end{aligned} \tag{1}$$

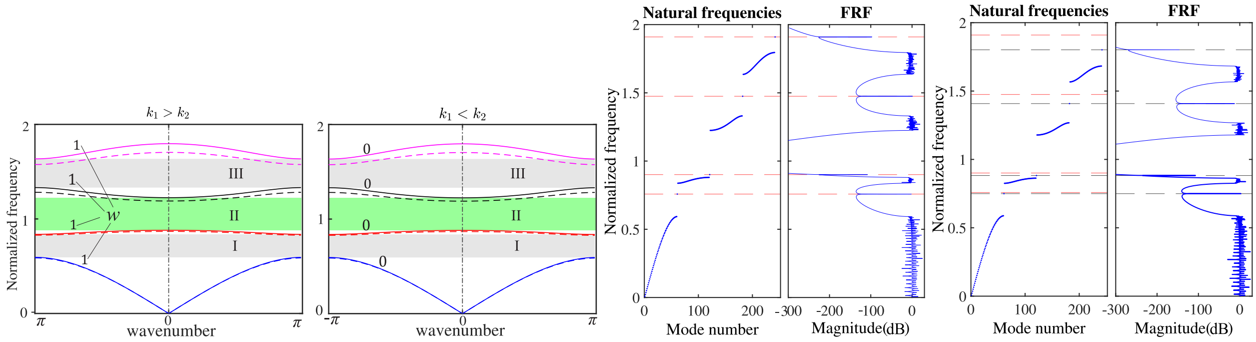
and

$$m_b \ddot{u}^p_{(\zeta)b} + k_b \left(u^p_{(\zeta)b} - u^p_{(\zeta)a}\right) + j_b \left(\ddot{u}^p_{(\zeta)b} - \ddot{u}^p_{(\zeta)a}\right) = 0, \tag{2}$$

where $\zeta = 2, 3, ..., \mathcal{N}-1$, $u_{(\varepsilon)a}$ and $u_{(\varepsilon)b}$, $\varepsilon = 1, 2, ..., \mathcal{N}$ are displacements of outer m_a and inner m_b masses, respectively k_ε, k_b are the springs stiffnesses, while j_a, j_b are the inerter parameters connecting outer and inner masses, respectively. It should be noted that for the adopted notation, the repeating unit cell of the last outer mass is connected to the next unit cell through the spring $k_{\mathcal{N}+1} = k_1$, where equations for the first and the last mass-in-mass sub-systems can be written accordingly. We note that the values of outer and inner masses, inerter parameters and stiffness of inner mass springs are assumed to be the same for different mass-in-mass sub-systems. Size of a unit cell depends on the number of different springs and mass-in-mass sub-units. The system of equations for the one-dimensional finite chain constructed from two sub-lattices connected at interface can be obtained by following the notations given in (Fig.1).

Results and discussion

Topological properties of locally resonant acoustic lattice can be examined through dispersion and eigenstate analysis of a representative unit cell based on the topological invariant called the winding number (here denoted as w) or by using



(a) Band structure of a diatomic unit cell (b) Finite chain $k_1 > k_2$ (c) Finite chain inerters $k_1 > k_2$

Figure 2: Band structure, natural frequencies and frequency response function of the one-dimensional locally resonant acoustic chain with diatomic mass-in-mass unit cells.

the the bulk-edge correspondence. Dispersion curves for the representative unit cell can be obtained as a solution of the eigenvalue problem based on Eqs.1 and 2 by considering the stiffnesses $k_1 = k(1+\gamma)$ and $k_2 = k(1-\gamma)$, with k denoting the mean stiffness and γ is the dimensionless parameter. Natural frequencies and frequency response function of the finite chain can be obtained from the corresponding system of equations by considering the $N = 30$ unit cell on each side of the interface. Fig. 2a shows dispersion curves of a diatomic unit cell with (dashed lines) and without (full lines) inerters when $\gamma = 0.5$, $m_2/m_1 = 0.5$, $m_2/m_1 = 0.5$, $k = 10 \times 10^3$[N/m], $k_b = 8 \times 10^3$[N/m] and $j_a = j_b = 0.02$. Four bands and three band gaps can be noticed, where II band gap highlighted in green is of locally resonant origin. By comparing the lattices when $k_1 > k_2$ and $k_1 < k_2$ does not change the band structure but it changes the topology of eigenvectors i.e. the winding number of individual bands change its value from $w = 1$ to $w = 0$. This change physically means that interface modes can appear between the two lattice types. For the finite chain, we can confirm the existence of interface modes within the band gaps by observing its natural frequencies and frequency response function (Figs. 2b and 2c). In the diagram showing the natural frequencies, normalized with respect to the frequency of the local resonator, one can observe four interface modes. The presence of these modes is confirmed in the frequency response function diagram of the outer interface mass. Interface modes that appear inside the band gap II and above the last band are trivial since they will disappear if we change lattice configuration to low stiffness springs ($k_1 < k_2$) at the interface (see Fig. 1). The other two interface modes will remain in both configurations and they are robust and topologically protected. In this case, the effect of inertia amplification on interface states can be seen by comparing the interface mode frequencies of chains with (black dashed lines) and without (red dashed lines) inerters in Fig. 2b. The results reveal the shift of interface mode frequencies to lower values. This effect is especially pronounced at higher frequencies while shifting at lower frequency band gaps is smaller. The similar analysis can be performed for one-dimensional lattices with triatomic mass-in-mass unit cells with and without inerters, where quantized topological invariant winding number can be recovered only in the case when one of the springs connecting the outer masses is different from others.

Conclusions

Recent advances in discovering exotic metamaterials have provided a new insight for understanding the essential processes that create unique wave propagation phenomena and they have initiated a wide range of possible applications in industry and other fields of science. This study is a step forward towards a better understanding of the role of inerters in passive control and tuning of interface modes in locally resonant acoustic chains. Parametric study has demonstrated a significant shifting of interface states when inertia amplification effect is introduced while the topological properties of bands were preserved. This opens new possibilities for application of inerters in more complex lattices or periodic structures capable of generating interface modes.

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References

[1] Xin, L., Siyuan, Y., Harry, L., Minghui, L., Yanfeng, C. (2020) Topological mechanical metamaterials: A brief review. *Curr. Opin. Solid State Mater. Sci.* **24**: 100853.

[2] Montgomery, S. M., Wu, S., Kuang, X., Armstrong, C. D., Zemelka, C., Ze, Q., ..., Qi, H. J. (2020) Magneto-mechanical metamaterials with widely tunable mechanical properties and acoustic bandgaps. *Adv. Funct. Mater.* **31**: 2005319.

[3] Lv, X. F., Chuang, K. C., Erturk, A. (2020) Tunable elastic metamaterials using rotatable coupled dual-beam resonators. *J. App. Phys.* **126**: 035107.

[4] Smith M. C. (2020) The inerter: a retrospective. *Annu. Rev. Control Robot. Auton. Syst.* **3**: 361–391.

[5] Cajić, M., Christensen, J., Adhikari, S. (2021) Tuning of topological interface modes in an elastic beam array system with inerters. *Int. J. Mech. Sci.* **205**: 106573.