Price-Rising Competition: a Higher Market Price When a Monopoly Faces a Small Entrant

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Abstract

The UK retail electricity market revealed (i) the co-occurrence between a declining market concentration and an increasing price and (ii) price differentiation between incumbents and small suppliers. We construct an infinite sequential game in which a monopoly faces a small entrant and find an equilibrium where both players are not motivated to deviate. However, this equilibrium imposes a condition on the values of model parameters. If the condition is unmet, the interactions between the two players will be unending. Nonetheless, we suggest that two states where at least one player is not motivated to deviate could be a place to settle, but the choice depends on which player is more concerned about market stability. Besides, the two observations are found in equilibrium and two states. Finally, we argue that the finding is jointly contributed by four features: small entrant with a lower marginal cost and constrained capacity, switching costs, and barriers to entry. If any feature is relaxed, the entry of the small firm will not lead to a rising market price.

1 Introduction

It is widely accepted that increased competition brings downward pressure on prices, but this standard view has been challenged. An increase in the number of firms in the market may lead to a higher market price through two channels. The first channel is related to consumers' search costs. An increase in the number of firms in the market makes consumers more difficult to find the lowest price, lowering their incentives to

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search and leading to a higher equilibrium market price (Satterthwaite 1979; Stiglitz 1987; Schulz and Stahl 1996; Janssen and Moraga-González 2004).[1] The second channel is related to the presence of loyal and switching consumer groups. Assuming each seller are facing these two groups of consumers, an increase in the number of sellers reduces the share of switching consumer faced by each seller, increasing the incentive to exploit the loyal consumers through a higher price (Rosenthal 1980).

In the UK retail electricity market, the market share of six large incumbent suppliers declined dramatically from 99 per cent in 2012 to 70 per cent in 2019, after the entry of small suppliers (Ofgem 2020a).[2] During the same period, the retail electricity price increased from £118.5 per MWh to £171.1 per MWh (BEIS 2020).[3] The co-occurrence of a declining market concentration and a rising price contrasts with the concept that increased competition should bring downward pressure on the price. Moreover, from the Consolidated Segmental Statements provided by these six incumbent suppliers, the retail price they charged was £180.8 per MWh in 2019, higher than the market price of £171.1 per MWh, suggesting that small suppliers charged lower prices than these incumbents.[4] Therefore, price differentiation exists between incumbents and small suppliers, although they provide an identical product, i.e. electricity.[5]

We suggest that four features in the UK retail electricity market have jointly contributed to these two observations. The first two features are relevant to the fact that small suppliers are exempted from several environmental obligations, such as the Feed-in Tariff scheme and the Energy Company Obligation scheme.[6] These exemptions give small suppliers cost advantages as incumbents need to pass the costs of these environmental obligations to consumers through higher electricity prices.[7]

While exemptions bring small suppliers cost advantages, they have to limit their capacity below the threshold. The cost advantage will disappear if small suppliers increase their capacities above the threshold. Our analysis will consider large firms with small suppliers with constrained capacity. Early studies discuss constrained capacity in a duopoly and find Cournot-like behaviour remains (Levitan and Shubik 1972; Kreps and Scheinkman 1983).

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[1] A recent study about search costs suggests that a reduction in search costs raises market price, see Choi et al. (2018).

[2] The six large energy firms were British Gas, EDF, E.ON, Npower, Scottish Power, and SSE, known as the Big Six, up to 2019Q4. The associated Herfindahl–Hirschman Index decreased from 1761 (2012Q1) to 970 (2019Q4) in the same interval.

[3] After deducting the fuel costs, network costs, and environmental obligations, the residual price remained increasing. For example, from their Consolidated Segmental Statements, six incumbent suppliers had this residual price increased from £23.89 in 2013 to £30.73 in 2019. The rising costs of environmental obligations were discussed by Shao et al. (2022).

[4] The price gap between large incumbents and other suppliers were also indicated by the regulator (Ofgem 2020b).

[5] Another stream of studies suggests that product differentiation can explain the increase in price when competition increased (Chen and Riordan 2008).

[6] The Feed-in Tariff scheme was introduced in 2010 to support small-scale renewable generation and the Energy Company Obligation was introduced in 2013 to tackle fuel poverty and help reduce carbon emissions.

[7] An empirical study by Foster et al. (2008) suggests that entrants are more productive than incumbents based on evidence from the US manufacturing sector.

Third, consumers who previously purchased from one firm may have switching costs for several reasons, such as uncertainty about the quality of the new supplier and brand loyalty (Klemperer 1995). Unlike a market for goods, the utility market is a market of contracts of service, in which switching costs may be more significant due to low consumer engagement (i.e. consumers continue to receive service from the chosen supplier without taking action). The switching costs allow incumbents to exploit non-switching captive consumers. Klemperer (1987) illustrates a two-period model with two symmetric firms and suggests that firms may compete more aggressively in the first period and then charge a higher price on captive consumers, given the switching costs. Farrell and Shapiro (1988) consider a two-period overlapping generation model in which the incumbent specialises in serving its captive consumers and leave uncaptive to its rivals, allowing less efficient entrants.

Two studies discussed both constrained capacity and switching costs. Herk (1993) shows that the existence of switching costs supports Cournot behaviour in capacity-constrained duopoly competition. Farrell and Klemperer (2007) mentioned that, when there are switching costs, the entry of a small-scale new entrant should be easy as it can take uncaptive consumers and the incumbent has captive consumers to exploit. However, the impact of switching costs on the dynamics of the competition between incumbents and small entrants remained unexploited, and the rising market price after the entry of small suppliers remains puzzling.

Fourth, barriers to entry remain high in the retail market. For example, one main contributing factor is the difficulty of accessing wholesale energy. The six large energy firms are all vertically integrated as they have both generation and retailing activities. Together, they generate about 70% of total electricity generation (CMA 2016).

This study aims to explain (i) the co-occurrence of a declining market concentration and a rising price and (ii) the price differentiation between incumbents and small suppliers. With assumptions reflecting the four features discussed above, the model starts with a monopoly earning monopoly profit. After the entry of a small supplier, its actions depend on the condition of undermining, which compares the price gap and the switching costs.

If the condition of undermining is not met, the small supplier will take consumers who previously had no service, and the monopoly's profit is untouched. The situation is referred to as *Cooperation*. In contrast, if the condition of undermining is met, the small supplier will undermine the monopoly by taking its consumers. After that, a sequence of actions by the monopoly and the small supplier is expected. Nonetheless, there are two situations where at least one player is not motivated to deviate. At *State One*, the small supplier is not motivated to deviate but the monopoly may receive a higher profit by reducing its price and taking consumers back. At *State Two*, the monopoly is not motivated to deviate but the small supplier has the motivation to raise its price. *Equilibrium* is reached when State One coincides with State Two. Regarding the two observations, price differentiation is found in all four situations, but the rising prices after the entry of the small suppliers are found in State One, State Two, and Equilibrium.

Further, Equilibrium imposes an additional condition on the values of model parameters. If such a condition is unmet, there will be unending interactions between the monopoly and the small supplier. Nonetheless, we suggest that State One and State Two can be two possible places for players to stay on, but the choice depends on which player is more concerned about market stability.

This paper will be constructed in the following way. Section 2 explains the theoretical model and Sect. 3 conducts simulation exercises. Section 4 provides discussions and Sect. 5 concludes the paper.

2 The Model

We construct an infinite sequential game in which a monopoly faces the entry of a small supplier. We first consider an existing supplier as a monopoly in the market, and then a small supplier with a lower marginal cost and a constrained capacity enters the market. In addition, consumers face switching costs when they switch suppliers, and no other supplier enters the market (barriers to entry).

There are six structural parameters in the model: the parameters of the demand curve (a and b), the marginal costs of the monopoly and the small supplier (c and c_s), the capacity of the small supplier ($\bar{q}_s$), and the fractional parameter of the switching costs (μ). All other variables in the model can be expressed in terms of these six structural parameters.

2.1 The Monopoly Alone

Consider an existing supplier as a monopoly, with a constant marginal cost c. The market demand function is

$$p = a - bq \leftrightarrow q = \frac{a - p}{b} \tag{1}$$

where p is the price and q is the quantity. The monopoly's profit maximisation problem is

$$\max_q \pi = (p - c)q = (a - bq - c)q \tag{2}$$

The first-order condition gives the monopoly quantity q_m as

$$\frac{\partial \pi}{\partial q} = 0 \Rightarrow a - 2bq - c = 0 \Rightarrow q_m = \frac{a - c}{2b} \tag{3}$$

and the monopoly price p_m is solved as

$$p_m = a - bq = a - b\frac{a - c}{2b} \Rightarrow p_m = \frac{a + c}{2} \tag{4}$$

At the monopoly equilibrium, the monopoly profit π_m is

$$\pi_m = \left(p_m - c\right)q_m = \left(\frac{a + c}{2} - c\right)\frac{a - c}{2b} = \frac{(a - c)^2}{4b} \tag{5}$$

Figure 1 shows the market with the monopoly alone and the equilibrium in which the monopoly maximises its profit. When the monopoly moves up or down from this equilibrium, its profit is lower.[8]

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[8] Appendix 1 shows that the profit is lower when the monopoly moves further away from the monopoly equilibrium.

Fig. 1 The market with a monopoly alone
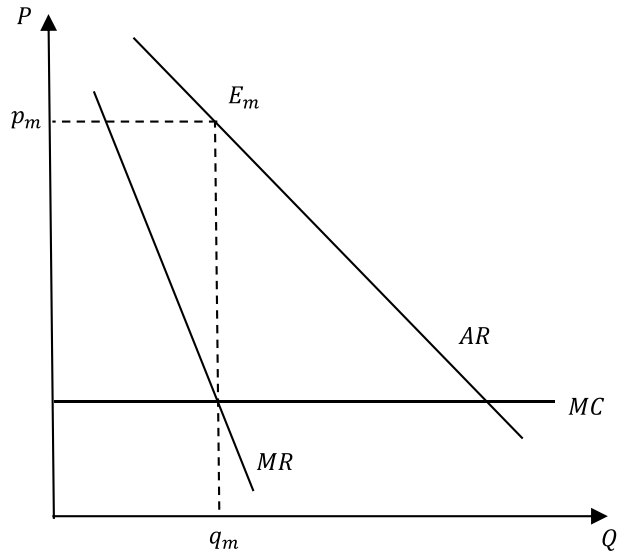
2.2 Consumers' Willingness To Pay and the Switching Costs

Unlike a market for goods, the utility market is a market for contracts of service in which consumers continue to receive service from the chosen supplier until they switch to another supplier, which is associated with switching costs.

At the monopoly equilibrium, there are two groups of consumers in the market. The first group, with the quantity q_m, receives service from the monopoly. As they have received service already, there are switching costs if they switch to another supplier. The second group, to the right of the point q_m, does not receive service from the monopoly, so they actively engage in the market without switching costs.

Consider p_i^w is the willingness to pay by individual i, represented by each level of the price corresponding to each point on the demand curve. Given the demand function for the market, Eq. (1), the willingness to pay ranges as

$$p_i^w \sim (0, a) \tag{6}$$

where zero is the minimum (the intercept point between the demand curve and the horizontal axis) and a is the maximum (the intercept point between the demand curve and the vertical axis). We assume that idiosyncratic switching costs are positively related to the willingness to pay, $\gamma_i'\left(p_i^w\right) > 0$. That is, a consumer who is willing to pay a higher price has higher switching costs. For simplicity, we assume that the switching cost is a fraction of the willingness to pay,
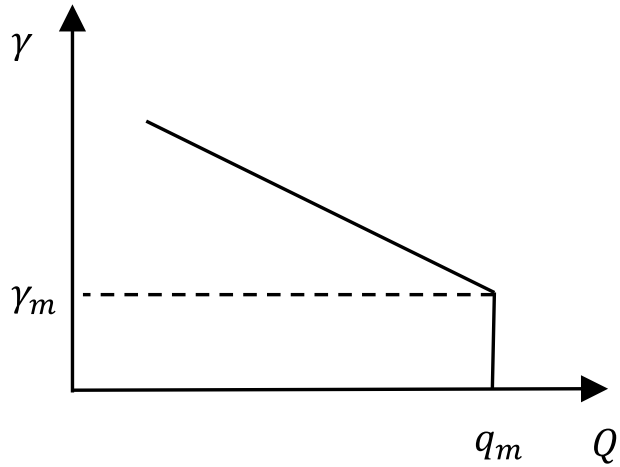
$$\gamma_i = \mu p_i^w \tag{7}$$

where μ is the constant fractional parameter, with a range of $0 < \mu < 1$.

Therefore, given the demand curve, Eq. (1), the curve of switching costs is

$$\gamma = \mu p = \mu a - \mu b q \text{ if } q < q_m \tag{8A}$$

Fig. 2 The curve of switching costs for consumers

$$\gamma = 0 \ if \ q > q_m \tag{8B}$$

which shows positive switching costs for consumers with service (the first group) and zero for consumers without service (the second group). The curve of switching costs is illustrated in Fig. 2, with a turning point at the marginal consumer, who has the lowest switching costs in the first group as $\gamma_m = \mu p_m$.

2.3 Consumers' Decision If There Is a New Supplier

Imagine that a new supplier enters the market and offers a lower price at p_s. Consumers in the second group (without service) will purchase the service from the small supplier if their willingness to pay is higher than the price,

$$p_i^w > p_s \tag{9}$$

Consumers in the first group are facing two options. First, if the consumer stays with the monopoly, the surplus is the difference between the willingness to pay and the monopoly price, $(p_i^w - p_m)$. Second, if the consumer switches to the new supplier, the surplus is the difference between the willingness to pay and the price after deducting the switching costs, $(p_i^w - p_s - \gamma_i)$.

Therefore, the consumers' decision depends on the price charged by the new supplier p_s and the switching costs γ_i. Consumers will stay with the monopoly if this option gives a higher surplus,

$$(p_i^w - p_m) > (p_i^w - p_s - \gamma_i) \leftrightarrow \gamma_i > \delta = p_m - p_s \tag{10}$$

which indicates that the price gap is insufficient to cover the switching costs. The price gap between these two prices is denoted as $\delta = p_m - p_s$. In contrast, consumers will switch to the new supplier if this option gives a higher surplus,

$$(p_i^w - p_m) < (p_i^w - p_s - \gamma_i) \leftrightarrow \gamma_i < \delta = p_m - p_s \tag{11}$$

which indicates that the price gap is large enough to cover the switching costs.

Fig. 3 The situation of Cooperation
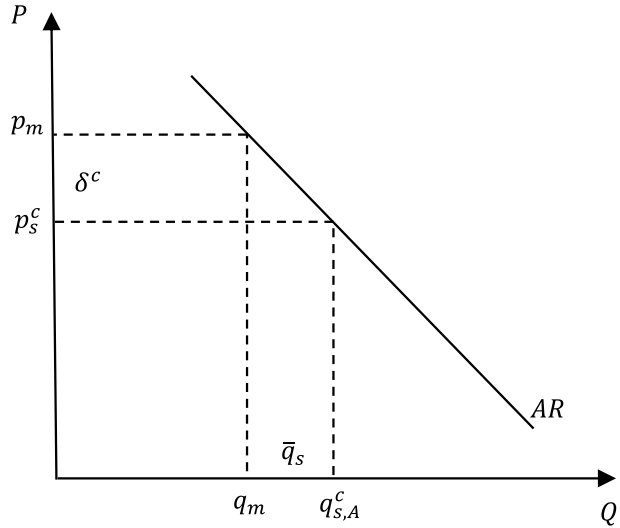
2.4 The Monopoly and the New Small Supplier

Assume the new supplier has a lower marginal cost c_s and a constrained capacity $\bar{q}_s$. For simplicity in the analysis, we assume that the capacity of the small supplier is below a certain level so that it does not behave like a monopoly on the residual demand.[9]

If the small supplier sets a price above or equal to the monopoly price p_m, there will be no demand from any consumer. Consumers in the first group will stay with the monopoly, and consumers in the second group will stay without service. Therefore, to gain consumers, the only option for the small supplier is to set a price below the monopoly price p_m.

2.4.1 Cooperation

If the small supplier aims to attract consumers without service (from the second group), it will set a price to attract those with the highest willingness to pay in this group until its capacity is met. In this case, as shown in Fig. 3, the consumers served by the small supplier ranged from q_m to $q_{s,A}^c$, equal to its capacity $\bar{q}_s$,

$$\bar{q}_s = q_{s,A}^c - q_m \tag{12}$$

where $q_{s,A}$ is the total consumers served by both the monopoly and the small supplier. Given the demand function, Eq. (1) and the quantity $q_{s,A}$, we can solve for the price set by the small supplier p_s as

$$p_s^c = a - b\left(q_m + \bar{q}_s\right) \tag{13}$$

Therefore, the profit received by the small supplier is
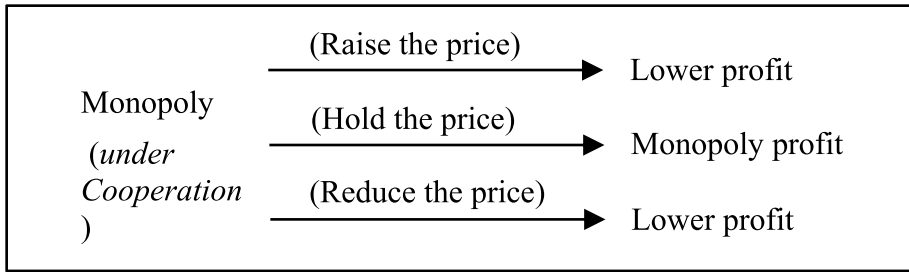
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[9] The condition is showed in Appendix 2.

Fig. 4 The monopoly's choice under Cooperation

$$\pi_s^c = \left(p_s^c - c_s\right)\left(q_{s,A}^c - q_m\right) = \left(p_s^c - c_s\right)\overline{q}_s \tag{14}$$

and the gap between the monopoly price and the price set by the small supplier is

$$\delta^c = p_m - p_s^c = p_m - \left[a - b\left(q_m + \overline{q}_s\right)\right] = b\overline{q}_s \tag{15}$$

In this situation, the monopoly is not undermined as the small supplier takes consumers who were previously not served, as shown in Fig. 3. This situation is referred to as Cooperation, in which the aggregate market price is lower than the monopoly price, i.e. the price is lower after the entry of the small supplier.

Here, we discuss the possible actions from the monopoly and the new entrant at Cooperation. First, the monopoly has three options. The monopoly still earns the monopoly profit if it holds the price. Raising or reducing the price will lower its profit as it moves up from the monopoly equilibrium or charges a lower price to its existing consumers. Therefore, the monopoly is not motivated to deviate from Cooperation, and the decision is illustrated in Fig. 4.

Second, regarding the incentive for the small supplier to deviate from Corporation, we will explain that its action depends on if the condition of undermining is met or not.

2.4.2 The Condition of Undermining Is Not Met

As shown by Eq. (10) and Eq. (11), consumers' switching decision is affected by the switching costs γ and the gap between the price charged by the monopoly and the small supplier, δ.

Using the function of switching costs Eq. (8A) and the monopoly quantity q_m, the switching costs for the marginal consumer who has the lowest willingness to pay in the first group is

$$\gamma_m = \mu p_m = \mu a - \mu b q_m \tag{16}$$

The fractional parameter μ decides the size of the switching costs and leads to two possible scenarios.

In the first scenario, when this fractional parameter is high, the switching costs are high, so the price gap is less than the switching costs of the marginal consumer in the first group,
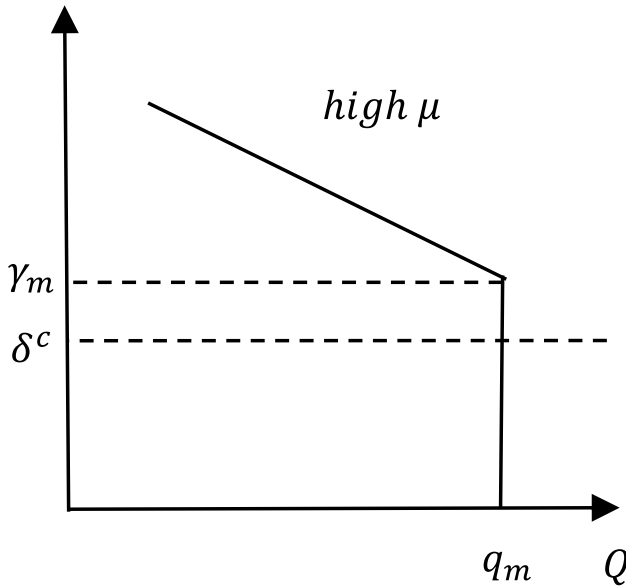
$$\delta^c < \gamma_m \tag{17}$$

Fig. 5 The condition of undermining is met when the fractional parameter μ is high
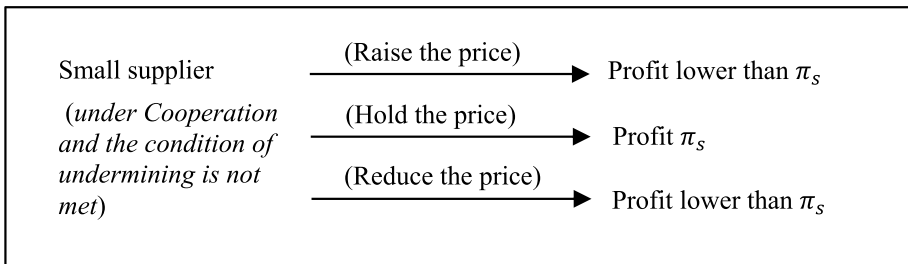

**Fig. 6** The small supplier's choice under Cooperation and the condition of undermining is not met

That is, none of the consumers with the monopoly is attracted by the small supplier, so the condition of undermining is not met, as shown in Fig. 5.

When the condition of undermining is not met, the decision of the small supplier is illustrated in Fig. (6). If the small supplier holds the price, the profit is π_s, as shown in Eq. (14). If the small supplier reduces the price, its profit is lower as the number of consumers remains the same, given its constrained capacity. On the other hand, if the small supplier raises the price, it will lose some consumers but not attract any consumers from the monopoly, leading to a lower profit.[10]

Therefore, when the condition of undermining is not met, the monopoly and the small supplier are not motivated to deviate from Cooperation. At this equilibrium, there is price differentiation, but the market price is lower than after the entry of the small supplier.

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[10] Appendix 3 shows the condition that the small supplier is not willing to continually increase its price when the quantity of consumers it serves is reduced.

Fig. 7 The condition of under-mining is met when the fractional parameter μ is low
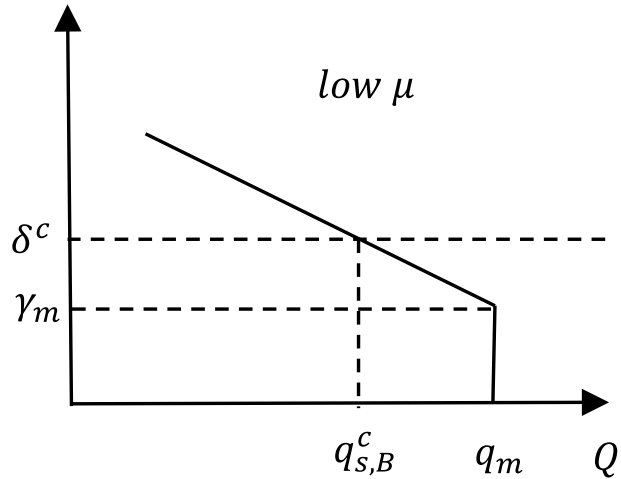
2.4.3 The Condition of Undermining Is Met

In the second scenario, when this fractional parameter is low, the switching costs are low, so the price gap is greater than the switching costs of the marginal consumer in the first group,

$$\delta^c > \gamma_m \tag{18}$$

That is, some consumers with the monopoly are attracted by the small supplier as they receive a higher surplus if they switch, so the condition of undermining is met, as shown in Fig. 7.

Given the price gap Eq. (15) and the switching costs Eq. (16), the condition of undermining in Eq. (18) can be written as

$$\delta^c > \gamma_m \rightarrow b\bar{q}_s > \mu p_m \tag{19}$$

Currently, the small supplier serves consumers ranging from q_m to $q_{s,A}^c$, as shown in Fig. 3. Besides, as shown in Fig. 7, consumers ranging from $q_{s,B}^c$ to q_m in the first group will gain if they switch. Therefore, the quantity of consumers who demand the service from the small supplier is greater than its capacity,

$$q_{s,A}^c - q_{s,B}^c > \bar{q}_s \tag{20}$$

In this case, there is an opportunity for the small supplier to undermine the monopoly, i.e. attract consumers from the monopoly. As shown in Fig. 8, the small supplier can raise its price to increase its profit, taking advantage of a higher willingness to pay from the consumers in the first group. In contrast, its profit will be lower if it reduces the price.

Therefore, when the condition of undermining is met, the monopoly is not motivated to deviate from Cooperation, but the small supplier is motivated to attract consumers from the monopoly and earn a higher profit.
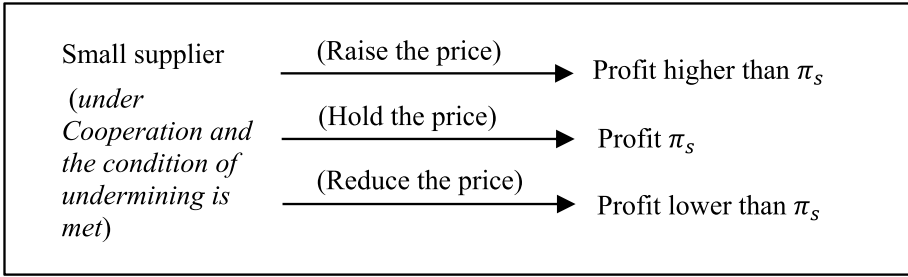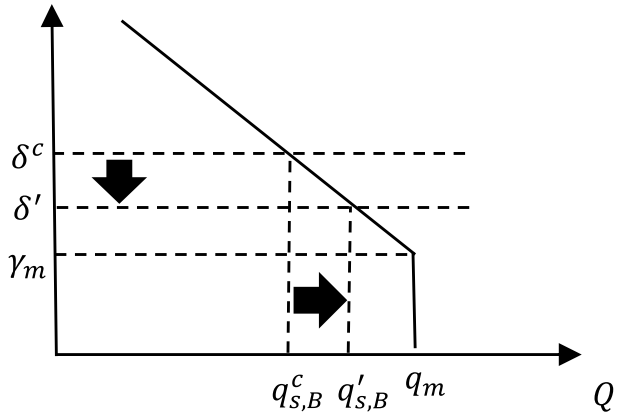
Fig. 8 The small supplier's choice under Cooperation and the condition of undermining is met

Fig. 9 A rising price by the small supplier reduces the number of consumers from the first group

2.4.4 The Process of Undermining

Assume the condition of undermining is met. The small supplier begins to raise its price to increase profits by attracting consumers with a higher willingness to pay. However, the rising price affects the number of consumers from both groups. First, as shown in Fig. 9, a reduced price gap leads to fewer consumers in the first group who are willing to switch. Second, as shown in Fig. 10, the number of consumers from the second group also declines because of the higher price.

The small supplier will stop raising its price at p'_s, with the price gap $\delta' = p_m - p'_s$. That is, the small supplier will increase the price until the demand from both groups is equal to its capacity,

$$q'_{s,A} - q'_{s,B} = \bar{q}_s \tag{21}$$

Using the demand function Eq. (1), we can find the expression for $q'_{s,A}$,

$$p'_s = a - b q'_{s,A} \leftrightarrow q'_{s,A} = \frac{a - p'_s}{b} = \frac{a - p_m + \delta'}{b} \tag{22}$$

Fig. 10 A rising price by the small supplier reduces the number of consumers from the second group
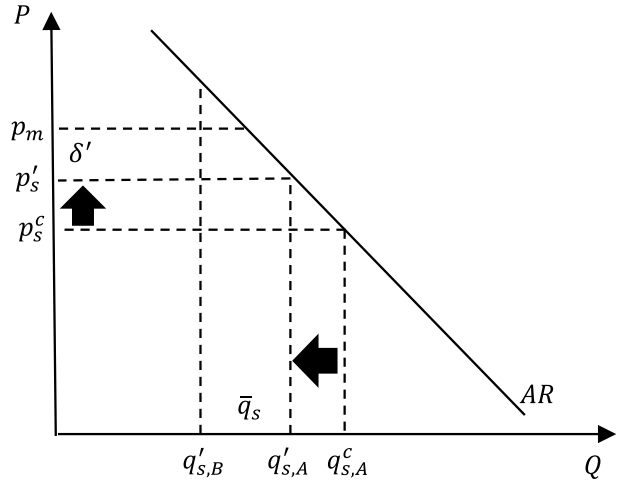
where p_m is the monopoly price. Using the switching costs function Eq. (8A), we can find the expression for $q'_{s,B}$,

$$\delta' = \mu a - \mu b q'_{s,B} \leftrightarrow q'_{s,B} = \frac{\mu a - \delta'}{\mu b} \qquad (23)$$

Both Eq. (22) and Eq. (23) depend on the price gap δ'. After replacing $q'_{s,A}$ and $q'_{s,B}$ into Eq. (21), we have,

$$q'_{s,A} - q'_{s,B} = \bar{q}_s \leftrightarrow \frac{a - p_m + \delta'}{b} - \frac{\mu a - \delta'}{\mu b} = \bar{q}_s \qquad (24)$$

Given the fixed capacity, $\bar{q}_s$, we solve the price gap δ' as,

$$\delta' = \frac{\mu}{1+\mu} b\bar{q}_s + \frac{\mu}{1+\mu} p_m \qquad (25)$$

The price set by the small firm p'_s as

$$p'_s = p_m - \delta' = \frac{1}{1+\mu} p_m - \frac{\mu}{1+\mu} b\bar{q}_s \qquad (26)$$

and the profit of the small firm is

$$\pi'_s = \left(p'_s - c_s\right)\bar{q}_s \qquad (27)$$

To confirm that the small supplier increases its price, we compare the price gap δ' in Eq. (25) and the previous price gap δ^c in Eq. (15), as

$$\delta' - \delta^c = \frac{\mu}{1+\mu} b\bar{q}_s + \frac{\mu}{1+\mu} p_m - b\bar{q}_s = \frac{1}{1+\mu}\left(\mu p_m - b\bar{q}_s\right) = \frac{1}{1+\mu}\left(\gamma_m - \delta\right) < 0 \qquad (28)$$

Equation (28) holds since Eq. (19), which is the condition for undermining. Therefore, since the price gap is less than the previous price gap, the price charged by the small supplier and its profit increased,

$$p_s' > p_s^c \rightarrow \pi_s' > \pi_s^c \tag{29}$$

Therefore, when the condition of undermining is met, some consumers in the first group are willing to switch to the small supplier. The small supplier undermines the monopoly by raising the price from p_s^c to p_s' and taking consumers from the monopoly. By doing so, the small supplier receives a higher profit from a higher price, $p_s' > p_s^c$, given that its capacity and marginal costs remain the same.

For further analysis, the switching costs of two marginal consumers with the small supplier at the point $q_{s,A}'$ and $q_{s,B}'$ are the following. Among all consumers with the small supplier, the marginal consumers at the point $q_{s,A}'$ has the lowest switching costs $\mu p_s'$,

$$\gamma_{s,A}' = \mu p_s' \tag{30}$$

In contrast, the marginal consumer at the point $q_{s,B}'$ has the highest switching costs δ',

$$\gamma_{s,B}' = \mu\left(a - bq_{s,B}'\right) = \delta' \tag{31}$$

2.5 Reactions from the Monopoly

After being undermined by the small supplier, the monopoly has three options: holding, raising, or reducing its price. For the monopoly, the number of consumers who switched to the small supplier is $\left(q_m - q_{s,B}'\right)$ and the number of captive consumers with it is $q_{s,B}'$.

2.5.1 Holding Its Price

Without any action, the monopoly's profit is $\pi_{m,hold}'$, which is less than the monopoly profit as part of its consumers switched to the small supplier,

$$\pi_{m,hold}' = (p_m - c)q_{s,B}' < \pi_m = (p_m - c)q_m \tag{32}$$

2.5.2 Rising Its Price

The second option for the monopoly is raising the price to explore captive consumers still with it. The strategy is to raise the price to the level that equals the lowest willingness to pay from its captive consumers. As shown in Fig. 11, using the demand equation Eq. (1) and the quantity of captive consumers $q_{s,B}'$, the higher price charged by the monopoly is

$$p_{m,h}' = a - bq_{s,B}' \tag{33}$$

Then its profit is higher than that of holding the price,

$$\pi_{m,high}' = \left(p_{m,high}' - c\right)q_{s,B}' > \pi_{m,hold}' = (p_m - c)q_{s,B}' \tag{34}$$

Fig. 11 The monopoly raises its price from p_m to $p'_{m,h}$
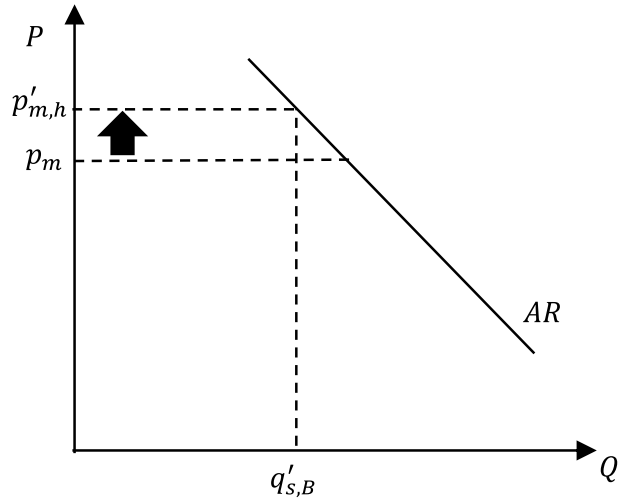
After the monopoly increases its price from p_m to $p'_{m,h}$, the price gap is widened from δ' to δ'',

$$\delta'' = p'_{m,h} - p'_s > \delta' = p_m \tag{35}$$

The widened price gap suggests that the small supplier now has another opportunity to attract consumers with an even higher willingness to pay, i.e. consumers ranging from $q''_{s,B}$ to $q'_{s,B}$. This is illustrated in Fig. 12.

The demand for service from the small supplier exceeds its capacity again as,

$$q'_{s,A} - q''_{s,B} > \bar{q}_s \tag{36}$$

Fig. 12 The impact of the widened price gap due to a higher price from the monopoly. Note that the marginal consumer with service becomes $q'_{s,A}$, instead of q_m
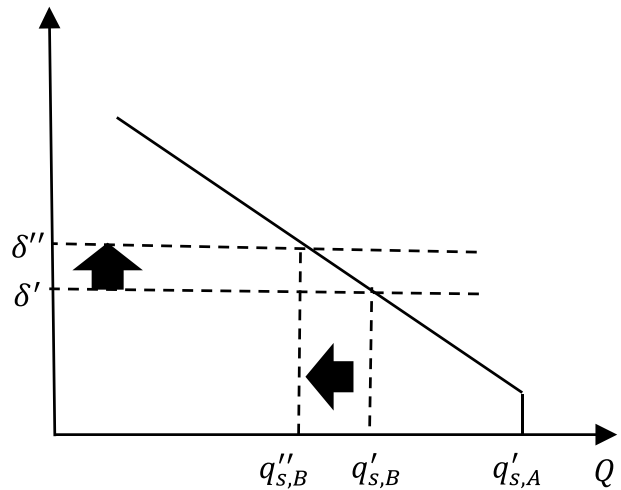
Then, by repeating the process of undermining, the small supplier raises the price to reduce the demand and serves a group of consumers with a slightly higher willingness to pay. Meanwhile, consumers who have the lowest willingness to pay are given up by the small supplier.

After the second round of undermining, the monopoly can raise its price further on the captive consumers still with it, and further undermining actions from the small supplier may be expected. As long as the condition of undermining is met, the small supplier is always motivated to undermine the monopoly. These upward one-way sequential actions may happen until the monopoly reaches zero profit when it charges the price high enough to drive out all its consumers,

$$\lim_{p_{m,high} \to a} \pi_{m,high} \to 0 \tag{37}$$

Therefore, raising prices continuously in sequential actions is not an optimal strategy. At some point, the monopoly should find it better to reduce its price and regain consumers from the small supplier.

2.5.3 Reducing Its Price

The third option for the monopoly is reducing its price to attract consumers back, as shown in Fig. 13 and Fig. 14, with three stages.

During the first stage, the monopoly reduces the price from p_m to p_s', which is the price charged by the small supplier. At the end of the first stage, the price charged by the monopoly is

$$p_{m,l1}' = p_s' \tag{38}$$

and the reverse price gap $\hat{\delta}$ (the gap between the price charged by the small supplier and the monopoly as the monopoly is trying to attract consumers back) is

$$\hat{\delta}_{l1} = p_s' - p_{m,l1}' = p_s' - p_s' = 0 \tag{39}$$

During this stage, the monopoly cannot attract additional consumers. Consumers served by the small supplier are unwilling to switch due to the switching costs. Meanwhile, consumers without service have a willingness to pay lower than p_s'. Therefore, during this

Fig. 13 The market when the monopoly reduces its price
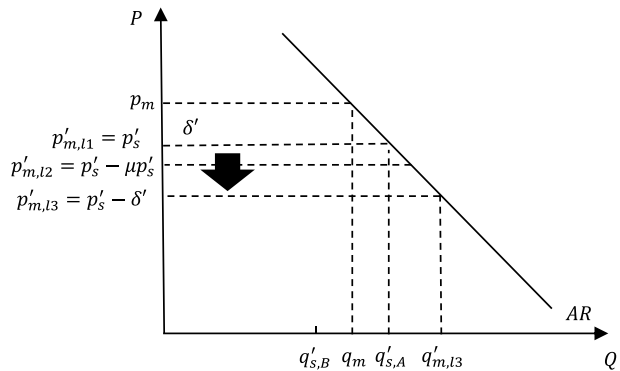
stage, the monopoly reduces the price charged to its captive consumers, leading to a lower profit. The profit at the end of this stage is

$$\pi'_{m,l1} = \left(p'_{m,l1} - c\right) \bullet q'_{s,B} \tag{40}$$

In the second stage, the monopoly continues reducing the price further below p'_s, and begins to attract consumers without service, i.e. consumers to the right of $q'_{s,A}$. This stage lasts until the price is reduced to the level below p'_s by the amount $\mu p'_s$, before attracting any consumers from the small supplier. In this stage, all consumers with the small supplier are still not interested as the reverse price gap is not large enough to cover the switching costs. Note that the lowest switching costs is $\mu p'_s$, indicated by Eq. (30).

At the end of the second stage, the price charged by the monopoly is

$$p'_{m,l2} = p'_s - \mu p'_s \tag{41}$$

The reverse price gap as

$$\widehat{\delta}_{l2} = p'_s - p'_{m,l2} = \mu p'_s \tag{42}$$

and the monopoly's profit is

$$\pi'_{m,l2} = \left(p'_{m,l2} - c\right) \bullet q'_{s,B} + \left(p'_{m,l2} - c\right) \bullet \left(\frac{a - p'_{m,l2}}{b} - q'_{s,A}\right) \tag{43}$$

in which the first term is the profit from captive consumers and the second term is the profit from attracting consumers without service previously.

During the third stage, the monopoly continues reducing the price to the level below p'_s by the amount of δ'. From Eq. (31), δ' is the switching costs of consumers at $q'_{s,B}$, who has the highest switching costs among consumers receiving service from the small supplier. Therefore, consumers with the small supplier, ranging from $q'_{s,B}$ to $q'_{s,A}$, are attracted by the monopoly gradually in ascending order of their switching costs, from right to left.

Fig. 14 The reverse price gap when the monopoly reduces its price
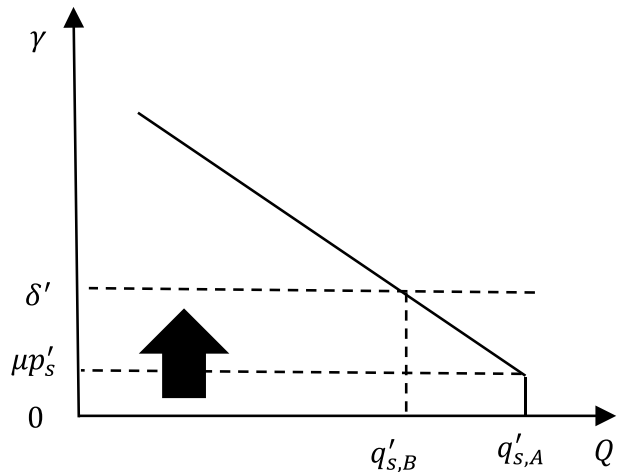
At the end of this stage, the price charged by the monopoly is

$$p'_{m,l3} = p'_s - \delta' = p_m - 2\delta' \tag{44}$$

That is, previously small supplier sets $p'_s = p_m - \delta'$ to attract this consumer, the monopoly needs to set $p_{m,l} = p'_s - \delta'$ to attract this consumer back. The reverse price gap is given as

$$\widehat{\delta}'_{l3} = p'_s - p'_{m,l3} = \delta' \tag{45}$$

During the third stage, consumers are attracted back from right to left on the demand curve as the monopoly reduces its price. The curve of switching costs Eq. (9A) can be rearranged as

$$q_{m,l} = \frac{\mu a - \gamma}{\mu b} = \frac{\mu a - \widehat{\delta}}{\mu b} = \frac{\mu a - \left(p'_s - p_{m,l}\right)}{\mu b} \tag{46}$$

which identifies the quantity of consumers given the price charged by the monopoly, as illustrated in Fig. 15. The difference between $q'_{s,A}$ and $q_{m,l}$ is the quantity of consumers who were attracted back by the monopoly, given its price $p_{m,l}$.

At the end of this stage, the monopoly's profit is

$$\pi'_{m,l3} = \left(p'_{m,l3} - c\right) \cdot q'_{s,B} + \left(p'_{m,l3} - c\right) \cdot \left(\frac{a - p'_{m,l3}}{b} - q'_{s,A}\right) + \left(p'_{m,l3} - c\right)\left(q'_{s,A} - \frac{\mu a - \left(p'_s - p'_{m,l3}\right)}{\mu b}\right) \tag{47}$$

in which the first term is the profit from captive consumers (0 to $q'_{s,B}$), the second term is the profit from consumers without service previously ($q'_{s,A}$ to $q'_{m,l3}$), and the third term is the profit from consumers switched from the small suppliers ($q'_{s,B}$ to $q'_{s,A}$).

The process of price reduction by the monopoly results in a piecewise function for its profit for three intervals of its price: from p_m to p'_s, from p'_s to $p'_s - \mu p'_s$, from $p'_s - \mu p'_s$ to $p'_s - \delta'$, respectively. The location of the maximum depends on the value of structural parameters. For simplicity, we assume that the monopoly compares the profits at the end of

Fig. 15 Identifying the quantity of consumers attracted by the monopoly using the curve of switching costs, given the reverse price gap
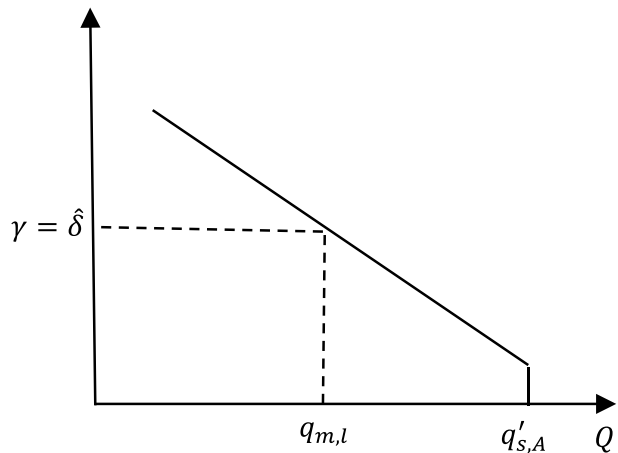
each interval. Based on our simulation results in Sect. 3, we consider that the profit at the end of the third interval is more likely to give a higher profit. Therefore, we assume that, if the monopoly reduces its price after being undermined, it will reduce to the level that takes all consumers from the small supplier.

Therefore, the small supplier lost all consumers and will have to reduce its price to attract consumers. The small supplier is always capable of setting a lower price given its lower marginal costs. These downward one-way sequential actions may happen until the monopoly reaches zero profit as its price is low enough to equal its marginal costs,

$$\lim_{p_{m,low} \to c} \pi_{m,low} \to 0 \tag{48}$$

Therefore, reducing prices continuously in sequential actions is not an optimal strategy. At some point, the monopoly should find it better to return to the monopoly equilibrium by charging a higher price to its consumers and giving up those with a low willingness to pay.

2.5.4 Summary

Figure 16 summarises the monopoly's payoff from its reaction after being undermined by the small supplier. If the monopoly holds the price, the profit is $\pi'_{m,hold}$, which is lower than the monopoly profit. If the monopoly raises the price on its captive consumers, the profit is $\pi'_{m,h}$, which is higher than the profit from holding the price. If the monopoly reduces its price to take all consumers from the small supplier, the profit is $\pi'_{m,l}$.

After being undermined, the monopoly may raise or reduce its price as a response, but the one-way price movement (up or down) is not optimal as the monopoly ends with zero profit (lose all consumers if up or the profit margin falls to zero if down). Therefore, at some point, the monopoly is expected to change the direction of the price movement to gain a higher profit. Nonetheless, the small supplier always responds as long as the condition of undermining is met.

2.6 State One

In this section, we propose a situation known as State One, in which the small supplier is not motivated to deviate, as shown in Fig. 17 and Fig. 18. At State One, the monopoly serves consumers up to $q^*_{s,B}$ with a price at p^*_m and the small supplier serves consumers at its full capacity ranging from $q^*_{s,B}$ to $q^*_{s,A}$ with a price p^*_s,
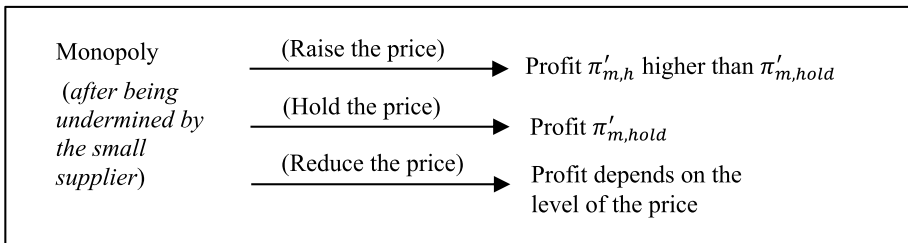
Fig. 16 The responses of the monopoly after being undermined by the small supplier
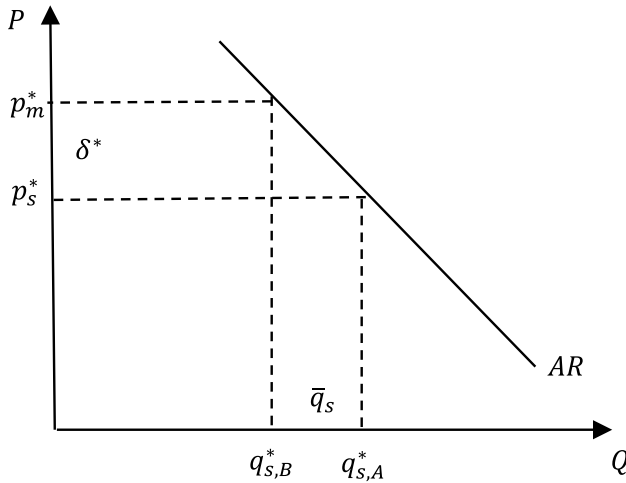
Fig. 17 The market at State One

Fig. 18 The switching costs function at State One
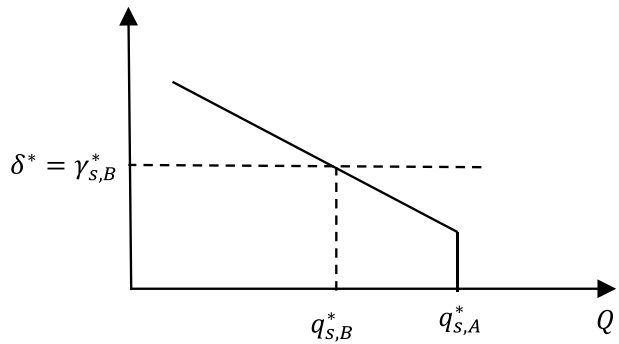
$$q_{s,A}^* - q_{s,B}^* = \bar{q}_s \tag{49}$$

State One is achieved when the optimal price gap, $\delta^* = p_m^* - p_s^*$, is equal to the switching costs $\gamma_{s,B}^*$ of the marginal consumer with the monopoly at $q_{s,B}^*$.

State One can be solved as the following. Using the demand function Eq. (1), $q_{s,A}^*$ is expressed as

$$p_s^* = a - bq_{s,A}^* \Rightarrow q_{s,A}^* = \frac{a - p_s^*}{b} = \frac{a - p_m^* + \delta^*}{b} \tag{50}$$

Using the switching costs function Eq. (9A), $q_{s,B}^*$ is expressed as

$$\delta^* = \gamma_{s,B}^* = \mu a - \mu b q_{s,B}^* \rightarrow q_{s,B}^* = \frac{\mu a - \delta^*}{\mu b} \tag{51}$$

Using the demand equation Eq. (1), and the full capacity equation Eq. (49), the optimal gap is solved as

$$\delta^* = p_m^* - p_s^* = \left(a - bq_{s,B}^* \right) - \left(a - bq_{s,A}^* \right) = b\overline{q}_s \tag{52}$$

Substituting the optimal price gap Eq. (52) into Eq. (51), the quantity served by the monopoly $q_{s,B}^*$ is solved as

$$q_m^* = q_{s,B}^* = \frac{\mu a - b\overline{q}_s}{\mu b} > 0 \tag{53}$$

which is greater than zero if

$$\mu a > b\overline{q}_s \tag{54}$$

which means that the switching costs of the consumer who has the highest willingness to pay are greater than the optimal gap, so at least one consumer stays with the monopoly.

After having the quantity, the price set by the monopoly price is

$$p_m^* = a - bq_{s,B}^* \rightarrow p_m^* = a - b\frac{\mu a - b\overline{q}_s}{\mu b} = \frac{b\overline{q}_s}{\mu} \tag{55}$$

which shows that the price charged by the monopoly is positively related to the capacity of the small supplier $\overline{q}_s$ but negatively related to the fractional parameter of the switching costs μ. Using the condition of undermining Eq. (19), the price charged by the monopoly p_m^* is higher than the monopoly price p_m,

$$b\overline{q}_s > \mu p_m \rightarrow \frac{b\overline{q}_s}{\mu} > p_m \rightarrow p_m^* > p_m \tag{56}$$

It can also be shown that the price charged by the small supplier is

$$p_s^* = p_m^* - \delta^* = \frac{b\overline{q}_s}{\mu} - b\overline{q}_s = \left(\frac{1 - \mu}{\mu} \right) b\overline{q}_s \tag{57}$$

Figure 19 summarises actions at State One. If the small supplier reduces the price, its profit will be lower, as the number of consumers it serves remains the same. On the other hand, if it raises the price, the reduced price gap will be lower than the switching costs of any consumers with the monopoly, so it will not attract any consumers with a higher willingness to pay but only lose consumers on the lower end.

For the monopoly, if it holds the price, the profit is

$$\pi_{m,hold}^* = \left(p_m^* - c \right) q_{s,B}^* \tag{58}$$

If the monopoly raises the price to $p_{m,h}^*$, the profit is

$$\pi_{m,h}^* = \left(p_{m,h}^* - c \right) q_{s,h}^* < \pi_{m,hold}^* \tag{59}$$

which is lower than the profit from holding the price, as it moves further away from the monopoly equilibirum. In contrast, the monopoly may receive higher profit from reducing its price. Using the profit function from price reduction Eq. (47), if the monopoly reduces the price to take all consumers from the small supplier, the profit is
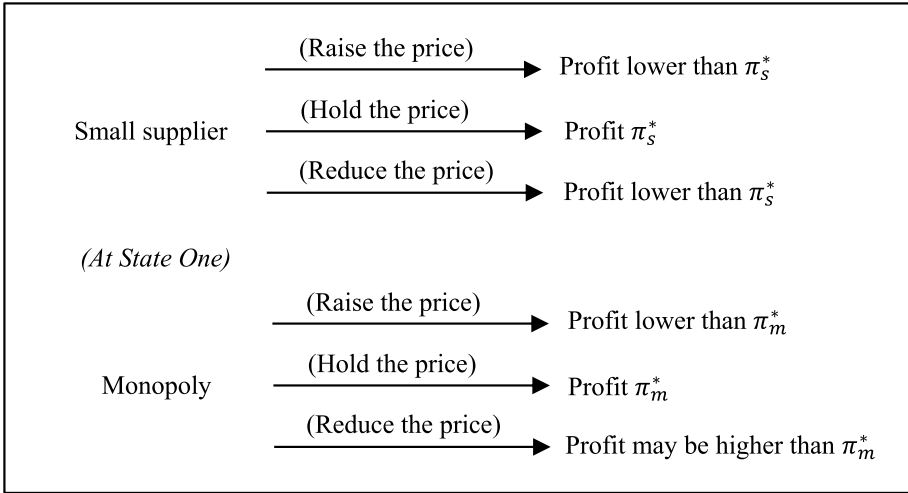
Fig. 19 The responses of the monopoly and the small supplier at State One

$$\pi^*_{m,l3} = \left(p^*_{m,l3} - c\right) \bullet \left(q^*_{s,B} + \frac{a - p^*_{m,l3}}{b} - \frac{\mu a - \left(p^*_s - p^*_{m,l3}\right)}{\mu b}\right) \tag{60}$$

At State One, the small supplier is not motivated to deviate, but the monopoly may receive a higher profit from reducing its price. However, several considerations may prevent the monopoly from reducing the price. First, this is the only situation where the small supplier is unwilling to deviate. If the monopoly, as the market leader, is concerned about market stability, it may be unwilling to deviate because the small supplier will always respond. Second, if a price war is triggered, in which the small supplier cuts prices immediately, the monopoly cannot attract any consumers back when its price is reduced, leading to lower profits only. Third, the small supplier has lower marginal costs, so it will not be squeezed out of the market in any price war because it can undercut any price charged by the monopoly.

2.7 State Two

Assume that State One is achieved. Although there are considerations, the small supplier knows that the monopoly may reduce the price and attract consumers back for a higher profit. However, the small supplier can prevent this from happening if it reduces its price to a level that makes the monopoly indifferent between holding and reducing the price, but at the cost of a lower profit to the small supplier.

At State One, the marginal consumer with the small supplier, who has the highest willingness to pay, is at $q^*_{s,B}$ with the switching costs δ^*. The monopoly can attract this consumer back when it reduces its price to

$$p^*_{m,l3} = p^*_s - \delta^* \tag{61}$$

Fig. 20 The market at State Two
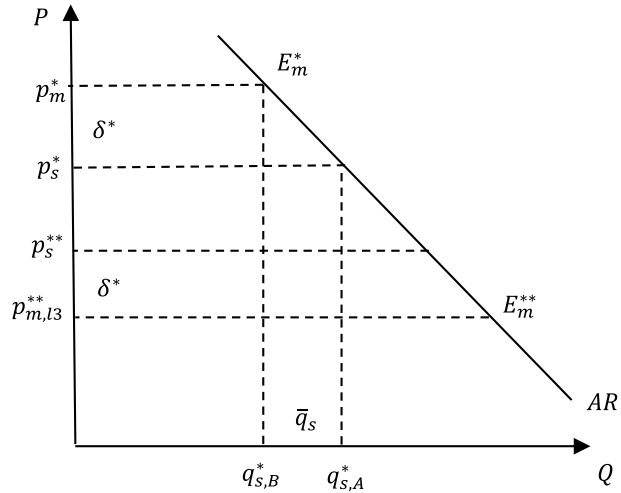
However, if the small supplier reduces its price from p_s^* to p_s^{**}, the monopoly needs to reduce its price to a lower level to attract this consumer back,

$$p_{m,l3}^{**} = p_s^{**} - \delta^* < p_{m,l3}^* \tag{62}$$

As this lower price $p_{m,l3}^{**}$ is also applied to its other consumers, the monopoly's profit is not necessarily higher if it reduces its price to such a low level to attract all consumers from the small supplier.

If the monopoly reduces its price, a specific price level produces the same profit as staying at State One. After knowing this specific price level, the small supplier has the targeted level for its price reduction if it does not wish the monopoly to deviate from State One. State Two is achieved after the small supplier reduces its price to this targeted level p_s^{**}, as shown in Fig. 20.

State Two can be identified in the following way. First, if the monopoly holds the price at p_m^*, its profit is

$$\pi_m^* = (p_m^* - c)q_{s,B}^* = \left(\frac{b\overline{q}_s}{\mu} - c\right) \bullet \frac{\mu a - b\overline{q}_s}{\mu b} \tag{63}$$

Through calculation,[11] the price level that produces the same profit (after taking all consumers from the small supplier) for the monopoly is

$$p_{m,l3}^* = a + c - \frac{b\overline{q}_s}{\mu} \tag{64}$$

Then the price set by the small supplier is

$$p_s^{**} = p_{m,l}^* + \delta^* = a + c + \left(\frac{\mu - 1}{\mu}\right)b\overline{q}_s \tag{65}$$

---

[11] Appendix 4 calculates the two price levels that produce the same level of profit.

Figure 21 summarises actions at State Two. The monopoly is not motivated to reduce its price to attract all consumers from the small supplier as the profit remains the same. To achieve State Two, the small supplier needs to reduce its price and sacrifice its profit. The small supplier is motivated to raise its price back for a higher profit, but considerations such as market stability may prevent it from doing so.

2.8 Equilibrium

At State One, the small supplier is not motivated to deviate. At State Two, the monopoly is not motivated to deviate. Therefore, an equilibrium can be achieved when these two States coincide together. That is, the price charged by the small supplier at State One is exactly the price making the monopoly unprofitable from reducing its price,

$$p_s^E = p_s^* = p_s^{**} \rightarrow \left(\frac{1-\mu}{\mu} \right) b\bar{q}_s = a + c - \frac{b\bar{q}_s}{\mu} + b\bar{q}_s \rightarrow \frac{a+c}{2} = \frac{1-\mu}{\mu} b\bar{q}_s \quad (66)$$

which imposes a condition on the values of structural parameters. When Eq. (66) holds, there is an Equilibrium in which both the small supplier and the monopoly are unwilling to deviate, as shown in Fig. 22.

This Equilibrium can be solved in the following sequence. The equilibrium price gap is

$$\delta^E = b\bar{q}_s \quad (67)$$

Since the marginal consumer at $q_{s,B}^E$ has switching costs δ^E, so the quantity with the monopoly can be solved using the switching costs function as

$$q_m^E = \frac{\mu a - \delta^E}{\mu b} = \frac{\mu a - b\bar{q}_s}{\mu b} \quad (68)$$
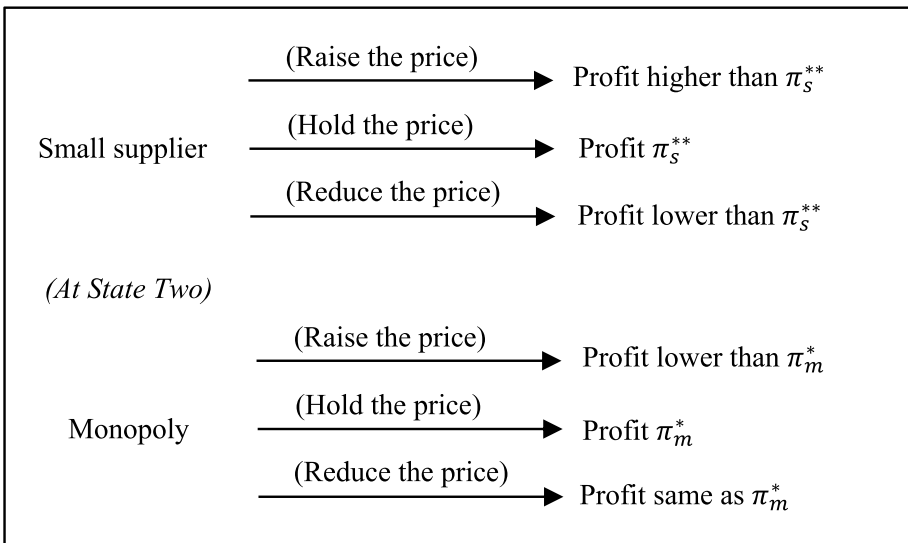
Fig. 21 The responses of the monopoly and the small supplier at State Two

Fig. 22 The market at Equilibrium
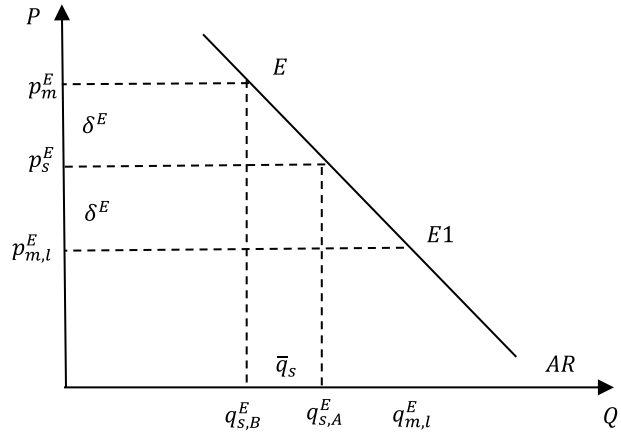
Then using the demand function, the price charged by the monopoly is

$$p_m^E = a - bq_m^E = \frac{b\bar{q}_s}{\mu} \tag{69}$$

Using the equilibrium price gap to solve the price charged by the small supplier as

$$p_s^E = p_m^E - \delta^E = \frac{b\bar{q}_s}{\mu} - b\bar{q}_s = \frac{1-\mu}{\mu} b\bar{q}_s \tag{70}$$

Substituting Eq. (66) into Eq. (70), it can be shown that the price charged by the small supplier at Equilibrium is the monopoly price,

$$p_s^E = \frac{1-\mu}{\mu} b\bar{q}_s = \frac{a+c}{2} = p_m \tag{71}$$

which implies that both the monopoly and the small supplier serve the same number of consumers as the monopoly quantity,

$$q_m^E = q_m \tag{72}$$

3 Simulation

3.1 Monopoly and Cooperation

We first simulate the market with the monopoly alone and Cooperation. In the benchmark scenario, we consider the constant of the demand function as $a = 100$, the slope of the demand function as $b = 2$, the fractional parameter of the switching costs as $\mu = 0.2$, the capacity of the small supplier as $\bar{q}_s = 8$, the marginal costs of the monopoly as $c = 5$, and the marginal costs of the small supplier as $c_s = 4$. For clarity, a monetary sign £ is added to all monetary values.

Table 1 shows that, when only the monopoly is in the market, it charges the price of £52.5 to 23.8 consumers, giving a profit of £1128.1. After the entry of the small supplier, the simulation for Cooperation is also shown in Table 1. If the small supplier only takes consumers without service, it charges the price of £36.5 to 8 consumers, giving a profit of £260.0.

When only the monopoly is in the market, the price is £52.5. However, after the entry of the small supplier, the (quantity-weighted) market price is reduced to £48.5 because the small supplier offers a lower price to consumers who previously had no service. The six additional scenarios with varying values of parameters show similar results.

3.2 Small Supplier Undermines and the Monopoly's Reactions

When the condition of undermining is met, the small supplier begins to undermine the monopoly. Table 2 shows the simulation after the small supplier undermines the monopoly for the first time. The small supplier increases its price to £41.1 and still serves 8 consumers. The monopoly's consumers fall from 23.8 to 21.5 because the small supplier takes 2.3 consumers. The small supplier receives a profit of £296.7, higher than £260.0 under Cooperation. Therefore, the small supplier is motivated to undermine the monopoly.

After being undermined by the small supplier, the monopoly has three options. First, by holding the price, its profit is £1019.3. If the monopoly charges a higher price on the

Table 1 The simulation results at (i) the monopoly alone and (ii) cooperation

Scenarios	Structural parameters						Monopoly alone			Cooperation			
	a	b	μ	$\bar{q}_s$	c	c_s	p_m	q_m	π_m	p_s^c	q_s^c	π_s^c	p_{agg}^c
Benchmark	100	2	0.2	8	5	4	52.5	23.8	1128.1	36.5	8	260.0	48.5
Lower b	100	1.9	0.2	8	5	4	52.5	25.0	1187.5	37.3	8	266.4	48.8
Higher b	100	2.1	0.2	8	5	4	52.5	22.6	1074.4	35.7	8	253.6	48.1
Lower μ	100	2	0.18	8	5	4	52.5	23.8	1128.1	36.5	8	260.0	48.5
Higher μ	100	2	0.22	8	5	4	52.5	23.8	1128.1	36.5	8	260.0	48.5
Lower $\bar{q}_k$	100	2	0.2	7.5	5	4	52.5	23.8	1128.1	37.5	7.5	251.3	48.9
Higher $\bar{q}_k$	100	2	0.2	8.5	5	4	52.5	23.8	1128.1	35.5	8.5	267.8	48.0

Table 2 The simulation results at (i) the small supplier undermines and (ii) the monopoly's reactions

Scenarios	Small supplier undermines					Monopoly's reaction				
	p_m'	q_m'	p_s'	q_s'	π_s'	$\pi_{m,hold}'$	$\pi_{m,h}'$	$\pi_{m,l1}'$	$\pi_{m,l2}'$	$\pi_{m,l3}'$
Benchmark	52.5	21.5	41.1	8	296.7	1019.3	1117.6	774.3	712.5	867.4
Lower b	52.5	22.9	41.2	8	297.7	1089.6	1179.4	830.8	763.0	919.5
Higher b	52.5	20.1	41.0	8	295.6	955.7	1061.3	723.3	666.8	820.3
Lower μ	52.5	21.0	42.1	8	304.4	996.3	1112.7	777.1	729.9	909.8
Higher μ	52.5	21.9	40.1	8	289.2	1041.5	1121.5	770.7	693.2	823.0
Lower $\bar{q}_k$	52.5	21.9	41.3	7.5	279.4	1039.1	1121.1	793.0	728.0	875.0
Higher $\bar{q}_k$	52.5	21.0	40.9	9.5	313.8	999.5	1113.5	755.7	697.0	859.8

captive consumers, its profit is higher at £1117.6. Alternatively, if the monopoly decides to reduce the price, the profit is at £774.3, £712.5, and £867.4 at the end of each of the three stages (attract none; attract consumers without service; attract consumers from the small suppliers). By comparing these payoffs, raising the price on the captive consumers gives a higher profit. However, the higher price set by the monopoly increases the price gap, allowing the small supplier to undermine further.

3.3 State One

Table 3 shows the simulation at State One, where the small supplier is not motivated to deviate. At State One, the monopoly charges the price of £80 to 10 consumers, giving a profit of £750. The small supplier charges the price of £64 to 8 consumers, giving a profit of £480. The monopoly charges a higher price and provides a lower quantity than the monopoly equilibrium. The small supplier charges a lower price than the monopoly, and the quantity-weighted market price is £72.9, higher than the monopoly price of £52.5, suggesting that the market price is higher after the entry of the small supplier.

Table 3 also shows the monopoly's profit if the monopoly reacts. The monopoly will lose consumers if it raises the price, leading to a lower profit (moving further away from the monopoly position). In contrast, if the monopoly reduces the price, its profit will be £590.0, £757.7, and £1118.0 at the end of each of the three stages. Specifically, the monopoly needs to reduce its price to £48 (£64—£16) to take all consumers from the small supplier. By comparing these payoffs, reducing its price to attract consumers back from the small supplier gives a higher profit to the monopoly. Nonetheless, if the monopoly reduces its price, the small supplier will reduce its price further to regain consumers.

3.4 State Two

Table 4 shows the simulation at State Two, where the monopoly is not motivated to deviate. To achieve State Two, the small supplier reduces its price from £64 at State One to

Table 3 The simulation results at State One

Scenarios	State One						Monopoly's reaction				
	p_m^*	q_m^*	p_s^*	q_s^*	π_s^*	p_{agg}^*	$\pi_{m,hold}^*$	$\pi_{m,h}^*$	$\pi_{m,l1}^*$	$\pi_{m,l2}^*$	$\pi_{m,l3}^*$
Benchmark	80.0	10.0	64.0	8	480.0	72.9	750.0	747.2	590.0	757.7	1118.0
Lower b	76.0	12.6	60.8	8	454.4	70.1	896.8	894.4	704.8	830.5	1162.4
Higher b	84.0	7.6	67.2	8	505.6	75.4	601.9	598.9	473.9	683.6	1072.3
Lower μ	88.9	5.6	72.9	8	551.1	79.4	466.0	462.4	377.2	663.6	1118.5
Higher μ	72.7	13.6	56.7	8	421.8	66.8	923.6	921.5	705.4	780.1	1058.8
Lower $\bar{q}_k$	75.0	12.5	60.0	7.5	420.0	69.4	875.0	872.7	687.5	795.5	1100.0
Higher $\bar{q}_k$	85.0	7.5	68.0	8.5	544.0	76.0	600.0	596.7	472.5	706.4	1127.0

Table 4 The simulation results at State Two

Scenarios	State Two						Monopoly's reaction				
	p_m^*	q_m^*	$p_{s,l}^*$	q_s^*	π_s^*	p_{agg}^*	$\pi_{m,hold}^*$	$\pi_{m,h}^*$	$\pi_{m,l1}^{**}$	$\pi_{m,l2}^{**}$	$\pi_{m,l3}^{**}$
Benchmark	80.0	10.0	41.0	8	296.0	62.7	750.0	747.2	590.0	647.3	750.0
Lower b	76.0	12.6	44.2	8	321.6	63.7	896.8	894.4	704.8	750.9	896.8
Higher b	84.0	7.6	37.8	8	270.4	60.3	601.9	598.9	473.9	542.4	601.9
Lower μ	88.9	5.6	32.1	8	224.9	55.4	466.0	462.4	377.2	454.8	466.0
Higher μ	72.7	13.6	48.3	8	354.2	63.7	923.6	921.5	705.4	742.2	923.6
Lower $\bar{q}_k$	75.0	12.5	45.0	7.5	307.5	63.8	875.0	872.7	687.5	728.0	875.0
Higher $\bar{q}_k$	85.0	7.5	37.0	8.5	280.5	59.5	600.0	596.7	472.5	548.3	600.0

£41, giving a reduced profit of £296.0. If the monopoly holds its price, its profit is £750.0. Instead, if the monopoly reduces the price, its profit will be at £590.0, £647.3, and £750.0 at the end of each of the three stages. In particular, the monopoly needs to reduce its price to £25 (£41-£16) to take all consumers from the small supplier, but the monopoly receives the same profit as that of holding the price, so it is not motivated to deviate. Besides, the aggregate price is £62.7, which is higher than the monopoly price, suggesting that the market price is higher after the entry of the small supplier.

At State Two, the small supplier reduces its price and earns a lower profit than State One, so it may not be willing to reduce its price. However, its profit (£296.0) at State Two is higher than that under Cooperation (£260.0) and similar to that after undermining the monopoly for the first time (£296.7).

3.5 Equilibrium

As discussed in Sect. 2.8, Equilibrium can be found when Eq. (66) holds. Table 5 shows Equilibrium for three scenarios with different values of parameters. For illustration, in Case 1, given the values of parameters, the monopoly price is £62.4, and the monopoly

Table 5 The simulation results at Equilibrium

	Structural parameters						Monopoly alone		
	a	b	μ	$\bar{q}_s$	c	c_s	p_m	q_m	π_m
Case 1	120	1.95	0.2	8	4.8	4	62.4	29.5	1701.4
Case 2	80	1.6	0.24	9	11.2	4	45.6	21.5	739.6
Case 3	200	2.8	0.15	6.5	6.2	4	103.1	34.6	3353.4
	Equilibrium						Monopoly's reaction		
	p_m^E	q_m^E	p_s^E	q_s^E	π_s^E	p_{agg}^E	$\pi_{m,hold}^E$	$\pi_{m,h}^E$	$\pi_{m,l3}^E$
Case 1	78.0	21.5	62.4	8	467.2	73.8	1576.6	1575.0	1576.6
Case 2	60.0	12.5	45.6	9	374.4	54.0	610.0	608.2	610.0
Case 3	121.3	28.1	103.1	6.5	643.9	117.9	3234.7	3233.4	3234.7

quantity is 29.5, giving a profit of £1701.4 to the monopoly when it is alone in the market. At Equilibrium, the monopoly charges a price of £78.0 to 21.5 consumers, giving a profit of £1576.6. The monopoly is not motivated to raise or reduce its price.

Meanwhile, the small supplier charges a price of £62.4 to 8 consumers, giving a profit of £467.2. As discussed previously, the small supplier is not motivated to deviate. At Equilibrium, the monopoly and the small supplier charge different prices, and the quantity-weighted market price is £73.8, higher than the price of £62.4 before the small supplier enters the market.

The small supplier charges exactly the monopoly price, suggesting that the number of consumers served by both the monopoly and the small supplier is the same as those served by the monopoly before the entry of the small supplier. Similar results are found in both Case 2 and Case 3, with various values of parameters to hold Eq. (66).

4 Discussion

4.1 The Monopoly and the Small Supplier

We start with a monopoly as a benchmark model and then introduce a small supplier with a lower marginal cost and a constrained capacity. After the entry of the small supplier, we assume that the small supplier charges a lower price and takes the consumers who previously had no service without touching the consumers and the profit of the monopoly. This situation is referred to as Cooperation.

The monopoly is not motivated to deviate from Cooperation but the small supplier's choice depends on if the condition of undermining is met. The condition of undermining compares the price gap between the monopoly and the small supplier and the switching cost of the marginal consumer with the monopoly (the consumer has the lowest willingness to pay).

If the price gap is less than the switching costs of the marginal consumer, the condition of undermining is not met. In this case, the small supplier is not able to attract any consumers with the monopoly, so it is not motivated to deviate. Therefore, Cooperation can be considered an equilibrium because both the monopoly and the small supplier are unwilling to deviate. At Cooperation, there is price differentiation, but the quantity-weighted market price is lower after the entry of the small supplier.

In contrast, if the price gap is greater than the switching costs of the marginal consumer, the condition of undermining is met because at least one consumer with the monopoly is willing to switch to the small supplier. In this case, for the small supplier, the demand for its service is greater than its capacity of supply, so it is motivated to charge a higher price and serve consumers with a higher willingness to pay.

After being undermined by the small supplier, the monopoly can raise or reduce its price. The monopoly can charge a higher price to its captive consumers without losing any of them, so the profit is greater than that from holding the price. If the monopoly raises the price on the captive consumers, the price gap increases, making some of its consumers find the price from the small supplier attractive. Then the small supplier has another opportunity to undermine the monopoly again. If the monopoly continues to raise its price on its

captive consumers after successive rounds of undermining, its profit will approach zero as the number of captive consumers approaches zero.

Alternatively, the monopoly can reduce its price to retake consumers in three stages: attracting no one, attracting consumers without service, and attracting consumers from the small supplier. The monopoly will reduce the price if the profit from the price reduction is higher than that from raising the price. After losing all its consumers, the small supplier will reduce its price further to regain consumers, and the price war may continue. If the price is continuously reduced, the monopoly's profit will approach zero as the price approaches the marginal costs.

Regardless the monopoly decides to raise or lower its price, the small supplier is expected to respond, which leads to further action from the monopoly. A sequence of responses is expected in the market, but the one-way price movement is not an optimal strategy, as the monopoly's profit will be higher if it changes the direction of price movement at some point.

4.2 State One, State Two, and Equilibrium

The small supplier always undermines the monopoly when the condition of undermining is met. When the price gap is slightly lower than the switching costs of the marginal consumer with the monopoly, the small supplier is not able to attract consumers from the monopoly anymore. This situation is referred to as State One, where the small supplier has no motivation to deviate, but the monopoly may receive a higher profit by reducing its price and taking consumers back from the small supplier.

If the small supplier sets its price lower, the monopoly has to reduce its price further to take back consumers. The reduced price by the monopoly is applied to all its consumers, possibly leading to a lower profit. Therefore, the small supplier can reduce its price to a specific level which makes the monopoly indifferent between holding its price and reducing its price to take back consumers. This situation is referred to as State Two, where the monopoly is not motivated to deviate, but the small supplier may raise back its price to earn a higher profit.

The analysis of State One and State Two suggests the existence of Equilibrium, in which both the monopoly and the small supplier have no motivation to deviate from it. This Equilibrium is achieved when State One coincides with State Two. That is, the price charged by the small supplier at State One is exactly the specific level that makes the monopoly indifferent between holding and reducing its price in State Two.

However, this Equilibrium imposes an additional condition on the values of structural parameters. If the condition is not met, the interactions between the monopoly and the small supplier will be unending. However, we suggest that, as at least one player is not motivated to deviate, State One and State Two can be two possible places to settle. If the monopoly, as a market leader, is more concerned about market stability, it may accept a lower profit at State One, as the small supplier will not respond further. In contrast, if the small supplier, as a new firm, is more concerned about market stability, it may settle at State Two for a lower profit, as the monopoly will not respond further.

4.3 Price Differentiation and the Aggregate Price

Our analysis was motivated by two observations in the UK retail electricity market. One observation is that there was price differentiation between incumbents and small suppliers. In all four situations of our analysis (Cooperation, State One, State Two, and Equilibrium), the monopoly charges a higher price than the small supplier. Another observation is that the price level rose after the entry of small suppliers. From our analysis, after the entry of the small supplier, the market price is lower at Cooperation, but is higher at State One, State Two, and Equilibrium.

5 Conclusion

There were two observations in the UK retail electricity market: (i) the co-occurrence of a declining market concentration and a rising price and (ii) the price differentiation between incumbents and small suppliers. To explain these two observations, we construct an infinite sequential game in which an existing monopoly faces the entry of a small supplier with a lower marginal cost and a constrained capacity.

When the small supplier enters the market, suppose it charges a lower price and takes the consumers who previously had no service without touching the consumers and the profit of the monopoly. The situation is *Cooperation*. When the condition of undermining is not met, i.e. the price gap is less than the switching costs of the marginal consumers with the monopoly, the small supplier cannot attract consumers from the monopoly so it is not motivated to deviate. In this case, *Cooperation* can be considered an equilibrium, in which there is price discrimination but the market price is lower after the entry of the small supplier.

When the condition of undermining is met, i.e. the price gap is greater than the switching costs of the marginal consumers, the small supplier is motivated to take consumers from the monopoly. After being undermined by the small supplier, the monopoly can raise its price on captive consumers or reduce its price to attract consumers back, depending on which option provides a higher profit. After the choice made by the monopoly, the small supplier will adjust its price further as a response, and then a sequence of actions by the monopoly and the small supplier is expected. Nonetheless, there are two situations where at least one of them is not motivated to deviate.

After successive rounds of undermining, the condition of undermining becomes marginally unmet, so the small supplier cannot attract consumers from the monopoly anymore. This situation is *State One*, in which the small supplier is not motivated to deviate but the monopoly may receive a higher profit by reducing its price and taking consumers back. However, the small supplier can reduce its price to a level which makes the monopoly indifferent between holding and reducing its price, and this situation is *State Two*, where the monopoly is not motivated to deviate but the small supplier may raise back the price for a higher profit.

State One coincides with State Two when the price charged by the small supplier at State One is exactly the specific level that makes the monopoly indifferent between holding and reducing its price. This situation is *Equilibrium*, in which both the monopoly and the small supplier are not motivated to deviate. The observations of price differentiation and rising prices after the entry of the small supplier are all found at State One, State Two, and Equilibrium.

However, Equilibrium imposes a condition on the values of parameters in the model. If the condition is not met, State One and State Two can be two possible places to settle during the unending interactions between the monopoly and the small supplier, but the choice depends on which player is more concerned about market stability.

Our analysis has been based on four features, which jointly help our model reproduce the observations: lower marginal costs and constrained capacity from the small supplier, switching costs, and barriers to entry. If any of these assumptions are relaxed, price discrimination will not be observed and the aggregate market price will remain the same or, more likely, decrease.

First, if the small supplier does not have lower marginal costs, it becomes vulnerable to price competition from the monopoly. The monopoly may reduce the price to force the small supplier out of the market and then return to its monopoly position. During the competition, the market price is lower than the monopoly price.

Second, if the capacity of the small supplier is not constrained, the small supplier may reduce its price below the marginal cost of the monopoly, given its cost advantage. This action may force the monopoly out of the market, and the small supplier becomes the new monopoly. As the small supplier has a lower marginal cost, a lower monopoly price and a higher monopoly quantity are expected at the new monopoly equilibrium.

Third, if the switching costs do not exist, the small supplier can charge the same (precisely, marginally lower) price to take consumers from the monopoly. This will end up at the Stackelberg-type equilibrium in which the monopoly will take the fixed supply from the small supplier as given and find the quantity to maximise its profit. In this case, there is no price differentiation and the price level is lower than the monopoly price. In addition, if the first three assumptions are relaxed, there will be two similar competitors in the market (i.e. duopoly) with the Cournot or Bertrand competition.

Fourth, if there are no barriers to entry, a larger number of small suppliers may enter the market, lowering the price to their marginal cost and forcing the monopoly out of the market. The market may be featured as a perfect competition with a price equal to the marginal cost of small suppliers.

Appendix 1. The monopoly's profit when it moves away from the monopoly equilibrium

When the monopoly moves away from this equilibrium, its profit is lower. To show this, the profit function is rewritten as

$$\pi = (p - c)q = (p - c)\frac{a - p}{b} = -\frac{p^2}{b} + \frac{(a + c)p}{b} - \frac{ca}{b} \tag{73}$$

The first derivative with respect to the price is

$$\frac{d\pi}{dp} = -\frac{2}{b}p + \frac{a + c}{b} \tag{74}$$

gives

$$\frac{d\pi}{dp} > 0 \, if \, p < \frac{a + c}{2} \tag{75}$$

$$\frac{d\pi}{dp} < 0 \, if \, p > \frac{a+c}{2} \tag{76}$$

which suggest that, when the price is below (above) the monopoly price, a lower (higher) price leads to a lower profit.

Appendix 2. The condition that the small supplier does not behave like a monopoly on the residual demand

Given the monopoly in the market, the residual demand function for the small supplier is

$$p = p_m - bq \tag{77}$$

If the small supplier only takes consumers without service, its profit maximisation problem is

$$\max_{q_s} \pi_s = (p_s - c_s)q_s = (p_m - bq_s - c_s)q_s \tag{78}$$

The first-order condition gives the optimal quantity as

$$\frac{\partial \pi}{\partial q_s} = 0 \Rightarrow p_m - 2bq_s - c_s = 0 \Rightarrow q_s^* = \frac{p_m - c_s}{2b} \tag{79}$$

Since we have the monopoly price,

$$p_m = \frac{a+c}{2} \tag{80}$$

The optimal quantity for the small supplier becomes

$$q_s^* = \frac{p_m - c_s}{2b} = \frac{\frac{a+c}{2} - c_s}{2b} = \frac{a + c - 2c_s}{4b} \tag{81}$$

If the capacity of the small supplier is greater than this value, it can behave as a monopoly on the residual demand and leave some spare capacity. However, for simplicity, we assume that the small supplier has a capacity lower than this level as

$$\overline{q}_s < \frac{a + c - 2c_s}{4b} \tag{82}$$

Appendix 3. The condition that the small supplier has no motivation to raise its price further when demand for its service is equal to its supply capacity

In Sect. 2.4.4, there is a possibility that the small supplier may continue to raise its price by sacrificing the demand for its service. Here we show the condition that the small supplier is not motivated to do that.

The small supplier's profit function can be written as,

$$\pi'_s = \left(p'_s - c_s\right)\left(q'_{s,A} - q'_{s,B}\right) = \left(p_m - \delta' - c_s\right)\left(\frac{a - p_m + \delta'}{b} - \frac{\mu a - \delta'}{\mu b}\right) \tag{83}$$

The first-order derivative with respect to the price gap δ' is

$$\frac{\partial \pi'_s}{\partial \delta'} = \frac{\partial p'_s}{\partial \delta'}\left(q'_{s,A} - q'_{s,B}\right) + \frac{d\left(q'_{s,A} - q'_{s,B}\right)}{\partial \delta'}\left(p'_s - c_s\right) \tag{84}$$

If this first-order derivative is greater than zero, the small supplier's profit is lower if it charges a higher price (i.e. the gap is lower).

The first-order derivative can be written as

$$\frac{\partial \pi'_s}{\partial \delta'} = -\left(q'_{s,A} - q'_{s,B}\right) + \left(\frac{1}{b} + \frac{1}{\mu b}\right)\left(p'_s - c_s\right) \tag{85}$$

Substituting Eq. (22) and Eq. (23) into it gives

$$\frac{\partial \pi'_s}{\partial \delta'} = \frac{(2\mu + 1)p_m - 2(\mu + 1)\delta' - (1 + \mu)c_s}{\mu b} \tag{86}$$

Substituting Eq. (25) into it gives

$$\frac{\partial \pi'_s}{\partial \delta'} = \frac{\frac{a+c}{2} - 2\mu b \bar{q}_s - (1 + \mu)c_s}{\mu b} \tag{87}$$

The condition that the first-order derivative is greater than zero is

$$\frac{\frac{a+c}{2} - (1 + \mu)c_s}{2\mu b} > \bar{q}_s \tag{88}$$

Using Eq. (82) from Appendix 2, we can rewrite the condition as

$$\frac{\frac{a+c}{2} - (1 + \mu)c_s}{2\mu b} > \frac{a + c - 2c_s}{4b} \tag{89}$$

$$(1 - \mu)\frac{(a + c)}{2} > c_s \tag{90}$$

$$p_m - \mu p_m > c_s \tag{91}$$

Equation (91) holds when c_s is zero and is highly likely to hold when c_s is small. Note that the left term is the difference between the willingness to pay p_m and the switching costs μp_m, for the marginal consumer at q_m.

Appendix 4. Find the two price levels that give the same profit to the monopoly

Except for the monopoly equilibrium, there are two price-quantity combinations on the demand curve to produce the same profit for the monopoly. If the monopoly reaches this point after attracting all consumers from the small supplier, it will not be motivated. To find this point, we rewrite the demand curve as

$$\pi_m^x = \left(p_m^x - c\right)q_m^x = \left(p_m^x - c\right)\left(\frac{a - p_m^x}{b}\right) \tag{92}$$

where π_m^x is the specific level, p_m^x and q_m^x are the price-quantity combination. Assume that the profit is the same as the profit received by the monopoly at State One. Equating (83) to the profit from Eq. (63), which gives

$$\pi_m^x = \pi_m^* \rightarrow \left(p_m^x - c\right)\left(\frac{a - p_m^x}{b}\right) = \left(\frac{b\bar{q}_s}{\mu} - c\right) \bullet \frac{\mu a - b\bar{q}_s}{\mu b} \tag{93}$$

which can be written as a quadratic equation,

$$\left(p_m^x\right)^2 - (a + c)p_m^x + \left(a + c - \frac{b\bar{q}_s}{\mu}\right)\frac{b\bar{q}_s}{\mu} = 0 \tag{94}$$

Its solution can be found as

$$p_m^x = \frac{(a + c) \pm \sqrt{\left(a + c - 2\frac{b\bar{q}_s}{\mu}\right)^2}}{2} \tag{95}$$

And the two solutions are

$$p_m^{x1} = \frac{b\bar{q}_s}{\mu} \tag{96}$$

and

$$p_m^{x2} = a + c - \frac{b\bar{q}_s}{\mu} \tag{97}$$

Therefore, as Eq. (96) gives the price at State One, Eq. (97) gives another price that produces the same level of profit it the monopoly reduces its price to attract all consumers from the small supplier.

Appendix 5

Table 6 A list of variables in the model

Structural parameters			
a	Constant of the demand equation	b	Slope of the demand equation
c	Marginal costs of the monopoly	c_s	Marginal costs of the small supplier
$\bar{q}_s$	Capacity of the small supplier	μ	Fractional parameter on switching costs
Monopoly alone			
p_m	Monopoly price	q_m	Monopoly quantity
π_m	Monopoly profit	γ_m	Switching costs of the consumer at q_m
Cooperation			
p_s^c	Price charged by the small supplier	π_s^c	Profit of the small supplier
δ^c	Price gap at Cooperation	$q_{s,B}^c$	Quantity served by the monopoly
$q_{s,A}^c$	Quantity served by both the monopoly and the small supplier		
p_{agg}^c	Quantity-weighted aggregate price		
After the small supplier undermines the monopoly			
p_s'	Price charged by the small supplier	π_s'	Profit of the small supplier
δ'	Price gap after undermining	$q_{s,B}'$	Quantity served by the monopoly
$q_{s,A}'$	Quantity served by both the monopoly and the small supplier		
$\gamma_{s,A}'$	Switching costs of the consumer at $q_{s,A}'$	$\gamma_{s,B}'$	Switching costs of the consumer at $q_{s,B}'$
Reaction of the monopoly after being undermined			
$\pi_{m,hold}'$	Monopoly's profit if holds the price	$p_{m,high}'$	The price raised by the monopoly
$\pi_{m,h}'$	Monopoly's profit if raises the price	δ_h'	Price gap after raising the price
$q_{s,B}''$	Consumers who are not attracted by the increased price gap		
$p_{m,li}'$	The price reduced by the monopoly at the end of stage i ($i = 1, 2, 3$)		
$\widehat{\delta}_{li}'$	The inverse price gap at the end of stage i ($i = 1, 2, 3$)		
$\pi_{m,li}'$	The monopoly's profit at the end of stage i ($i = 1, 2, 3$)		
State One			

Table 6 (continued)

p_s^*	Price charged by the small supplier	π_s^*	Profit of the small supplier
δ^*	Price gap	$q_{s,B}^*$	Quantity served by the monopoly
$q_{s,A}^*$	Quantity served by both the monopoly and the small supplier		
$p_{m,li}^*$	The price reduced by the monopoly at the end of stage i ($i=1,2,3$)		
$\pi_{m,li}^*$	The monopoly's profit at the end of stage i ($i=1,2,3$)		
p_{agg}^*	Quantity-weighted aggregate price		
State Two			
p_s^{**}	Price charged by the small supplier		
$\pi_{m,li}^{**}$	The monopoly's profit at the end of stage i ($i=1,2,3$)		
p_{agg}^{**}	Quantity-weighted aggregate price		
Equilibrium			
p_s^E	Price charged by the small supplier	π_s^E	Profit of the small supplier
δ^E	Price gap	$q_{s,B}^E$	Quantity served by the monopoly
$q_{s,A}^E$	Quantity served by both the monopoly and the small supplier		
p_{agg}^E	Quantity-weighted aggregate price		

Declarations

Ethical Approval Ethics approval was not required for this study.

Informed Consent Informed consent was not required for this study.

Competing Interests The authors declare no competing interests.

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