Incorporating the Wilshire equations for time to failure and the minimum creep rate into a continuum damage mechanics for the creep strain of Waspaloy

Mark Evans*

Institute of Materials, Bay Campus, Swansea University, Fabian Way, Crymlyn Burrows, Wales, SA1 8EN, UK; m.evans@swansea.ac.uk; Tel.: 01792295748. Corresponding author.

ABSTRACT: In this paper, a new constitutive model is presented that combines the Wilshire equations with a modified Kachanov-Rabotnov continuum damage mechanics (CDM) to enable the prediction of uniaxial creep curves that contain both a primary and tertiary stage. Another advantage of this approach is that the Wilshire equations have been shown to accurately extrapolate the operational failure times and minimum creep rates from very short-term tests. This approach also removes the need to estimate the Wilshire time to strain equation at numerous different strains. A simple but multi-step procedure is also introduced for estimating the unknown parameters of this model. When applied to Waspaloy data, the model was shown to represent the shape of the experimental creep curves reasonably well 9especially at low and high strains) and provides reasonable creep curve predictions – with percentages errors averaging around 4-5%.

Keywords: Creep curves, Wilshire equations, Constitutive models

1. Introduction

The introduction of new materials has already supported major improvements in the efficiency and reliability of aeroengines [1]. Nevertheless, a combination of rising energy prices and global warming are now requiring further increases in engine efficiency to minimise fuel consumption and greenhouse gas emissions. One way to achieve this is through higher operational temperatures, but this requires materials with enhanced temperature capabilities such as the Nickel based superalloys – of which Waspaloy, is an example. Unfortunately, the 'materials development cycle' currently takes many years [2]. Long duration test programmes are needed to establish the tensile stresses which can be sustained over the planned design lives without creep failure occurring at the temperatures encountered during service. The Wilshire equation for time to failure has great potential for reducing the length of the development cycle because it has been shown in the literature to produce reliable failure time predictions for the operating conditions (or close to) of many materials using only very short term accelerated tests [3-12].

In order to prevent aeroengine blades rubbing against the engines outer casing, strain is also a very important design and material development criteria. Being able to realistically predict strain at given times is also very important for converting small punch test data into equivalent uniaxial test results given that the most promising way of doing this is via finite element models of the punch test. These finite element models require equations yielding incremental increases in strain with time and so require accurate predictions of all points along a uniaxial creep curve. The successful correlation of small punch and uniaxial test results will help release the full potential of the small punch test.

However, the literature is quite sparse on how to modify the Wilshire equations so as to be able to predict times to specified strains and therefore complete creep curves [13-14]. Most recently, Evans and Williams [15] have incorporated Artificial Neural Network technology (ANN) into the Wilshire methodology as a solution. However, the resulting model has no closed form expression and is quite cumbersome to implement. It involves numerous steps including modelling the Wilshire time to strain parameters as a function of strain using ANN's, entering a strain into these ANN's to get the Wilshire parameters and finally inserting these Wilshire parameters (and that strain) into the Wilshire time to strain equation to get a prediction for the time to that strain. Repeating for all strains up to the rupture strain yields the predicted creep curve. Such a process is not ideally suited to finite element modelling of the small punch creep test. Cano and Stewart [16] by passed this procedure by integrating the Wilshire equations into a CDM model, but the resulting model was not capable of modelling the primary stages of the creep process. Therefore, the aim of this paper is to overcome the limitations of these last two approaches by integrating the Wilshire equations for time to failure and minimum creep rates into a modified Kachanov-Rabotnov (K-R) creep continuum damage model. These modifications allow for primary creep and for failure to occur when the damage parameter is less than unity.

To achieve this objective the paper is structured as follows. The next section describes the uniaxial creep tests of the polycrystalline Nickel alloy Waspaloy that have been conducted at Swansea University. Section 3 reviews some CDM approaches to creep already in the literature. Section 4 outlines some statistics that can be used for evaluating the effectiveness of a creep model in predicting various creep properties. Section 5 outlines the proposed CDM model to be used in this paper, together with a description of how the unknown parameters of this model can be quantified. This model is then applied to the

Waspaloy data in section 6. The main findings are then summarised in the conclusions section.

2. The data

Thirty-one cylindrical test pieces were machined from an received Waspaloy bar, with a gauge length of 28mm and a diameter of 5mm. The chemical composition of this batch of material is shown in Table 1a. The material was heat treated for 4 h at 1353 K (water quenched), 4 h at 1123 K (air cooled) and 16 h at 1033 K (air cooled). This resulted in a uniform equiaxed structure of average grain diameter 45 μm. The microstructure contained uniform $\gamma^{/}$ particles of mean diameter 0.3 μm.

Table 1a Chemical composition (weight %)

Cr	Co	C	Mn	Si	Fe	Mo	Ti	Al	B	Zr	S	P	Cu
19.1	13.5	0.03	0.1	0.1	0.79	4.08	3.15	1.3	0.005	0.07	0.0025	0.01	0.1
Also 5 ppm of Ag, 10 ppm of Pb and 0.5 ppm of Bi with balance Ni.													

The tensile strength (σ_{TS}) values for this batch of material are shown in Table 1b. Normalisation of the stress using the tensile strength is done using these measured UTS values – no equation was required for interpolating the UTS as the test matrix contained no creep curves at any other temperature.

Table 1b Variation of Tensile Strength with temperature

Temperature (K)	873	923	973	1023
Tensile Strength (MPa)	1154	1120	975	827

The specimens were tested in tension over a range of stresses at 873K, 923K, 973K and 1023K using high precision in Andrade-Chalmers constant-stress machines [17]. Loads and stresses could be applied and maintained to an accuracy of 0.5%. In all cases, temperatures were controlled along the gauge lengths and with respect to time to better than ±1 K. The extensometer was capable of measuring tensile strain to better than 10^{-5}. Loading machines, extensometers and thermocouples were all calibrated with respect to NPL traceable standards. At 873K, eight specimens were placed on test over the stress range 1150 MPa to 700 MPa, at 923K seven specimens were placed on test over the stress range 1000 MPa to 550 MPa, at 973K nine specimens were placed on test over the stress range 950 MPa to 200 MPa and at 1023K seven specimens were tested over the stress range 700 MPa to 250 MPa. Up to 400 creep strain/time readings were taken during each of these tests. Because Waspaloy can serve at temperatures up to 920K for critical applications and 1040K for less demanding situations, the test programme covered stress ranges giving creep lives up to 5,500 h (around 19852000 s) at 873 to 1023 K. This data set has been published by Wilshire and Scharning [18] and Evans [19].

Analysis of the data obtained on constant load machines would proceed in the same way as outlined in this paper – all that is required is that the creep curves, minimum creep rates and failure times are all obtained at constant load. Converting the resulting predicted constant load creep curves into constant stress creep curves required for small punch modelling would require further manipulation of the predicted curves.

3. A brief review of some well-known CDM models

3.1. The Kachanov-Rabotnov (K-R) model

The Kachanov-Rabotnov (K-R) creep continuum damage model at constant temperature consists of a creep strain rate, $\dot{\varepsilon}$, and a damage evolution equation $\dot{w}$ [20,21]

$$\frac{dw}{dt} = \dot{w} = \frac{\delta\sigma^{\chi}}{(1-\phi)(1-w)^{\phi}} \tag{1a}$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = A\left(\frac{\sigma}{1-w}\right)^{n} \tag{1b}$$

where A and n are the Norton power law constants, σ is the constant stress associated with a uniaxial creep test, w is the K-R damage parameter that varies from 0 through to 1 during the creep test, and δ, χ, and ϕ are the tertiary creep damage constants. The constant χ must be greater than or equal to unity (but is typically set at 3). $\sigma/(1\text{-}w)$ is often referred to as the effective stress, representing the accelerating effect of damage accumulation on the initial stress. Assuming that failure occurs when w = 1, the definite integral of Eq. (1a) yields an expression for both the time to failure t_f and K-R damage

$$t_f = \frac{1}{\delta\sigma^{\chi}} \tag{1c}$$

$$w = 1 - [1 - \delta\sigma^{\chi}t]^{\frac{1}{\phi+1}} = 1 - \left[1 - \frac{t}{t_f}\right]^{\frac{1}{\phi+1}} \tag{1d}$$

If Eq. (1d) is inserted into Eq. (1b) and the indefinite integral taken, the form of the K-R uniaxial creep curve at stress σ and constant temperature emerges

$$\varepsilon = \frac{A\sigma^{n}}{1-\frac{n}{\phi+1}}\left\{1 - \left[1 - \frac{t}{t_f}\right]^{1-\frac{n}{\phi+1}}\right\} \tag{1e}$$

3.2. Modified Kachanov-Rabotnov (K-R) model

One issue with the K-R model is that it assumes failure occurs when w = 1, which in turn implies that the effective stress, creep rate and rate of damage accumulation are also all infinite at failure – a phenomenon that is not seen in uniaxial creep testing of metals for high temperature applications. This can be tackled within the K-R framework by introducing a critical damage parameter, $w_f \leq 1$, such that failure occurs when w reaches this quantity. Inserting w_f for w into Eq. (1d) when $t = t_f$ and solving for δ gives

$$\delta = \frac{1-[1-w_f]^{\phi+1}}{\sigma^{\chi}t_f} \tag{2a}$$

which when substituted back into Eq. (1d) gives a modified K-R damage as a function of time

$$w = 1 - \left[1 + \left([1-w_f]^{\phi+1} - 1\right)\frac{t}{t_f}\right]^{\frac{1}{\phi+1}} \tag{2b}$$

with the failure time equation remaining unchanged. Clearly, Eq. (2b) collapses to Eq. (1d) when $w_f = 1$. Notice also that Eq. (1b) can be written as

$$w = 1 - \frac{\sigma}{\left(\frac{\dot{\varepsilon}}{A}\right)^{\frac{1}{n}}} = \frac{\left(\frac{\dot{\varepsilon}}{A}\right)^{\frac{1}{n}} - \sigma}{\left(\frac{\dot{\varepsilon}}{A}\right)^{\frac{1}{n}}} \tag{3}$$

so that the K-R model implies that the minimum creep rate, $\dot{\varepsilon}_m = A\sigma^n$, occurs when there is no damage accumulation, i.e. when w = 0. This is most clearly seen in Eq. (1d) where when $\dot{\varepsilon} = \dot{\varepsilon}_m$, $\left(\frac{\dot{\varepsilon}}{A}\right)^{\frac{1}{n}} = \sigma$ so that w = 0. As soon as the creep strain rate is greater than the minimum creep strain rate, the term $\left(\frac{\dot{\varepsilon}}{A}\right)^{\frac{1}{n}}$ becomes larger than the equivalent stress, σ, and irreversible damage begins. Hence a clear limitation of the K-R model is that it does not account for primary creep.

3.3. The Hayhurst modification of the K-R model

Hayhurst et.al. [22] presented a solution to this problem by introducing a second damage variable – w_2. In this model the first damage parameter, w, represented dislocation softening and evolves from 0 through to 1. w_2 represented nucleation-controlled creep constrained to evolve from 0 to 1/3. In one version of this model, the creep rate and rate of accumulation in the new damage variable were dependent on stress through use of a hyperbolic sine function. A different approach that does not require the use of this additional damage variable was also proposed by Hayhurst et. al. [23] which simply involves adding the power expression t^m to Eqs. (1a,b) – where m is a material constant

$$\frac{dw}{dt} = \dot{w} = \frac{\delta\sigma^{\chi}t^{m}}{(1-\phi)(1-w)^{\phi}} \tag{4a}$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = A\left(\frac{\sigma}{1-w}\right)^{n} t^{m} \tag{4b}$$

Assuming that failure occurs when w = 1, the definite integral of Eq. (4a) yields an expression for both the time to failure t_f and damage w

$$t_f = \left[\frac{1+m}{\delta\sigma^{\chi}}\right]^{\frac{1}{m+1}} \tag{4c}$$

$$w = 1 - \left[1 - \left(\frac{t}{t_f}\right)^{m+1}\right]^{\frac{1}{\phi+1}} \tag{4d}$$

If Eq. (4d) is inserted into Eq. (4b) and the indefinite integral taken, the form of the uniaxial creep curve at stress σ and constant temperature emerges

$$\varepsilon = \frac{A\sigma^{n}}{\left(1-\frac{n}{\phi+1}\right)(m+1)}\left\{1 - \left[1 - \left(\frac{t}{t_f}\right)^{m+1}\right]^{1-\frac{n}{\phi+1}}\right\} \tag{4e}$$

It is clear from Eq. (4e) that the shape of the creep curve is such that it has a primary and tertiary component due to the fact that t/t_f has two power exponents - (m+1) and 1-n/(ϕ+1). This modified expression reduces to the original K-R model when m = 0, implying that primary creep is picked up via a negative value for m.

3.4. Power Law breakdown

The K-R, modified K-R and Hayhurst models discussed above all suffer from power law breakdown in that the form of the failure time equation is a power law (see Eqs. (1c,4c) and it is known that the parameter n is then not a material constant but varies with stress. Hayhurst et.al. [22] started to address this issue by using a Sinh rather than a power law relation to describe the role of stress on creep strain. More recently, the coupled Sinh creep damage constitutive model of Haque et. al. [24] consists of a creep strain rate and damage evolution equation as follows

$$\frac{dw}{dt} = \dot{w} = \frac{B[1-e^{-\phi}]\sinh\left(\frac{\sigma}{\sigma_t}\right)^{\chi}}{\phi} e^{\phi w} \tag{5a}$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = A\sinh\left(\frac{\sigma}{\sigma_s}\right) e^{\lambda w^{3/2}} \tag{5b}$$

where σ_t and σ_s are additional material constant and the parameters in Eqs. (5) have different meanings to the same parameters in the previously discussed models. Whilst the use of the Sinh function helps overcome power law breakdown, the definition of λ as $\ln(\dot{\varepsilon}_f/\dot{\varepsilon}_m)$ in Eq. (5b) means that the creep curves starts at time t = 0 with a creep rate equal to $\dot{\varepsilon}_m$ and from that point on the creep rate continues to increase, i.e. the model is for tertiary creep only ($\dot{\varepsilon}_f$ is the creep rate at failure).

An alternative approach is to overcome power law break down by making use of the Wilshire equations [3]. These equations have been shown to predict well both failure times and minimum creep rates over a wide range of stress and temperatures using just accelerated test data [4-12]. Cano et. al. [16] have recently developed a CDM version of this Wilshire approach in which they replace the Sinh function in Eqs. (5) with the Wilshire equations

$$\frac{dw}{dt} = \dot{w} = \frac{[1-e^{-\phi}]}{\phi}\frac{1}{t_f} e^{\phi w} \tag{6a}$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = \dot{\varepsilon}_m e^{\lambda w} \tag{6b}$$

where

$$t_f = \frac{\left[\frac{-\ln(\sigma/\sigma_{TS})}{k_1}\right]^{\frac{1}{u}}}{e^{\frac{-Q_c}{RT}}} \tag{6c}$$

$$\dot{\varepsilon}_m = \frac{\left[\frac{-\ln(\sigma/\sigma_{TS})}{k_2}\right]^{\frac{1}{v}}}{e^{\frac{Q_c}{RT}}} \tag{6d}$$

and u, v are materials constants, σ_{TS} is the tensile strength, R the universal gas constant, Q_c the activation energy for self-diffusion, and T the absolute temperature. Eqs. (6c,d) are the Wilshire equations for time to failure and minimum creep rates. In Eq. (6b), λ is set equal to $\ln(\dot{\varepsilon}_f/\dot{\varepsilon}_m)$ and in Eq. (6a) ϕ is set equal to $\ln(\dot{w}_f/\dot{w}_0)$ where $\dot{w}_f/\dot{w}_0$ is the ratio of the final to initial rates of damage accumulation. As such this is once again a model for tertiary creep only.

Another approach to solving the power law issue was presented by Liu and Murakami [25] who specified the damage rate and creep rate equations as

$$\frac{dw}{dt} = \dot{w} = \frac{B[1-e^{-\phi}]\sigma^{\chi}}{\phi} e^{\phi w} \qquad (7a)$$

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = A\sigma^{n} e^{q w^{3/2}} \qquad (7b)$$

where A, B, q, n, ϕ and χ are again material constants – again with different meanings to those in the K-R model.

4. A Proposed CDM models based on the Wilshire equations

4.1. Specification

First, rewrite the Wilshire equation for the minimum creep rate as

$$\dot{\varepsilon}_m = \left[\frac{1}{k_{2j}}\right]^{\frac{1}{v_j}} e^{-\frac{Q_{cj}}{RT}} [-\ln(\sigma/\sigma_{TS}]^{\frac{1}{v_j}} = A_j \tau^{n_j} \qquad (8a)$$

where $A_j = \left[\frac{1}{k_{2j}}\right]^{\frac{1}{v_j}} e^{-\frac{Q_{cj}}{RT}}$, $n_j = 1/v_j$ and $[-\ln(\sigma/\sigma_{TS}] = \tau$. In Eq. (8a), j = 1 when $\sigma/\sigma_{TS} \le \sigma_1^c$; j = 2 when $\sigma_1^c < \sigma/\sigma_{TS} \le \sigma_2^c$; …. ; j = p when $\sigma/\sigma_{TS} > \sigma_{p\text{-}1}^c$ and $\sigma_1^c < \sigma_2^c < \ldots\ldots\ldots < \sigma_{p\text{-}1}^c$. σ^c_j are critical values for the normalised stress and so fall between 0 and 1. In this approach, there are p creep regimes that occur in distinct ranges for the normalised stress and the p versions of Eq. (8a) then apply to each regime. Typically, p varies between 1 and 4 depending on the material being studied.

Next it is proposed that the creep rate ($\dot{\varepsilon}$) equation is of the form

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon} = (\dot{\varepsilon}_m)^{\rho}(1-w)^{n_j}\left(\frac{t}{t_f}\right)^{m_j} = \left(A_j\tau^{n_j}\right)^{\rho}(1-w)^{n_j}\left(\frac{t}{t_f}\right)^{m_j} \qquad (8b)$$

as n_j is negative in value and where m_j are the additional primary creep constants that will be negative in value in the presence of strong primary creep.

Re-arranging Eq. (8b) for damage gives

$$w = 1 - \left(\frac{\dot{\varepsilon}}{(\dot{\varepsilon}_m)^{\rho}}\right)^{\frac{1}{n_j}}\left(\frac{t}{t_f}\right)^{\frac{-m_j}{n_j}} \qquad (9a)$$

Assuming failure occurs when damage reaches some critical value, w_f, where $w_f \le 1$ then,

$$w_f = 1 - \left(\frac{\dot{\varepsilon}_f}{(\dot{\varepsilon}_m)^{\rho}}\right)^{\frac{1}{n_j}} \qquad (9b)$$

where $\dot{\varepsilon}_f$ is the strain rate at failure. When the minimum creep rate is reached at time t_m, the amount of accumulated damage is greater than zero

$$w_m = 1 - \left(\frac{\dot{\varepsilon}_m}{(\dot{\varepsilon}_m)^{\rho}}\right)^{\frac{1}{n_j}}\left(\frac{t_m}{t_f}\right)^{\frac{-m_j}{n_j}} \qquad (9c)$$

Next it is proposed that the CDM damage rate ($\dot{w}$) equation is of the form

$$\frac{dw}{dt} = \dot{w} = \frac{G_j \tau^{\chi_j}}{(1-w)^{\phi_j}} t^{m_j} \qquad (10a)$$

where G_j, χ_j and ϕ_j are additional tertiary damage constants. The indefinite integral of Eq. (10a) under the assumption that w = 0, when t = 0 gives

$$w = 1 - \left(1 - \frac{(1+\phi_j)G_j\tau^{\chi_j}}{m_j+1} t^{m_j+1}\right)^{\frac{1}{1+\phi_j}} \tag{10b}$$

Again, with failure occurring when $w = w_f$,

$$\frac{(1+\phi_j)G_j\tau^{\chi_j}}{m_j+1} = \frac{1-(1-w_f)^{1+\phi_j}}{t_f^{m_j+1}} \tag{10c}$$

Substituting Eq. (10c) into (10b) gives a simplified expression for damage

$$w = 1 - \left(1 + \left[(1-w_f)^{1+\phi_j} - 1\right]\left(\frac{t}{t_f}\right)^{m_j+1}\right)^{\frac{1}{1+\phi_j}} \tag{10d}$$

with the time to failure being given by

$$t_f = \left[\frac{(m_j+1)\left(1-(1-w_f)^{1+\phi_j}\right)}{(1+\phi_j)G_j\tau^{\chi_j}}\right]^{\frac{1}{m_j+1}} \tag{10e}$$

The Wilshire failure time equation (Eq. (6c)) can be written as

$$t_f = e^{\frac{Q_{cj}}{RT}}\left[\frac{1}{k_{1j}}\right]^{\frac{1}{u_j}} \tau^{\frac{1}{u_j}}$$

The failure time equation given by Eq. (10e) is therefore consistent with this Wilshire time to failure equation, with

$$1/u_j = -\chi_j/(m_j+1) \qquad \text{and} \qquad \frac{\frac{m_j+1}{\phi_j+1}[(1/k_{1j})1/u_j \exp(Q_{cj}/RT)](1+m_j)}{1-(1-w_f)^{1+\phi_j}} = 1/G_j \tag{10f}$$

Finally, taking the indefinite integral of Eq. (8b) (after substituting in Eq. (10b) or Eq. (10d) for w) and assuming ε = 0 when t = 0 (to determine the constant of integration) gives the following expressions for the uniaxial creep curve

$$\varepsilon = \Gamma\left\{1 - \left[\left(\frac{t}{t_f}\right)^{m_j+1}\left((1-w_f)^{1+\phi_j} - 1\right) + 1\right]^{\Delta_j}\right\}$$

or

$$\varepsilon = \Gamma\left\{1 - \left[\left(\frac{t}{\left[\frac{(m_j+1)\left(1-(1-w_f)^{1+\phi_j}\right)}{\left(1+\phi_j\right)G_j\tau^{\chi_j}}\right]^{\frac{1}{m_j+1}}}\right)^{m_j+1}\left((1-w_f)^{1+\phi_j} - 1\right) + 1\right]^{\Delta_j}\right\} \tag{11a}$$

where

$$\Gamma = \frac{(\dot{\varepsilon}_m)^{\rho}\, t_f}{\Delta_j\left[(1+m_j)-(1+m_j)(1-w_f)^{1+\phi_j}\right]} = \frac{(A_j \tau^{n_j})^{\rho}\left[\frac{(m_j+1)\left(1-(1-w_f)^{1+\phi_j}\right)}{(1+\phi_j)G_j\tau^{\chi_j}}\right]^{\frac{1}{m_j+1}}}{\Delta_j\left[(1+m_j)-(1+m_j)(1-w_f)^{1+\phi_j}\right]} \quad (11b)$$

and

$$\varepsilon_f = \Gamma\left\{1-\left[(1-w_f)^{1+\phi_j}\right]^{\Delta_j}\right\} \quad (11c)$$

and where $\Delta_j = 1 + \frac{\rho n_j}{1+\phi_j}$.

The normalised creep curve is then given by

$$\frac{\varepsilon}{\varepsilon_f} = \frac{1-\left[\left(\frac{t}{t_f}\right)^{m_j+1}\left((1-w_f)^{1+\phi_j}-1\right)+1\right]^{\Delta_j}}{1-\left[(1-w_f)^{1+\phi_j}\right]^{\Delta_j}} = \frac{1-\left[\left(\frac{t}{\left[\frac{(m_j+1)\left(1-(1-w_f)^{1+\phi_j}\right)}{(1+\phi_j)G_j\tau^{\chi_j}}\right]^{\frac{1}{m_j+1}}}\right)^{m_j+1}\left((1-w_f)^{1+\phi_j}-1\right)+1\right]^{\Delta_j}}{1-\left[(1-w_f)^{1+\phi_j}\right]^{\Delta_j}} \quad (11d)$$

4.2. Estimation

Estimation requires a mixture of linear and non-linear least squares. The first step involves estimating the Wilshire equation for the minimum creep rate to obtain values for v_j, Q_{cj} and k_{2j}. A hat symbol will be used to designate these as estimates. Details of such a linear least squares procedure are now well documented in, for example, Evans [15]. Then values for the parameters A_j and n_j in Eqs. (8b) are estimated as

$$\hat{A}_j = \left[\frac{1}{\hat{k}_{2j}}\right]^{\frac{1}{\hat{v}_j}} e^{-\frac{\hat{Q}_{cj}}{RT}} \qquad \text{and} \qquad \hat{n}_j = \frac{1}{\hat{v}_j} \quad (12a)$$

The second step involves testing the assumption that m_j and Δ_j in Eq. (11d) are temperature dependent. To do this, all the experimental creep curves in the jth creep regime at temperature 873K are normalised using the measured failure strains. Then w_f is set equal to 1 and non-linear least squares used to estimate the values for m_j and Δ_j in Eq. (11d) at this temperature and this creep regime. Standard Gauss-Newton algorithms can be used to minimise the sum of the squared differences between each actual normalised creep strain data point at 873K in the jth creep regime and that predicted by Eq. (11d). Repeat this process for all values of w_f in the range 0 – 1 using a simple grid search technique and choose the value for w_f to be that which gives the smallest such sum of squared differences. Again, let these estimates be denoted with the hat symbol - $\widehat{m_{jk}}$, $\widehat{w_{fk}}$ and $\widehat{\Delta_{jk}}$. The subscript k represents estimates for the kth temperature, and so for the data set in this paper k varies from 1 to 4. By repeating this process at all the other temperatures and creep regimes, plots of $\widehat{m_{jk}}$, $\widehat{w_{fk}}$ and $\widehat{\Delta_{jk}}$ against temperature for each creep regime can be made, and these will then reveal any dependency on stress and temperature.

If there is no dependency on temperature, then m_j and Δ_j can be estimated by minimising the sum of the squared differences between each actual normalised creep strain data point at all temperatures in the jth creep regime and that predicted by Eq. (11d). If there is dependency, then equations need to be found to model this.

The third step involves estimating the value for ρ. This parameter is simply the exponent in the Monkman-Grant relation [26]

$$(\dot{\varepsilon}_{\mathrm{m}})^{\rho} = \mathrm{C}\frac{\mathrm{t_f}}{\varepsilon_{\mathrm{f}}} \tag{12b}$$

Estimates for C and ρ are then easily found by regressing the log minimum creep rate against the log of the ratio of t_f to ε_f (1/ρ is the slope of this linear regression line). Let $\hat{\rho}$ be such an estimate. Assuming no temperature dependency, values for ϕ_j are then found as 1-{(1+ρn_j)/Δ_j}.

The fourth step involves estimating the Wilshire equation for times to failure to obtain values for $1u_j$ and k_{1j}. Then, and again using the hat symbol to denote parameter estimates, values for the parameters G_j and χ_j in Eqs. (10a) are given by

$$\hat{\chi_j} = -(\widehat{\mathrm{m_j}} + 1)/\widehat{u_j} \qquad \text{and} \qquad \frac{\frac{\widehat{\mathrm{m_j}}+1}{1+\widehat{\phi_j}}[(1/\widehat{\mathrm{k}_{1j}})1/\widehat{u_j}\exp(\widehat{\mathrm{Q}_{cj}}/\mathrm{RT})](1+\widehat{\mathrm{m_j}})}{1-(1-\widehat{\mathrm{w_f}})^{1+\widehat{\phi_j}}} = 1/\hat{\mathrm{G}}_{\mathrm{j}} \tag{12c}$$

5. Evaluation Statistics

Suppose a uniaxial test program is made up of h test conditions (such as one at σ = 700 MPa with T = 973K) and for each test condition some d_i strain-time readings made. Let ε_{it} be the experimental (or actual) value for strain measured at time t and $\hat{\varepsilon}_{it}$ the prediction made for these strains using the Wilshire CDM model described above. Then the accuracy of the predictions made for these experimental creep curves can be quantified using the mean percentage squared error (MPSE)

$$\mathrm{MPSE} = \frac{1}{h\sum_{i=1}^{h} d_i}\sum_{i=1}^{h}\sum_{t=1}^{d_i}[(\varepsilon_{it} - \hat{\varepsilon}_{it})/\varepsilon_{it}]^2 \cong \frac{1}{h\sum_{i=1}^{h} d_i}\sum_{i=1}^{h}\sum_{t=1}^{d_i}[e_{it}]^2 \tag{13}$$

where the $e_{it} = \ln(\varepsilon_{it}) - \ln(\hat{\varepsilon}_{it})$ are what Holdsworth et. al. [27] termed the residual log times. The approximation of this residual to the MPSE is better the smaller are the percentage errors (very close for an error of less than 10%). The standard deviation in these residuals (labelled $S_{A\text{-}RLT}$ by Holdsworth et. al.) can be calculated at each test condition as

$$s_{ei} = \frac{1}{d_i - 1}\sum_{t=1}^{d_i}[e_{it} - \bar{e}_i]^2 \qquad [14a]$$

where $\bar{e}_i$ is the mean residual log time at test condition i. Then, assuming these standard deviations are independent of test conditions, the standard deviation in the residuals over all conditions can be estimated as a weighted average of all the s_{ei}

$$s_e = \frac{\sum_{i=1}^{h}[(d_i - 1)s_{ei}]}{\sum_{i=1}^{h}(d_i - 1)} \tag{14b}$$

s_e and MPSE are different because the mean value for e may not be zero – as would be the case if the model systematically over or underestimates the strain.

If the residuals are assumed to be normally distributed (implying strains are log normally distributed), and the standard deviation for the residuals are independent of stress, the percentile (p) log strain at test condition i can be calculated

$$\ln(\varepsilon_i)_p = \ln(\hat{\varepsilon}_i) + s_e z_p \tag{15a}$$

where z_p is the pth percentile of the standard normal distribution. Because of the assumed log normality of the strains, the predicted log strain at test condition i, $\ln(\hat{\varepsilon}_i)$, is actually interpreted as the median (and therefore mean) log strain at that condition. Then, as an example, 99% of log strain values will be in the range

$$\ln(\hat{\varepsilon}_i) \pm s_e 2.58 \tag{15b}$$

and so 99% of the strain value will be in the range

$$\hat{\varepsilon}_i \, e^{\pm s_e 2.58} \tag{15c}$$

Holdsworth et. al. [27] have termed $e^{s_e 2.58}$ the Z-parameter and suggested it provides a means of quantifying model-fitting effectiveness. Ideally, for single-cast analysis, Z should not exceed 2, whereas Z in excess of 4 is unacceptable [28].

However, this interpretation of what is acceptable, assumes that the residual variation picked up by s_e (and thus Z) is all systematic in nature and so the result of a poorly fitting creep model. This will not always be the case. Granger and Newbold [29] have shown that

$$s_e^2 = (\beta - 1)^2 s_{\ln(\hat{\varepsilon})}^2 + s_v^2 \tag{16a}$$

where $s_{\ln(\hat{\varepsilon})}^2$ is the variance in the predicted log strains, β is the slope of the best fit line on a cross plot of $\ln(\varepsilon_{it})$ v $\ln(\hat{\varepsilon}_{it})$ and s_v^2 the variance in the residuals around this best fit line which is given by

$$\ln(\varepsilon_{it}) = \alpha + \beta \ln(\hat{\varepsilon}_{it}) + v_{it} \tag{16b}$$

So part of s_e^2 is caused by β differing from 1, and so by the best fit line being flatter or steeper than a 45^0 line on a scatter plot of $\ln(\varepsilon_{it})$ v $\ln(\hat{\varepsilon}_{it})$ This is clearly systematic bias that is caused by the used creep model itself, because in such a situation the creep model is then consistently over (or under) predicting $\ln(\varepsilon_{it})$ at low log strains followed by consistently under (or over) predicting at high log strains - depending on whether β is above or below 1. On the other hand, v_{it} is clearly random variation with s_v being the standard deviation and thus size of this random variation.

This suggests that a high value for Z would not be an indication of a creep model making large systematic prediction errors, provided β = 1. Rather, it would be due to a large value for s_v. In this extreme situation, all the variation being picked up by Z is purely random in nature and reflects the stochastic nature of creep in the material under investigation - which for some materials can result in substantial scatter. The size of this random variation is pre-determined, and no creep model can reduce it. Instead it is the result of things like microstructural variation in test samples, accuracy of test equipment etc. At the other extreme, a large value for Z would be an indicator of a poorly performing creep model if $s_v^2 = 0$ with $\beta \neq 1$.

Another issue with Z is that it does not pick up a poorly performing creep model that is the result of the model failing the predict the log strain even on the average. This can be seen by noting that the MPSE can also be worked out as

$$\text{MPSE} = \bar{e}^2 + s_e^2 = \bar{e}^2 + (\beta - 1)^2 s_{\ln(\hat{\varepsilon})}^2 + s_{\text{v}}^2 \qquad (17a)$$

where $\bar{e}$ is the mean residual over all tests conditions and times. Thus, a proportion of the MPSE is due to the creep model predicting the strain incorrectly on the average, which is clearly a systematic error - $U^M = \bar{e}^2/\text{MPSE}$. This is often referred to as the bias proportion. Another proportion of the MPSE is due to the regression parameter $\beta \neq 1$, which again is due to a poorly performing creep model – $U^R = (\beta - 1)^2 s_{\ln(\hat{\varepsilon})}^2/\text{MPSE}$. This is often called the new regression proportion. Finally, a proportion of the MPSE is due to $U^D = s_v^2/$ MPSE and is often called the random disturbance proportion. Granger and Newbold have shown that the last equation can be rewritten as

$$\text{MPSE} = \bar{e}^2 + (s_{\ln(\hat{\varepsilon})} - r s_{\ln(\varepsilon)})^2 + (1 - r^2)\, s_{\ln(\varepsilon)}^2 \qquad (17b)$$

where $s_{\ln(\varepsilon)}^2$ is the variance in the actual log strains and r the correlation between the actual and predicted log strains.

6. Results

6.1. The Wilshire equation for the minimum creep rate

The top half of Table 2 shows the least squares estimates made for the parameters of the Wilshire minimum creep rate equation.

Table 2. Estimated parameters of the Wilshire equations

	Wilshire minimum creep rate equation – Eq. (8a)	
Parameters	$\sigma/\sigma_{TS} > 0.726$ (j =1)	$\sigma/\sigma_{TS} \leq 0.726$ (j=2)
k_{2j}	42739791.637	6.6002
v_j	-0.8652	-5.3528
Q_{cj}(Jmol^{-1})	225,637	224,432
	Wilshire failure time equation – Eq. (6c)	
Parameters	$\sigma/\sigma_{TS} > 0.726$ (j =1)	$\sigma/\sigma_{TS} \leq 0.726$ (j=2)
k_{1j}	60448836.39	13.5687
u_j	0.8168	4.1565
Q_{cj}(Jmol^{-1})	201,529	213,689

This model is capable of explaining 95.14% of the variation in log minimum creep rates, and the parameters above and below $\sigma^c = 0.726$ are all statistically significantly different from each other at the 1% significance level. So, whilst the activation energies estimated either side of the break stress are only slightly different in value from each other (224 kJmol^{-1} v 226 kJmol^{-1}), this difference is nonetheless statistically significant at the 1% significance level. Fig. 1(a) provides a visualisation of this model by plotting ln(τ) against $\dot{\varepsilon}_m + Q_c/RT$. There is a clear break in the relationship around $\sigma^c = 0.726$ and all data points seem to fit tightly around the best fit line given by Eqs. (8a) - which is shown as the dashed line.

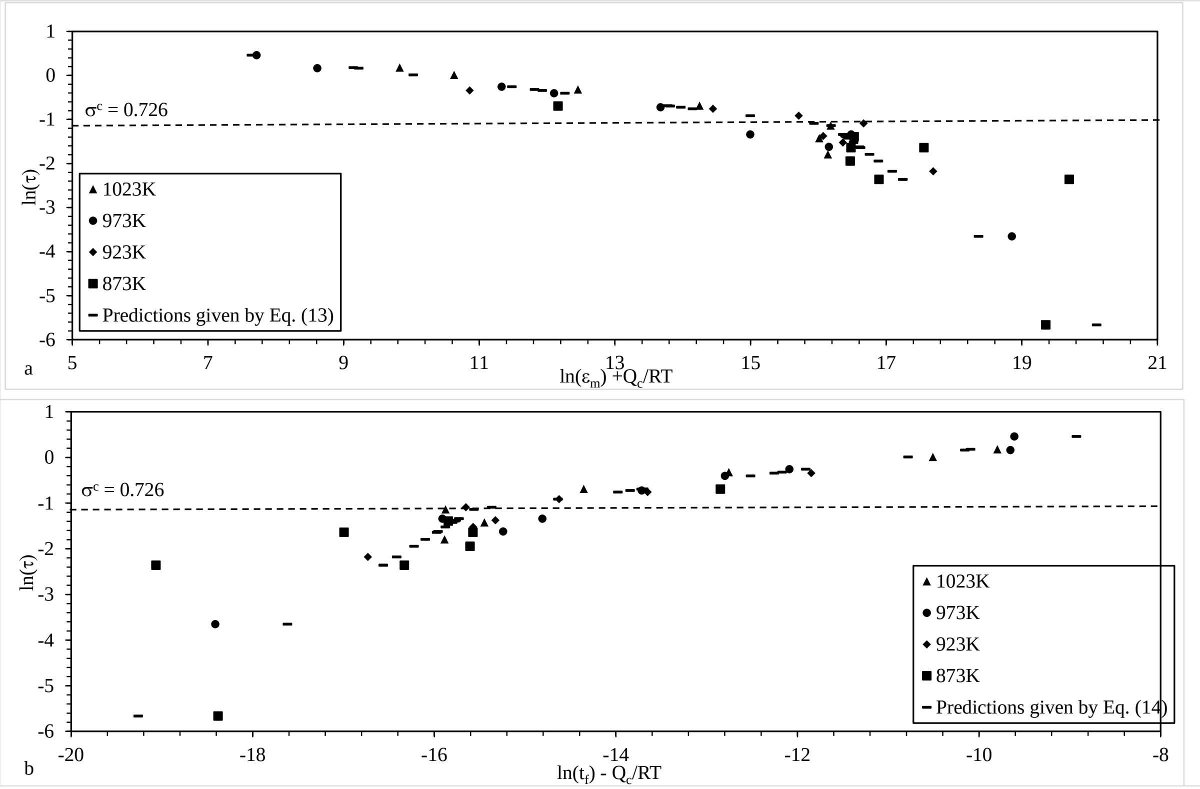

Fig. 1. Dependence of (a) $\ln[\dot{\varepsilon}_m \exp(Q^*_c/RT)]$ on $\ln[-\ln(\sigma/\sigma_{TS})]$ and b $\ln[t_f \exp(-Q^*_c/RT)]$ on $\ln[-\ln(\sigma/\sigma_{TS})]$ at various temperatures.

It therefore appears that the creep behaviour of Waspaloy is dependent on applied conditions, with two distinct regions corresponding to stresses above and below σ_Y (the yield stress which approximately corresponds to a normalised stress of 0.726). This change in creep behaviour is due to differing mechanisms of creep at different applied conditions. Whittaker et. al. [9] highlighted the dominance of diffusive climb at stresses below σ_Y with dislocation-dislocation interaction in the form of forest hardening limiting creep rates at higher stresses. They showed that geometrically necessary dislocation (GND) densities are higher at the grain boundaries in Waspaloy samples crept below σ_Y, where as GND densities were more uniformly spread through grains in samples crept above σ_Y.

6.2. The Wilshire equation for the time to failure

The bottom half of Table 2 shows the least squares estimates made for the parameters of the Wilshire failure time equation. This model is capable of explaining 93.87% of the variation in log times to failure, and the parameters (with the exception of Q_c) above and below σ^c = 0.726 are all statistically significantly different from each other at the 1% significance level. Fig. 1(b) provides a visualisation of this model by plotting ln(τ) against $t_f - Q_c/RT$. There is a clear break in the relationship around σ^c = 0.726 and all data the points seem to fit tightly around the best fit line given by Eqs. (6c) - which is shown as a dashed line.

6.3. The Monkman-Grant relation

Fig. 2 shows the least squares estimates of the Monkman – Grant relation, with this relation explaining 99.78% of the variation in log minimum creep rates. This relationship gives a good representation of the experimental data irrespective of the temperature, with ρ = 1/1.1263 = 0.8879 appearing to be independent of temperature.

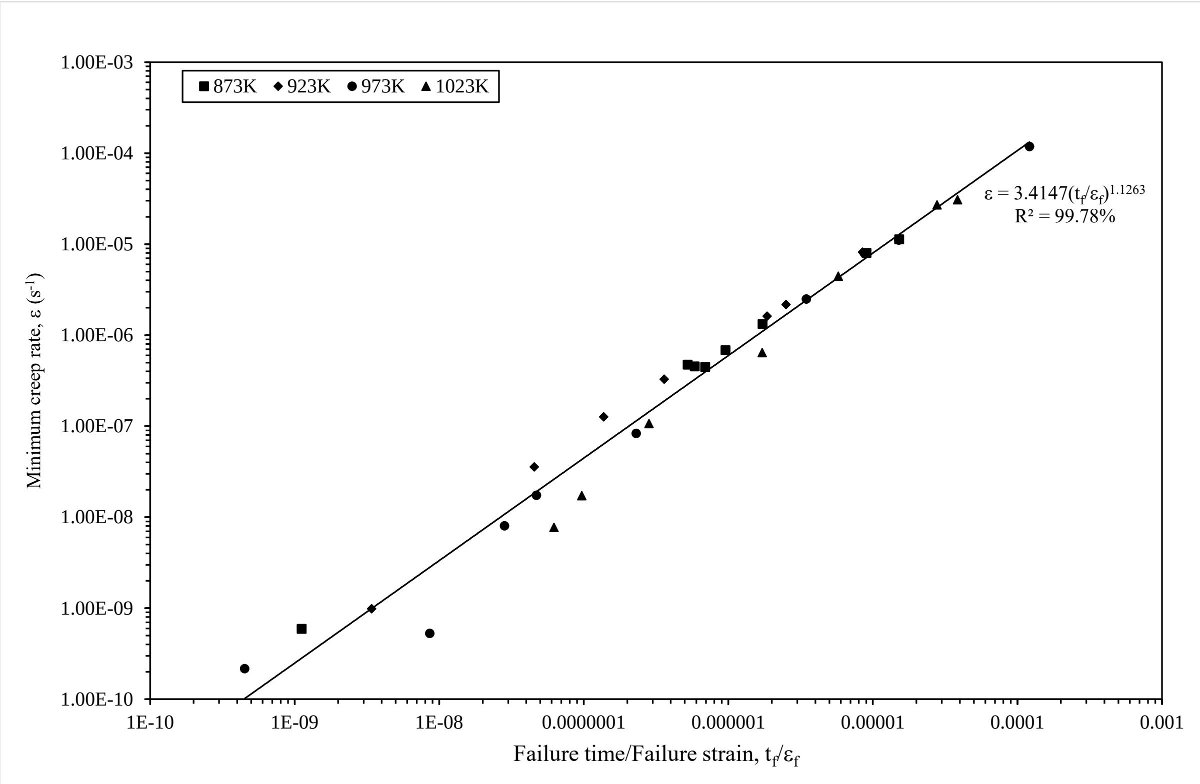

Fig. 2. Dependence of minimum creep rate on time to failure and failure strain.

6.4. Analysis at constant temperatures: checking the temperature dependency of Δ_j and m_j

The open circles in Fig. 3 show the experimentally measured creep curves obtained at 873K and for stresses of 900 MPa and above (900, 950, 1000,1050 and 1150 MPa – with one replication at 950 and 1050 MPa). This corresponds to the high stress regime identified in sub sections 4.1 and 4.2 above (j = 1). With a UTS of 1154 MPa at this temperature, these stresses all correspond to values for ln(τ) below -1.137. These creep curves have been normalised using the failure strains and times associated with these measured curves. Now Eq.(11d) can be thought of as a master normalised creep curve for all tests conditions at or above 840 MPa and at 873K (i.e. at or above σ/σ_{TS} = 0.726 at 873K). Table 3 shows the estimates made for the parameters m and Δ of this master curve using non-linear least squares.

Table 3 Parameters in the Wilshire CDM creep curve of Eq. (11a,b) and for use at a temperature of 873K with stresses above 840 MPa (i.e. j = 1).

Parameters	Calculation Method	Estimate
m_1		-0.3765
	Application of non-linear least squares to Eq. (11d)	
Δ_1		0.6592
n_1	Calculated from the values for v_1 in Table 2 using Eqs. (8a,12a)	-0.8652
A_1	Calculated from v_1, k_{21} and Q_{c1} in Table 2 using Eqs. (8a,12a)	6.8001E–07
G_1	Calculated from u_1, k_{11}, Q_{c1} in Table 2 and m_1, ϕ_1 using Eqs. (10f,12c)	1.5889E-06
χ_1	Calculated from m_1 and u_1 in Table 2 using Eqs. (10f,12c)	-0.7893
ρ	Exponent in the Monkman-Grant relation in Fig. 2	0.8879
ϕ_1	Derived from n_1, Δ_1 and ρ - see Eq. (11c)	1.2541

Using these values with w_f = 1 yields the master curve in Fig. 3 shown as the solid black curve. This curve appears to present the shape of the normalised experimental creep curves well.

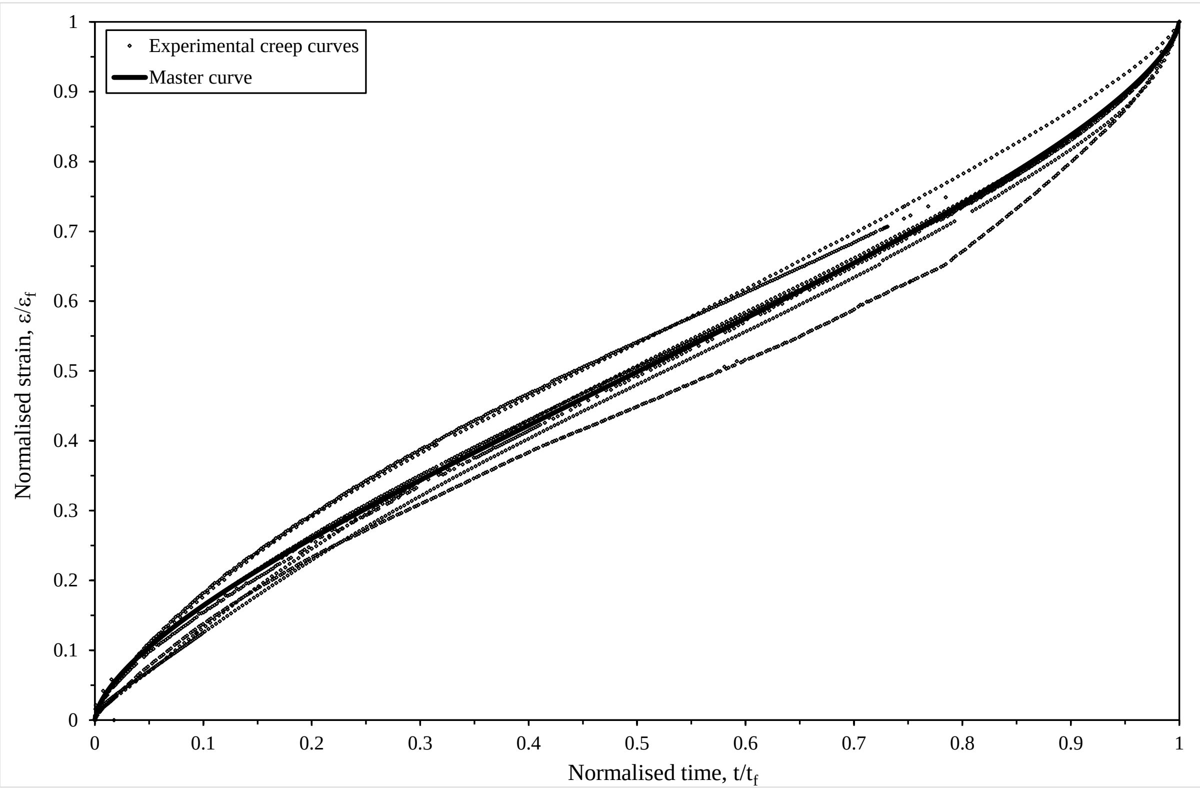

Fig. 3. Normalised experimental creep curves obtained at 873K and for stress at or above 900 MPa, together with the predicted master curve given by Eq. (11d).

This master curve can then be used to predict actual creep curves. This can be done by rescaling the master curve using the values for Γ in Eq. (11b), which in turn are calculated from the remaining parameters shown in Table 3. Fig. 4(a,b) shows some creep curves calculated in this way. In Fig. 4(a) the two creep curves measured at 873K and 950 MPa are shown together with the predicted creep curve for this test condition. The predicted curve is closer to one of the measured curves, but the figure also clearly demonstrates the large random or stochastic component of creep that is present for this alloy. In Fig. 4(b) the predicted creep curves is in between the two measured creep curves at 1050 MPa. Again, the stochastic nature of creep is clearly visualised.

The full accuracy of the predictions made for the recorded creep curves obtained at 873K and for stresses of 900 MPa or more is shown in Fig. 5. Here the actual strains are plotted against the predicted strains obtained using Eqs. (11a,b) on a (natural) log scale and so a perfect model would correspond to all data points falling on the shown 45^0 line. The Z parameter of 1.63 is below the minimum acceptable value suggested by Holdsworth et. al. and implies that 99% of all strains will fall within the range of 1.63 times the predicted strain. The shape of the creep curves appears to be well predicted at low and high strains, but at intermediate strains the predictions tend to underestimate the actuals creep strains. The mean percentage squared error (MPSE) is equal to 4.11% (with a root MPSE of just over 2%). Further, 4% of this MPSE is attributable to the squared difference between the average strain and the predicted strain. An additional 10.7% of this MPSE is attributable to the best fit line shown in Fig. 5 being a little steeper than the ideal 45^0 line, implying a small tendency of the Wilshire CDM model to systematically under predict at low strains and then over predict at higher strains. Both these components of the MPSE represent systematic errors made in predicting the actual creep curves at 873K and so this source of error amounts to some 15% of the MPSE. The remaining 85% of the MPSE is by deduction random (and so unpredictable) in nature and simply reflects the stochastic nature of creep in Waspaloy - as is evident in Figs (4).

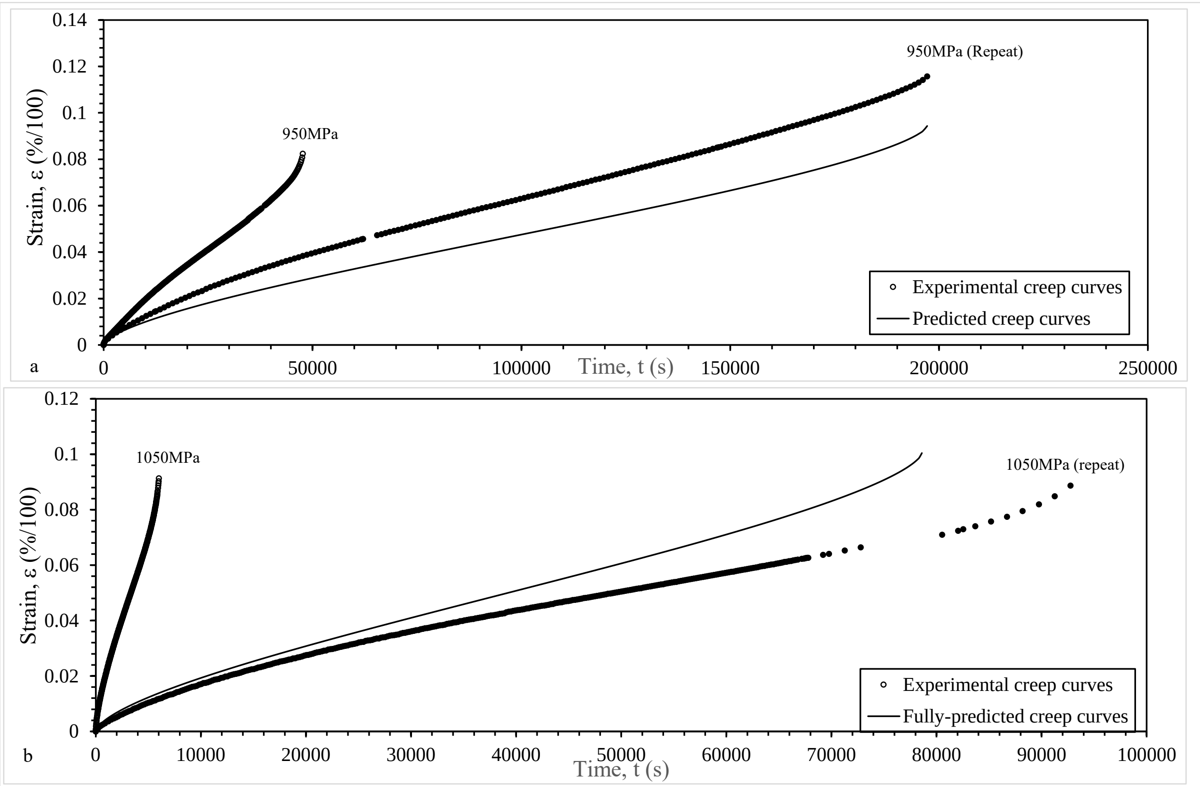

Fig. 4. Experimental creep curves obtained at 873K together with predicted curves given by Eqs. (11a,b) when (a) stress =950 MPa and (b) when stress = 1050 MPa.

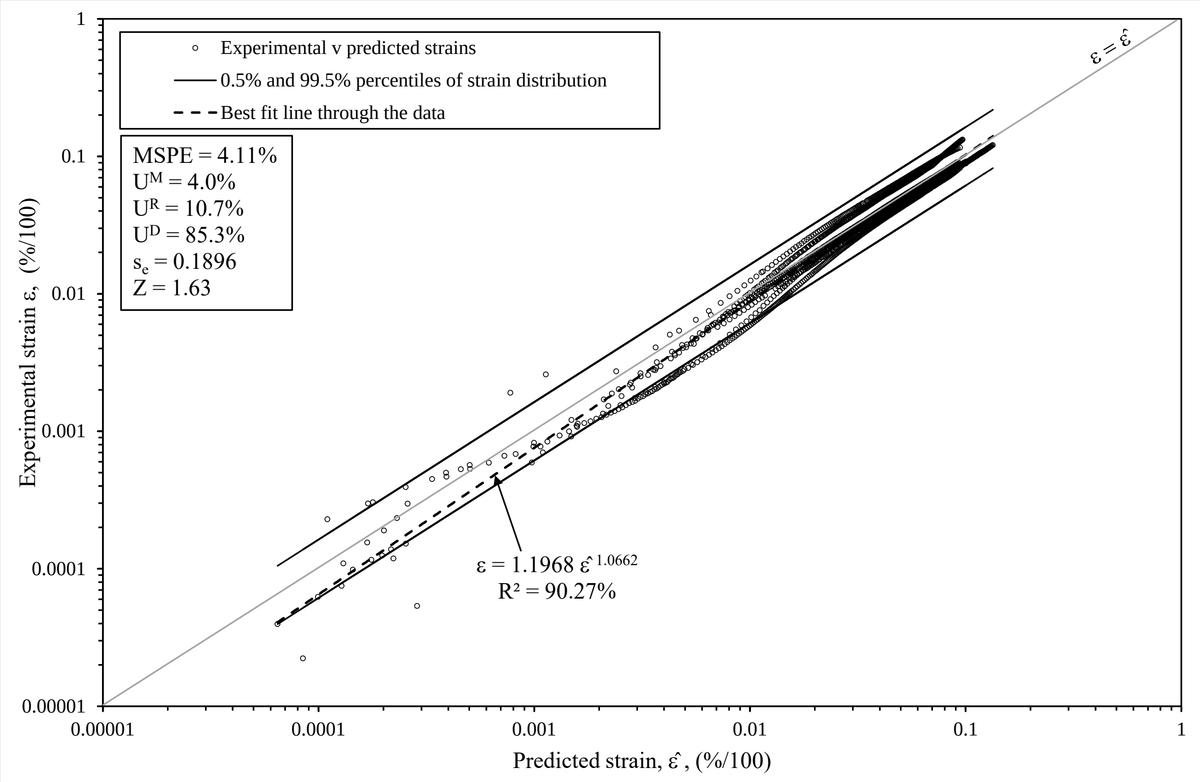

Fig.5. Plot of actual against predicted creep curves at 873K and at normalised stresses above 0.726.

Fig. 6 shows the results of repeating the above analysis at the other temperatures – both above and below the break point stress (as there is only 1 data point below σ^c at 873K no such an analysis could be done for these test conditions). It is clear from this figure that the parameter m is always higher in the lower stress regime (although the gap diminishes with decreasing temperature). In this low stress regime, m is also always positive implying there is no or very short-lived primary creep. In contrast, in the higher stress regime, m is always negative so that more pronounced primary creep occurs irrespective of the temperature. Irrespective of the stress regime, there is a tendency for m to reduce in size as the temperature decreases. Whilst ϕ is clearly higher in the low stress regime, there is no strong evidence to suggest that ϕ varies systematically with temperature. For the parameter Δ, there is again no strong evidence to suggest it varies either with stress or temperature.

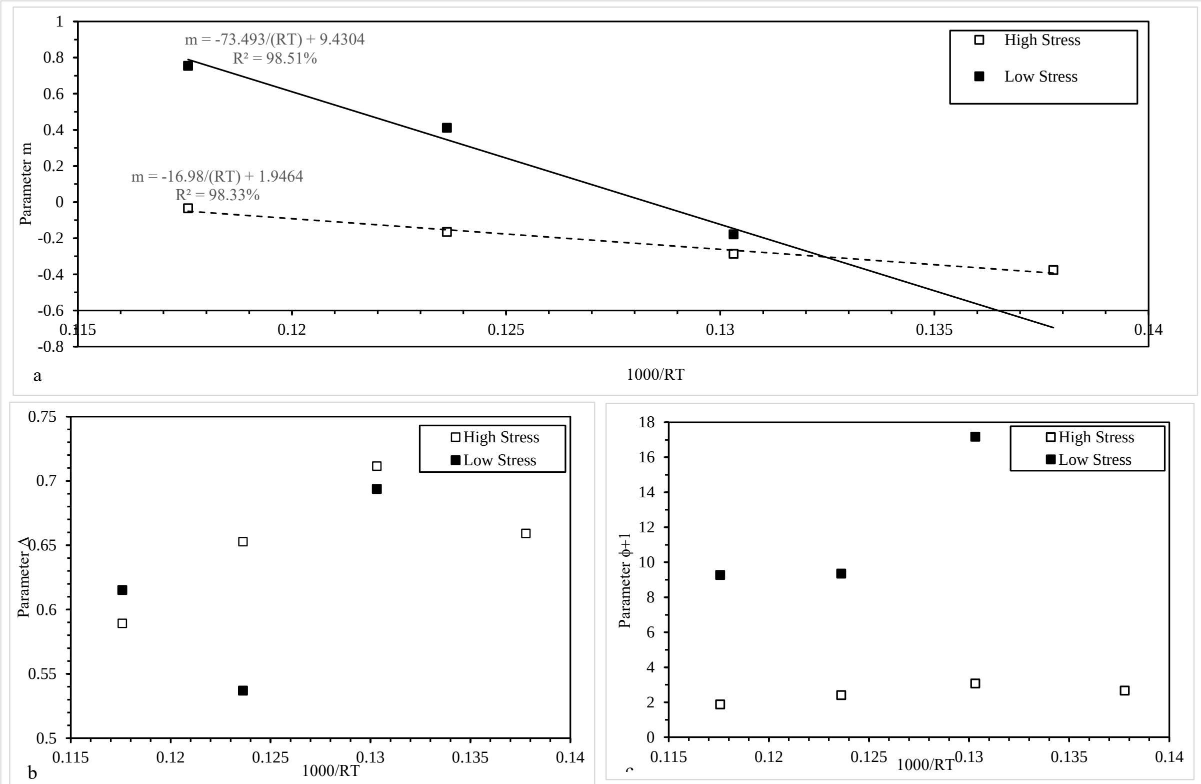

Fig. 6. Variations of (a) parameter m, (b) parameter Δ and (c) parameter ϕ with temperature and creep regime (low and high stress regimes).

Based on these results it seems sensible to assume that Δ and ϕ are independent of temperature, but m requires some additional modelling. In this paper, the dependence of m on temperature was modelled as a linear function of 1000/RT – as seen in Fig. 6(a)

6.5. Combining temperatures in the high stress regime .

The results above suggest the model can be used to predict creep curves at any temperature by treating Δ as fixed and modelling the parameter m using the linear expression shown in Fig. 6(a). Thus m+1 varied from 0.62 at 873K through to 0.97 at 1023K for normalised stresses in excess of 0.726. This implies that there is a well-defined primary creep stage present within the creep curves at these conditions, with this stage become less defined with increasing temperature. The open circles in Fig. 7 show the experimentally measured creep curves obtained at all temperatures and for all stresses in the high stress regime, (i.e. for σ^c > 0.726). These creep curves have been normalised using the failure strains and times associated with these measured curves. Using all such test conditions, and using w_f = 1, the non-linear least squares estimate for Δ_1 was found to be 0.6567 - and so not to different from the value obtained at 873K.

This Δ value and the temperature dependant values for m resulted in the master curves shown in Fig. 7 – represented as the solid grey curves and they appear to present the shape of the normalised experimental creep curves reasonably well. There is one master curve for each temperature as the parameter m changes with temperature.

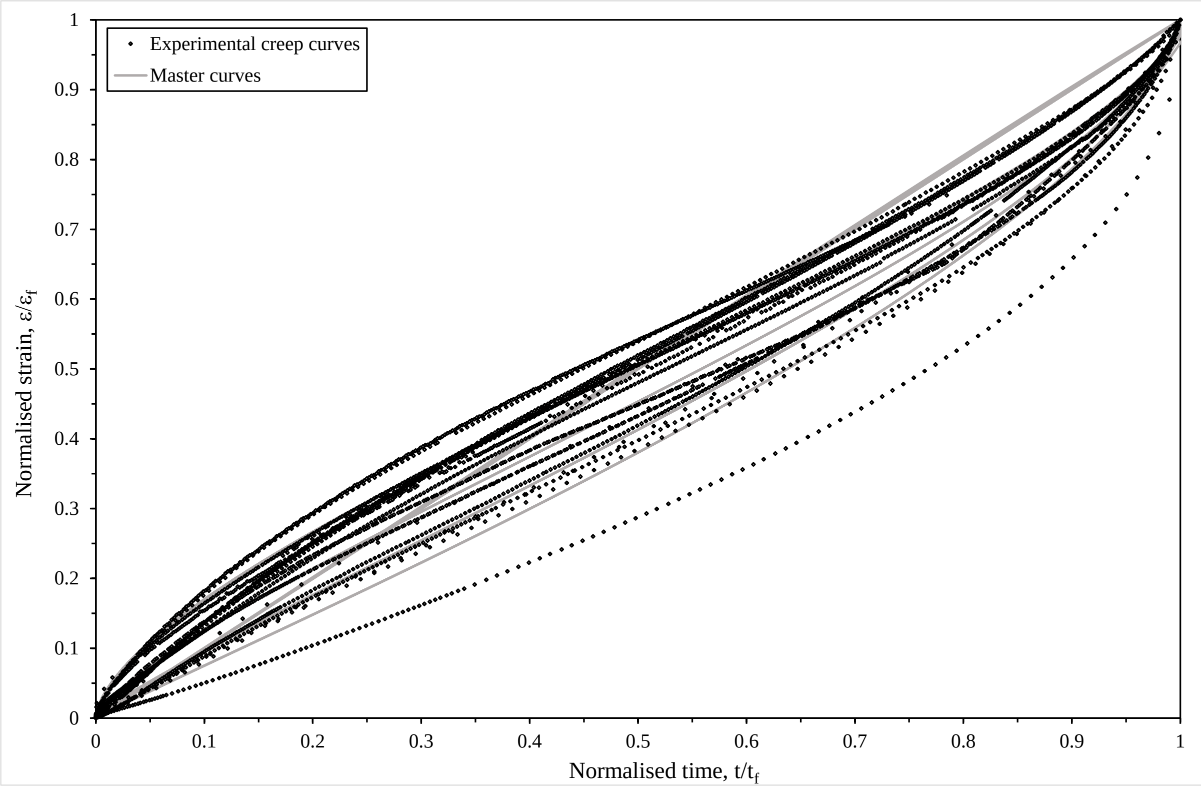

Fig. 7. Normalised experimental creep curves obtained at 873-1023K and for higher stresses corresponding to σ_c<0.726, together with the predicted master curves given by Eq. (11d).

Table 4 gives the parameters of Eq. (11a,b) that can be used to predict creep curves at the four shown temperatures and normalised stresses above 0.726. Fig. 8 shows one such predicted creep curve obtained at 973K and 750 MPa, together with the creep curves associated with the two specimens placed on test at this condition. Again, the highly stochastic nature of creep in this material is present and the predicted creep curve is a better fit at the earlier strains.

Table 4 Parameters in the Wilshire CDM creep curve of Eq. (11a,b) and for use at a temperatures of 873K, 923K, 973K and 1023K and with normalised stresses above 0.726 (i.e. j = 1).

	Estimates				
Parameters	All temperatures	873K	923K	973K	1023K
m_1	-	-0.393	-0.2663	-0.1526	-0.05
Δ_1	0.6567	-	-	-	-
n_1	-0.8652	-	-	-	-
A_1	-	1.2623E-07	6.8001E-07	3.0812E-06	1.2044E-05
G_1	-	8.0552E-05	5.5586E-05	5.3705E-05	6.6693E-05
χ_1	-	-0.4957	-0.5993	-0.6921	-0.7759
ρ	0.8879	-	-	-	-
ϕ_1	1.6065	-	-	-	-

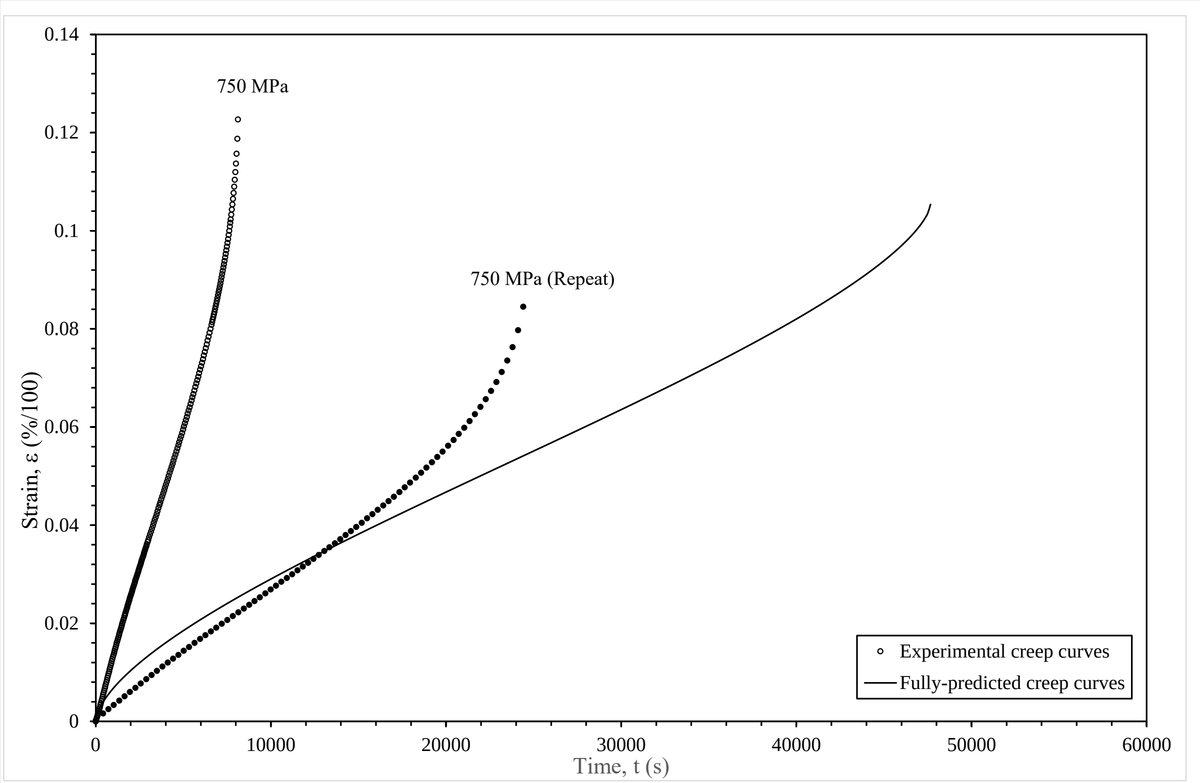

Fig. 8. Experimental creep curves obtained at 973K and 750 MPa, together with predicted curve given by Eqs. (11a,b).

The full accuracy of the predictions made for the recorded creep curves obtained at 873K – 1023K and for normalised stresses of 0.726 is shown in Fig. 9. Here the actual strains are plotted against the predicted strains obtained using Eqs. (11a,b) on a (natural) log scale and so a perfect model would correspond to all data points falling on the shown 45^0 line. The Z parameter of 1.79 is below the minimum acceptable value suggested by Holdsworth et. al. and implies that 99% of all strains will fall within the range of 1.79 times the predicted strain. The shape of the creep curves appears to be well predicted at low and high strains, but at intermediate strains the predictions tend to underestimate the actuals creep strains The mean percentage squared error (MPSE) is equal to 6.86% (with a root MPSE of just over 2.62%). Further, 4.5% of this MPSE is attributable to the squared difference between the average strain and the predicted strain. An additional 8.5% of this MPSE is attributable to the best fit line shown in Fig. 9 being a little steeper than the ideal 45^0 line, implying a small tendency of the Wilshire CDM model to systematically under predict low strains and then over predict at higher strains. Both these components of the MPSE represent systematic errors made in predicting the actual creep curves at these conditions and so this source of error amounts to some 13% of the MPSE. The remaining 87% of the MPSE is by deduction random (and so unpredictable) in nature and simply reflects the stochastic nature of creep in Waspaloy as was evident in Figs (4,8).

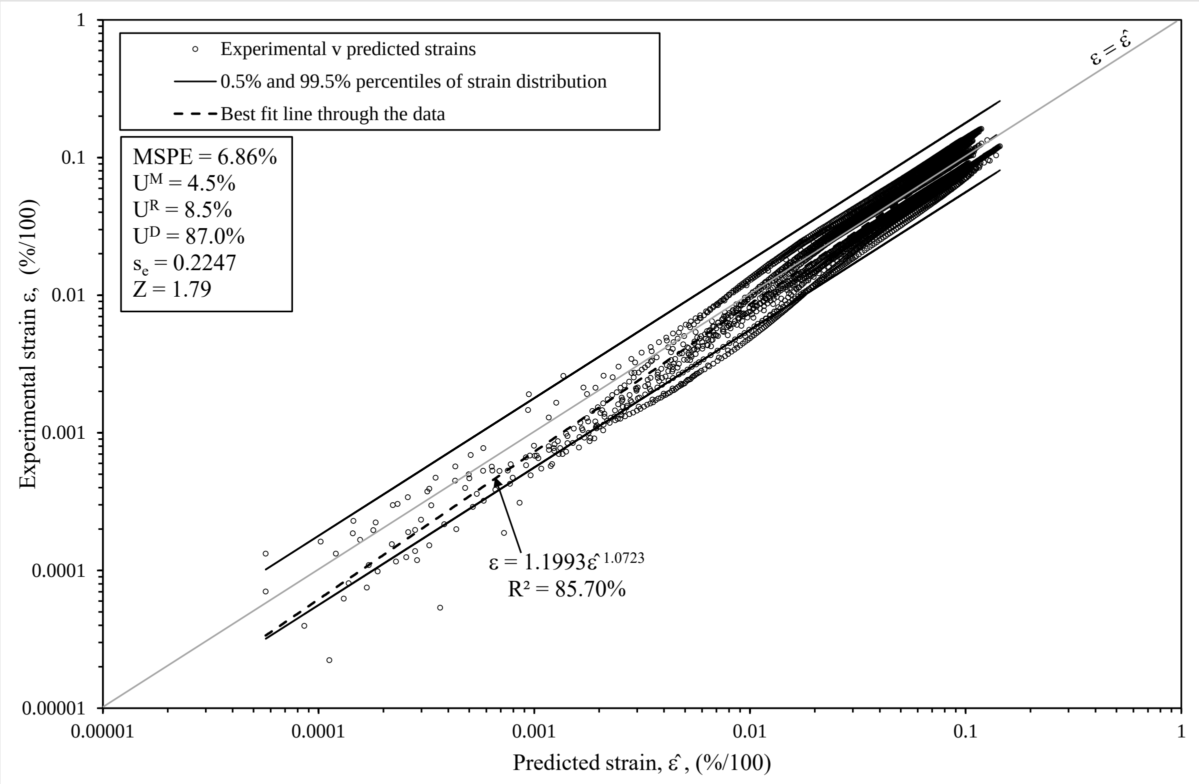

Fig. 9. Plot of actual against predicted creep curves at 873K – 1023K and at normalised stresses above 0.726.

7. Conclusions

This paper integrated the Wilshire failure time and minimum creep rate equations into a modified Kachanov-Rabotnov CDM creep model that was capable of modelling both primary and tertiary creep. The paper also presented an estimation strategy for the unknown parameters of this model. This CDM model was then applied to uniaxial creep data on Waspaloy. It was found that the Wilshire equations provided very good fits to the data on times to failure and minimum creep rates. Two different creep regimes were identified using these equations – with a change in creep regime appearing to take place at a normalised stress of 0.726. The activation energy, whilst statistically significantly different either side of this stress, was not very different in value. Following other authors, it was hypothesised that this break was due to a dominance of diffusive climb at stresses below σ_Y, with dislocation-dislocation interaction in the form of forest hardening limiting creep rates occurring at higher stresses.

The parameter m of the CDM model was found to be stress and temperature dependent, such that at the lower temperatures m is clearly negative implying well defined tertiary creep. This stage of creep then tends to disappear at higher temperatures (as m tends to zero at these temperatures). The parameter Δ was found to be broadly independent of the temperature. Finally, the CDM model was shown to be reasonably accurate at predicting the experimental creep curves. At 873K the root MPSE was just over 2%, whilst the model applied to all temperatures had a root MPSE of 2.6%. In both these illustrations the vast majority of the prediction errors were random in nature. The scatter, as measured by the Z parameters, were below 2 in both instances

Some areas for future research include the application of this model to other high temperature materials, to assessing the models ability to predict creep curve shape at conditions not used in estimating the models unknown parameters and the incorporation of the Wilshire equations into multiple damage parameter CDM models.

Data Statement

The data used in this paper has been in the public domain for a number of years as shown in the reference section. Data can be made available from the author upon request.

References

[1] P. Ruffles, Aerospace Structural Materials: Present and Future, (1995) The Institute of Materials London.

[2] Materials UK Energy Review: Report 1, Energy Materials – Strategic Research Agenda, 2007.

[3] B. Wilshire, A.J. Battenbough, Creep and creep fracture of polycrystalline copper. Materials Science and Engineering A, 443 (2007) 156-166.

[4] B. Wilshire, P. J. Scharning, Prediction of long term creep data for forged 1Cr-1Mo-0.25V steel, Materials Science and Technology, 24(1) (2008) 1-9.

[5] B. Wilshire, M. Whittaker, Long term creep life prediction for Grade 22 (2·25Cr—1Mo) steels, Materials Science and Technology, 27(3) (2011) 642-647.

[6] M. Evans, Incorporating specific batch characteristics such as chemistry, heat treatment, hardness and grain size into the Wilshire equations for safe life prediction in high temperature applications: An application to 12Cr stainless steel bars for turbine blade, Applied Mathematical Modelling, 40(23-24) (2016) 10342-10359.

[7] B. Wilshire, P. J. Scharning, A new methodology for analysis of creep and creep fracture data for 9–12% chromium steels, International Materials Reviews, 53(2) (2008) 91-104.

[8] M. T. Whittaker, M. Evans, B. Wilshire, Long-term creep data prediction for type 316H stainless steel. Materials Science and Engineering A, 552 (2012) 145-150.

[9] M. T. Whittaker, W. Harrison, C. Den , C. Rae , S. Williams, Creep Deformation by Dislocation Movement in Waspaloy, Materials, 10(1) (2017) 61.

[10] B. Wilshire, P. J. Scharning, Extrapolation of creep life data for 1Cr-0.5Mo Steel, Int. J. Press. Vessel. Pip., 85 (2008) 739–743.

[11] M. T. Whittaker, W. J. Harrison, R.J. Lancaster, S. Williams, An analysis of modern creep lifing methodologies in the Titanium alloy Ti6-4, Mater. Sci. Eng. A, 577 (2013) 14–119.

[12] W. Harrison, M. T. Whittaker, S. Williams, Recent Advances in Creep Modelling of the Nickel Base Superalloy, Alloy 720Li, Materials, 6 (2013) 1118–1137.

[13] Z. Abdallaha, K. Perkins, S. Williams, Advances in the Wilshire extrapolation technique—Full creep curve representation for the aerospace alloy Titanium 834, Materials Science and Engineering: A, 550(30) (2012) 176-182.

[14] V. Gray, M .T. Whittaker, Development and Assessment of a New Empirical Model for Predicting Full Creep Curves, Materials, 8, (2015) 4582-4592.

[15] M. Evans, T. Williams, Assessing the capability of the Wilshire equations in predicting uniaxial creep curves: an application to Waspaloy, Int. J. Pres. Ves. Pip. 172 (2019) 153–165.

[16] J. A. Cano, C. M. Stewart. A continuum damage mechanics (CDM) based Wilshire model for creep deformation, damage, and rupture prediction, Materials Science and Engineering: A, 799 (2021).

[17] R.W. Evans, B. Wilshire, Introduction to Creep, 1st Edition, (1993) Institute of Materials, London 23–28.

[18] B. Wilshire, P.J. Scharning, Theoretical and practical approaches to creep of Waspaloy, Materials Science & Technology, 25(2) (2009) 242-248.

[19] R.W. Evans, A constitutive model for the high-temperature creep of particle-hardened alloys based on the θ projection method, Proceedings of the Royal Society London - A, 456 (2000) 835-868.

[20] L.M. Kachanov, Rupture Time Under Creep Conditions, Int. J. Fract., 97 (1999) 11–18.

[21] Y.N. Rabotnov, Creep Problems in Structural Members, (1969) North-Holland Publishing, Amsterdam, The Netherlands.

[22] D.R. Hayhurst, B.F Dyson, J. Lin, Breakdown of the skeletal stress technique for lifetime prediction of notched tension bars due to creep crack growth. Engineering Fracture Mechanics, 49(5) (1994) 711–726.

[23] D.R. Hayhurst, Creep continuum damage mechanics: A unifying theme in high temperature design, High Temperature Structure Design, ESIS 12, Edited by L. H. Larsson, Mechanical Engineering Publications, London, 3 (1992) 17-334.

[24] M.S. Haque, C.M. Stewart, 2016, Finite-Element Analysis of Waspaloy using Sinh Creep-Damage Constitutive Model Under Triaxial Stress State, ASME J. Pressure Vessel Technol., 138(3), (2016) 031408.

[25] Y. Liu, S. Murakami, Damage localization of conventional creep damage models and proposition of a new model for creep damage analysis, JSME International Journal Series A, 41(1) (1998) 57–65.

[26] D. C. Dunand, B.Q. Han and A.M. Jansen: Monkman-Grant analysis of creep fracture in dispersion-strengthened and particulate-reinforced aluminium, *Metallurgical and Materials Transactions A*, 30(13) (1999) 829-838.

[27] S.R. Holdswortha, M. Askinsb, A. Bakerc, E. Gariboldi, S. Holmstro¨me, A. Klenkf, M. Ringelf, G. Merckling, R. Sandstromh, M. Schwienheeri, S. Spigarellij, Factors influencing creep model equation selection, International Journal of pressure Vessels and Piping, 5 (2008) 80–88.

[28] S.R. Holdsworth, Developments in the assessment of creep strain and ductility data, Mater. High Temp., 21(1) (2004) 125–32

[29] C. Granger, P. Newbold, Some comments on the evaluation of economic forecasts, *Applied Economics*, 5 (1973) 35-47.

[30] H. Theil, Applied Economic Forecasts, (1966) Amsterdam North Holland.

Appendix

Nomenclature	
T	Temperature (K)
σ	Stress (MPa)
σ_{TS}	Ultimate tensile strength (UTS)
$w, \dot{w}, w_f$	K-R damage parameter, rate of damage accumulation, damage at failure
τ	Transformed and normalised stress, $[-\ln(\sigma/\sigma_{TS}]$
σ^c	Critical values for the normalised stress in the Wilshire models
t	Time (s)
t_f	Time at failure
ε	Strain
ε_f	Strain at failure
$\acute{\varepsilon}$	Strain rate
$\acute{\varepsilon}_m$	Minimum creep rate
A, δ, ϕ, n, χ	Parameters of the K-R and Wilshire CDM models
m	Additional parameter of the Hayhurst extension of the K-R model
k_1, u	Parameters of the Wilshire failure time equation
k_2, v	Parameters of the Wilshire minimum creep rate equation
Q_c	Activation energy for self diffusion
C, ρ	Monkman- Grant parameters
G	Additional Parameter of the Wilshire CDM model
h	Number of separate test conditions in test matrix
m_i	Number of strain-time readings made at test condition i
ε_{it}	Strain recorded at time t under test condition i
$\hat{\varepsilon}_{it}$	Strain predicted by the Wilshire CDM model at time t under test condition i
e_{it}	Residual log times
MPSE	Mean percentage squared error
$\bar{e}$	Mean residual log timeover all test conditions
$\bar{e}_i$	Mean residual log time at test condition i
s_{ei}	Standard deviation in these residual log times at test condition i
s_e	Standard deviation in these residual log times at all test conditions
z_p	Percentile of the standard normal distribution
Z	Parameter quantifying effectiveness of a creep model to predict strain
$s^2_{\ln(\hat{\varepsilon})}$	Variance in the predicted log strains
S^2_v	Variance in the residual log times
$s^2_{\ln(\varepsilon)}$	Variance in the log strains
r	Correlation between actual and predicted log strains
α, β	Parameters of a best fit line for $\ln(\varepsilon)$ v $\ln(\hat{\varepsilon})$
U^M, U^R, U^D	Proportions of the MPSE
Γ, K	Variables for un-normalised the master normalised creep curve
$\ln(\varepsilon_i)_p$	Percentile of the log strain .
σ_t and σ_s	Additional material constants in the model by Haque [24]
$\dot{\varepsilon}_f$	Creep rate at failure
$\dot{w}_f/\dot{w}_0$	Ratio of the final to initial rates of damage accumulation