Modeling view-count dynamics for YouTube videos: A multi-modal perspective

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Abstract

Purpose:

Social media has become an important part of everyone's life. People use these platforms for varied purposes like for entertainment, seeking information, getting news, etc. These platforms have totally revolutionized the media industry because of the easiness in their accessibility. YouTube is one of the most used social media platforms. The Google owned website allows its users to upload and view videos on its platform. View-count patterns can be utilized in behavioral, social and management sciences. YouTube provides notification to the subscribers whenever a channel uploads a new video thereby making the channel subscribers the potential viewers of the video. And thus, they are the first to come to know about any new offering. But later on, the view-count also increases due to virality i.e. mass sharing of the content by the users on different social media platforms similar to word of mouth in the field of marketing. These different diffusion patterns should be carefully examined as they can help to inflate traffic and generate revenue.

Methodology:

YouTube's View-count grows majorly through virality. The pattern of view-count growth has generally been considered uni-modal in most of the available research in the field of

YouTube. In the present work, the growth process due to views through the subscribers and views due to word of mouth (virality) is presented. Considering that the impact of virality in view-count growth comes later in the video life cycle; the viewing patterns of both the segments have been mathematically modelled; independently.

Findings:

Different models have been proposed to capture the view-count growth pattern and how the impact of virality changes the view-count growth curve and thereby results into multi-modal curve structure. The proposed models have been verified on various view-count datasets of YouTube videos using SPSS (Statistical Package for the Social Sciences) and their ranks have been determined using weighted criteria-based approach. The results obtained clearly depicts the presence of many modes in the life cycle of view counts.

Originality/value: Till now, the literature is evident of video life cycle following bell shape curve. This study claims that the initial thrust is by subscribers and then the contribution in view count by people watching via word of mouth comes into picture and brings in another hump in the growth curve.

Keywords- Multi-modal curve, Subscribers, Video Life Cycle, View-count, Virality, YouTube.

1. Introduction

YouTube is an American video-sharing website headquartered in San Bruno, California. The service was created by three former PayPal employees – Chad Hurley, Steve Chen, and Jawed Karim – in February 2005. Google bought the site in November 2006 and it now operates as one of Google's subsidiaries. The site allows users to upload, view, rate, share, add to favorites, report and comment on videos, subscribe to other users, and it makes use of WebM, H.264/MPEG-4 AVC, and AdobeFlash Video technology to display a wide variety of user-generated and corporate media videos. Available content includes video clips, TV show clips, music videos, short and documentary films, audio recordings, movie trailers and other content such as video blogging, short original videos, and educational videos. The channels created by various users have been influencing many people in many ways be it sports, be it education or be it any area of mankind. Of late, a recent study by Arora and Lata (2020) describes how they can even be used to influence on destination visit intentions.

Most of the content on YouTube has been uploaded by individuals, but media corporations including CBS (Columbia Broadcasting System), the BBC (British Broadcasting Corporation), Vevo, and Hulu offer some of their material via YouTube as part of the YouTube partnership program. It is not just utilized for personal content sharing but rather it has been acting as a helping hand for Government organizations as well. A study by Bonson and Bednárová (2018) describes such exemplification. Unregistered users can only watch videos on the site, while registered users are permitted to upload an unlimited number of videos and add comments to videos. As claimed by Ashman et al. (2018), netigens like to post their comments, share and subscribe the channels and various others offerings of YouTube.

YouTube earns advertising revenue from Google AdSense, a program which targets ads according to site content and audience (Kumar et al. 2020). The vast majority of its videos are free to view, but there are exceptions, including subscription-based premium channels, film rentals, as well as YouTube Red, a subscription service offering ad-free access to the website and access to exclusive content made in partnership with existing users. Playback, Quality and formats, Uploading, 3D videos and 360° videos are some of the implicit features offered by You Tube as a part of Video technology. Similarly, Community, Content accessibility, Platforms are a part of User features of YouTube. The work by Lai and To (2015) provides a description about the ground theory approach for the same. Similarly another work by Liu (2014) provides an outlook towards the impact of social media cues and its effectiveness.

Out of many attributes of YouTube, View count has been one of the key attributes and has played a vital role in describing various things about YouTube's popularity. Sometimes these view counts are that huge that they are called as viral videos (Krijestorac et al. 2020). There have been several researches works for understanding the reason behind the large number of view count (Park et al. 2015; Jeon et al. 2020; Tafesse, 2020).

View-count predicting models explain the time dependent behavior of view-counts of a video. As far as prediction of view-count is concerned; various researchers have given their proposals in this framework; like the work by (Vaish et al. 2012; Richier et al. 2014a; Bauckhage et al. 2015; Aggrawal et al. 2020). A glimpse on how huge and consumption of YouTube was given by Cheng et al. (2008). They measured the effect of various factors on

view count. Zhou et al. (2010) studied the effect of recommendation system on view-count. Ding et al. (2011) demonstrated the uploading behavior of the uploaders. Vaish et al. (2012) used different attributes to calculate the virality index. Khan and Vong (2014) studied the view-count increment due to the impact of traffic coming from other social media platforms to YouTube. Richier et al. (2014b) proposed six different models to predict the view-count of various categories of videos. Three of them were for fixed population (viewers) models and three for growing population (viewers) models. Xu et al. (2015) showed that a video can have multiple popularity peaks throughout its life cycle. Goel et al. (2016) showed how virality is different from broadcasting. Their proposal showed that the popularity is usually driven by broadcasting even in the case of social media, which forces the producers to generate awareness about web series among viewers through promotional efforts.

Another set of researchers like Zhou et al. (2010) and Portilla et al. (2015); tried to find out the factors affecting the total view count. They found that large proportion of total view count is due to the occurrence of that video in recommendation list of any other video having large view-count. The literature is also evident of the fact that view count depends on lots of factor like content, popularity, uploader popularity, etc. (Bisht et al. 2019) Extending the work of Richier et al. (2014a; 2014b) and Aggrawal et al. (2018a) proposed three models which predicted the view-count of videos in three different scenario of population growth (viewers). They also considered repeat viewing along with exponential and linear growth in number of viewers. In their yet another work, Aggrawal et al. (2018b) gave a modeling and characterizing approach for viewers of You Tube videos. Bisht et al. (2019) applied ISM technique to find most influential attributes which influences other attributes causing viewcount. Irshad et al. (2019) gave an approach to model the popularity dynamics based on YouTube Viewers and Subscribers. Another work by Irshad et al. (2020) presents an approach for understanding active life span of YouTube Videos based on Changing Viewership-Rate. Work by Martin et al. (2020) describes about multi-mode perspective of information and management. Similarly, France et al. (2021) have provided an integrative decision support system framework for understanding online video channel management. In yet very recent work by Cao et al. (2021), the authors describe about the understanding of consumer's behavior by examining moderation effect of social media.

Nevertheless, all the existing models are incapable of analyzing the multi modal nature of the view-count growth curve. The variation in netizens viewing behavior requires a renewed

focus towards the segmented market structure which can directly affect the expected view count for YouTube. Previous studies are limited only towards describing the uni-modal nature of the view count growth curves; wherein in a real-life scenario, the diffusion of such a thing like video into the internet market may not have a perfect bell-shaped pattern. Consequently, it cannot be predicted by considering the earlier approaches given in literature. In today's era of competition; to build long-lasting relations and gain trust with its viewers, it becomes mandatory for YouTube to take into account different characteristics and behaviors of viewers in various segments of markets. Like, the concept of multisegmented market in management science (Anand et al. 2018) suggests the presence of dual market: an "early" market corresponding to the high needs and less price sensitivity and a "main" market corresponding to the relatively less needs and high price sensitivity; this work also discusses the presence of two contributors in the view count growth curve.

Reflecting on this research gap of multi-modality in the curve, the present study proposes the presence of two players; subscribers and normal viewers for contributing in the total view count growth of any video. Subscribers are different from normal viewers. They are mainly technophiles attracted to a particular video or channel for its competitive edge over other similar offerings in the segment; on the other hand, rest of the viewers are primarily more interested in the video because others have told them so. And as a matter of fact, this second category is enormous in number. The view counts generated through these contributors behaves differently as compared to the view count happened through subscribers. They are more calculative and are rationalists who weigh out interest in the given video before they make the final call. This reasoning accounts for entrance of word of mouth-based viewers late into the market. However, the existing view count models presume their entry at the earliest stages of the market. Hence, there is essentially sometime of consideration after a word of mouth-based viewer comes to know about the video and before he views it. In this paper, this dual internet market size modeling has been provided that is solved using the unified approach as available in management science. The numerical analysis is presented to demonstrate the practical applicability of the proposed diffusion models using the actual view count data of certain set of videos. Moreover, the proposed models have been further ranked using a multi criteria decision making technique named; weighted criteria approach. To the best of our knowledge, this is the earliest attempt to model the multi-modal nature of the view count growth curve.

The remainder of the paper is structured as follows. Section 2 reviews the relevant building blocks of the proposed work. It summarizes the various set of assumptions and mathematical modeling framework under consideration and some aspects of the literature available in the relevant field. Section 3 describes about the numerical illustration that has been carried out on 6 different sets for validation purpose. The section also provides the ranking results carried out on six data sets; wherein weighted criteria-based approach has been utilized for the same. Section 4 provides the overall discussion in which various research contributions. Implications for practice and limitations about the work followed by future scope have been described. Lastly Conclusions are presented in section 5.

2. The Building Blocks of Proposed Work

The methodology discussed in this work is based on following set of assumptions:

- Both the groups have their own potential viewers based on their respective viewing behavior.
- One viewing pattern is not influenced by the other, *i.e.* there is no cross-internet market influence.
- Market size (potential viewers) is fixed during the information diffusion process.
- There is a time lag between viewing through both the pedagogies.

Management Science is evident of the utility of Bass Model in not only just management domain but other domains like sociology, economics, and psychology to name some (Bass, 2004). Aggrawal et al. (2018a) presented an analogy from marketing science and proposed a framework which characterized the literary theory in terms of the view count. Their viewership computational model considered the association of two types of viewers in contributing to the overall view count. The model can be said to be developed based on the famous Bass Model (1969) wherein; the authors in their approach, assumed viewership to be initiated by certain number of viewers after the launch of the video in the internet market. Furthermore, they assumed the rate of viewing any video at a given time to comprise of two components that administrate the viewing process; the first factor constitutes the videos watched through external influence with an impact rate v_1 and the second factor represents the additional number of viewers who watch a video and under the influence of word of mouth (with rate v_2). Their View Count Process can be modelled in the following manner:

$$\frac{dV(t)}{dt} = v_1 \left(N - V(t) \right) + v_2 \frac{V(t)}{N} \left(N - V(t) \right) \tag{1}$$

Here v_1 represents the fraction of all viewers who are neoteric. The product v_2/N times V(T) reflects the pressure operating on followers as the number of early viewer's increases. After solving the equation (1) the closed form solution can be obtained as follows:

$$V(t) = N \left(\frac{1 - e^{-(\nu_1 + \nu_2)t}}{1 + \frac{\nu_2}{\nu_1} e^{-(\nu_1 + \nu_2)t}} \right)$$
 (2)

knowing the manner in which a video is able to generate popularity in terms of view count. Using the analogy from Marketing Science, a recent study by Irshad et al. (2019) describes about the alternative formulation of the aforesaid Aggrawal et al. (2018a) modelling framework. But in the present wok, a more general approach is required that is able to cater to the unified aspect of various scenarios that exist in the market. Using the hazard rate

approach as described in marketing science (Bass 1969; Bass 2004; Anand et al. 2016)

Their model so obtained had the ability to act as a forecasting tool that can help the firm in

$$\frac{dV(t)}{dt} = \left(\frac{f(t)}{1 - F(t)}\right) \left(N - V(t)\right) \tag{3}$$

literature, the following differential equation can be utilized to model the process:

The modelling framework obtained thereafter is very flexible and a closed form solution to the problem can be obtained using the initial condition at t = 0, V(t) = 0, as:

$$V(t) = N.(F(t)) \tag{4}$$

Where F(t) is the distribution function obtained through equation (4). As per the requirement, various distribution functions can be fit in in the above model and modelling can be done. If the hazard rate is considered as logistic function, one can obtain the same model as given by Irshad et al. (2019) and Aggrawal et al. (2018a). Thereby, this approach can be termed as a generic approach for modelling the view count process.

2.1. The Proposed View Count Modeling Framework

The uniqueness of the contribution in view count by both the groups; subscribers and normal viewers is worthy to elaborate. The present framework assumes that viewers are highly affected by the information that transfers from their own peer group rather than the same

information disseminated throughout the entire population (Dwivedi et al. 2008). It is fundamentally accepted fact and has been proven in the literature that once any diffusion process starts; it is slowly started by the primary group and then the spread happens through word of mouth. Thereby, here the authors have used the index s to define the notations of subscribers and the index s defines the main internet market (that contributes through word of mouth).

The modeling framework mentioned above can be utilized for understanding the process and mathematical equation can be presented as follows:

2.1.1. For view-counts through subscription

The information about the new product spreads with time (Anand et al. 2016). Similar ideology can be considered for the videos as well. As early viewers are usually the ones who have any type of subscription with them, the authors assume that the initially only the subscribers will contribute but later on others also enter into the contribution system. By definition of an S-shaped diffusion pattern, it is clear that the diffusion initially expands at a slow rate and later on, number of viewers' increases with time. Therefore, with this mindset it is justifiable to consider the subscription process to be logistic viz. S-shape. The S curve is a long-standing methodology used for prediction and hence, viewership process by this group can be best described by considering logistic distribution function, *i.e.* $\frac{f_s(t)}{1-F_o(t)} = \frac{b_s}{1+\beta_s e^{-b_s t}}$.

Here, $\frac{f_s(t)}{1-F_s(t)}$ denotes the hazard rate function that a subscriber will watch a video as a result of external and internal forces, N_s describes the market potential of the subscribers, $V_s(t)$ stands for the cumulative number of viewers by time t because of subscribers. By substituting the value of $\frac{f_s(t)}{1-F_s(t)}$ in Equation (3), and utilizing equation (4), the cumulative number of early market adopters can be given as:

$$V_{s}(t) = N_{s} \left(\frac{1 - e^{-b_{s}t}}{1 + \beta_{s} e^{-b_{s}t}} \right)$$
 (5)

2.1.2. For view-counts through word of mouth

In marketing science, it has been widely studied that the buyers of the main market are generally more utilitarian and much interested in the applicability of the innovation (Dwivedi et al. 2008, Williams et al. 2009). They appraise the benefits of adopting a product and also wait until the utility of the product override its price before entering into the market (Rogers, 1962). In line with this, here also it can be understood that, a video on a channel is being largely watched by people who get to know about a video through word of mouth. Based on this assumption it can be considered that the accountability to the view count process can take more or less time vis-a-vis subscribers depending upon the video's availability in internet market. To address the heterogeneity of people contributing to the process, the authors have considered different types of S-shaped distribution functions. Based on aforesaid discussion, the authors have assumed, $\frac{f_w(t)}{1-F_v(t)}$ as the rate at which main internet market viewer will

watch video as a result of external and internal forces and N_w defines the market potential of the main market (through WOM); $V_w(t)$ describes the cumulative number of viewers of the main contributors by time t.

The viewing process through subscribers can be understood by expression obtained in equation (5). But for reading the main contribution i.e. from word of mouth, the authors have employ a new parameter τ for incorporating the delay that the promotors through word of mouth take in comparison to the subscribers. Hence, the contribution of view counts being generated through word of mouth after a certain time τ can be represented in the following way:

$$\frac{dV_{w}(t-\tau)}{dt} = \frac{f_{w}(t-\tau)}{1 - F_{w}(t-\tau)} \left[N_{w} - V_{w}(t-\tau) \right]$$

$$\tag{6}$$

If τ equals 0, Equation (6) is equivalent to Equation (3). Let $t' = t - \tau$, and so, equation (6) can be rewritten as

$$\frac{dV_{w}(t')}{dt} = \frac{f_{w}(t')}{1 - F_{w}(t')} [N_{w} - V_{w}(t')]$$
(7)

$$V_{w}(t') = N_{w} \cdot \left(F_{w}(t')\right) \tag{8}$$

It is imperative to note that there can be different scenarios and patterns in which the view count could behave for the population watching the video through word of mouth. Hence, it is imperative to consider various distributions to study them. Some of them, considered for this study are mentioned below:

Case 1: In case when word of mouth follows exponential distribution; i.e. $1-\exp(-b.t')$ Using this in equation (8), the total number of viewers by time t' is found as:

$$V_{w}(t') = N_{w} \left(1 - e^{-b_{w}t'} \right) \tag{9}$$

Case 2: Considering F (t) to follow the logistic distribution; and using it in Equation (8), the corresponding total number of viewers by WOM can be given as:

$$V_{w}(t') = N_{w} \left(\frac{1 - e^{-b_{w}t'}}{1 + \beta_{w}e^{-b_{w}t'}} \right)$$
 (10)

Case 3: Considering F (t) to follow the gamma distribution $T \sim \gamma(b_w, \beta_w)$; and using it in Equation (8), the corresponding total number of viewers by WOM can be given as:

$$V_{w}(t') = N_{w} \left(1 - \left(1 - \Gamma(t', b_{w}, \beta_{w}) \right) \right) \tag{11}$$

Case 4: Considering F (t) to follow the normal distribution $T \sim N(\mu, \sigma^2)$; and using it in Equation (8), the corresponding total number of viewers by WOM can be given as:

$$V_{w}(t') = N_{w} \left(1 - \left(1 - \varphi(t', \mu, \sigma) \right) \right) \tag{12}$$

The authors now, define a function $L_w(t)$ as the cumulative number of viewers at time t, which starts from the initial time point 0, as follows:

$$L_{w}(t) = \begin{cases} V_{w}(t-\tau) & \text{for } t \ge \tau, \\ 0 & \text{for } t < \tau. \end{cases}$$
(13)

The different values of function $V_w(t)$ have been taken from equation (9)-(12).

2.1.3. Total view count modeling

Using unified modeling approach, the dual information innovation diffusion-based modeling framework has been formulated. By adding the cumulative number of viewers through subscribers and through WOM; by time t, the cumulative number of view counts at any given time as:

$$V(t) = V_{c}(t) + L_{cc}(t)$$
(14)

Here it is noted that the market potential of early market N_s and main market N_w has been obtained from the market potential of total market, N. Assuming, θ defining the proportion of the early market in the population of the total market; as such,

$$N_s = \theta N \text{ And } N_w = (1 - \theta)N \text{ where } (0 \le \theta \le 1)$$
 (15)

Therefore, all the aforesaid cases can be represented in the form of various models that have been presented in the Table (1) given below:

Table 1. Model Description

Model	V(t)
I	$V(t) = \theta N \left(\frac{1 - e^{-b_s t}}{1 + \beta_s e^{-b_s t}} \right) + (1 - \theta) N \left(1 - e^{-b_w (t - \tau)} \right)$
II	$V(t) = \theta N \left(\frac{1 - e^{-b_s t}}{1 + \beta_s e^{-b_s t}} \right) + (1 - \theta) N \left(\frac{1 - e^{-b_w (t - \tau)}}{1 + \beta_w e^{-b_w (t - \tau)}} \right)$
III	$V(t) = \theta N \left(\frac{1 - e^{-b_s t}}{1 + \beta_s e^{-b_s t}} \right) + (1 - \theta) N \left(1 - \left(1 - \Gamma((t - \tau), b_w, \beta_w) \right) \right)$
IV	$V(t) = \theta N \left(\frac{1 - e^{-b_s t}}{1 + \beta_s e^{-b_s t}} \right) + (1 - \theta) N \left(1 - \left(1 - \varphi((t - \tau), \mu, \sigma) \right) \right)$

3. Model Illustration

The validation of the proposed modeling has been performed on six different videos; which pertains to different episodes of web series. Daily view-count data has been collected on manual basis for each video under consideration. The details of the data collected have been shown in table 2. These videos have been chosen from entertainment category out of different varieties available on YouTube. The duration for data collection is not fixed for each video; like the DS 1 was collected for 66 days whereas DS 2 was collected for 58 days and so on as described in table 2 whereas data has been collected daily (approximately after 24 hours) for every video. Along with their time frames; their description is also provided for reference.

To solve the defined problems; this study has determined the unknown parameters of the proposed model through the non-linear least square (NLLS) method (Srinivasan and Mason, 1986) for all the six videos. The calculated parameters for all the six data sets are shown in table (3-8). And, the values goodness of fit has been shown in table (9).

Table 2. Data Description

S. No.	Video Title	URL	Data Collection
			Period (Days)
DS 1	What's Your Status Web Series E01 -	https://www.youtube.com/watc	66
	Sunday Cheers!	h?v=2J2yXSLgKko	
DS 2	What's Your Status Web Series E02 -	https://www.youtube.com/watc	58
	January Cheers!	h?v=CY7K2VFyUeo	
DS 3	What's Your Status Web Series E03 -	https://www.youtube.com/watc	45
	June Cheers!	h?v=3Qbd81Lf2RU	
DS 4	Awkward Conversations With Parents	https://www.youtube.com/watc	66
	Web Series E01 - Condom TSP	h?v=VSqqLt2nCGs	

DS 5	Awkward Conversations With Parents	https://www.youtube.com/watc	58
	Web Series E02 - Girlfriend TSP	<u>h?v=U-wQTOVjnUI</u>	
DS 6	Awkward Conversations With Parents	https://www.youtube.com/watc	50
	Web Series E03 - Wet Dreams TSP	h?v=RhdNjZCFUzI	

Table 3. Parameter Estimates for DS 1

Parameter	Model I	Model II	Model III	Model IV
N	3301.67	2936.524	4015.109	2945.537
b_s	0.99	0.99	0.99	0.99
$b_{\scriptscriptstyle w}$	0.039	0.01	0.576	-
$oldsymbol{eta}_s$	5.98	3.779	6.867	2.499
$oldsymbol{eta}_{w}$	-	11.123	0.012	-
σ	-	-	-	19.39
μ	-	-	-	22.838
θ	0.284	0.126	0.252	0.103
τ	22.167	31.704	26.5	8.514

Table 4. Parameter Estimates for DS 2

Parameter	Model I	Model II	Model III	Model IV
N	1554.361	1441.703	1534.8	1430.956
b_s	0.99	0.99	0.99	0.99
$b_{_{\scriptscriptstyle W}}$	0.052	0.983	0.872	-
$oldsymbol{eta}_s$	3.309	1.771	3.512	1.66
$oldsymbol{eta}_{\scriptscriptstyle w}$	-	8.761	0.052	-
σ	-	-	-	14.576
μ	-	-	-	2.232
θ	0.392	0.245	0.416	0.243
τ	15.166	21.747	17.206	20.204

Table 5. Parameter Estimates for DS 3

Parameter	Model I	Model II	Model III	Model IV
N	1853.593	1717.443	2529.763	1716.653
b_{s}	0.99	0.99	0.99	0.99
$b_{_{\scriptscriptstyle W}}$	0.067	0.959	0.525	-

$oldsymbol{eta_s}$	3.4	1.994	1.327	4.538
$oldsymbol{eta}_{\scriptscriptstyle w}$	-	6.539	0.011	-
σ	-	-	-	10.886
μ	-	-	-	3.619
θ	0.404	0.182	0.25	0.262
τ	7	10.601	6.623	9.281

Table 6. Parameter Estimates for DS 4

Parameter	Model I	Model II	Model III	Model IV
N	3516.618	3406.274	3610.177	3394.538
b_s	0.99	0.99	0.99	0.99
$b_{_{\scriptscriptstyle W}}$	0.074	0.99	0.699	-
$oldsymbol{eta_s}$	3.583	4.02	2.971	4.496
$oldsymbol{eta}_{\scriptscriptstyle W}$	-	6.806	0.047	-
σ	-	-	-	11.018
μ	-	-	-	4.215
θ	0.376	0.143	0.332	0.207
τ	7.01	11.248	6.13	9.056

Table 7. Parameter Estimates for DS 5

Parameter	Model I	Model II	Model III	Model IV
N	3465.486	3387.244	3630.539	3383.195
b_{s}	0.99	0.99	0.99	0.99
$b_{_{\scriptscriptstyle W}}$	0.091	0.99	0.475	-
$oldsymbol{eta_s}$	3.773	3.4	3.708	3.595
$oldsymbol{eta}_{\scriptscriptstyle w}$	-	5.45	0.03	-
σ	-	-	-	8.89
μ	-	-	-	6.317
θ	0.599	0.396	0.571	0.45
τ	7.65	9.501	9.01	5.239

Table 8. Parameter Estimates for DS 6

Parameter	Model I	Model II	Model III	Model IV
N	2906.553	2859.954	3219.843	2855.187
b_s	0.99	0.99	0.99	0.99
$b_{_{\scriptscriptstyle w}}$	0.089	0.99	0.382	-
$oldsymbol{eta}_s$	1.028	0.967	0.948	0.993
$oldsymbol{eta_{\scriptscriptstyle w}}$	-	5.481	0.011	-
σ	-	-	-	8.857
μ	-	-	-	1.93
θ	0.727	0.663	0.645	0.679
τ	5.564	9.765	6.89	9.377

The performance of the various models can be compared using different comparison criteria like MSE, RMSPE, Variance, Bias and R-Square.

Table 9. Comparison Criteria for proposed Models

Dataset	Criteria	Model I	Model II	Model III	Model IV
DS 1	R-Square	0.969	0.978	0.967	0.978
	Variance	151.749	128.571	157.074	127.311
	Bias	12.822	6.702	14.874	0.248
	M.S.E.	22185.703	16145.272	23634.570	15962.362
	R.M.S.P.E	152.290	128.746	157.776	127.311
DS 2	R-Square	0.990	0.989	0.989	0.988
	Variance	36.718	39.170	43.038	40.470
	Bias	1.964	0.998	9.737	0.840
	M.S.E.	1313.364	1504.854	1535.873	1607.504
	R.M.S.P.E	36.770	39.183	44.125	40.479
DS 3	R-Square	0.977	0.966	0.983	0.965
	Variance	4373.949	6423.917	3196.746	6673.117
	Bias	4.507	4.477	3.016	4.862
	M.S.E.	4215.800	6221.019	3098.422	6453.906
	R.M.S.P.E	4373.951	6423.918	3196.748	6673.119
DS 4	R-Square	0.982	0.974	0.984	0.972
	Variance	113.914	138.072	106.460	143.318
	Bias	7.737	8.355	6.524	8.623
	M.S.E.	12600.158	18565.573	11034.361	20005.604
	R.M.S.P.E	114.176	138.325	106.660	143.577
DS 5	R-Square	0.956	0.958	0.954	0.957
	Variance	22116.164	20911.466	22907.230	21325.837
	Bias	11.207	9.453	11.180	9.759
	M.S.E.	21358.055	20282.833	22137.320	20672.452

	R.M.S.P.E	22116.167	20911.469	22907.232	21325.839
DS 6	R-Square	0.969	0.960	0.970	0.959
	Variance	69.007	78.355	66.948	78.860
	Bias	4.352	4.668	0.318	4.594
	M.S.E.	4705.156	6074.143	4481.766	6155.571
	R.M.S.P. E	69.144	78.494	66.949	78.994

From Table 9, it is apparent that all the four models are performing fairly well on the six considered datasets. Figures 1-6 represents the accuracy of the predicting models with respect to the original data.

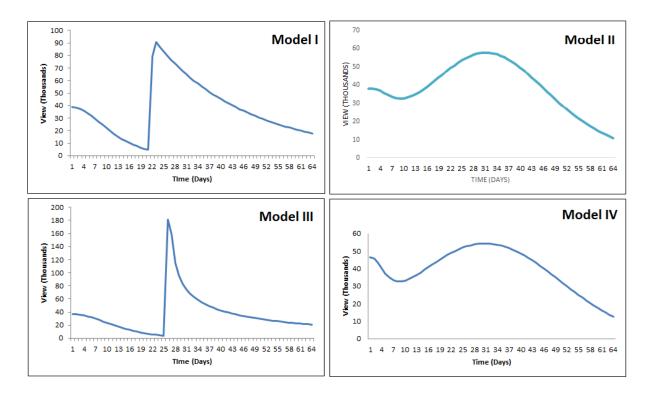


Figure 1. Graphical Representation of the proposed models on DS 1

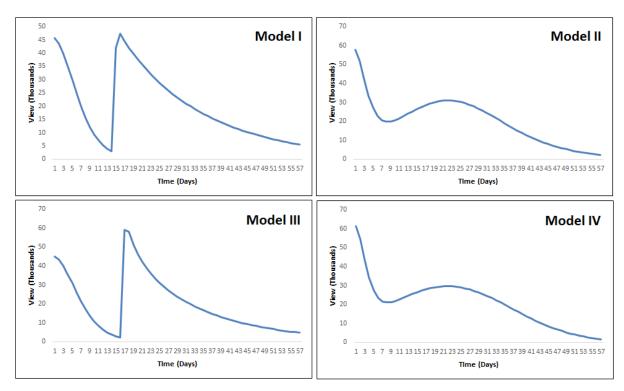


Figure 2. Graphical Representation of the proposed models on DS 2

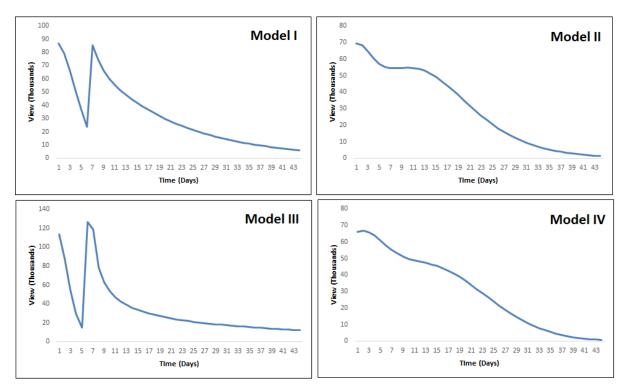


Figure 3. Graphical Representation of the proposed models on DS 3

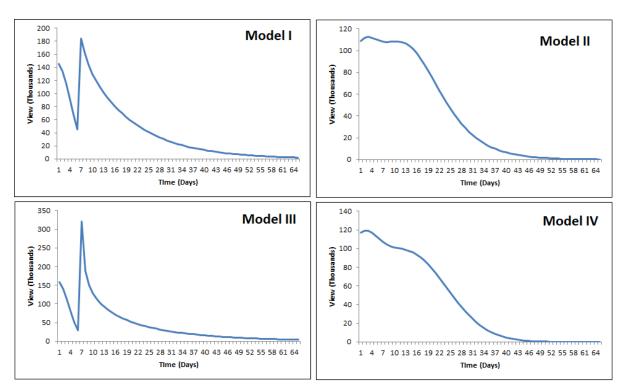


Figure 4. Graphical Representation of the proposed models on DS 4

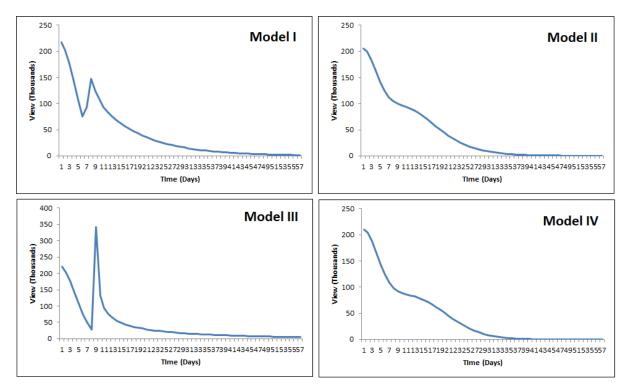


Figure 5. Graphical Representation of the proposed models on DS 5

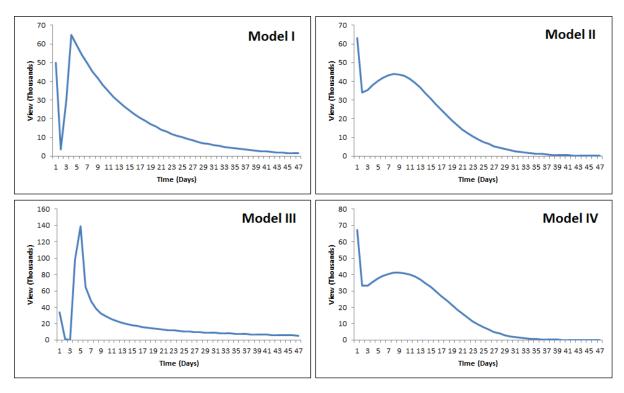


Figure 6. Graphical Representation of the proposed models on DS 6

In Figures 1-6, there are two distinct peaks in the predicted view count data for each model. As soon as a video is released, the subscribers get a notification from YouTube. Over the next few days, the number of per day viewers continues to rise which eventually leads to the first hump/peak in the graph. A saturation level is achieved as most of the subscribers have viewed the video. Once the video gains popularity due to mass sharing across various social media platforms by subscribers, word-of-mouth affect comes into picture and the per day view count starts increasing once again. As can be seen in Table 3-8, the second market of viewers came into the picture at time point τ for each model and dataset. The rising view count leads to the second visible peak in the values.

Looking at the results from table 9, and all the figures, it becomes difficult to ascertain which model is performing better for different data sets. In order to find out the same, weighted Criteria Approach given by Aggrawal et al. (2018b) and Bhatt et al. (2017) has been used. As described by them; "Weighted criteria approach is a ranking tool which helps to determine the best fit among various models on the basis of the comparison parameters for each dataset". And so, ranking of models is done the basis of this algorithm wherein; smaller permanent value of model represents good rank as compared to the bigger permanent value of the model. So, all permanent values can be compared and ranks for each model can be determined. The analysis done on DS 2 has been shown below in table 10 (and the rest of the

dataset analysis is shown in Appendix (Table A.4-A.8)). Based on the set of matrixes obtained; the overall result for all the six data sets has been shown in table 11.

Table 10: Weighted Criteria Matrix for DS 2

					R-			
Model	Variance	Bias	M.S.E.	R.M.S.P.E	Square	Total	Division	Rank
M-I	0.0000	0.2481	0.0000	0.0000	0.0000	0.2481	1.9636	1
M-II	15.2003	0.0178	979.6846	12.8534	0.4945	1008.2506	534.9072	4
M-III	43.0376	9.7370	1161.8451	44.1253	0.4945	1259.2395	295.8411	2
M-IV	24.0319	0.0000	1607.5041	20.4131	0.9880	1652.9371	533.5326	3

Table 11: Final ranks for the proposed models on different data sets

	DS 1	DS 2	DS 3	DS 4	DS 5	DS 6
M-I	3	1	2	2	2	2
M-II	2	4	3	3	1	3
M-III	4	2	1	1	4	1
M-IV	1	3	4	4	3	4

4. Discussion

Using tables 3-8, it can be seen that all the proposed models have a reasonably good estimate and are closely related to each other. Table 9 presents the performance of these proposed models on the different types of data sets under consideration. Furthermore, table 11 clearly depicts the overall ranks of the various proposed models on considered datasets using weighted criteria approach. From the results, one can clearly note that for DS 1; Model IV is performing the best. i.e., when the view count through word of mouth is accounted through normal distribution. So, this data set is more suitable to study or do prediction via Model IV. Similarly, for DS 2; one can see that Model I perform best as compared to other models under consideration. This shows that for DS 2, the multi modal nature can be best modeled and predicted when view count is governed by word of mouth following exponential distribution. Model III performs best for around 3 data sets; DS 3, DS 4 and DS 6; that is when the contribution of view counts being generated through word of mouth follows gamma distribution. On careful examination, we can see that Model II performs best for DS 5; that is when view count through word of mouth follows logistic distribution.

As a matter of fact, all the available models in literature have considered the bell shape curve for estimating the number of view counts, but by using the methodical way of understanding the growth curve as presented in this study, one can very will see that a video goes through hands of series of players who contribute to the view counts and thus represents the multimodal structure of the information diffusion curve. The common characteristic shown by majority of the available models is either exponential or S-shape in nature. The work presented here, describes utility of two contributors; subscribers following logistic distribution and the other set of viewers via word of mouth following various distribution functions that shows the manner in which view count is generated.

4.1. Research contributions

In the proposed work, a novel concept was observed and discussed with respect to viewing patterns of YouTube videos. Two viewers group were identified i.e., the subscribers of the channel and the viewers due to word of mouth. Subscribers are notified as soon as the video is uploaded, hence, a fraction of potential viewers see the video instantly and cause the initial hump in the viewing pattern. With time, these viewers spread information about the video across the different social media platforms like Facebook, Twitter, Reditt, Instagram, WhatsApp, etc. via the *share* feature of YouTube (Algharabat, 2017). This phenomenon of information sharing by word of mouth takes time in increasing the view count. Hence, there are two different peaks because of this time lag wherein; the second hump is due to the views obtained by the mass sharing. It is also observed that the viewership due to word of mouth is dependent on the channel subscribers i.e., more the video is shared by the subscribers more views are garnered through word of mouth. Thus, increasing the channel subscribers would result in higher view-count which would eventually result in social as well as economic benefits due to YouTube's advertisement-based revenue model.

The proposed modeling framework has considered different functional forms to cater to the different possible viewership patterns of the view counts generated through word of mouth and by considering the logistic pattern for subscribers. The models were able to identify the proportion of view-counts by each group as well as determined the time when the effect of word of mouth on view-count comes into play. The models were validated on six view-count datasets collected manually from YouTube. The graphical illustration and the comparison criteria were used to demonstrate the efficient working of the proposed models which validate our claim.

4.2. *Implications for Practice*

As and when anything is updated or uploaded on YouTube; it reaches to every netizen; but in a different manner. Likewise, the subscribers are the first ones to have the knowledge and privilege of knowing these facts and are the initial set of contributors for view count. Once the initiate; the count starts increasing and here comes a saturation time till when the subscribers will be able to contribute. Usually, all the discussed and earlier works in the area of mathematical modeling pertaining to view counts have revolved around uni-modal nature of this diffusion pattern. But as a matter of fact, what through this work we have tried to present is that there is a difference in the time frame of how this information diffuses to the rest of the segment of the internet market which comes to know about any video through word of mouth. So, when this segment comes to know about the offering, they also participate and contribute to view counts. The view count again starts to increase for that video after a certain halt and produces a new hump like structure thereby bringing in multimodal behavior in the growth curve. So, assuming that the view count follows uni-modal structure would bring in distortion in the accountability and its prediction. Also, we are aware that this has direct linkage with the economic perspective as YouTube works on advertisement-based revenue model. And if the prediction of its total viewers and view counts will not be estimated properly, it might affect their decision making.

In this work, the subscribers have been modeled using logistic distribution. Also, it has been taken into assumption that since different people behave differently under different circumstances. So, different types and scenarios have been catered to undertake different scenarios for the contributors through word of mouth. Similarly, exponential distribution has been taken to showcase the behavior governed through word-of-mouth contributors. This distribution has a constant rate and is broadly used in modeling the diffusion pattern. As a simplistic case, it can be said that the word-of-mouth spreads in the internet market in a constant manner. Another distribution that has been considered is logistic distribution. This is the most widely used distribution function to model the diffusion pattern. It describes the diffusion of information amongst the viewers and follows an S-shaped pattern for cumulative number of viewers. Now as per the nomenclature, the internet market attracts the viewers and they start contributing to the view counts through word of mouth. After gathering sufficient information, the number starts to increase with time. As time goes by, it helps to again gain a

peak value which the initial set of subscribes had left at. Another distribution that has been considered is view counts being generated through word of mouth following gamma distribution. This distribution defines the heterogeneous behavior of the target users (here viewers) with respect to the intensity to watch a video. It considers a consistent rate in terms of the propensity to watch a video. Yet another type of scenario can be understood through normal distribution. This distribution can also be used to describe the information diffusion process amongst the netizens because of learning effects of the target audience by the people spreading information through word of mouth. Therefore, in nutshell, it can be claimed that after the view count has been governed through subscribers; there comes another growth pattern due to the rising popularity of the video and which might influence the remaining netizens to watch the video.

A common characteristic shown by majority of the discussed models is either exponential or S-shape in nature. These models consider various distribution functions that shows the manner in which view count is generated based on symmetric, asymmetric and flexible pattern. Together, when clubbed with the logistic distribution being considered for the subscription, the total view count shows a multi modal behavior.

4.3. Limitations and Future Research Directions

The current work revolves around the data collected from entertainment zone, may be some different outlook would be there in case other zoners are also taken into consideration. Furthermore, the proposed set of assumptions describes the presence of two types of players; subscribers and normal viewers (who watch after a time lag). Although we have catered to many possible prospects of the pattern through which word of mouth diffusion can be understood, in future prospects, we would like to work on some more distributions for the same.

5. Conclusions

Through this work, the authors have shown the possibility of existence of multi modes in the growth curve of a YouTube video's life cycle. The work presents the role of subscribers as the front runners in contributing to the total view counts followed by the viewers who contribute after a time gap. This time lag is due to the fact that subscribers are the initial set of people who get an update about the particular offering on a channel that they have

subscribed. Therefore, with them the initial diffusion of the information starts. It is only after them, the actual diffusion amongst the rest of the internet market starts. This second diffusion wave has been modeled using various distribution functions like that of exponential, logistic, gamma and normal. When clubbed with the subscribers, the actual view count results in a dump shaped pattern and that too many times wherein the humps apart from the first ones can be understood generating through the word of mouth. The various set of models so obtained have been ranked for their performance on the different data sets using weighted criteria-based approach.

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Appendix

Steps involved in weighted criteria method calculation for dataset 1 is shown in table A.1-A.4.

Table A.1: Comparison Criteria Matrix for DS 1

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square
M-I	151.749	12.822	22185.703	152.290	0.969
M-II	128.571	6.702	16145.272	128.746	0.978
M-III	157.074	14.874	23634.570	157.776	0.967
M-IV	127.311	0.248	15962.362	127.311	0.978
Min	127.311	0.248	15962.362	127.311	0.967
Max	157.074	14.874	23634.570	157.776	0.978

Table A.2: Criteria Rating Matrix for DS 1

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square
M-I	0.17889492	0.14029874	0.18884615	0.180085792	0.18182
M-II	0.95766922	0.55871186	0.97615939	0.952922647	1
M-III	0	0	0	0	0
M-IV	1	1	1	1	1

Table A.3: Weighted Matrix for DS 1

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square	Total
M-I	0.82110508	0.85970126	0.81115385	0.819914208	0.81818	4.1301
M-II	0.04233078	0.44128814	0.02384061	0.047077353	0	0.5545
M-III	1	1	1	1	1	5.0000
M-IV	0	0	0	0	0	0.0000

Table A.4: Weighted Criteria Matrix for DS 1

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square	Total	Division	Rank
M-I	124.6021	11.0233	17996.0188	124.8648	0.7928	18257.3018	4420.5940	3
M-II	5.4425	2.9577	384.9131	6.0610	0.0000	399.3743	720.1944	2
M-III	157.0736	14.8743	23634.5703	157.7763	0.9670	23965.2615	4793.0523	4
M-IV	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1

Similarly the calculations for DS2- DS6 is done and final weighted criteria matrix is shown for each dataset in tables A.5-A.8 except for DS2 which is already shown in table 10.

Table A.5: Weighted Criteria Matrix for DS 3

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square	Total	Division	Rank
M-I	1481.1485	3.6414	1403.8644	1481.1503	0.3257	4370.1303	2031.2391	2
M-II	5963.4234	3.5447	5789.2511	5963.4245	0.9123	17720.5560	3917.5739	3
M-III	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1
M-IV	6673.1174	4.8621	6453.9055	6673.1192	0.9650	19805.9692	3961.1938	4

Table A.6: Weighted Criteria Matrix for DS 4

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square	Total	Division	Rank
M-I	23.0362	4.4679	2199.1703	23.2460	0.1637	2250.0840	1698.7789	2
M-II	118.4214	7.2885	15585.4946	118.6449	0.8117	15830.6610	3715.6537	3
M-III	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1
M-IV	143.3175	8.6235	20005.6043	143.5767	0.9720	20302.0941	4060.4188	4

Table A.7: Weighted Criteria Matrix for DS 5

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square	Total	Division	Rank
M-I	13349.9276	11.2071	12383.2895	13349.9331	0.4780	39094.8352	11893.5922	2
M-II	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1
M-III	22907.2295	11.0052	22137.3198	22907.2323	0.9540	67963.7407	13635.3204	4
M-IV	4427.7768	1.6998	4343.1870	4427.7770	0.2393	13200.6798	12577.7746	3

Table A.8: Weighted Criteria Matrix for DS 6

Model	Variance	Bias	M.S.E.	R.M.P.S.E	R-Square	Total	Division	Rank
M-I	11.9274	4.0359	627.9612	12.6019	0.0881	656.6145	435.7722	2
M-II	75.0342	4.6685	5778.6453	75.2381	0.8727	5934.4589	1242.4077	3
M-III	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1
M-IV	78.8600	4.5162	6155.5712	78.9937	0.9590	6318.9001	1268.0978	4