TURNING TURTLE: SCALING RELATIONSHIPS AND SELF-RIGHTING ABILITY IN CHELYDRA SERPENTINA

Ilan M. Ruhr[1‡], Kayleigh A. R. Rose[2‡], William I. Seller[3], Dane A. Crossley, II[4], & Jonathan R. Codd[1*]

[1]School of Biological Sciences, University of Manchester, UK; [2]Department of Biosciences, Swansea University, Swansea, UK; [3]Department of Earth and Environmental Sciences, University of Manchester, UK; [4]Department of Biological Sciences, University of North Texas, USA

***CORRESPONDING AUTHOR**

jonathan.codd@manchester.ac.uk

1. SUPPLEMENTARY MATERIALS

(A) DERIVATION OF SCALING PREDICTIONS

With geometric similarity mass scales as the product of the three linear dimensions, hence:

$$\textbf{(1)} \quad \text{MASS} \propto \text{LENGTH}^3$$

Whilst we are primarily interested in the relationship with mass, it is easier to work out the scaling relationships using length as the base term:

$$\textbf{(2)} \quad \text{LENGTH} \propto \text{MASS}^{1/3}$$

Force in vertebrates is generated by the contraction of skeletal muscle and the force is proportional to the cross-sectional area. Thus:

$$\textbf{(3)} \quad \text{FORCE} \propto \text{LENGTH}^2$$

Torque is defined as the force multiplied by the moment arm length. Thus:

$$\textbf{(4)} \quad \text{TORQUE} \propto \text{LENGTH}^3$$

The energy required to overcome the potential hill is proportional to the body mass multiplied by the height change required. Thus:

$$\textbf{(5)} \quad \text{ENERGY} \propto \text{LENGTH}^4$$

This energy is conserved so must be equivalent to the kinetic energy produced by the animal pushing off the substrate. This kinetic energy equivalent is proportional to the mass multiplied by the velocity squared so we can calculate the equivalent velocity. Note this is a simplification based on projectile mechanics and assumes that the animal starts from rest and that the centre of mass has zero velocity at its highest point.

$$\textbf{(6)} \quad \text{MASS} \times \text{VELOCITY}^2 \propto \text{LENGTH}^4$$

And substitution with equation **(1)**:

$$\textbf{(7)} \quad \text{VELOCITY} \propto \text{LENGTH}^{1/2}$$

Since impulse equals change of momentum and we are assuming a starting velocity of zero:

$$\textbf{(8)} \quad \text{IMPULSE} = \text{MASS} \times \text{VELOCITY}$$

And substitution with equations **(1)** and **(7)** gives:

$$\textbf{(9)} \quad \text{IMPULSE} \propto \text{LENGTH}^{7/2}$$

If we assume a constant force (or indeed that the shape of the force/time graph remains the same with body mass), then impulse is equal to force multiplied by time, and substitution in equation **(3)** we get:

$$\textbf{(10)} \quad \text{TIME} \times \text{LENGTH}^2 \propto \text{LENGTH}^{7/2}$$

which simplifies to:

(11) $\text{TIME} \propto \text{LENGTH}^{3/2}$

Finally, the mean power is defined as the energy divided by the time and using equation **(5)**:

(12) $\text{MEAN POWER} \propto \text{LENGTH}^{4}/\text{LENGTH}^{3/2}$

which simplifies to:

(13) $\text{MEAN POWER} \propto \text{LENGTH}^{5/2}$

Using equation **(1)** we can convert all these length relationships to mass relationships simply by dividing the exponent by 3. Hence, we can derive the mass relationships given in the paper:

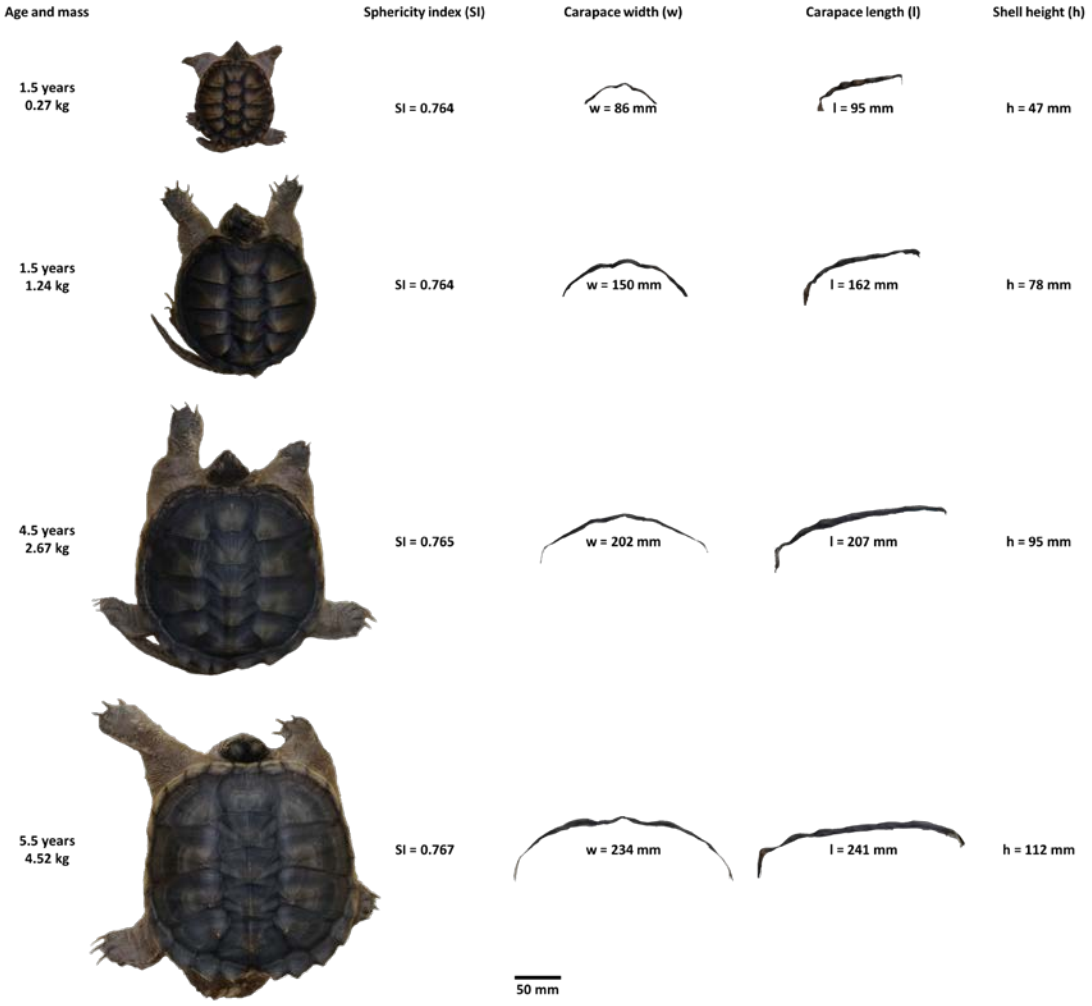
(14) $\text{FORCE} \propto \text{MASS}^{2/3}$

(15) $\text{ENERGY} \propto \text{MASS}^{4/3}$

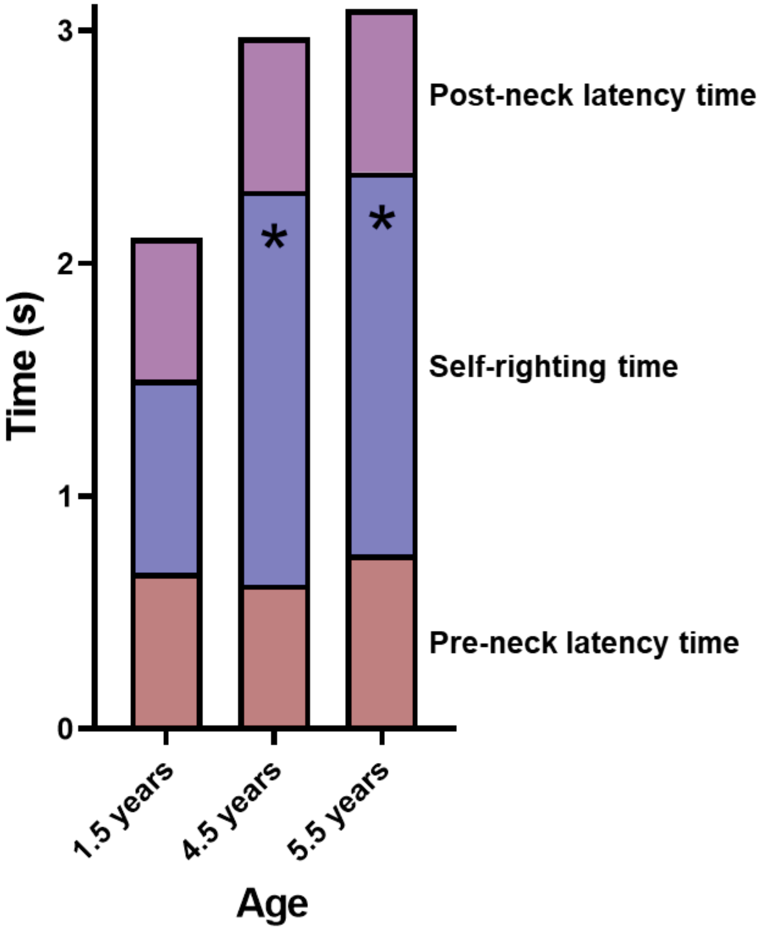
(16) $\text{TIME} \propto \text{MASS}^{3/6}$

(17) $\text{MEAN POWER} \propto \text{MASS}^{5/6}$

2. SUPPLEMENTARY FIGURES AND TABLES



Supplementary Figure S1. Representative images of growth of female common snapping turtles, throughout ontogeny. Shell dimensions (carapace width, carapace length, and shell height) grow in proportion to one another, so that sphericity indices (a measure of how domed a shell is) remains constant with age. Cross-sections and longitudinal slices of each shell are shown, to illustrate carapace width and length, respectively. Representative slices of shell height could not be obtained.



Supplementary Figure S2. A stacked bar chart showing the self-righting manoeuvre as three separate time periods. The pre-neck latency time (red sections) was calculated as the length of time between placing a turtle on its backs and the turtle's head making contact with the ground (to begin the self-righting maneuver). The self-righting time, during neck force was applied (blue sections), was calculated as the length of time a turtle used its neck during the self-righting maneuver. The post-neck latency time (purple sections) was calculated as the length of time between a turtle lifting its head off the ground and completing the self-righting maneuver (when all four limbs made contact with the ground). Generalized linear models, followed by Sidak post-hoc tests for multiple comparisons, revealed that only self-righting time differed between the groups. Our analyses show that the youngest turtles self-right about twice as fast as the two oldest cohorts (see Table S1, for the full details of statistical analysis). Significant differences in self-righting time from the 1.5-year age group are indicated by asterisks (*).

Supplementary Table S1. Test statistics of individual generalized linear models comparing self-righting time, body and shell morphometric, and self-righting biomechanics of snapping turtles (N = 33) from three age groups (1.5 y, 4.5 y, and 5.5 y). One-tailed Sidak post-hoc tests were used for multiple comparisons, to determine differences between the age groups. N-values: n = 26, 1.5 y; n = 4, 4.5 y; and n = 3, 5.5 y. Abbreviations: degrees of freedom, d.f.; confidence interval, CI; Self-righting time is the duration of time when neck force is applied.

Variable	Pairwise comparison	d.f.	P-value	Lower 95% CI	Upper 95% CI
Self-righting time (s)	1.5 y vs. 4.5 y	1	0.023	-1.709	-0.01
	1.5 y vs. 5.5 y	1	0.036	-1.683	0.059
	4.5 y vs. 5.5 y	1	0.464	-0.978	1.073
Body mass (kg)	1.5 y vs. 4.5 y	1	$P \leq 0.001$	-2.042	-1.014
	1.5 y vs. 5.5 y	1	$P \leq 0.001$	-3.387	-2.507
	4.5 y vs. 5.5 y	1	$P \leq 0.001$	-1.908	-0.929
Neck length (mm)	1.5 y vs. 4.5 y	1	$P \leq 0.001$	-41.427	-20.304
	1.5 y vs. 5.5 y	1	$P \leq 0.001$	-64.55	-44.681
	4.5 y vs. 5.5 y	1	$P \leq 0.001$	-31.615	-15.885
Carapace length (mm)	1.5 y vs. 4.5 y	1	$P \leq 0.001$	-66.501	-32.637
	1.5 y vs. 5.5 y	1	$P \leq 0.001$	-91.086	-60.053
	4.5 y vs. 5.5 y	1	$P \leq 0.001$	-38.095	-13.905
Carapace width (mm)	1.5 y vs. 4.5 y	1	$P \leq 0.001$	-64.794	-29.14
	1.5 y vs. 5.5 y	1	$P \leq 0.001$	-94.064	-59.287
	4.5 y vs. 5.5 y	1	$P \leq 0.001$	-44.926	-14.491
Shell height (mm)	1.5 y vs. 4.5 y	1	$P \leq 0.001$	-32.058	-16.719
	1.5 y vs. 5.5 y	1	$P \leq 0.001$	-43.884	-30.026
	4.5 y vs. 5.5 y	1	$P \leq 0.001$	-18.403	-6.73
Max neck force (N kg^{-1})	1.5 y vs. 4.5 y	1	0.222	-1.451	4.472
	1.5 y vs. 5.5 y	1	0.143	-1.162	6.017
	4.5 y vs. 5.5 y	1	0.313	-2.773	4.608
Kinetic-energy equivalent (J kg^{-1})	1.5 y vs. 4.5 y	1	$P \leq 0.001$	-0.657	-0.136
	1.5 y vs. 5.5 y	1	0.057	-1.975	0.164
	4.5 y vs. 5.5 y	1	0.149	-1.467	0.449
Power-output equivalent (W kg^{-1})	1.5 y vs. 4.5 y	1	0.463	-0.104	0.114
	1.5 y vs. 5.5 y	1	0.054	-0.538	0.041
	4.5 y vs. 5.5 y	1	0.054	-0.544	0.038