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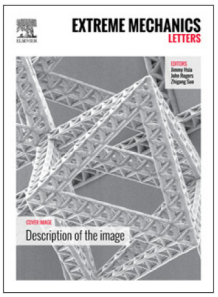

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Anisotropy tailoring in geometrically isotropic multi-material lattices

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Abstract

This article proposes the concept of anisotropy tailoring in multi-material lattices based on a mechanics-based bottom-up framework. It is widely known that isotropy in a mono-material lattice can be obtained when the microstructure has an isotropic geometry. For example, regular hexagonal lattices with a unit cell comprised of six equal members and equal internal angle of 120^o each, show isotropy in the elastic properties. Such limited microstructural configuration space for having isotropy severely restricts the scope of many multi-functional applications such as space filling in 3D printing. We first demonstrate that there are multiple structural geometries in mono-material lattices that could lead to isotropy. It is shown that the configuration space for isotropy can be expanded by multiple folds when more than one intrinsic material is introduced in the unit cell of a lattice. We explicitly demonstrate different degrees of anisotropy in regular geometrically isotropic lattices by introducing the multi-material architecture. The contours of achieving minimum anisotropy, maximum anisotropy and a fixed value of anisotropy are presented in the design space consisting of geometric and multi-material parameters. Proposition of such multi-material microstructures could essentially expand the multi-functional design scope significantly, offering a higher degree of flexibility to the designer in terms of choosing (or identifying) the most suitable microstructural geometry. An explicit theoretical characterization of the contours of anisotropy along with physical insights underpinning the configuration space of multi-material and geometric parameters will accelerate the process of its potential exploitation in various engineered multi-functional materials and structural systems across different length-scales with the demand of any specific degree of anisotropy but limitation in the micro-structural geometry.

Keywords: Anisotropy tailoring, Multi-material lattices, Elastic properties, Lattice materials, Honeycomb microstructures, Periodic structures

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1. Introduction

Lattice based periodic material microstructures provide an unprecedented opportunity to artificially engineer the global mechanical properties of materials based on multi-functional demands of modern structural systems by identifying (or designing) the intrinsic material distribution and structural geometry at micro-scale. In such microstructured materials the tailorable effective macro-scale mechanical properties (such as equivalent elastic moduli, deformation, buckling, energy absorption, vibration and wave propagation characteristics with modulation features) are defined by the structural configuration along with intrinsic material properties of the constituent members. Even though a limited number of natural systems can show few unusual properties (for example, lightweight bone structures, and auxeticity in various crystalline materials), the scope of having multiple desired mechanical properties to the optimum level in a single natural material is rare. Most of the naturally occurring materials cannot exhibit one or more of the fascinating multi-functional properties like negative Poisson's ratio, extremely lightweight characteristics, negative stiffness, pentamode material characteristics (meta-fluid), programmable constitutive laws etc., which can be achieved by an intelligent and intuitive microstructural design [1–9]. For example, the conventional positive effective Poisson's ratio in a hexagonal lattice can be converted to negative by considering the cell angle θ in figure 1 as negative (as indicated in figure 1(C-D) of the supplementary material), or to a 'zero' value by intuitively designing the hexagonal unit cell [10, 11]. Recently buckling-induced instability has been utilized to obtain and modulate unusual mechanical properties like enhanced energy absorption capacity and strain rate dependent constitutive behaviour [12–14]. Besides static properties, various unusual and useful properties can be realized in metamaterials under dynamic condition, such as negative bulk modulus [15], negative mass density [16], negative Young's modulus [17], negative shear modulus [18] and elastic cloaks [19]. These novel class of artificially engineered materials with tailorable bespoke properties have tremendous potential for applications in futuristic multi-functional aerospace, mechanical, civil, electronics and biomedical systems.

In lattice metamaterials two material properties are involved at two different length-scales. One is the intrinsic material(s) which is actually the material of the constituting elements (such as the connecting beam members shown in figure 1) at micro-scale. These materials are either naturally occurring monolithic materials or their alloys and compounds. The intrinsic mechanical properties depend on the chemical composition, atomic and molecular structure of that material. The second set of material properties correspond to the effective macro-scale behaviour of the entire lattice. Such

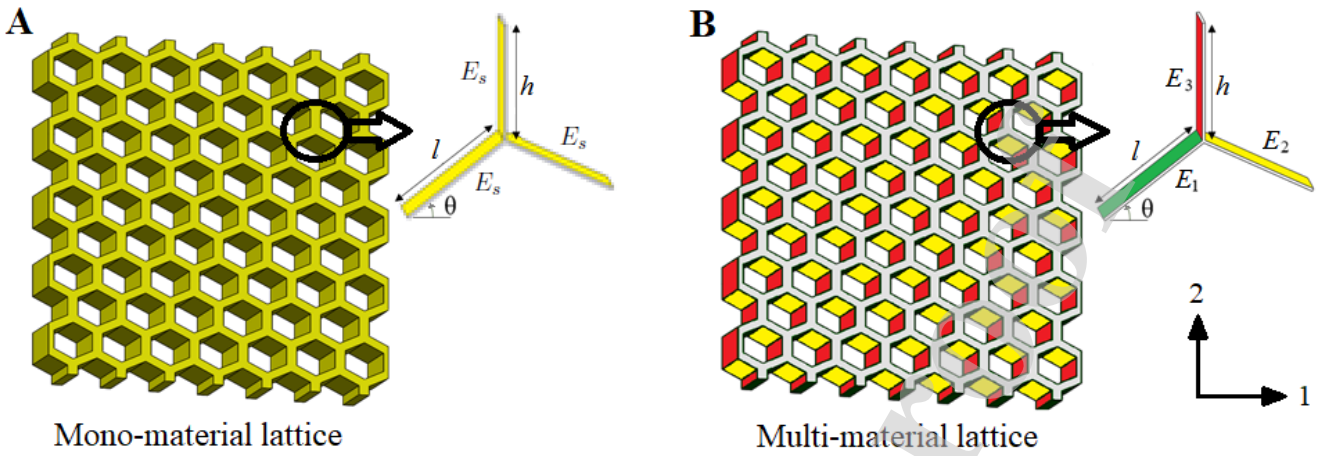

Figure 1: Multi-material microstructure. **(A)** Mono-material hexagonal lattice where all the members in a unit cell are made of same material **(B)** Multi-material hexagonal lattice where the three constituting members in a unit cell are made of three different materials (as indicated by separate colours). The respective unit cells with dimensions are shown using insets. Unit cell of a multi-material lattice has three different intrinsic material properties corresponding to the three constituting beam elements (E_1, E_2 and E_3). In case of unit cell of the mono-material lattice $E_1 = E_2 = E_3 = E_s$. In the mono-material and multi-material lattice configurations shown here, if θ becomes negative, the lattice will become re-entrant that can show auxeticity i.e. negative Poisson's ratio (refer to figure 1(C-D) of the supplementary material). The fundamental mechanics of such lattices are normally scale-independent across micro to macro scales. From a multi-scale point of view, even though there exists a lattice structure at microscale, the mechanical properties at macroscale could be idealized to have equivalent values of a bulk continuum-like solid for further analysis. Therefore, for a structural analysis at macroscale, one only needs to consider the effective macroscale mechanical properties without bothering about the microstructural geometry.

macro-scale properties depend on the intrinsic material properties as well as the microstructural geometry of the lattice, the compound effect of which presents us with a tremendous opportunity to achieve unprecedented properties that are not available in conventional monolithic materials. A unit cell based approach is often adopted to model periodic microstructures leading to a set of effective elastic moduli at macro-scale, equivalent to an idealized continuous solid material [20–25]. The fundamental mechanics for lattices being scale-independent in most cases, the research findings in this context are normally applicable for a wide range of materials and structural forms. Two dimensional lattices of hexagonal form can be found across nano to macro scale covering various natural and artificial systems in abundance (such as nanostructures of graphene, hBN etc., core of sandwich panels, microstructure of multiple woods and bones, microstructure of metamaterials, space filling pattern for 3D printing etc.) [26–32, 32–34]. In fact hexagonal tessellations can be proved to be the most efficient 2D space-filling pattern. Moreover, from a geometric view-point, it can be noted that a hexagonal lattice can effectively be converted to rectangular, rhombic and re-entrant configurations as special cases. Such widespread relevance of hexagonal lattices has led to our current focus on this form of microstructure in the present article while selecting a lattice configuration to demonstrate the concepts

of anisotropy tailoring.

Most of the investigations for creating mechanical metamaterials focus on the microstructural geometry for modulation of physical properties, rather than the constituent intrinsic materials. Thus, one single material, suitable for manufacturing and having adequate mechanical properties, are normally adopted as the constituent material [35]. However, recent advances in the area of multi-material additive manufacturing [36–38] have propelled the rationale for creation of a new class of metamaterial microstructures, where two or more intrinsic materials could be used to form the unit cell. Such multi-material microstructures could essentially expand the multi-functional design scope significantly, offering a higher degree of flexibility to the designer in terms of choosing (or identifying) the most suitable microstructural geometry. The Majority of investigations related to multi-functional metamaterials try an inverse identification approach to identify the intrinsic materials and their corresponding volume fractions based on numerical algorithms [39–41]. Multi-material microstructures have shown to be a pathway for achieving different unprecedented multi-physical properties like zero and negative thermal expansion along with other necessary mechanical attributes [42, 43]. However, physically insightful analytical formulations of such systems leading to tailorable elastic properties are limited in the current literature. Anisotropy tailoring is an important design parameter for various mechanical systems under static and dynamic conditions, where different stiffness components are necessary along different directions. Thus it is necessary to develop the capability of achieving different degrees of anisotropy in microstructured materials with sufficient options of microstructural configurations, from which a designer can choose the most suitable one based on various other functional demands and manufacturing constraints. Here we aim to present a physics-based analytical framework for deriving the theoretical contours of microstructures to achieve different degrees of anisotropy in multi-material lattices.

It is widely known that isotropy in a mono-material hexagonal lattice can be obtained when the microstructure has an isotropic geometry. For example, regular hexagonal lattices with a unit cell comprised of six equal members and equal internal angle of 120^o each, show isotropy in the elastic properties. Such limited microstructural configuration space for having isotropy restricts the scope of many multi-functional applications such as space filling in 3D printing. In this article we first aim to demonstrate that there could be multiple structural geometries in mono-material lattices that could lead to isotropy. It will be shown that the design space for isotropy can be expanded by multiple folds when more than one intrinsic material is introduced in the unit cell of a lattice. Further we will explore the possibility of having different degrees of anisotropy in regular geometrically isotropic

lattices by introducing multi-material configurations. The theoretical contours of achieving minimum anisotropy, maximum anisotropy and any fixed value of anisotropy will be systematically presented in the design space consisting of geometric and multi-material parameters. In essence, we aim to decouple the long-standing conventional understanding between relationship of effective (an)isotropy and (an)isotropic structural geometry by showing that both isotropy and anisotropy (of certain specific degree) can be achieved in a set of microstructural configurations in multi-material lattices. Though we would concentrate on hexagonal lattices in this article, the basic concepts are general and it would be applicable to other two and three dimensional lattice geometries.

2. Effective elastic moduli of multi-material lattices

In principle, multi-material lattices could have two different forms. In the first form, the unit cell could have multiple intrinsic materials and it may be tessellated to construct a lattice. In the second form the lattice may be constructed of different unit cells where two or more unit cells have different intrinsic material properties. The second form of lattice cannot be modelled using the conventional unit cell based approach as it does not lead to a periodic structure in the true sense. In the present article, our focus is on the first form of multi-material lattice where the desirable periodicity can be achieved.

Effective elastic moduli of multi-material lattices would be different from the mono-material lattice with the same structural geometry. To present adequate insights concerning the anisotropy tailoring in multi-material lattices, computational models for Young's moduli of such lattices are required. On the basis of a unit cell (consisting of three beam-like members connected at a single point, refer to Figure 1) based approach, closed-form analytical expressions for the effective Young's moduli in two orthogonal directions are derived as a function of the intrinsic material properties and structural geometry. In the analytical derivation, only bending deformation is considered, which is most predominant for thin-walled lattices with axially rigid members. In most of the advanced mechanical systems including space filling in 3D printing, thin-walled lattices are preferred due to the fact that it leads to a lightweight design.

In this section we will focus on the two effective Young's moduli $\bar{E}_1$ and $\bar{E}_2$ since they are functions of the multi-material configuration (i.e. E_1 and E_2). It may be noted that the two in-plane Poisson's ratios are not dependent on the material properties even in case of multi-material lattices (note that the Poisson's ratio's are reported to be not dependent on the intrinsic material properties in case of mono-material lattices [20]). Expressions of the two effective Young's moduli for a multi-material

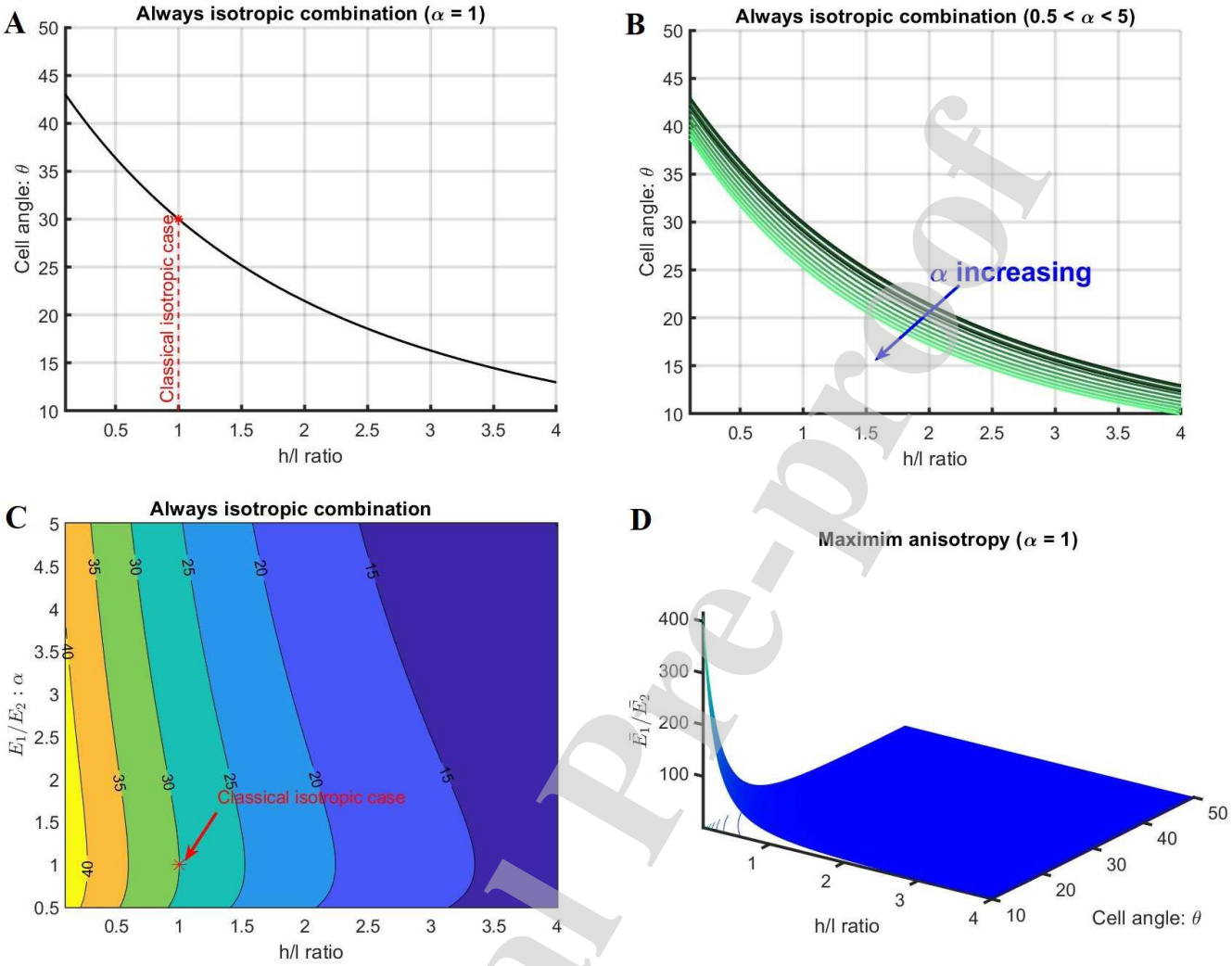

Figure 2: Minimum and maximum anisotropy in multi-material lattices. **(A)** The combination of the geometric parameters, namely, the cell angle θ and $\frac{h}{l}$ resulting in effective isotropy of a mono-material lattice at macro-scale. The classical case when $h/l = 1$, $\theta = 30^o$ and $\alpha = 1$ is shown by a star. **(B)** The combination of the cell angle θ and $\frac{h}{l}$ resulting in effective isotropy of multi-material lattices with $\alpha = \frac{E_1}{E_2}$ varying from 0.5 to 5. **(C)** Isotropy contour of the cell angle θ for multi-material lattices as a function of the multi-material parameter (α) and $\frac{h}{l}$. The point corresponding to the classical isotropic case when $h/l = 1$, $\theta = 30^o$ and $\alpha = 1$ is shown by a star. **(D)** Maximum degree of effective anisotropy (corresponding to $\alpha = 1$) as a function of the geometric parameters θ and $\frac{h}{l}$.

lattice (refer to the supplementary material for detailed derivation) can be written as

$$\bar{E}_1 = 2E_s\,(t/l)^3\,\frac{\cos\theta}{(h/l+\sin\theta)\sin^2\theta(1+\alpha)} \tag{1}$$

$$\bar{E}_2 = E_s\,(t/l)^3\,\frac{(h/l+\sin\theta)(1+\alpha)}{2\alpha\cos^3\theta} \tag{2}$$

where, $\frac{E_1}{E_2} = \alpha$ and $E_1 = \alpha E_2 = E_s$ (E_1 and E_2 are the intrinsic Young's moduli of the two slant members, refer to figure 1). Here we define degree of anisotropy as $q = \frac{\bar{E}_1}{\bar{E}_2}$. Note that $\bar{E}_1$ and $\bar{E}_2$ are macro-scale properties of the lattice, while E_1 and E_2 are micro-scale properties of the intrinsic

materials. It can be noticed from the above expressions that the effective in-plane elastic properties of a multi-material hexagonal lattice depend only on the intrinsic Young's modulus of the two slant members, whereas material properties of the vertical member has no contribution. This observation is similar to the case of mono-material hexagonal lattices [20]. Here t denotes thickness of the cell walls. The other geometric parameters involved in the expressions of $\bar{E}_1$ and $\bar{E}_2$ (such as h, l and θ) are indicated in figure 1.

For $\alpha = 1$, the expressions of elastic moduli for multi-material lattice (refer to equations 1 and 2) reduce to the traditional expressions of mono-material lattice [20] as a special case. This observation provides an exact analytical validation of the derived formulae. To validate the proposed expressions of $\bar{E}_1$ and $\bar{E}_2$ for different other values of α, we adopt a finite element based approach as discussed in section 2.3 of the supplementary material. The finite element analysis results are presented in figure 3 of the supplementary material along with the corresponding analytical results (obtained based on equations 1 and 2) for different values of α. A good agreement between the results of two forms of analysis can be noticed. It is also interesting to note the increasing level of deviation between $\bar{E}_1$ and $\bar{E}_2$ for higher values of α, essentially corroborating the preliminary evidence of the possibility of anisotropy tailoring based on multi-material parameters. The exact analytical validation for $\alpha = 1$ and finite element based numerical validation for different values of α generate necessary confidence to utilize the proposed analytical formulae of $\bar{E}_1$ and $\bar{E}_2$ for demonstrating the aspect of anisotropy tailoring further. Three different cases will be discussed systematically in the following sections leading to: I. isotropy (i.e. minimum anisotropy), II. maximum anisotropy and III. a fixed value of anisotropy. The theoretical contours in the design space of multimaterial and geometrical parameters of a unit cell will be presented considering all these three cases including insightful numerical results for demonstration. It can be noted that we have used the term 'contour' in the context of the current article to describe the microstructural configuration and their variations to achieve different degrees of (an)isotropy.

2.1. Isotropy or minimum anisotropy

In the case of $\alpha \neq 1$, an interplay among h/l, θ and α may lead to isotropy of the lattice. In literature, it is widely mentioned that $\bar{E}_1 = \bar{E}_2$ when $h/l = 1$, $\theta = 30^o$, $\alpha = 1$ [20]. The value of Young's modulus would be the same, not only in any two orthogonal directions, but in all directions for an isotropic lattice. Here we show the possibility of unexplored isotropy contours of lattice materials with multi-material as well as mono-material (i.e. $\alpha = 1$) configurations. Using the condition of $\bar{E}_1 = \bar{E}_2$

based on the Equations 1 and 2, we get

$$\left[\frac{4\alpha}{(1+\alpha)^2}\right]\frac{\cos^4\theta}{\sin^2\theta} = (h/l + \sin\theta)^2 \tag{3}$$

From on the above equation, we investigate two possible cases depending on the value of α.

First we consider the case of $\alpha = 1$, that is, the lattice is made of a single material. In such case Equation 3 reduces to

$$\frac{\cos^4\theta}{\sin^2\theta} = (h/l + \sin\theta)^2 \tag{4}$$

One of the solutions of Equation 4 is

$$h/l + \sin\theta = \frac{\cos^2\theta}{\sin\theta} \tag{5}$$

The above equation leads to

$$\sin\theta = \frac{1}{4}\left[-\left(\frac{h}{l}\right) \pm \sqrt{\left(\frac{h}{l}\right)^2 + 8}\right] \tag{6}$$

Equation 6 provides a set of solutions involving h/l and θ that will lead to isotropy in mono-material hexagonal lattices (refer to Figure 2A). Out of this set, only one solution is widely acknowledged in the scientific literature corresponding to the case of $h/l = 1$ and $\theta = 30^o$. It is important to note that the possibility of physical structural configuration of a honeycomb should always be kept in mind. For example, the two solutions of Equation 6 corresponding to $h/l = 1$ are $\theta = 30^o$ and $\theta = -90^o$, among which the latter one is physically impossible.

The second solution of Equation 4 is

$$h/l + \sin\theta = -\frac{\cos^2\theta}{\sin\theta} \tag{7}$$

The above equation leads to

$$\sin\theta = -\frac{l}{h} \tag{8}$$

Here h and l are both positive; thus θ must be negative, i.e. the solution corresponds to auxetic configurations. The possibility of physical structural configuration of a honeycomb should be kept in mind. For example, the following inequality must be satisfied for an auxetic hexagonal configuration ($\theta \leqslant 0$)

$$h \geqslant 2l\sin\theta \tag{9}$$

A closed-form solution for Young's moduli can be readily obtained corresponding to Equation 8 as

$$\bar{E_1} = \bar{E_2} = -\frac{E_s}{\sin\theta\cos\theta}\left(\frac{t}{l}\right)^3 \tag{10}$$

Note that θ is negative in the above equation (i.e. corresponding to the auxetic configuration) and subsequently $\bar{E_1}$ and $\bar{E_2}$ are positive.

The issue of geometrically isotropic lattice requires further explanation to convey the thought of the authors with more clarity. By geometrically isotropic lattice, we refer to a particular microstructural geometry of mono-material lattices where isotropy in the Young's modulus can be achieved at macroscale. In the context of hexagonal lattice microstructures, it is a general wisdom in the field of lattice metamaterials that such isotropy in the elastic properties can be achieved when the cell angle $\theta = 30^o$ and $h = l$. Thus, for a hexagonal lattice, we refer to this particular microstructural configuration as the commonly known 'geometrically isotropic' configuration. However, in the preceding paragraphs, we have shown that there are multiple other solutions for an isotropic lattice (including re-entrant microstructure) with mono-material configuration besides the conventionally reported solution corresponding to $h/l = 1$ and $\theta = 30^o$. The domain of such an isotropic solution can be increased by many folds if we consider multi-material configuration. In case of multi-material lattices, we rewrite Equation 3 as

$$\frac{4\alpha}{(1+\alpha)^2} = \frac{1}{K} \tag{11}$$

where $\frac{1}{K} = \frac{\sin^2\theta}{\cos^4\theta}\left(\frac{h}{l} + \sin\theta\right)^2$ is a function of only structural geometry. The Equation 11 leads to the condition of isotropy in multi-material lattices as

$$\alpha = \frac{E_1}{E_2} = -1 + 2K \pm 2\sqrt{-K + K^2} \tag{12}$$

where, $K^2 - K \geqslant 0$ and the honeycomb structure should be physically achievable. The above equation provides a non-dimensional measure of the multi-material parameters (α) in terms of the microstructural geometry. Figure 2B and 2C show the isotropy contour in a restricted domain of the multi-material and geometric parameters.

2.2. Maximum anisotropy

In this section, we aim to maximize $q = \left(\frac{\bar{E}_1}{\bar{E}_2}\right)$ and obtain the relationship among the multi-material and geometric parameters of the lattice. Based on Equations 1 and 2, we can write

$$q = \left(\frac{\bar{E}_1}{\bar{E}_2}\right) = \frac{\alpha}{(1+\alpha)^2} m \tag{13}$$

Here $m = \frac{4\cos^4\theta}{(h/l+\sin\theta)^2\sin^2\theta}$, which represents a purely microstructural geometric function. For maximizing the value of q, we solve the differential equation $\frac{dq}{d\alpha} = 0$ for α. The solution comes out to be $\alpha = 1$, which yields $\left(\frac{d^2q}{d\alpha^2}\Big|_{\alpha=1} < 0\right)$, indicating maxima. Thus, the condition of maximum isotropy can be obtained in the special case of mono-material lattice (i.e. $E_1 = E_2$). The corresponding value of

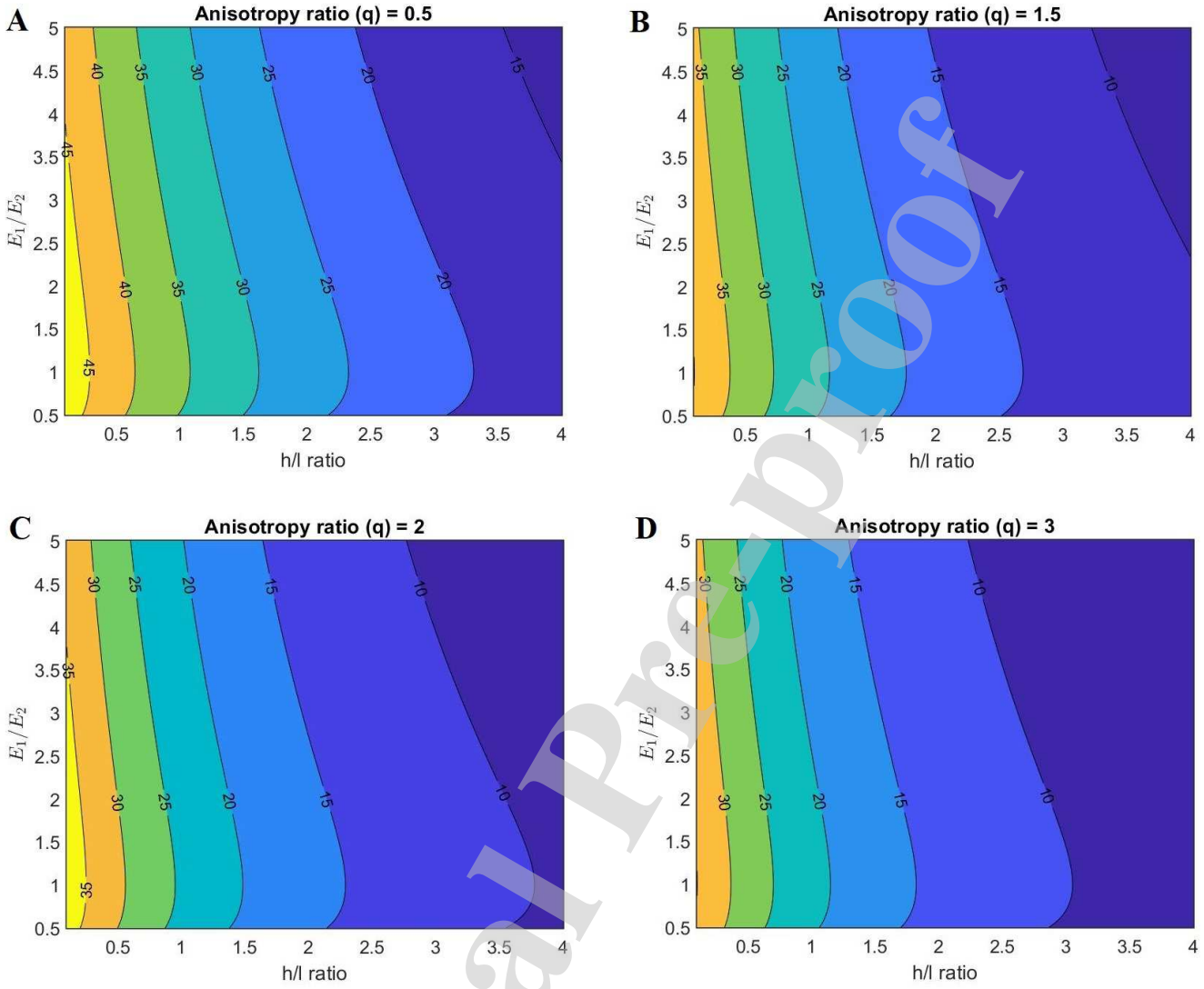

Figure 3: Realization of fixed degree of anisotropy. **(A)** Anisotropy contour for $q = 0.5$. **(B)** Anisotropy contour for $q = 1.5$. **(C)** Anisotropy contour for $q = 2$. **(D)** Anisotropy contour for $q = 3$. Here the fixed degree of anisotropy values ($q = \frac{\bar{E}_1}{\bar{E}_2}$) are realized as a function of the intrinsic multi-material parameter $\alpha = \frac{E_1}{E_2}$ and geometric parameters θ and $\frac{h}{l}$. The contour lines in this figure show the values of the cell angle θ.

maximum anisotropy is given by

$$(q^{\alpha}_{max}|_{\alpha=1}) = \frac{\cos^4\theta}{\left(\frac{h}{l} + \sin\theta\right)^2 \sin^2\theta} \tag{14}$$

From the above expression, it can be found that q is purely a function of the geometric parameters, where it increases monotonically as $\frac{h}{l}$ and θ decrease (refer to figure 2D).

2.3. Fixed anisotropy

In this section we try to find out the ratio $\frac{E_1}{E_2} = \alpha$ which will lead to a desired degree of anisotropy q. Thus, for the condition of $\frac{\bar{E}_1}{\bar{E}_2} = q$, we can write (refer to Equation 13)

$$\frac{\alpha}{(1+\alpha)^2} = \frac{q}{m} \tag{15}$$

The above equation leads to the solution in terms of multi-material parameters as

$$\alpha = \frac{E_1}{E_2} = \frac{1}{2}\left[-\left(2 - \frac{q}{m}\right) \pm \sqrt{\left(2 - \frac{q}{m}\right)^2 - 4}\right] \tag{16}$$

Considering that E_1 and E_2 are both positive, in addition to the constraint of a physically achievable lattice structure, we get the following two conditions for real solutions from the above equation

$$\left[-\left(2 - \frac{q}{m}\right) \pm \sqrt{\left(2 - \frac{q}{m}\right)^2 - 4}\right] > 0 \tag{17}$$

$$\left(2 - \frac{q}{m}\right)^2 - 4 \geqslant 0 \tag{18}$$

Figure 3 shows the contours of θ for achieving different degrees of anisotropy (q) in terms of the geometric and multi-material parameters. It is interesting to note that the maximum value of anisotropy occurs corresponding to $\alpha = \frac{E_1}{E_2} = 1$, which is in agreement with our findings corresponding to the case of maximum isotropy as presented in the preceding section.

As explained in the introduction section, a hexagonal lattice could show negative Poisson's ratio when the cell angle θ becomes negative (refer to figure 1(C-D) of the supplementary material). This fact is evident from the closed-form expressions of Poisson's ratios (refer to equations 17 and 20 of the supplementary material) and the physical geometric constraints of a hexagonal re-entrant structure. However, from the expressions it is noted that the Poisson's ratios are not dependent on the multi-material parameters, which are the main focus of this study. The current paper primarily deals with degree of anisotropy as a function of the two Young's moduli. For this reason, we have not presented any numerical results specifically for the auxetic configurations (i.e. negative cell angle). The closed form formulae for effective Young's moduli of multi-material lattices are valid for any value of cell angle (positive, or negative), meaning that the proposed concept of anisotropy tailoring can also be directly applied to auxetic configurations. In fact, we have shown a particular family of structural configurations to achieve isotropy that is only valid for auxetic structures (refer to equation (8)). It can be noted in this context that the current analytical derivation and the following investigation are carried out in the linear regime of elastic analysis.

In summary, we have discussed the aspect of anisotropy tailoring in this section through the intro-

duction of multi-material periodic systems. We start by considering a special case of hexagonal lattice, wherein the results reveal that there are multiple structural geometries in mono-material configurations that could lead to isotropy besides the conventionally considered microstructure with $h/l = 1$, $\theta = 30^o$, $\alpha = 1$ for this purpose. It is shown that the design space for isotropy can be expanded by multiple folds when more than one intrinsic material is introduced in the unit cell of a lattice. We explicitly demonstrate different degrees of anisotropy in regular geometrically isotropic lattices by introducing the multi-material architecture in the design domain of geometric and material parameters. Noteworthy is that the physical models at industry-scale of such multi-material lattices are viably manufacturable following the tremendous recent advancements in additive manufacturing technology. Though we have primarily concentrated on two dimensional hexagonal lattices in this article, the concept of anisotropy tailoring using multiple intrinsic materials in the unit cell could potentially be extended to other lattice forms and three dimensional microstructures.

3. Conclusions and perspective

This article presents physics-based insights on the possibilities of having anisotropy in geometrically isotropic lattices and isotropy in geometrically anisotropic lattices. Novelty and impact of the paper lie in both conceptual development and analytical formulation. We have systematically demonstrated the aspect of tailoring anisotropy in lattice microstructures by effectively decoupling the existing knowledge on the relationship between (an)isotropy and microstructural geometry. The theoretical contours of having minimum anisotropy (i.e. isotropy), maximum anisotropy and any fixed value of anisotropy are presented in closed-form in terms of the geometric and multi-material parameters. The conventional wisdom of having limited microstructural configuration space for obtaining isotropy (or a specific degree of anisotropy) restricts the scope of many multi-functional applications such as space filling in 3D printing. Anisotropy tailoring is an important design parameter for various mechanical systems under static and dynamic conditions, where different stiffness components are necessary along different directions. Thus, it is necessary to develop the capability of achieving different degree of effective anisotropy in microstructured materials with sufficient options of microstructural configurations, from which a designer can choose the most suitable one based on various functional demands along with geometrical design and manufacturing constraints. The disseminated generic concepts of this article on anisotropy tailoring would be crucial in innovating next-generation of multi-functional materials across different length-scales without the constraint of fixed micro-structural geometry.

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References

[1] Zadpoor, A. A., 2016. Mechanical meta-materials. Materials Horizons 3 (5), 371–381.

[2] Fleck, N. A., Deshpande, V. S., Ashby, M. F., 2010. Micro-architectured materials: past, present and future. Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences 466 (2121), 2495–2516.

[3] Hunt G. W., Dodwell T. J., 2019. Complexity in phase transforming pin-jointed auxetic lattices. Proceedings of the Royal Society A 475 20180720.

[4] Khakalo S., Balobanov V., Niiranen J., 2019. Modelling size-dependent bending, buckling and vibrations of 2D triangular lattices by strain gradient elasticity models: Applications to sandwich beams and auxetics. International Journal of Engineering Science 127 33 - 52.

[5] Cummer, S. A., Christensen, J., Alù, A., 2016. Controlling sound with acoustic metamaterials. Nature Reviews Materials 1 (3), 16001.

[6] Lai, Y., Wu, Y., Sheng, P., Zhang, Z.-Q., 2011. Hybrid elastic solids. Nature materials 10 (8), 620.

[7] Wang H., Zhao D., Jin Y., Wang M., Mukhopadhyay T., You Z., 2020. Modulation of multi-directional auxeticity in hybrid origami metamaterials. Applied Materials Today 20 100715.

[8] Mukhopadhyay T., Ma J., Feng H., Hou D., Gattas J. M., Chen Y., You Z., 2020. Programmable stiffness and shape modulation in origami materials: Emergence of a distant actuation feature, Applied Materials Today, 19 100537.

[9] Mukhopadhyay T., Adhikari S., Alu A., 2019. Theoretical limits for negative elastic moduli in subacoustic lattice materials, Physical Review B, 99 094108.

[10] Mukhopadhyay, T., Adhikari, S., 2016. Effective in-plane elastic properties of auxetic honeycombs with spatial irregularity. Mechanics of Materials 95, 204 – 222.

[11] Mukhopadhyay, T., Adhikari, S., 2017. Stochastic mechanics of metamaterials. Composite Structures 162, 85 – 97.

[12] Wadee M. A., Phillips A. T. M., Bekele A., 2020. Effects of Disruptive Inclusions in Sandwich Core Lattices to Enhance Energy Absorbency and Structural Isolation Performance. Frontiers in Materials 7 DOI: 10.3389/fmats.2020.00134.

[13] Janbaz S., Narooei K., van Manen T., Zadpoor A.A., 2020. Strain rateâĂŞdependent mechanical metamaterials. Science Advances 6(25), p.eaba0616.

[14] Champneys A. R., Dodwell T. J., Groh R. M. J., Hunt G.W., Neville R. M., Pirrera A., Sakhaei A. H., Schenk M., Wadee M. A., 2019. Happy catastrophe: recent progress in analysis and exploitation of elastic instability. Frontiers in Applied Mathematics and Statistics 5 DOI: 10.3389/fams.2019.00034.

[15] Li, J., Chan, C. T., 2004. Double-negative acoustic metamaterial. Phys. Rev. E 70, 055602.

[16] Liu, Z., Zhang, X., Mao, Y., Zhu, Y. Y., Yang, Z., Chan, C. T., Sheng, P., 2000. Locally resonant sonic materials. Science 289 (5485), 1734–1736.

[17] Adhikari S., Mukhopadhyay T., Shaw A., Lavery N.P., 2020. Apparent negative values of YoungâĂŹs moduli of lattice materials under dynamic conditions. International Journal of Engineering Science 150 103231.

[18] Wu, Y., Lai, Y., Zhang, Z.-Q., 2011. Elastic metamaterials with simultaneously negative effective shear modulus and mass density. Phys. Rev. Lett. 107, 105506.

[19] Milton, G. W., Briane, M., Willis, J. R., 2006. On cloaking for elasticity and physical equations with a transformation invariant form. New Journal of Physics 8 (10), 248.

[20] Gibson, L., Ashby, M. F., 1999. Cellular Solids Structure and Properties. Cambridge University Press, Cambridge, UK.

[21] Mukhopadhyay T., Adhikari S., Alu A., 2019. Probing the frequency-dependent elastic moduli of lattice materials. Acta Materialia 165 654-665.

[22] Zschernack C., Wadee M. A., Vollmecke C., 2016. Nonlinear buckling of fibre-reinforced unit cells of lattice materials. Composite Structures 136 217 - 228.

[23] Mukhopadhyay, T., Adhikari, S., Batou A., 2019. Frequency domain homogenization for the viscoelastic properties of spatially correlated quasi-periodic lattices. International Journal of Mechanical Sciences 150 784-806.

[24] Zhang H., Wang Y., Kang Z., 2019. Topology optimization for concurrent design of layer-wise graded lattice materials and structures. International Journal of Engineering Science 138 26 - 49.

[25] Wagner M. A., Lumpe T. S., Chen T., Shea K., 2019. Programmable, active lattice structures: Unifying stretch-dominated and bending-dominated topologies. Extreme Mechanics Letters 29 100461.

[26] Mukhopadhyay, T., Adhikari, S., 2017. Effective in-plane elastic moduli of quasi-random spatially irregular hexagonal lattices. International Journal of Engineering Science 119 142-179.

[27] Ding H., Zhen Z., Imtiaz H., Guo W., Zhu H., Liu B., 2019. Why are most 2D lattices hexagonal? The stability of 2D lattices predicted by a simple mechanics model. Extreme Mechanics Letters 32, 100507.

[28] Mukhopadhyay, T., Mahata, A., Adhikari, S., Zaeem, M. A., 2017. Effective elastic properties of two dimensional multiplanar hexagonal nanostructures. 2D Materials 4 (2), 025006.

[29] Mukhopadhyay T., Adhikari S., 2016. Free vibration analysis of sandwich panels with randomly irregular honeycomb core. Journal of Engineering Mechanics 142(11) 06016008.

[30] Gupta K. K., Mukhopadhyay T., Roy A., Dey S., 2020. Probing the compound effect of spatially varying intrinsic defects and doping on mechanical properties of hybrid graphene monolayers, Journal of Materials Science & Technology, 50 44 âĂŞ 58.

[31] Chandra Y., Mukhopadhyay T., Adhikari S., Figiel F., 2020. Size-dependent dynamic characteristics of graphene based multi-layer nano hetero-structures, Nanotechnology, 31 145705.

[32] Mahata A., Mukhopadhyay T., 2018. Probing the chirality-dependent elastic properties and crack propagation behavior of single and bilayer stanene, Physical Chemistry Chemical Physics, 20 22768âĂŤ22782.

[33] Mukhopadhyay T., Mahata A., Adhikari S., Asle Zaeem M., 2018. Probing the shear modulus of two-dimensional multiplanar nanostructures and heterostructures, Nanoscale, 10 5280 âĂŞ 5294.

[34] Mukhopadhyay T., Mahata A., Adhikari S., Asle Zaeem M., 2018. Effective mechanical properties of multilayer nano-heterostructures, Nature Scientific Reports, 7 15818.

[35] Tancogne-Dejean T. and Mohr D., 2018. Elastically-isotropic elementary cubic lattices composed of tailored hollow beams. Extreme Mechanics Letters 22 13 – 18.

[36] Bandyopadhyay A., Heer B., 2018. Additive manufacturing of multi-material structures. Materials Science and Engineering: R: Reports 129 1 - 16.

[37] Chen D., Zheng X., 2018. Multi-material additive manufacturing of metamaterials with giant, tailorable negative Poisson's ratios. Scientific reports 8 1–8.

[38] Skylar T., 2014. 4D printing: multi-material shape change. Architectural Design 84 116–121.

[39] Vogiatzis P., Chen S., Wang X., Li T., Lifeng W., 2017. Topology optimization of multi-material negative Poisson's ratio metamaterials using a reconciled level set method. Computer-Aided Design 83 15 - 32.

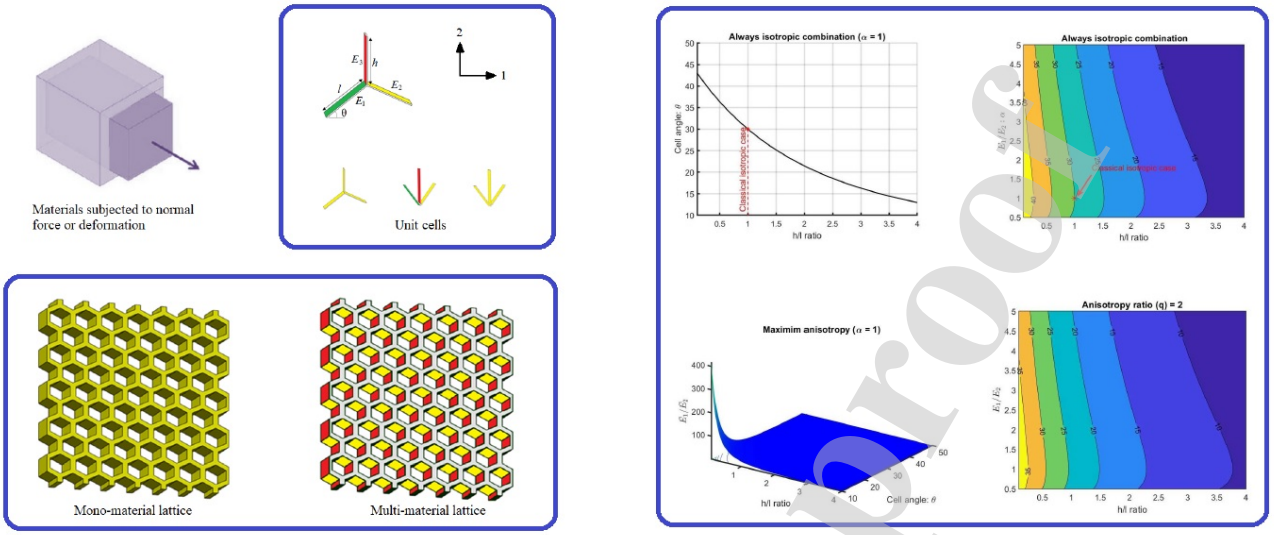
[40] Kang, D., Park, S., Son, Y., Yeon, S., Kim, S. H., Kim, I., 2019. Multi-lattice inner structures for high-strength and light-weight in metal selective laser melting process. Materials & Design 175, 107786.

[41] Mirzaali, M. J., Caracciolo, A., Pahlavani, H., Janbaz, S., Vergani, L., Zadpoor, A. A., 2018. Multi-material 3D printed mechanical metamaterials: Rational design of elastic properties through spatial distribution of hard and soft phases. Applied Physics Letters 113, 241903.

[42] Boatti E., Vasios N., Bertoldi K., 2017. Origami metamaterials for tunable thermal expansion. Advanced Materials 29 1700360.

[43] Xu H., Pasini D., 2016. Structurally efficient three-dimensional metamaterials with controllable thermal expansion. Scientific reports 6 34924.

Anisotropy tailoring in multi-material lattices (minimum, maximum and any fixed value of anisotropy)



Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: