

# $M$ -strings and $\text{AdS}_3$ solutions to M-theory with small $\mathcal{N} = (0, 4)$ supersymmetry

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**ABSTRACT:** We construct a general class of (small)  $\mathcal{N} = (0, 4)$  superconformal solutions in M-theory of the form  $\text{AdS}_3 \times S^3/\mathbb{Z}_k \times \text{CY}_2$ , foliated over an interval. These solutions describe M-strings in M5-brane intersections. The M-strings support  $(0, 4)$  quiver CFTs that are in correspondence with our backgrounds. We compute the central charge and show that it scales linearly with the total number of M-strings. We introduce momentum charge, thus allowing for a description in terms of M(atrrix) theory. Upon reduction to Type IIA, we find a new class of solutions with four Poincaré supercharges of the form  $\text{AdS}_2 \times S^3 \times \text{CY}_2 \times \mathcal{I}$ , that we extend to the massive IIA case. We generalise our constructions to provide a complete class of  $\text{AdS}_3$  solutions to M-theory with  $(0, 4)$  supersymmetry and  $\text{SU}(2)$  structure. We also construct new  $\text{AdS}_2 \times S^3 \times M_4 \times \mathcal{I}$  solutions to massive IIA, with  $M_4$  a 4d Kähler manifold and four Poincaré supercharges.

**KEYWORDS:** AdS-CFT Correspondence, M-Theory

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**1 Introduction**

Two-dimensional  $\mathcal{N} = (0, 4)$  CFTs play a prominent role in the microscopic description of 5d black holes [1–6]. They are also central in the description of 6d (1,0) CFTs deformed away from the conformal point. In fact, when the M5-branes are separated in an extra transverse direction one gets a theory of interacting strings. These strings support a (0, 4) supersymmetric quiver gauge theory, whose elliptic genus has been shown to capture the full supersymmetric partition function of the 6d theory [7, 8].

M2-branes suspended between parallel M5-branes lead to strings on their boundaries. We refer to them as  $M$ -strings [9]. For M5-branes probing  $A$ -type singularities, the case that will concern us in this paper, these strings are referred as  $M_A$ -strings [7]. They support 2d (0, 4) quiver gauge theories with unitary gauge groups. Other general configurations of  $M$ -strings can be obtained for M5-branes probing D-type singularities, or “end of the space” M9-branes. These support quiver gauge theories involving symplectic and orthogonal gauge groups, and exceptional gauge groups, respectively [8]. More general configurations can be obtained beyond the realm of M-theory, using F-theory [10, 11]. In all cases, once the quiver gauge theory is specified the elliptic genus can be computed using localisation.

Explicit AdS<sub>3</sub> holographic duals to 2d (0,4) quiver gauge theories were however quite rare in the literature, with known examples reducing to intersections of D1-D5 branes [12] with KK-monopoles [13–16] or D9-branes [17]. The recent results in [18] significantly contributed to fill this gap.<sup>1</sup>

The geometries constructed in [18] are AdS<sub>3</sub> × S<sup>2</sup> × M<sub>4</sub> foliations over an interval, with M<sub>4</sub> either a CY<sub>2</sub> (class I) or a 4d Kähler (class II) manifold. They are solutions to massive Type IIA supergravity involving D2-D4-D6-NS5-D8 brane configurations. They preserve small (i.e. with only one SU(2) R-symmetry)  $\mathcal{N} = (0, 4)$  supersymmetries and possess an SU(2) structure. The dual CFTs of the first class were studied in [34, 35]. They arise in the infrared limit of (0,4) quiver gauge theories containing two families of unitary gauge groups,  $\prod_{i=1}^n \text{SU}(k_i) \times \text{SU}(\tilde{k}_i)$ . The gauge group SU( $k_i$ ) is associated to  $k_i$  D2-branes while the gauge group SU( $\tilde{k}_i$ ) is associated to  $\tilde{k}_i$  D6-branes, wrapped on the CY<sub>2</sub>. Both D2 and D6 branes are stretched between NS5-branes. On top of this, there are D4 and D8 branes that provide flavour groups to both types of gauge groups, and render the field theory anomaly-free.

The uplift of these solutions to M-theory provides explicit holographic duals to the 2d (0,4) quiver gauge theories with unitary gauge groups supported by  $M_A$ -strings. We will see that they are AdS<sub>3</sub> × S<sup>3</sup>/Z <sub>$k$</sub>  × CY<sub>2</sub> foliations over an interval that still realise small (0,4) superconformal symmetry. This will be one of the main results in this paper. Our class of solutions extends previous results in the literature, which took more restricted ansätze for the fluxes [36]. Furthermore, we are able to show that they are in one to one correspondence with 2d quiver CFTs describing  $M_A$ -strings. The CFTs arise as infrared fixed points of 2d field theories living on M2-branes and M-theory Kaluza-Klein monopoles (wrapped on the CY<sub>2</sub>, and thus behaving effectively as 2-branes) stretched between parallel M5-branes. This set-up realises two families of unitary gauge groups, supported by flavour groups coming from extra M5-branes that render the quivers non-anomalous. Our field theories generalise quivers constructed in the literature [8]. The key ingredient is that we are able to obtain them within controlled string theory set-ups with known holographic duals. They provide examples for 2d (0,4) quiver gauge theories for which the elliptic genus can be computed.

The contents of the paper are distributed as follows. In section 2 we summarise the main properties of the backgrounds of the form AdS<sub>3</sub> × S<sup>2</sup> × CY<sub>2</sub> foliated over an interval constructed in [18]. We focus our attention on compact Calabi-Yau 2-folds. We give a brief account of the 2d (0,4) quiver CFTs that are dual to these solutions [34, 35]. In section 3 we perform the uplift of the sub-class of solutions with vanishing Romans’ mass to eleven dimensions. We construct the explicit 2d quivers dual to these backgrounds and compute the central charge, both holographically and field theoretically, finding agreement in the holographic limit. Furthermore, we show that the central charge scales linearly with the total number of  $M_A$ -strings of the configuration. This identifies the latter with the defining degrees of freedom of our theories, and allows us to reinterpret with generality previous results obtained in more restricted scenarios [13]. In section 4 we construct

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<sup>1</sup>See the papers [19–33] for more general AdS<sub>3</sub> solutions with different amounts of supersymmetries.

new  $\text{AdS}_3/\mathbb{Z}_k \times \text{S}^3 \times \text{CY}_2$  solutions in M-theory, foliated over an interval, preserving four Poincaré supersymmetries. We achieve this through a double analytical continuation. The new solutions are associated to M2-M5-M5' brane intersections with momentum charge, and provide a holographic description for the superconformal quantum mechanics (SCQM) that arises in the low energy limit. These SCQMs generalise quantum mechanical descriptions of M-branes in the context of M(atr)ix theory studied in the literature [37–42]. In section 5 we construct a new family of  $\text{AdS}_2 \times \text{S}^3 \times \text{CY}_2 \times \text{I}$  solutions to Type IIA with four Poincaré supercharges, upon reduction from M-theory. These solutions are associated to D0-F1-D4-D4' brane intersections. We naturally extend them to backgrounds of massive IIA supergravity, upon double analytical continuation from the solutions summarised in section 2. In section 6 we present our conclusions and future lines of research motivated by our results. Appendix A summarises the class I and class II solutions presented in [18]. In appendix B we present the most general class of  $\text{AdS}_3 \times \text{S}^2 \times \text{M}_4$  solutions to M-theory with (0,4) supersymmetries and SU(2) structure. Appendix C contains more general  $\text{AdS}_2 \times \text{S}^3 \times \text{M}_4$  solutions to massive IIA where  $\text{M}_4$  is a 4d Kähler manifold. The geometries studied in the main body of the paper are special cases of those in the appendices. It would be interesting to understand the holographic dual to these more general backgrounds.

## 2 Review of the $\text{AdS}_3 \times \text{S}^2 \times \text{CY}_2$ solutions to massive Type IIA

In [18] the most general class of  $\text{AdS}_3 \times \text{S}^2$  solutions to massive IIA supergravity with small (0,4) supersymmetry and SU(2) structure was presented. These solutions are foliations of  $\text{AdS}_3 \times \text{S}^2 \times \text{M}_4$  over an interval, with  $\text{M}_4$  either a  $\text{CY}_2$  or a 4d Kähler manifold. The first type of solutions were referred to as class I. The second, which contain as a particular case the T-duals of the solutions found in [11], were referred to as class II. The backgrounds in class I for which the symmetries of the  $\text{CY}_2$  are respected by the solution constitute a particularly interesting subclass for which the full family of 2d (0,4) dual CFTs can be identified [34, 35]. This subclass of solutions is the focus of our main interest in this work. From them we will construct a general class of  $\text{AdS}_3 \times \text{S}^3/\mathbb{Z}_k \times \text{CY}_2$  solutions to M-theory, to which we will associate 2d (0,4) quiver CFTs supported by  $M_A$ -strings. The uplifts of the most general solutions in class I and class II will be presented in appendix B. Our solutions provide altogether a complete classification of  $\text{AdS}_3$  solutions to M-theory with (0,4) supersymmetries and SU(2) structure.

We begin our analysis by reviewing the class I geometries constructed in [18], with the further restriction that the symmetries of the Calabi-Yau 2-fold are respected by the full solution. This requires the Calabi-Yau to be compact, and therefore we will take it to be either  $T^4$  or  $K3$ . The NS sector of this subclass of solutions reads,

$$\begin{aligned}
 ds^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}} \left( ds_{\text{AdS}_3}^2 + \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 + (u')^2} ds_{\text{S}^2}^2 \right) + \sqrt{\frac{\widehat{h}_4}{h_8}} ds_{\text{CY}_2}^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u} d\rho^2, \\
 e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4} \sqrt{u}} \sqrt{4\widehat{h}_4 h_8 + (u')^2}, \quad H_{(3)} = \frac{1}{2} d \left( -\rho + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2} \right) \wedge \widehat{\text{vol}}_{\text{S}^2}.
 \end{aligned}
 \tag{2.1}$$

Here  $\Phi$  is the dilaton,  $H_{(3)}$  the NS three-form and the metric is given in string frame. A prime denotes a derivative with respect to  $\rho$ .

The RR sector reads

$$\begin{aligned}
 F_{(0)} &= h'_8, & F_{(2)} &= -\frac{1}{2} \left( h_8 - \frac{h'_8 u' u}{4h_8 \widehat{h}_4 + (u')^2} \right) \widehat{\text{vol}}_{S^2}, \\
 F_{(4)} &= - \left( d \left( \frac{u' u}{2\widehat{h}_4} \right) + 2h_8 d\rho \right) \wedge \widehat{\text{vol}}_{\text{AdS}_3} - \partial_\rho \widehat{h}_4 \widehat{\text{vol}}_{\text{CY}_2}.
 \end{aligned}
 \tag{2.2}$$

Higher RR fluxes are related to these as  $F_{(6)} = -\star F_{(4)}$ ,  $F_{(8)} = \star F_{(2)}$ ,  $F_{(10)} = -\star F_{(0)}$ , where  $\star$  is the ten-dimensional Hodge-dual operator. Supersymmetry holds when

$$u'' = 0, \tag{2.3}$$

which makes  $u$  a linear function of  $\rho$ . In turn, the Bianchi identities of the fluxes impose

$$h''_8 = 0, \quad \widehat{h}''_4 = 0, \tag{2.4}$$

which make  $h_8$  and  $\widehat{h}_4$  also linear functions. The particular configurations reviewed above are independent of the  $\text{CY}_2$ -fold coordinates and  $\widehat{h}_4$  has support on the  $\rho$  coordinate only. The supersymmetry and Bianchi identities are satisfied for  $u$ ,  $h_8$ ,  $\widehat{h}_4$  arbitrary linear functions in  $\rho$ . This is the above mentioned restriction we adopt with respect to [18]. We shall keep this restriction hereafter, with the exception of the material in the appendices.

The magnetic components of the Page fluxes  $\widehat{F} = F \wedge e^{-B_{(2)}}$  are given by

$$\widehat{F}_{(0)} = h'_8 \tag{2.5}$$

$$\widehat{F}_{(2)} = -\frac{1}{2} \left( h_8 - (\rho - 2\pi j) h'_8 \right) \widehat{\text{vol}}_{S^2} \tag{2.6}$$

$$\widehat{F}_{(4)} = -\widehat{h}'_4 \widehat{\text{vol}}_{\text{CY}_2} \tag{2.7}$$

$$\widehat{F}_{(6)} = \frac{1}{2} \left( \widehat{h}_4 - (\rho - 2\pi j) \widehat{h}'_4 \right) \widehat{\text{vol}}_{\text{CY}_2} \wedge \widehat{\text{vol}}_{S^2}, \tag{2.8}$$

where we have included large gauge transformations in  $B_{(2)}$  of parameter  $j$ , such that

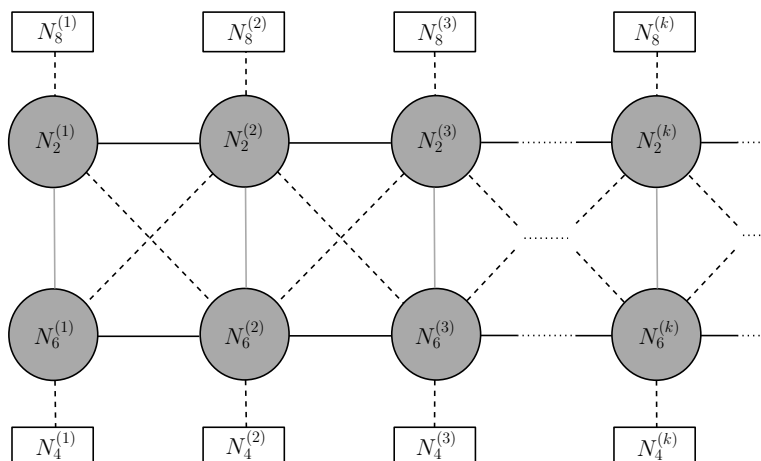
$$B_{(2)} = \frac{1}{2} \left( 2\pi j - \rho + \frac{u u'}{4\widehat{h}_4 h_8 + (u')^2} \right) \wedge \widehat{\text{vol}}_{S^2}. \tag{2.9}$$

## 2.1 Brief description of the 2d dual CFTs

Associated to the Page fluxes there is a D2-D4-D6-D8-NS5 brane system, depicted in table 1. The 2d CFTs that live on these brane set-ups were analysed in [34, 35], to which the reader is referred for more details. They are described by  $(0, 4)$  superconformal quivers with gauge groups associated to stacks of D2 and D6 branes (the latter wrapped on the  $\text{CY}_2$  manifold), both stretched between NS5 branes. Being the extension of the D2 and D6 branes finite in the  $\rho$  direction, the field theory living on their intersection is effectively two dimensional at low energies. These quivers are rendered non-anomalous with adequate flavour groups at each node, coming from D4 and D8 branes. Figure 1 illustrates their

	0	1	2	3	4	5	6	7	8	9
D2	x	x					x			
D4	x	x						x	x	x
D6	x	x	x	x	x	x	x			
D8	x	x	x	x	x	x		x	x	x
NS5	x	x	x	x	x	x				

**Table 1.**  $\frac{1}{8}$ -BPS brane intersection underlying the  $\text{AdS}_3$  solutions.  $(x^0, x^1)$  are the directions where the 2d CFT lives,  $(x^2, \dots, x^5)$  span the  $\text{CY}_2$ , on which the D6, the D8 and the NS5 branes are wrapped,  $x^6$  is the direction along the  $\rho$ -interval, and  $(x^7, x^8, x^9)$  are the transverse directions on which the  $\text{SO}(3)_R$  symmetry is realised.



**Figure 1.** Generic quiver field theory whose IR is holographic dual to the solutions reviewed in this section. The solid black line represents a  $(4,4)$  hypermultiplet, the grey line a  $(0,4)$  hypermultiplet and the dashed line a  $(0,2)$  Fermi multiplet. The degrees of freedom at each node are  $(4,4)$  vector multiplets.

general structure. The quivers can be divided into two long linear quivers consisting on  $(4,4)$  gauge groups connected horizontally by  $(4,4)$  bifundamental hypermultiplets, coupled through  $(0,4)$  hypermultiplets (vertically) and  $(0,2)$  Fermi multiplets (in the diagonals). The flavour degrees of freedom couple through  $(0,2)$  Fermi multiplets with its corresponding gauge node. These couplings render the quiver  $(0,4)$  supersymmetric.

Let us see how the cancellation of gauge anomalies works. For a given  $\text{SU}(N_2^{(i)})$  gauge group we are concerned with the contributions to the anomaly coming from the  $(0,4)$  hypermultiplets that connect it to the  $\text{SU}(N_6^{(i)})$  gauge node and with the various  $(0,2)$  Fermi multiplets. The  $(0,4)$  hypermultiplets in the bifundamental representation are composed of two  $(0,2)$  chiral multiplets, which contribute to the gauge anomaly a factor of 1. In turn, the  $(0,2)$  Fermi multiplets in the fundamental or bifundamental representations contribute a factor of  $-\frac{1}{2}$ . Putting these together, we have that for a  $\text{SU}(N_2^{(i)})$  gauge group the gauge anomaly cancellation condition is

$$2N_6^{(i)} = N_6^{(i-1)} + N_6^{(i+1)} + N_8^{(i)}. \tag{2.10}$$

This becomes

$$2N_2^{(i)} = N_2^{(i-1)} + N_2^{(i+1)} + N_4^{(i)}, \quad (2.11)$$

for  $SU(N_6^{(i)})$  gauge groups. The reader is referred to [35] for more details.

In the next section we study the M-theory lift of the backgrounds in (2.1)–(2.2).

### 3 New $AdS_3 \times S^3/\mathbb{Z}_k \times CY_2$ solutions in M-theory

Let us consider the uplift to eleven dimensions of the solutions discussed in the previous section. To perform this lift we need  $F_{(0)} = 0$ , which according to (2.2) imposes the function  $h_8$  to be a constant. Thus, the IIA brane configuration that we lift consists on intersecting D2-D4-D6-NS5 branes. The restriction to vanishing Romans' mass implies that the number of D6-branes ( $N_6$ ) must remain constant between all pairs of NS5-branes. In the lift to eleven dimensions this number becomes a modding parameter of the geometry, associated with KK-monopole charge.

Once this lift is performed, we obtain a class of  $AdS_3 \times S^3/\mathbb{Z}_k \times CY_2$  solutions to M-theory foliated over an interval. They preserve  $\mathcal{N} = (0, 4)$  supersymmetry. These solutions read

$$ds_{11}^2 = \Delta \left( \frac{u}{\sqrt{\widehat{h}_4 h_8}} ds_{AdS_3}^2 + \sqrt{\frac{\widehat{h}_4}{h_8}} ds_{CY_2}^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u} d\rho^2 \right) + \frac{h_8^2}{\Delta^2} ds_{S^3/\mathbb{Z}_k}^2, \quad (3.1)$$

$$G_{(4)} = -d \left( \frac{uu'}{2\widehat{h}_4} + 2\rho h_8 \right) \wedge \widehat{vol}_{AdS_3} + 2h_8 d \left( -\rho + \frac{uu'}{4\widehat{h}_4 h_8 + u'^2} \right) \wedge \widehat{vol}_{S^3/\mathbb{Z}_k} - \partial_\rho \widehat{h}_4 \widehat{vol}_{CY_2}, \quad (3.2)$$

$$\Delta = \frac{h_8^{1/2} (4\widehat{h}_4 h_8 + u'^2)^{1/3}}{2^{2/3} \widehat{h}_4^{1/6} u^{1/3}},$$

where  $k = h_8 = N_6$ . The quotiented 3-sphere is written as an  $S_z^1$  Hopf fibration over an  $S^2$ ,

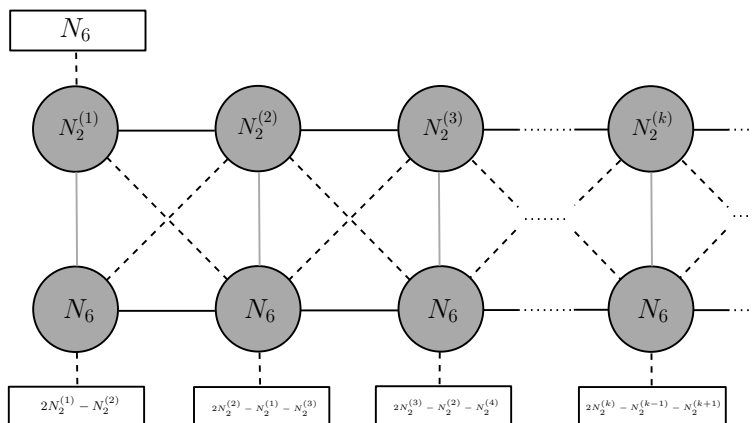
$$ds_{S^3/\mathbb{Z}_k}^2 = \frac{1}{4} \left[ \left( \frac{dz}{k} + \eta \right)^2 + ds_{S^2}^2 \right] \quad \text{with} \quad d\eta = \widehat{vol}_{S^2}. \quad (3.3)$$

In the previous solutions the symmetries  $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$  and  $SU(2)$  are realised geometrically on the  $AdS_3$  and the quotiented 3-sphere, respectively.

The dual quivers associated to this class of solutions are depicted in figure 2. The gauge anomaly is automatically cancelled for the  $SU(N_2^{(i)})$  gauge groups, once an extra  $SU(N_6)$  flavour group is added to the first node, while for the  $SU(N_6)$  gauge groups the condition (2.11) has been enforced. In what follows, we concentrate on the backgrounds in (3.1)–(3.2). In appendix B we discuss the lift to eleven dimensions of the more general backgrounds constructed in [18].

#### 3.1 Brane set-up

In the new class of solutions given by (3.1)–(3.2), the number of Type IIA D6-branes became the orbifold parameter in  $S^3/\mathbb{Z}_k$ ,  $k = N_6 = h_8$ , and thus corresponds to KK-monopole



**Figure 2.** Generic quiver field theories dual to the  $\text{AdS}_3$  solutions with vanishing Romans’ mass.

	0	1	2	3	4	5	6	7	8	9	10
M2	x	x					x				
M5	x	x						x	x	x	x
KK	x	x	x	x	x	x	x				z
M5’	x	x	x	x	x	x					

**Table 2.**  $\frac{1}{8}$ -BPS brane intersection underlying the  $\text{AdS}_3 \times S^3/\mathbb{Z}_k$  solutions in M-theory. The directions  $(x^0, x^1)$  are those where the 2d dual CFT lives,  $(x^2, \dots, x^5)$  span the  $\text{CY}_2$ ,  $x^6$  is the ‘field space’ direction and  $(x^7, x^8, x^9)$  are the transverse directions on which the  $\text{SO}(3)_R$  symmetry is realised. Finally,  $x^{10}$  is the extra eleventh direction, which spans the  $S^1/\mathbb{Z}_k \subset S^3/\mathbb{Z}_k$  and plays the role of Taub-NUT direction of the KK-monopole.

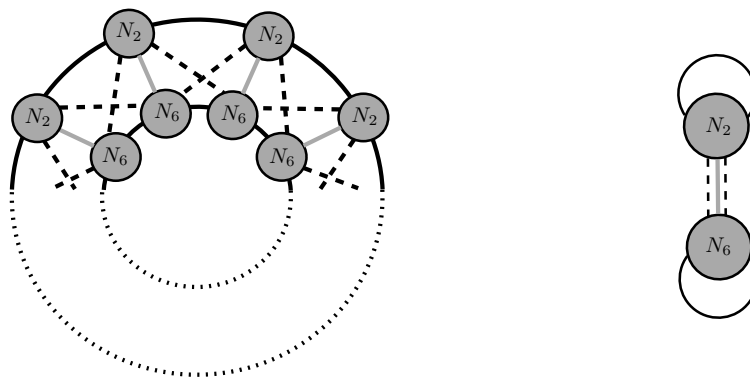
charge. The D2-branes became M2-branes. Their charge in the interval  $\rho \in [2\pi j, 2\pi(j+1)]$  is obtained by integrating the Page flux  $\widehat{G}_{(7)} = G_{(7)} - G_{(4)} \wedge C_{(3)}$ . The component of  $\widehat{G}_7$  relevant to calculate the number of M2 branes is given by

$$\widehat{G}_{(7)} = 2h_8 \left( \widehat{h}_4 - (\rho - 2\pi j) \widehat{h}'_4 \right) \widehat{\text{vol}}_{S^3/\mathbb{Z}_k} \wedge \widehat{\text{vol}}_{\text{CY}_2}. \tag{3.4}$$

The D4-branes of the Type IIA solution became M5-branes. Their presence is captured by non-trivial flux of  $G_4$  through the  $\text{CY}_2$ . Finally, the NS5 branes became M5’-branes, whose charge is given by a non-trivial flux of  $G_4$  through the  $(\rho, S^3/\mathbb{Z}_k)$  cycle. Therefore, the D2-D4-D6-NS5 branes underlying the Type IIA solutions become M2-M5-KK-M5’ branes, intersecting as shown in table 2. The KK-monopoles (wrapped on the  $\text{CY}_2$ ) and the M2 branes are stretched between parallel M5’-branes and there are extra M5-branes providing for flavour groups. This describes  $M_A$ -strings, supplemented with extra M5-branes. The corresponding dual quivers are the ones depicted in figure 2, with upper row nodes associated to M2-branes and lower row nodes to KK-monopoles. The M5-branes provide for extra flavour groups that render the quivers non-anomalous (and the supergravity equations of motion satisfied).

Our new solutions in M-theory (3.1)–(3.2), provide for explicit  $\text{AdS}_3$  geometries that can be used to study these quivers holographically. It would be interesting to see these





**Figure 3.** Left: Generic quiver field theory whose IR limit is holographic dual to the  $AdS_3$  solutions with  $I = S^1$ . Right: Quiver field theory for  $M = 1$ . In the right quiver, the  $(0,4)$  hypermultiplets combine with two  $(0,2)$  Fermi multiplets to produce  $(4,4)$  hypermultiplets. Supersymmetry is thus enhanced to  $(4,4)$ .

geometries emerging in the near horizon limit of intersecting M2-M5-MKK-M5' brane systems. This is currently under investigation [43].

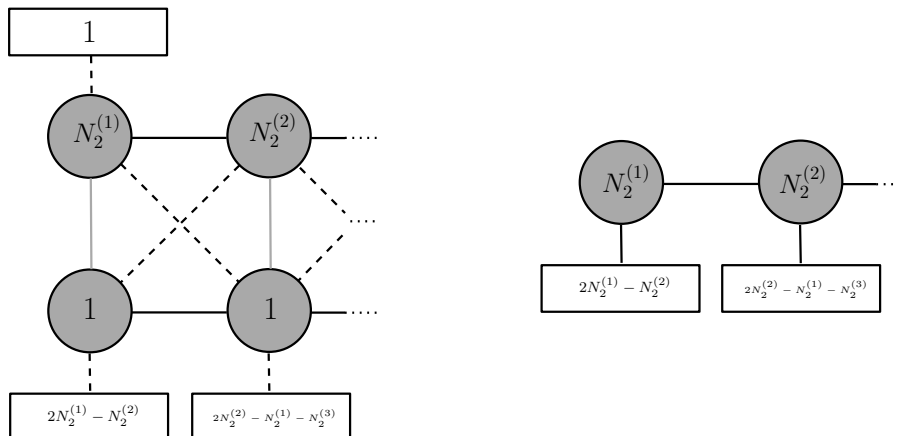
Note that when  $u' = 0$  the M5-branes wrapped on  $AdS_3 \times S^3/\mathbb{Z}_k$  support self-dual strings on their worldvolumes, coupled to the (self-dual) 3-form field

$$C_{(3)} = -2\rho h_8(\widehat{\text{vol}}_{AdS_3} + \widehat{\text{vol}}_{S^3/\mathbb{Z}_k}). \tag{3.5}$$

They arise from M2-branes ending on the M5-branes. These solutions provide then for fully backreacted near horizon geometries for OM theory [44], the theory conjectured to be the UV completion of the  $(2,0)$  theory with constant background 3-form field living on the M5-branes [45]. In our explicit set-up the 3-form depends on the positions of the M5-branes in the  $\rho$ -direction. Extra intersecting M5'-branes and KK-monopoles further reduce the supersymmetries by a half.

An interesting particular case contained in our class of solutions is when  $\mathcal{I} = S^1$ , for which  $\widehat{h}'_4 = u' = 0$ . This case was discussed in [7] (see also [46]). In this case the background [47] is the uplift of the T-dual of the  $AdS_3 \times S^3/\mathbb{Z}_M \times CY_2$  geometry that describes D1-D5 branes in a  $A_{M-1}$  singularity, introduced by  $M$  KK-monopoles. The IIB KK-monopoles become the M5'-branes in M-theory, with their Taub-NUT charge provided by the Type IIB D5-branes. The associated 2d quivers are those on the left of figure 3. When  $M = 1$  supersymmetry is enhanced to  $(4,4)$ , the brane system becomes a M2-M5' brane intersection and the associated quiver becomes the one on the right.

Another interesting case is when  $k = 1$  and there is just one KK-monopole stretched between the M5'-branes. The resulting quivers are depicted in figure 4 (left). In M-theory one KK-monopole is equivalent to no modding, and therefore the brane system reduces to the M2-M5-M5' brane intersection included in table 2. This intersection is still 1/8-BPS. These brane intersections might play a role as brane set-ups where 2d defect CFTs could be realised, in connection with the phenomenon of deconstruction [48]. Indeed, our quivers generalise (by the inclusion of flavours) the 2d defect CFTs living in D3-D3'-KK intersections studied in [49], which deconstruct 4d  $\mathcal{N} = 2$  CFTs living in M5-brane



**Figure 4.** Left: 2d (0,4) quiver CFT dual to the  $\text{AdS}_3 \times \text{S}^3 \times \text{CY}_2 \times \text{I}$  solution. Right: 4d  $\mathcal{N} = 2$  quiver CFT with flavours.

intersections. One might expect that the 2d quivers depicted on the left of figure 4 could emerge through a similar mechanism as the one described in [49], by coupling the 4d  $\mathcal{N} = 2$  CFT depicted on the right with an Abelian field theory. It would be interesting to explore this possibility.

### 3.2 Central charge

In this section we compute the (right moving) central charge of the CFTs dual to our solutions. We consider generic quivers such as the ones depicted in figure 2, that we terminate by adding adequate flavour groups, rendering the quiver non-anomalous, with large but finite length (see [35] for more details). One possibility is the completed quiver depicted in figure 5. The corresponding functions  $h_8$  and  $\hat{h}_4(\rho) = \Upsilon h_4(\rho)$  are given by,

$$h_8 = N_6, \quad 0 \leq \rho \leq 2\pi(P+1).$$

$$\hat{h}_4(\rho) = \Upsilon \begin{cases} \frac{\beta_0}{2\pi} \rho & 0 \leq \rho \leq 2\pi \\ \alpha_j + \frac{\beta_j}{2\pi} (\rho - 2\pi j) & 2\pi j \leq \rho \leq 2\pi(j+1), \\ \alpha_P - \frac{\alpha_P}{2\pi} (\rho - 2\pi P) & 2\pi P \leq \rho \leq 2\pi(P+1), \end{cases} \quad (3.6)$$

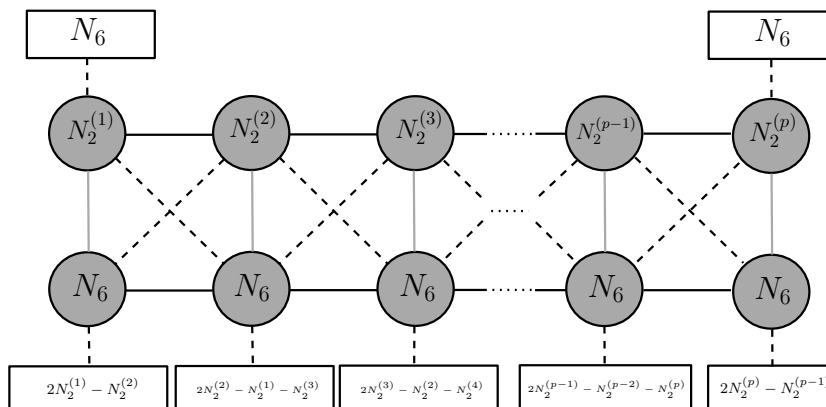
with  $\alpha_j = \sum_{r=0}^{j-1} \beta_r$  and  $j = 1, \dots, P-1$ . We have  $\hat{h}_4(0) = \hat{h}_4(2\pi(P+1)) = 0$ . At these values of  $\rho$ , the asymptotic analysis indicates the presence of M5-branes extended on  $\text{AdS}_3 \times \text{S}^3/\mathbb{Z}_k$  and the space terminates. In what follows, we choose  $\Upsilon$  such that  $\Upsilon \text{vol}_{\text{CY}_2} = 16\pi^4$ .

The numbers of M2 and M5 branes at each  $2\pi j \leq \rho \leq 2\pi(j+1)$  interval, with  $j = 1, \dots, P$ , are given by

$$N_2^{(j)} = \frac{1}{(2\pi)^6} \int_{\text{CY}_2 \times \text{S}^3/\mathbb{Z}_k} \hat{G}^{(7)} = \frac{2}{(2\pi)^6} (\hat{h}_4 - (\rho - 2\pi j) \hat{h}_4') \text{vol}_{\text{CY}_2} \text{vol}_{\text{S}^3} = \alpha_j \quad (3.7)$$

and

$$N_5^{(j)} = \frac{1}{(2\pi)^3} \int_{\text{CY}_2} G^{(4)} = \begin{cases} \beta_j & 2\pi j \leq \rho \leq 2\pi(j+1); \quad j = 0, \dots, P-1 \\ -\alpha_P & 2\pi P \leq \rho \leq 2\pi(P+1). \end{cases} \quad (3.8)$$



**Figure 5.** Completed quiver field theories whose IR limits are holographic duals to the  $\text{AdS}_3 \times \text{S}^3/\mathbb{Z}_k$  solutions in M-theory.  $N_2^{(j)}$  refer to M2-brane charges and  $N_6 = k$  to the constant, KK-monopole charge. M5-branes provide for the  $2N_2^{(i)} - N_2^{(i-1)} - N_2^{(i+1)}$  flavour groups that render the quiver non-anomalous.

Notice that  $\beta_{j-1} - \beta_j = 2N_2^{(j)} - N_2^{(j-1)} - N_2^{(j+1)}$  is the number of flavours at each  $2\pi j \leq \rho \leq 2\pi(j+1)$  interval, with  $j = 1, \dots, P-1$ , and there are extra  $\alpha_P + \beta_{P-1} = 2N_2^{(P)} - N_2^{(P-1)}$  flavours at the  $2\pi P \leq \rho \leq 2\pi(P+1)$  interval, as depicted in figure 5.

We proceed now with the computation of the holographic central charge. The interesting result we shall obtain is that the central charge is proportional to the total number of  $M_A$ -strings and is also related to the action of the  $M_5'$ -branes.

The central charge being directly proportional to the number of  $M_A$ -strings indicates that the fundamental degrees of freedom of this theory should be understood as  $M_A$ -strings. On the other hand, the relation between the action of  $M_5'$ -branes and the central charge indicates that these branes (that provide a boundary condition for the membranes to end) capture on their world-volumes the dynamics of the lower dimensional branes. This is a non-trivial fact, already encountered in [50]. It would be of interest to reproduce it in other holographic systems to fully understand its origin.

The right-moving central charge of the  $\text{AdS}_3 \times \text{S}^2 \times \text{CY}_2$  solutions constructed in [18] was computed in [35]. The expression found there remains valid upon uplift to eleven dimensions. In terms of the ten dimensional Newton's constant we have,

$$c = \frac{3\pi}{2G_N} \text{vol}_{\text{CY}_2} \int d\rho h_8 \hat{h}_4. \quad (3.9)$$

This gives, for the functions  $\hat{h}_4$  and  $h_8$  displayed above

$$c = \frac{3}{\pi} h_8 \int_0^{2\pi(P+1)} d\rho h_4 = 6h_8 \sum_{j=1}^P \alpha_j = 6h_8 \sum_{j=1}^P N_2^{(j)} = 6kN_2 = 6N_{M_A}, \quad (3.10)$$

where  $N_{M_A}$  stands for the total number of  $M_A$ -strings in the configuration, taking into account the orbifolding by  $\mathbb{Z}_k$ . This result emphasises the fact that the  $M_A$ -strings holographically capture the degrees of freedom of the conformal field theory. This suggests

that the  $M_A$ -strings actually are the degrees of freedom of the strongly coupled conformal field theory. Notice the factor of “6” in eq. (3.10). This factor is fixed on purely algebraic grounds. See for example [51].

We show next that this result can be reproduced from the action describing the M5'-branes of the configuration, where the M2-branes end, realising the  $M_A$ -strings introduced in [7, 9].

The M5'-branes on which the M2-branes end, span the  $(t, x^1, \text{CY}_2)$  directions of the geometry, and are positioned along the  $\rho$ -interval at  $\rho = 2\pi j$ . Their worldvolume effective action is given by

$$S_{M5'(j)} = T_{M5'} \int d^6\xi \sqrt{\det g} = \frac{1}{4} T_{M5'} \text{vol}_{\text{CY}_2} \int \left( \hat{h}_4 h_8 + \frac{1}{4} u'^2 \right) \cosh r dt dx^1. \quad (3.11)$$

For an M5'-brane located at  $\rho = 2\pi j$  and  $r = 0$  this becomes

$$S_{M5'(j)} = \frac{1}{4(2\pi)^4} \text{vol}_{\text{CY}_2} \text{vol}_{\mathbb{R}} \left( \hat{h}_4 (2\pi j) h_8 + \frac{1}{4} u'^2 \right) = \frac{1}{4} \text{vol}_{\mathbb{R}} \left( \alpha_j h_8 + \frac{u'^2}{4\Upsilon} \right). \quad (3.12)$$

Summing the contributions of all M5'-branes we have, to leading order in  $P$ ,

$$S_{M5'} = \sum_{j=1}^P S_{M5'(j)} \sim h_8 \sum_{j=1}^P \alpha_j = N_{M_A}. \quad (3.13)$$

This reproduces the scaling of the central charge to leading order within the context of the M5'-branes effective action.

Our discussion in the previous subsection about the interpretation of the  $M_A$ -strings as self-dual strings when  $u' = 0$  suggests that we should also be able to reproduce the scaling of the central charge from the M5-branes effective action, where the M2-branes realise self-dual strings. However, to check this we would need an action for non-Abelian M5-branes.

### 3.2.1 Field theory calculation

Finally, we check for consistency that the previous central charge coincides with the field theory result for long quivers with large ranks — the regime in which we can trust the supergravity solutions (see [35]). At the conformal point the central charge is related to the two point  $U(1)_R$  current correlation function (see for example [52]), such that

$$c = 6(n_{hyp} - n_{vec}), \quad (3.14)$$

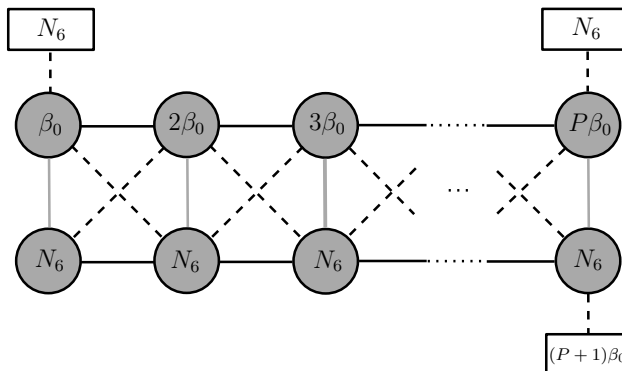
where  $n_{hyp}$  is the number of  $\mathcal{N} = (0, 4)$  hypermultiplets and  $n_{vec}$  the number of  $\mathcal{N} = (0, 4)$  vector multiplets of the quiver in the UV description.

For the quivers considered in figure 5, we find

$$n_{hyp} = \sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} + (P-1)N_6^2 + N_6 \sum_{j=1}^P N_2^{(j)}, \quad (3.15)$$

and

$$n_{vec} = \sum_{j=1}^P \left( (N_2^{(j)})^2 - 1 \right) + P(N_6^2 - 1). \quad (3.16)$$



**Figure 6.** 2d (0,4) quiver CFT with gauge groups of linearly increasing ranks.

After defining  $N_2 = \sum_{j=1}^P N_2^{(j)}$ , this gives for the central charge

$$c = 6(n_{hyp} - n_{vec}) = 6 \left( N_6 N_2 + \sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} - \sum_{j=1}^P (N_2^{(j)})^2 - N_6^2 + 2P \right). \quad (3.17)$$

Now, the net contribution to this expression of the term  $\left( \sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} - \sum_{j=1}^P (N_2^{(j)})^2 \right)$  is subleading when compared with the contribution of  $N_6 N_2$ . This hierarchy occurs when the number of gauge nodes is large (for long quivers). As a consequence, to leading order in the number of nodes  $P$ , we find

$$c = 6N_6 N_2 + \mathcal{O}(P). \quad (3.18)$$

The only situation in which the two competing terms above scale similarly in  $P$ , is when there are no intermediate flavour groups, i.e. when  $N_2^{(j)} = j\beta_0$  for  $i = 1, \dots, P$ . This particular situation corresponds to the quiver with gauge groups of linearly increasing ranks, depicted in figure 6. Next we show that the contribution of  $\left( \sum_{j=1}^{P-1} N_2^{(j)} N_2^{(j+1)} - \sum_{j=1}^P (N_2^{(j)})^2 \right)$  is indeed subleading with respect to that of  $N_6 \sum_{j=1}^P N_2^{(j)}$ . To do this, we should impose that  $N_6$  is much bigger than  $\beta_0$ . This is required to have a trustable supergravity background — see the analysis in section 4.4 of [35]. The central charge then reads

$$c = 6 \left[ N_6 N_2 + \beta_0^2 \left( \sum_{j=1}^{P-1} j(j+1) - \sum_{j=1}^P j^2 \right) - N_6^2 + 2P \right]. \quad (3.19)$$

Keeping in mind that  $N_2 = \beta_0 \sum_{j=1}^P j = \beta_0 \frac{P(P+1)}{2}$  we get

$$c = 3\beta_0(N_6 - \beta_0)P^2 + \mathcal{O}(P) \sim 3\beta_0 N_6 P^2. \quad (3.20)$$

We used that  $N_6$  must be much larger than  $\beta_0$  for the supergravity background to be trustable.

One can now easily check that this is in agreement with the holographic calculation. In fact, the function  $\widehat{h}_4$  representing the quiver with linearly increasing ranks is

$$\widehat{h}_4(\rho) = \Upsilon h_4(\rho) = \Upsilon \begin{cases} \frac{\beta_0}{2\pi} \rho & 0 \leq \rho \leq 2\pi P \\ \frac{\beta_0 P}{2\pi} (2\pi(P+1) - \rho) & 2\pi P \leq \rho \leq 2\pi(P+1), \end{cases} \quad (3.21)$$

	0	1	2	3	4	5	6	7	8	9	10
M0	x	z									
M2	x	x					x				
M5	x	x						x	x	x	x
M5'	x	x	x	x	x	x					

**Table 3.**  $\frac{1}{8}$ -BPS brane intersection underlying the  $\text{AdS}_3/\mathbb{Z}_k \times \text{S}^3$  solutions in M-theory.  $x^1$  is the direction of propagation of the wave.  $(x^2, \dots, x^5)$  span the  $\text{CY}_2$ ,  $x^6$  is the direction along the  $\rho$ -interval,  $(x^7, x^8, x^9, x^{10})$  are the transverse directions on which the  $\text{SO}(4)$  symmetry is realised. The presence of the wave renders the dual CFT one-dimensional.

with  $h_8 = N_6$ . Using our expression for the central charge (3.9) we find,

$$c = \frac{3}{\pi} N_6 \left( \int_0^{2\pi P} \frac{\beta_0}{2\pi} \rho d\rho + \int_{2\pi P}^{2\pi(P+1)} \frac{\beta_0}{2\pi} (2\pi(P+1) - \rho) d\rho \right) = 3N_6\beta_0 P^2 \left( 1 + \frac{1}{P} \right), \quad (3.22)$$

in coincidence with eq. (3.20) when long quivers are considered.

Any other situation with intermediate (many, but sparse enough) flavour groups will work along similar lines, showing the validity of eq. (3.18). This shows that the holographic calculation and the field theoretical one coincide for long quivers with large enough ranks.

This closes our analysis of the backgrounds in equations (3.1)–(3.2). In the next section we present a new branch of  $\text{AdS}_3$  solutions in M-theory.

## 4 Double analytic continuation

A double analytic continuation in the  $\text{AdS}_3 \times \text{S}^3/\mathbb{Z}_k \times \text{CY}_2 \times \text{I}$  solutions presented in (3.1)–(3.2), gives rise to a second class of solutions in which the  $\text{AdS}_3$  subspace is quotiented instead of the  $\text{S}^3$ . These solutions preserve the same amount of supersymmetries. The KK-monopoles turn into M0-branes, or waves, with the Taub-NUT direction of the KK-monopoles turning into the direction of propagation of the waves. The solutions are then associated to the M0-M2-M5-M5' brane intersections depicted in table 3. The double analytic continuation of the background given in (3.1) works as follows. The  $\text{AdS}_3$  and  $\text{S}^3$  factors can be swapped as

$$\text{AdS}_3 \rightarrow -\text{S}^3, \quad \text{S}^3 \rightarrow -\text{AdS}_3. \quad (4.1)$$

In order to get a spacetime with the correct signature the  $u, \hat{h}_4, h_8$  functions need to be also analytically continued, as follows

$$u \rightarrow -iu, \quad \hat{h}_4 \rightarrow i\hat{h}_4, \quad h_8 \rightarrow ih_8, \quad (4.2)$$

together with  $\rho \rightarrow i\rho$ .

Applying this set of transformations to the solutions to M-theory discussed in (3.1)–(3.2) gives rise to the following new solutions

$$ds_{11}^2 = \frac{h_8^2}{\Delta^2} ds_{\text{AdS}_3/\mathbb{Z}_k}^2 + \Delta \left( \frac{u}{\sqrt{\hat{h}_4 h_8}} ds_{\text{S}^3}^2 + \sqrt{\frac{\hat{h}_4}{h_8}} ds_{\text{CY}_2}^2 + \frac{\sqrt{\hat{h}_4 h_8}}{u} d\rho^2 \right) \quad (4.3)$$

	0	1	2	3	4	5	6	7	8	9
D0	x									
F1	x					x				
D4	x	x	x	x	x					

**Table 4.**  $\frac{1}{8}$ -BPS intersection involving D0, D4 and F1 branes. A  $\mathcal{N} = 4$  supersymmetric quantum mechanics lives in the common  $x^0$  direction.  $(x^1, \dots, x^4)$  span the directions on the  $CY_2$ .  $x^5$  is the direction along the  $\rho$ -interval. The  $(x^6, \dots, x^9)$  directions enjoy an  $SO(4)$  rotational symmetry, of which an  $SU(2)$  is the R-symmetry of the  $SU(1,1|2)$  supergroup and another  $SU(2)$  a global symmetry.

$$G_{(4)} = -d \left( -\frac{uu'}{2\widehat{h}_4} + 2\rho h_8 \right) \wedge \widehat{\text{vol}}_{S^3} - 2h_8 d \left( \rho + \frac{uu'}{4\widehat{h}_4 h_8 - u'^2} \right) \wedge \widehat{\text{vol}}_{\text{AdS}_3/\mathbb{Z}_k} - \partial_\rho \widehat{h}_4 \widehat{\text{vol}}_{CY_2} \quad (4.4)$$

$$\Delta = \frac{h_8^{1/2} (4\widehat{h}_4 h_8 - u'^2)^{1/3}}{2^{2/3} \widehat{h}_4^{1/6} u^{1/3}} \quad (4.5)$$

where  $k = h_8$  and the quotiented  $\text{AdS}_3$  subspace is written as a Hopf fibration over  $\text{AdS}_2$ ,

$$ds_{\text{AdS}_3/\mathbb{Z}_k}^2 = \frac{1}{4} \left[ \left( \frac{dz}{k} + \eta \right)^2 + ds_{\text{AdS}_2}^2 \right] \quad \text{with} \quad d\eta = \widehat{\text{vol}}_{\text{AdS}_2}. \quad (4.6)$$

#### 4.1 Dual quantum mechanics

Due to the momentum charge, the previous class of solutions is dual to a 1d superconformal quantum mechanics (SCQM). Holographically, they are thus essentially different from the  $\text{AdS}_3 \times S^3/\mathbb{Z}_k$  solutions on which the double analytic continuation was performed. From the isometries of the background, we see that the  $\mathcal{N} = 4$  SCQM must preserve  $\mathfrak{su}(1,1|2)$  superconformal algebra, whose bosonic sub-algebra is  $\mathfrak{sl}(2) \oplus \mathfrak{su}(2)$  [53].<sup>2</sup>

A particular solution that can be used to provide some hint on the nature of the dual quantum mechanics is the one for which  $\mathcal{I} = S^1$ . This is the background that follows from setting  $\widehat{h}'_4 = u' = 0$  in (4.3)–(4.5). This solution is associated to a M0-M2-M5' brane intersection, and is the uplift to M-theory of the T-dual of the  $\text{AdS}_3/\mathbb{Z}_M \times S^3 \times CY_2$  geometry that describes D1-D5 branes with  $M$  units of momentum along the Hopf-fibre direction of  $\text{AdS}_3$ . T-dualising on the Hopf-fibre gives rise to D0-branes, D4-branes and F1-strings, as shown in table 4, which upon uplift give the M0-M2-M5' brane intersection included in table 3. When  $M = 1$  supersymmetry is enhanced to 2d (4,4) and the brane intersection becomes the M2-M5' brane set-up discussed in section 3.1. The associated quiver is the one depicted on the right of figure 3. Switching on momentum charge allows for a quantum mechanical description of this system within M(atrrix) theory, upon taking the Sen-Seiberg limit [54, 55]. The  $\text{AdS}_3/\mathbb{Z}_M \times S^3 \times CY_2 \times S^1$  solution (or its  $\text{AdS}_2$  reduction to Type IIA) provides for an alternative holographic description of this quantum mechanics.

<sup>2</sup>Note that one of the  $SU(2)$  isometry groups of the 3-sphere is a global symmetry.

More general  $\text{AdS}_3/\mathbb{Z}_k \times \text{S}^3 \times \text{CY}_2 \times \text{I}$  solutions in our class should be dual to M(atrrix) quantum mechanics describing M2-M5-M5' brane intersections. M5-M5' brane intersections were discussed in this context in [42], but these give rise to 4d SCFTs in their common worldvolume, and are therefore different from the intersections considered in this paper. The M(atrrix) theory description of the 2d SCFTs living in our M-brane intersections is currently under investigation [56].

Connections between  $\text{AdS}_2$  solutions and M(atrrix) theory have been discussed in the literature in various contexts (see for instance [50, 57–60]). The results in [50] are particularly interesting regarding our system. Indeed, the system depicted in table 4 can be thought of as the result of adding F1-branes to the 1/4-BPS D0-D4 brane system discussed in [50], dual to a  $\text{AdS}_2 \times \text{S}^3 \times \text{S}^4$  geometry foliated over an interval. The D0-D4 brane system describes in M(atrrix) theory longitudinal M5-branes, in terms of a  $U(k)$  gauge theory with hypermultiplets in the adjoint representation and  $N$  fundamentals [39]. This quantum mechanics is the reduction on a circle of the quiver CFT dual to the D1-D5 system (depicted on the right of figure 3).<sup>3</sup> Alternatively, one could think of our system in terms of a 1/4-BPS D0-F1 brane system with extra D4-branes. 1/4-BPS D0-F1 branes are dual to a  $\text{AdS}_2 \times \text{S}^7$  geometry foliated over an interval [50, 61]. Our solutions can be interpreted in these set-ups as the fully backreacted supergravity solutions that arise when F1-strings are placed in the  $\text{AdS}_2 \times \text{S}^3 \times \text{S}^4$  geometry dual to the D0-D4 brane system, or D4-branes are placed in the  $\text{AdS}_2 \times \text{S}^7 \times \text{I}$  solutions dual to the D0-F1 brane system, uplifted to eleven dimensions.

## 5 New $\text{AdS}_2 \times \text{S}^3 \times \text{CY}_2$ solutions to massive Type IIA

The  $\text{AdS}_3/\mathbb{Z}_k \times \text{S}^3 \times \text{CY}_2 \times \text{I}$  solutions to M-theory presented in (4.3)–(4.4), can be reduced on the Hopf-fibre of  $\text{AdS}_3$ . This produces a new class of  $\text{AdS}_2 \times \text{S}^3 \times \text{CY}_2 \times \text{I}$  solutions to massless Type IIA supergravity. These solutions are associated to D0-F1-D4-D4' brane systems, preserve four Poincaré supersymmetries and have an  $SU(2)$  structure. In fact, one can check that they are just the double analytic continuation of the  $\text{AdS}_3 \times \text{S}^2 \times \text{CY}_2 \times \text{I}$  solutions reviewed in section 2, when restricted to the massless case. Therefore, these backgrounds can be extended straightforwardly to the massive case. In this section we present this new class of solutions. We leave their detailed study to our forthcoming publication [56].

Performing the analytic continuation explained in the previous section on the class of solutions given by (2.1)–(2.2) we find a NS sector,

$$\begin{aligned}
 ds^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}} \left( \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 - (u')^2} ds_{\text{AdS}_2}^2 + ds_{\text{S}^3}^2 \right) + \sqrt{\frac{\widehat{h}_4}{h_8}} ds_{\text{CY}_2}^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u} d\rho^2, \\
 e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4} \sqrt{u}} \sqrt{4\widehat{h}_4 h_8 - (u')^2}, \quad H_{(3)} = -\frac{1}{2} d \left( \rho + \frac{uu'}{4\widehat{h}_4 h_8 - (u')^2} \right) \wedge \widehat{\text{vol}}_{\text{AdS}_2}.
 \end{aligned}
 \tag{5.1}$$

---

<sup>3</sup>In the limit in which the  $\text{CY}_2$  is taken to be very large, such that the group associated to the D5-branes becomes global.



The RR sector reads

$$\begin{aligned}
 F_{(0)} &= h'_8, & F_{(2)} &= -\frac{1}{2} \left( h_8 + \frac{h'_8 u' u}{4h_8 \widehat{h}_4 - (u')^2} \right) \widehat{\text{vol}}_{\text{AdS}_2}, \\
 F_{(4)} &= \left( -d \left( \frac{u' u}{2\widehat{h}_4} \right) + 2h_8 d\rho \right) \wedge \widehat{\text{vol}}_{\text{S}^3} - \partial_\rho \widehat{h}_4 \widehat{\text{vol}}_{\text{CY}_2}.
 \end{aligned}
 \tag{5.2}$$

The background in equations (5.1)–(5.2) solves the equations of motion provided that  $u'' = \widehat{h}_4'' = h_8'' = 0$ . The last two conditions come from the Bianchi identities for the RR sector. Note that we must have  $4\widehat{h}_4 h_8 - (u')^2 > 0$ , in order for the metric to be of the correct signature and the dilaton to be real.

The Page fluxes are given by

$$\widehat{F}_{(0)} = F_{(0)} = h'_8, \tag{5.3}$$

$$\widehat{F}_{(2)} = F_{(2)} - F_{(0)} B_{(2)} = -\frac{1}{2} (h_8 - \rho h'_8) \widehat{\text{vol}}_{\text{AdS}_2}, \tag{5.4}$$

$$\widehat{F}_{(4)} = F_{(4)} = -\widehat{h}'_4 \widehat{\text{vol}}_{\text{CY}_2} - \left( 2h_8 - \left( \frac{uu'}{2\widehat{h}_4} \right)' \right) \widehat{\text{vol}}_{\text{S}^3} \wedge d\rho, \tag{5.5}$$

$$\widehat{F}_{(6)} = \frac{1}{2} (\widehat{h}_4 - \rho \widehat{h}'_4) \widehat{\text{vol}}_{\text{AdS}_2} \wedge \widehat{\text{vol}}_{\text{CY}_2} + \left( \left( \frac{u(\rho u' - u)}{4h_4} \right)' - \rho h_8 \right) \widehat{\text{vol}}_{\text{AdS}_2} \wedge \widehat{\text{vol}}_{\text{S}^3} \wedge d\rho, \tag{5.6}$$

$$\widehat{F}_{(8)} = \left( 2\widehat{h}_4 - \left( \frac{uu'}{2h_8} \right)' \right) \widehat{\text{vol}}_{\text{CY}_2} \wedge \widehat{\text{vol}}_{\text{S}^3} \wedge d\rho. \tag{5.7}$$

The class of solutions given by (5.1) and (5.2) provide a new class of backgrounds to Type IIA with four Poincaré supersymmetries and SU(2)-structure, which are warped products of  $\text{AdS}_2 \times \text{S}^3 \times \text{M}_4$  over an interval, with  $\text{M}_4$  a Calabi-Yau 2-fold. The  $\text{AdS}_2 \times \text{S}^3$  subspace realises an  $\text{SL}(2, \mathbb{R}) \times \text{SO}(4)$  isometry group. As mentioned above, one of the SU(2)'s in  $\text{SO}(4) \cong \text{SU}(2) \times \text{SU}(2)$  is a global symmetry, so the R-symmetry is that of the SU(1, 1|2) supergroup. This identifies the superconformal group of the associated dual quantum mechanics. As in section 2, we have restricted ourselves to the case in which the symmetries of the  $\text{CY}_2$  are respected by the full solution. We will construct the most general class of solutions with SU(2) structure in appendix C, where we will relax this condition on the class I solutions in [18] and analytically continue the solutions in class II. Note that a more general class of solutions with the same supersymmetry can in principle be obtained taking an identity structure instead of the SU(2)-structure considered here.

### 5.1 Brane set-up

Associated to the Page fluxes we find the following quantised charges,

$$N_8 = 2\pi \int_{\mathcal{I}_\rho} d\widehat{F}_{(0)}, \quad d\widehat{F}_{(0)} = h_8'' d\rho. \tag{5.8}$$

According to (5.8), we have a natural definition of D8-branes as objects localised in the  $\rho$  direction. This, in turn, leads to the fact that D8-branes are not dissolved into fluxes, and effectively behave as flavour branes. They span the  $\text{AdS}_2 \times \text{S}^3 \times \text{CY}_2$  sub-manifold.

	0	1	2	3	4	5	6	7	8	9
D0	x									
F1	x					x				
D4	x						x	x	x	x
D4'	x	x	x	x	x					
D8	x	x	x	x	x		x	x	x	x

**Table 5.**  $\frac{1}{8}$ -BPS brane intersection underlying the  $\text{AdS}_2 \times \text{S}^3$  solutions in Type IIA.  $(x^1, \dots, x^4)$  span the  $\text{CY}_2$  and  $(x^6, x^7, x^8, x^9)$  are the transverse directions on which the  $\text{SO}(4)$  symmetry group is realised. A  $\mathcal{N} = 4$  supersymmetric quantum mechanics lives in the common  $x^0$  direction.

From the expression for  $\widehat{F}_{(4)}$  in (5.5) we identify two manifolds supporting  $\widehat{F}_{(4)}$ -fluxes. These are  $\widetilde{\mathcal{M}}_4 = \text{CY}_2$  and  $\mathcal{M}_4 = \text{S}^3 \times \mathcal{I}$ . There are therefore two quantised charges, associated to D4 and D4' branes

$$\begin{aligned}
 N_4 &= \frac{\text{vol}_{\text{CY}_2}}{(2\pi)^3} \int_{\mathcal{I}} d\rho \widehat{h}_4'', \\
 N_{4'} &= \frac{\text{vol}_{\text{S}^3}}{(2\pi)^3} \int_{\mathcal{I}} d\rho \left( 2h_8 - \left( \frac{uu'}{2h_4} \right)' \right).
 \end{aligned}
 \tag{5.9}$$

Given that

$$d\widehat{F}_{(4)} = \widehat{h}_4'' d\rho \wedge \widehat{\text{vol}}_{\text{CY}_2}, \tag{5.10}$$

the D4-branes provide localised sources, and are therefore flavour branes. They are extended on  $\text{AdS}_2 \times \text{S}^3$ . In turn, the D4'-branes are dissolved into fluxes and therefore do not provide additional physical sources. They are colour branes and extend on  $(t, \text{CY}_2)$ .

Finally, there is D0 brane charge,

$$N_0 = \frac{1}{(2\pi)^7} \text{vol}_{\text{CY}_2} \text{vol}_{\text{S}^3} \int_{\mathcal{I}} d\rho \left( 2\widehat{h}_4 - \left( \frac{uu'}{2h_8} \right)' \right). \tag{5.11}$$

Given that  $d\widehat{F}_8$  vanishes identically the D0-branes are colour branes. On top of this there are F1-strings, associated to the electric components of  $H_{(3)}$ , in (5.1). These F1 extend on  $\text{AdS}_2$ .

The brane set-up associated to the quantised charges is summarised in table 5. Note that this is exactly what is obtained reducing the M-brane configuration in the previous section, with the addition of extra D8-branes, not present in M-theory. Similar brane intersections have been discussed in [62], in connection with  $\text{AdS}_2 \times \text{S}^3 \times \text{S}^3$  geometries warped over a strip. In this reference the dual SCQM was interpreted in terms of D0-F1-D4 brane defects inside the 5d  $\text{Sp}(N)$  fixed point theory dual to the  $\text{AdS}_6$  Brandhuber-Oz background [63]. It is likely that a similar interpretation is at place for our brane system [56].

## 6 Conclusions

In this paper we have presented and studied new families of solutions preserving four Poincaré supersymmetries.

The first is a new class of solutions to M-theory preserving  $\mathcal{N} = (0, 4)$  (small) supersymmetry, of the type  $\text{AdS}_3 \times \text{S}^3/\mathbb{Z}_k \times \text{CY}_2$  foliated over an interval. These solutions are holographic duals to 2d (small)  $\mathcal{N} = (0, 4)$  SCFTs supported by  $M_A$ -strings in M5-brane intersections. We identified the precise quivers in correspondence with the functions defining the backgrounds. Through the computation of the central charge we have checked the compliance with the holographic dictionary and the identification of  $M_A$ -strings as the defining degrees of freedom of our theories. We calculated the central charge of a two dimensional  $\mathcal{N} = (0, 4)$  QFT that flows to a conformal sigma model. To perform our calculation we used a brane intersection of M2 and M5 branes and the geometry they generate. If we were to think of our supergravity solutions as near horizons of higher dimensional black holes, our central charge would be identified with the entropy of these black holes. Notice that these ideas are similar to those in [1]. In fact, the entropy of black holes was calculated in [1] using a 2d  $\mathcal{N} = (0, 4)$  sigma model. Even when the brane configuration considered in [1] is different from the one we discussed here, the ideology is resemblant.

Through analytic continuation, we have constructed a second family of new solutions for which the modding is performed on the Hopf fibre of  $\text{AdS}_3$ . These solutions are holographically dual to SCQM, which are the M(atr)ix theory descriptions of M2-M5-M5' brane intersections, upon Sen-Seiberg limit. We have postponed a more detailed analysis of these supersymmetric quantum mechanics to our forthcoming publication [56]. We have shown that the subclasses of  $\text{AdS}_3 \times \text{S}^3/\mathbb{Z}_k$  and  $\text{AdS}_3/\mathbb{Z}_k \times \text{S}^3$  M-theory solutions with  $u' = 0$ , contain self-dual strings. Therefore, they provide with explicit fully backreacted  $\text{AdS}_3 \times \text{S}^3$  OM-theory [45] backgrounds [44].

Upon reduction, we have constructed a third new class of solutions to Type IIA supergravity with four Poincaré supercharges, of the type  $\text{AdS}_2 \times \text{S}^3 \times \text{CY}_2$  foliated over an interval. These solutions should be holographic duals to the quantum mechanical systems described above, in the regime of validity of the Type IIA description. We have extended these solutions to the massive case noticing that they are related through analytic continuation to the  $\text{AdS}_3 \times \text{S}^2 \times \text{CY}_2$  solutions to massive Type IIA supergravity constructed in [18]. The dual quantum mechanics is under investigation in [56]. Three appendices extend our solutions to the most general class of  $\text{AdS}_3 \times \text{S}^2$  solutions to M-theory with (0,4) supersymmetries and  $\text{SU}(2)$  structure, and to new  $\text{AdS}_2 \times \text{S}^3 \times \text{M}_4$  solutions to massive IIA where  $\text{M}_4$  is a Kähler manifold. It would be interesting to understand the holographic duals for the general backgrounds presented there.

There is an interesting connection between our work and holographic duals of defect CFTs constructed in the literature. It was shown in [64] that a subclass of the solutions in [18] allowed for an interpretation in terms of 2d D2-D4 defects in the 6d (1,0) CFTs living in D6-NS5-D8 brane intersections. Key to this realisation was the identification of a mapping between these solutions and the  $\text{AdS}_7$  solutions to massive Type IIA supergravity constructed in [65]. In the same vein, one would expect that a similar interpretation should be possible for our  $\text{AdS}_3$  M-theory solutions, this time in terms of 2d M2-M5 defects in the 6d (1,0) CFTs living in M5'-branes probing A-type singularities. In this direction, it would be interesting to show whether our solutions bear any relation to the flows constructed in [66, 67], which are asymptotically locally  $\text{AdS}_7$  in the UV. Along

closely related lines,  $\text{AdS}_2 \times \text{S}^3 \times \mathbb{R}$  flows to asymptotically locally  $\text{AdS}_6$  in the UV have been constructed in [68, 69], and interpreted as 1d defects in the 5d  $\text{Sp}(N)$  fixed point theory dual to the  $\text{AdS}_6$  Brandhuber-Oz solution [63]. We would expect that our  $\text{AdS}_2 \times \text{S}^3$  solutions to massive Type IIA bear a relation to these, thus allowing for an interpretation as D0-F1-D4 brane defects within the  $\text{Sp}(N)$  fixed point theory living in D4'-D8 branes. These issues are currently under investigation [43].

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## A Review of the general $\text{AdS}_3 \times \text{S}^2 \times \text{M}_4$ solutions in [18]

In this appendix we summarise the generic backgrounds found in [18]. These backgrounds were divided in two classes: class I, for which the  $\text{M}_4$  is a Calabi-Yau 2-fold, and class II, for which  $\text{M}_4$  is a general 4d Kähler manifold. The particular case in class I in which the full solution respects the symmetries of the Calabi-Yau manifold, and therefore the Calabi-Yau manifold needs to be compact, was discussed in section 2. This is the case that concerned us in the main body of the paper. In these appendices we complete the analysis by providing the most general solutions in M-theory with (0,4) supersymmetries and  $\text{SU}(2)$  structure. In appendix B we present the uplift to M-theory of the most general solutions in class I and of the solutions in class II. In appendix C we construct  $\text{AdS}_2 \times \text{S}^3 \times \text{M}_4$  solutions to massive Type IIA supergravity through double analytical continuation of the class I and class II solutions.

We start reviewing the most general class I backgrounds in [18].

**Class I:  $\text{M}_4 = \text{CY}_2$ .** The explicit form of the NS sector of the solutions referred to as class I in [18] is given by,

$$\begin{aligned}
 ds^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}} \left( ds_{\text{AdS}_3}^2 + \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 + (u')^2} ds_{\text{S}^2}^2 \right) + \sqrt{\frac{\widehat{h}_4}{h_8}} ds_{\text{CY}_2}^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u} d\rho^2, \\
 e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4} \sqrt{u}} \sqrt{4\widehat{h}_4 h_8 + (u')^2}, \\
 H_{(3)} &= \frac{1}{2} d \left( -\rho + \frac{uu'}{4\widehat{h}_4 h_8 + (u')^2} \right) \wedge \widehat{\text{vol}}_{\text{S}^2} + \frac{1}{h_8} d\rho \wedge H_2.
 \end{aligned} \tag{A.1}$$

Here  $\Phi$  is the dilaton,  $H_{(3)}$  the NS three-form and the metric is given in string frame. A prime denotes a derivative with respect to  $\rho$ . The two-form  $H_2$  is defined on the  $\text{CY}_2$  as

we specify below. The RR sector reads

$$\begin{aligned}
 F_{(0)} &= h'_8, \quad F_{(2)} = -H_2 - \frac{1}{2} \left( h_8 - \frac{h'_8 u' u}{4h_8 \widehat{h}_4 + (u')^2} \right) \widehat{\text{vol}}_{\text{S}^2}, \\
 F_{(4)} &= - \left( d \left( \frac{u' u}{2\widehat{h}_4} \right) + 2h_8 d\rho \right) \wedge \widehat{\text{vol}}_{\text{AdS}_3} - \partial_\rho \widehat{h}_4 \widehat{\text{vol}}_{\text{CY}_2} - \frac{h_8}{u} (\widehat{\star}_4 d_4 \widehat{h}_4) \wedge d\rho \\
 &\quad - \frac{u' u}{2(4\widehat{h}_4 h_8 + (u')^2)} H_2 \wedge \widehat{\text{vol}}_{\text{S}^2},
 \end{aligned} \tag{A.2}$$

where  $\widehat{\star}_4$  is the Hodge dual on the  $\text{CY}_2$ . Higher RR fluxes are related to these as  $F_{(6)} = -\star F_{(4)}$ ,  $F_{(8)} = \star F_{(2)}$ ,  $F_{(10)} = -\star F_{(0)}$ , where  $\star$  is the ten-dimensional Hodge-dual operator. Supersymmetry holds when

$$u'' = 0, \quad H_2 + \widehat{\star}_4 H_2 = 0, \tag{A.3}$$

which makes  $u$  a linear function of  $\rho$ .  $H_2$  is defined in terms of three functions  $g_{1,2,3}$  on the  $\text{CY}_2$  and the vielbein on  $\text{M}_4$ ,  $\widehat{e}^i$ ,

$$H_2 = g_1(\widehat{e}^1 \wedge \widehat{e}^2 - \widehat{e}^3 \wedge \widehat{e}^4) + g_2(\widehat{e}^1 \wedge \widehat{e}^3 + \widehat{e}^2 \wedge \widehat{e}^4) + g_3(\widehat{e}^1 \wedge \widehat{e}^4 - \widehat{e}^2 \wedge \widehat{e}^3). \tag{A.4}$$

Hence, the Bianchi identities of the fluxes impose

$$\begin{aligned}
 h''_8 &= 0, \quad dH_2 = 0 \\
 \frac{h_8}{u} \nabla_{\text{CY}_2}^2 \widehat{h}_4 + \partial_\rho^2 \widehat{h}_4 + \frac{2}{h_8^3} (g_1^2 + g_2^2 + g_3^2) &= 0.
 \end{aligned} \tag{A.5}$$

In the particular case when  $H_2$  vanishes and  $\widehat{h}_4$  has support on the  $\rho$  coordinate we find that the supersymmetry and Bianchi identities are satisfied for  $u$ ,  $h_8$ ,  $\widehat{h}_4$  arbitrary linear functions in  $\rho$ . We are then in the case reviewed in section 2 and lifted to eleven dimensions in section 3.

**Class II:  $\text{M}_4 = \text{Kähler}$ .** We now summarise the details of the class II backgrounds in [18]. These are warped products of the form  $\text{AdS}_3 \times \text{S}^2 \times \text{M}_4 \times \text{I}$ , where  $\text{M}_4$  is a family of Kähler four-manifolds with metrics that depend on the interval coordinate  $\rho$ , and with an integrable complex structure that is  $\rho$ -independent. These solutions have the following NS sector

$$\begin{aligned}
 ds^2 &= \frac{u}{\sqrt{hw^2 - v^2}} \left[ ds_{\text{AdS}_3}^2 + \frac{hw^2 - v^2}{4(hw^2 - v^2) + (u')^2} ds_{\text{S}^2}^2 \right] + \frac{\sqrt{hw^2 - v^2}}{u} \left[ \frac{u}{hw} ds_{\text{M}_4}^2 + d\rho^2 \right], \\
 H_{(3)} &= \frac{1}{2} d \left( -\rho + \frac{uu'}{4(hw^2 - v^2) + (u')^2} \right) \wedge \text{vol}_{\text{S}^2} + d \left( \frac{v}{wh} \widehat{J} \right), \\
 e^{-\Phi} &= \frac{wh^{\frac{1}{2}} \sqrt{4(hw^2 - v^2) + (u')^2}}{2\sqrt{u}(hw^2 - v^2)^{\frac{1}{4}}}.
 \end{aligned} \tag{A.6}$$

The functions  $u, v$  and  $w$  depend on  $\rho$ , while  $h$  has support in  $\rho$  and  $M_4$ .  $\widehat{J}$  is a two-form defined on the Kähler manifold.<sup>4</sup> The RR fluxes are given by,

$$\begin{aligned}
 F_{(0)} &= v', \\
 F_{(2)} &= -\frac{w^2}{u} d\rho \wedge \widehat{\star}_4(d_4 h \wedge \widehat{J}) - \partial_\rho(w\widehat{J}) + \frac{vv'}{hw} \widehat{J} - \frac{1}{2} \left( v - \frac{v'u u'}{4(hw^2 - v^2) + (u')^2} \right) \text{vol}_{S^2}, \\
 F_{(4)} &= \frac{1}{2} \text{vol}_{\text{AdS}_3} \wedge \left( d \left( \frac{v u u'}{h w^2 - v^2} \right) + 4v d\rho \right) + \frac{v}{2h} \left( \frac{v v'}{h w^2} - \partial_\rho \log(v^{-1} h w^2) \right) \widehat{J} \wedge \widehat{J} \\
 &\quad - \frac{v w}{u} d\rho \wedge \widehat{\star}_4 d \log h + \frac{1}{2} \left( \frac{u u'}{4(hw^2 - v^2) + (u')^2} F_2 + \frac{h w^2 - v^2}{h w} \widehat{J} \right) \wedge \text{vol}_{S^2}. \quad (\text{A.7})
 \end{aligned}$$

Here  $d_4 = \partial + \bar{\partial}$ , with  $\partial, \bar{\partial}$  defined as the Dolbeault operators, expressed in terms of complex coordinates on  $M_4$ .

Supersymmetry and the Bianchi identities (away from localised sources) hold by the following conditions,

$$\begin{aligned}
 u'' &= 0, & \partial_\rho \left( \frac{\widehat{g}^{\frac{1}{2}}}{h} \right) &= 0, & i\partial\bar{\partial} \log h &= \widehat{\mathfrak{R}} \\
 \text{and } v'' &= 0, & 2i\partial\bar{\partial} h &= \partial_\rho^2(w\widehat{J}).
 \end{aligned} \quad (\text{A.8})$$

The quantity  $\widehat{g}$  is the determinant of the metric and  $\widehat{\mathfrak{R}}$  the Ricci form on  $M_4$ .

## B New $\text{AdS}_3 \times S^2 \times M_4$ solutions in M-theory

In this appendix we consider the uplift to eleven dimensions of the most general solutions in class I and the solutions in class II reviewed in the previous appendix. Our backgrounds provide the most general class of  $\text{AdS}_3 \times S^2$  solutions in M-theory with (0,4) supersymmetries and  $SU(2)$  structure. Note that a more general class of solutions with (0,4) SUSY can in principle be obtained taking an identity structure instead of the  $SU(2)$ -structure considered here. We will focus separately on the class I and class II backgrounds. In both cases, conditions must be imposed to allow the lift to eleven dimensions.

**Lift of the class I backgrounds.** We consider the class I geometries first. Imposing that  $F_{(0)} = 0$  to allow the lift of the solutions described by equations (A.1)–(A.2), we find the eleven dimensional configurations,

$$\begin{aligned}
 ds_{11}^2 &= \Delta \left( \frac{u}{\sqrt{\widehat{h}_4 h_8}} ds_{\text{AdS}_3}^2 + \sqrt{\frac{\widehat{h}_4}{h_8}} ds_{\text{CY}_2}^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u} d\rho^2 \right) + \frac{h_8^2}{4\Delta^2} \left( ds_{S^2}^2 + (D\tilde{\psi})^2 \right), \\
 G_{(4)} &= - \left( d \left( \frac{u u'}{2\widehat{h}_4} \right) + 2h_8 d\rho \right) \wedge \widehat{\text{vol}}_{\text{AdS}_3} - \partial_\rho \widehat{h}_4 \widehat{\text{vol}}_{\text{CY}_2} - \frac{u u'}{2(\widehat{h}_4 h_8 + (u')^2)} H_2 \wedge \widehat{\text{vol}}_{S^2} \\
 &\quad - \frac{h_8}{u} \star_4 d_4 \widehat{h}_4 \wedge d\rho + \frac{h_8}{2} \left[ \frac{1}{2} d \left( -\rho + \frac{u u'}{4\widehat{h}_4 h_8 + (u')^2} \right) \wedge \widehat{\text{vol}}_{S^2} + \frac{1}{h_8} d\rho \wedge H_2 \right] \wedge D\tilde{\psi}, \quad (\text{B.1})
 \end{aligned}$$

<sup>4</sup>The interested reader can find a detailed explanation in [18].

where we have defined the following expressions,

$$\begin{aligned}
 H_2 &= -d\mathcal{A}, \\
 D\tilde{\psi} &= d\tilde{\psi} + \tilde{\mathcal{A}} + \omega \quad \text{with} \quad d\omega = \widehat{\text{vol}}_{S^2}, \\
 \Delta &= \frac{h_8^{1/2}(4\widehat{h}_4 h_8 + u'^2)^{1/3}}{2^{2/3}\widehat{h}_4^{1/6}u^{1/3}},
 \end{aligned} \tag{B.2}$$

with  $\tilde{\psi} = \frac{2}{h_8}\psi$  and  $\tilde{\mathcal{A}} = \frac{2}{h_8}\mathcal{A}$ . In (B.2) we have assumed that  $dH_2 = 0$  holds globally, allowing us to globally define  $H_2 = -d\mathcal{A}$ . Notice that the connection  $\tilde{\mathcal{A}} + \omega$  makes the fibre over the  $S^2$  and the  $\text{CY}_2$  non trivial. The uplift to eleven dimensions preserves the  $\mathcal{N} = (0, 4)$  supersymmetry of the Type IIA solutions, as well as their  $\text{SU}(2)$ -structure.

Considering a sub-class of solutions –with  $H_2 = 0$  and  $\widehat{h}_4 = \widehat{h}_4(\rho)$ – we obtain  $\text{AdS}_3 \times S^3/\mathbb{Z}_k \times \text{CY}_2$  solutions to M-theory with  $(0, 4)$  (small) supersymmetry, warped over an interval. These were the solutions written in equations (3.1)–(3.2).

**Lift of the class II backgrounds.** To allow for a lift to M-theory, we impose that  $F_{(0)} = 0$ . Considering  $v' = 0$  and uplifting the solution described by equations (A.6)–(A.7) we find,

$$\begin{aligned}
 ds_{11}^2 &= \Delta \left[ ds_{\text{AdS}_3}^2 + \frac{hw^2 - v^2}{hwu} \left( ds_{M_4}^2 + \frac{hw}{u} d\rho^2 \right) \right] + \frac{u^2 w^2 h}{4(hw^2 - v^2)\Delta^2} \left[ ds_{S^2}^2 + \frac{v^2}{w^2 h} (D\tilde{\psi})^2 \right] \\
 G_{(4)} &= \frac{1}{2} \text{vol}_{\text{AdS}_3} \wedge \left[ d \left( \frac{vuu'}{hw^2 - v^2} \right) + 4v d\rho \right] - \frac{v}{2h} (\partial_\rho \log(v^{-1}hw^2)) \widehat{J} \wedge \widehat{J} \\
 &\quad - \frac{vw}{u} d\rho \wedge \widehat{\star}_4 d \log h + \frac{1}{2} \left( \frac{uu'}{4(hw^2 - v^2) + (u')^2} J_2 + \frac{hw^2 - v^2}{hw} \widehat{J} \right) \wedge \text{vol}_{S^2} \\
 &\quad + \frac{v}{2} \left[ \frac{1}{2} d \left( -\rho + \frac{uu'}{4(hw^2 - v^2) + (u')^2} \right) \wedge \text{vol}_{S^2} + d \left( \frac{v}{wh} \widehat{J} \right) \right] \wedge D\tilde{\psi},
 \end{aligned} \tag{B.3}$$

where we have defined the following expressions,

$$\begin{aligned}
 D\tilde{\psi} &= d\tilde{\psi} + \tilde{\mathcal{J}} + \eta \quad \text{with} \quad d\eta = \widehat{\text{vol}}_{S^2}, \\
 J_2 &= d\tilde{\mathcal{J}} = -\frac{w^2}{u} d\rho \wedge \widehat{\star}_4 (d_4 h \wedge \widehat{J}) - \partial_\rho (w\widehat{J}), \\
 \Delta &= \left( \frac{uw\sqrt{h}\sqrt{4(hw^2 - v^2) + u'^2}}{2(hw^2 - v^2)} \right)^{2/3}.
 \end{aligned} \tag{B.4}$$

Here  $\tilde{\psi} = \frac{2}{v}\psi$  and  $\tilde{\mathcal{J}} = \frac{2}{v}\mathcal{J}$ . As before, the connection  $\tilde{\mathcal{J}} + \eta$  makes the fibre over the  $S^2$  and the  $M_4$  non trivial. In order to find this uplift we have assumed that  $dJ_2 = 0$  holds globally, allowing us to globally define  $J_2 = d\tilde{\mathcal{J}}$ .

The uplift to eleven dimensions preserves the  $\mathcal{N} = (0, 4)$  supersymmetry of the Type IIA solutions, as well as their  $\text{SU}(2)$ -structure.

## C New $\text{AdS}_2 \times S^3 \times M_4$ solutions in massive Type IIA

Applying the set of transformations discussed around equation (4.2) to the previous M-theory solutions gives rise to  $\text{AdS}_2 \times S^3 \times M_4$  solutions with 4 Poincaré supercharges and

SU(2) structure. In these solutions the AdS<sub>2</sub> is non-trivially fibred. These solutions give upon reduction to Type IIA the double analytical continuation of the class I and class II solutions reviewed in appendix A. Thus, by acting with these rules directly on these sets of solutions we can generalise the backgrounds to the massive case. We present these backgrounds in this appendix. The AdS<sub>2</sub> × S<sup>3</sup> × M<sub>4</sub> M-theory solutions arise upon uplift when F<sub>(0)</sub> = 0.

**Class I backgrounds.** A sub-class of these solutions was presented in section 5. Here we generalise this class to the case in which there is a dependence of the fluxes on the CY<sub>2</sub>. Performing the double analytical continuation

$$u \rightarrow -iu, \quad \widehat{h}_4 \rightarrow i\widehat{h}_4, \quad h_8 \rightarrow ih_8, \quad \rho \rightarrow i\rho, \quad (\text{C.1})$$

together with

$$AdS_3 \rightarrow -S^3, \quad S^2 \rightarrow -AdS_2, \quad (\text{C.2})$$

in the most general solutions in class I, given by equations (A.1), (A.2), we arrive at

$$\begin{aligned} ds^2 &= \frac{u}{\sqrt{\widehat{h}_4 h_8}} \left( \frac{\widehat{h}_4 h_8}{4\widehat{h}_4 h_8 - (u')^2} ds_{AdS_2}^2 + ds_{S^3}^2 \right) + \sqrt{\frac{\widehat{h}_4}{h_8}} ds_{CY_2}^2 + \frac{\sqrt{\widehat{h}_4 h_8}}{u} d\rho^2, \\ e^{-\Phi} &= \frac{h_8^{3/4}}{2\widehat{h}_4^{1/4} \sqrt{u}} \sqrt{4\widehat{h}_4 h_8 - (u')^2}, \\ H_{(3)} &= -\frac{1}{2} d \left( \rho + \frac{uu'}{4\widehat{h}_4 h_8 - (u')^2} \right) \wedge \widehat{\text{vol}}_{AdS_2} + \frac{1}{h_8} d\rho \wedge H_2. \end{aligned} \quad (\text{C.3})$$

The RR sector reads

$$\begin{aligned} F_{(0)} &= h'_8, \quad F_{(2)} = -H_2 - \frac{1}{2} \left( h_8 + \frac{h'_8 u' u}{4h_8 \widehat{h}_4 - (u')^2} \right) \widehat{\text{vol}}_{AdS_2}, \\ F_{(4)} &= \left( -d \left( \frac{u' u}{2\widehat{h}_4} \right) + 2h_8 d\rho \right) \wedge \widehat{\text{vol}}_{S^3} - \frac{h_8}{u} \widehat{\star}_4 d_4 h_4 \wedge d\rho - \partial_\rho \widehat{h}_4 \widehat{\text{vol}}_{CY_2} \\ &\quad + \frac{u' u}{2(4\widehat{h}_4 h_8 - (u')^2)} H_2 \wedge \widehat{\text{vol}}_{AdS_2}. \end{aligned} \quad (\text{C.4})$$

These backgrounds generalise the solutions in section 5 to the case in which H<sub>2</sub> ≠ 0 and ∇<sub>CY<sub>2</sub></sub>  $\widehat{h}_4 \neq 0$ .

Supersymmetry holds when

$$u'' = 0, \quad H_2 + \widehat{\star}_4 H_2 = 0. \quad (\text{C.5})$$

H<sub>2</sub> is defined in terms of three functions g<sub>1,2,3</sub> on the CY<sub>2</sub> and the vielbein on M<sub>4</sub>,  $\widehat{e}^i$ , as in eq. (A.4). The Bianchi identities of the fluxes impose

$$\begin{aligned} h''_8 &= 0, \quad dH_2 = 0 \\ \frac{h_8}{u} \nabla_{CY_2}^2 \widehat{h}_4 + \partial_\rho^2 \widehat{h}_4 + \frac{2}{h_8^3} (g_1^2 + g_2^2 + g_3^2) &= 0. \end{aligned} \quad (\text{C.6})$$

Note that it must be that 4 $\widehat{h}_4 h_8 - (u')^2 > 0$ , in order for the metric to be of the correct signature and the dilaton to be real.



**Class II backgrounds.** In this case we consider the following analytical continuation

$$u \rightarrow -iu, \quad v \rightarrow iv, \quad w \rightarrow iw, \quad \rho \rightarrow i\rho \quad (\text{C.7})$$

together with

$$AdS_3 \rightarrow -S^3, \quad S^2 \rightarrow -AdS_2, \quad (\text{C.8})$$

of the class II solutions reviewed in appendix A. The NS sector of the background we get reads

$$\begin{aligned} ds^2 &= \frac{u}{\sqrt{hw^2 - v^2}} \left[ \frac{hw^2 - v^2}{4(hw^2 - v^2) - (u')^2} ds_{AdS_2}^2 + ds_{S^3}^2 \right] + \frac{\sqrt{hw^2 - v^2}}{u} \left[ \frac{u}{hw} ds_{M_4}^2 + d\rho^2 \right], \\ e^{-\Phi} &= \frac{wh^{1/2} \sqrt{4(hw^2 - v^2) - (u')^2}}{2u^{1/2}(hw^2 - v^2)^{1/4}}, \quad H_{(3)} = \frac{1}{2} d \left( -\rho - \frac{uu'}{4(hw^2 - v^2) - (u')^2} \right) \wedge \widehat{\text{vol}}_{AdS_2}. \end{aligned} \quad (\text{C.9})$$

The RR sector is given by

$$\begin{aligned} F_{(0)} &= v', \\ F_{(2)} &= -\frac{w^2}{u} d\rho \wedge \widehat{\star}_4 (d_4 h \wedge \widehat{J}) - \partial_\rho w \widehat{J} + \frac{vv'}{hw} \widehat{J} - \frac{1}{2} \left( v + \frac{v'uu'}{4(hw^2 - v^2) - (u')^2} \right) \wedge \widehat{\text{vol}}_{AdS_2}, \\ F_{(4)} &= -\frac{1}{2} \widehat{\text{vol}}_{S^3} \wedge \left( d \left( \frac{vuu'}{hw^2 - v^2} \right) - 4v d\rho \right) + \frac{v}{2h} \left( \frac{vv'}{hw^2} - \partial_\rho \log(v^{-1}hw^2) \right) \widehat{J} \wedge \widehat{J} \\ &\quad - \frac{vw}{u} d\rho \wedge \widehat{\star}_4 d \log h + \frac{1}{2} \left( -\frac{uu'}{4(hw^2 - v^2) - (u')^2} F_{(2)} + \frac{hw^2 - v^2}{hw} \widehat{J} \right) \wedge \widehat{\text{vol}}_{AdS_2}. \end{aligned} \quad (\text{C.10})$$

Here  $d_4 = \partial + \bar{\partial}$ , with  $\partial, \bar{\partial}$  defined as the Dolbeault operators, expressed in terms of complex coordinates on  $M_4$ . Supersymmetry and the Bianchi identities hold by the conditions,

$$\begin{aligned} u'' &= 0, \quad \partial_\rho \left( \frac{\widehat{g}^{\frac{1}{2}}}{h} \right) = 0, \quad i\partial\bar{\partial} \log h = \widehat{\mathfrak{R}} \\ \text{and} \quad v'' &= 0, \quad 2i\partial\bar{\partial} h = \partial_\rho^2 (w\widehat{J}). \end{aligned} \quad (\text{C.11})$$

The backgrounds presented in this appendix provide the most general class of  $AdS_2 \times S^3$  solutions to massive Type IIA supergravity with 4 Poincaré supercharges and  $SU(2)$  structure.

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