Table 1: Descriptive statistics of weekly observed continuously compounded yields implied by US Treasury bonds (1995-2017) for all maturities from one year to twenty years. Yields are measured in decimal terms.

Descriptive statistics for the in sample period from 4th January 1995 to 31st May 2006										
Maturity	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
Average	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Stdev	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Max	0.01	0.01	0.01	0.02	0.02	0.03	0.03	0.03	0.03	0.03
Min	0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Skewness	-0.51	-0.45	-0.38	-0.29	-0.18	-0.08	-0.01	0.05	0.13	0.20
Kurtosis	-1.20	-1.07	-1.00	-0.98	-1.00	-1.04	-1.06	-1.03	-0.94	-0.85
Maturity	11 years	12 years	13 years	14 years	15 years	16 years	17 years	18 years	19 years	20 years
Average	0.05	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Stdev	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Max	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Min	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
Skewness	0.25	0.28	0.28	0.26	0.23	0.19	0.17	0.15	0.15	0.16
Kurtosis	-0.78	-0.71	-0.66	-0.63	-0.60	-0.57	-0.54	-0.52	-0.50	-0.50
Descriptive statistics for the out of sample period 7th June 2006 to 29th November 2017										
Maturity	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
Average	0.01	0.01	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03
Stdev	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Max	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01
Min	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Skewness	1.79	1.73	1.58	1.38	1.16	0.96	0.79	0.65	0.54	0.44
Kurtosis	1.78	1.68	1.36	0.95	0.52	0.10	-0.29	-0.62	-0.88	-1.09
Maturity	11 years	12 years	13 years	14 years	15 years	16 years	17 years	18 years	19 years	20 years
Average	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
Stdev	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Max	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Min	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Skewness	0.35	0.29	0.23	0.19	0.16	0.14	0.13	0.11	0.10	0.09
Kurtosis	-1.24	-1.32	-1.37	-1.40	-1.41	-1.41	-1.41	-1.41	-1.41	-1.41

Table 2: Results of Extended Kalman Filter (EKF) estimation for US yields

Model	**$A_0(3)$**		**$A_0(4)$**		**$A_0(5)$**		**$A_0(6)$**		**$A_0(10)$**		**$A_0(15)$**		**$A_0(20)$**	
σ_1	0.0003	**	0.0004	**	0.0005	**	0.0006	**	0.0008	**	0.0013	***	0.0013	**
κ_1	0.0742	**	0.1504	**	0.2346	**	0.3104	**	0.4476	**	0.7236	***	0.9773	**
κ_1*	0.0243	**	0.0365	**	0.0000		0.0001		0.0000		0.0000		0.0000	
μ_1	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0	
μ_1*	0.0000		0.0000		22.4700	**	0.0004		0.0000		0.0099	***	0	
h	0.0009	**	0.0005	**	0.0004	**	0.0003	**	0.0002	**	0.0001	***	0.0001	**
lk	129,837		140,367		146,118		149,658		157,737		164,491		-168,207	
Root mean squared errors (RMSE) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	0.0022	0.0018	0.0012	0.0011	0.0011	0.0009	0.0010	0.0008	0.0010	0.0008	0.0011	0.0008	0.0011	0.0008
10 years	0.0016	0.0014	0.0014	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0012	0.0013	0.0013	0.0013	0.0013
20 years	0.0013	0.0017	0.0013	0.0014	0.0013	0.0013	0.0013	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012
Avg RMSE	0.0015	0.0014	0.0013	0.0013	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012
Predictive variance (PV) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	-3.30	-3.80	-0.34	-0.75	-0.06	-0.11	0.02	-0.01	0.01	-0.01	0.00	0.00	0.00	0.00
10 years	-0.65	-0.16	-0.20	-0.01	-0.09	-0.02	-0.06	0.01	-0.03	0.01	0.00	0.00	0.01	0.00
20 years	-0.21	-0.88	-0.20	-0.30	-0.12	-0.15	-0.06	-0.09	0.02	0.02	0.01	0.02	0.01	0.01
Global PV	-0.43	-0.47	-0.14	-0.14	-0.08	-0.05	-0.04	-0.03	-0.01	0.00	0.00	0.00	0.00	0.00
Predictive variance (PV) for one week ahead EKF predicted yields during the "great" US recession (last quarter of 2007 to second quarter of 2009).														
Maturity		Out sample		Out sample		Out sample		Out sample		Out sample		Out sample		Out sample
1 year		0.00		0.14		0.07		0.07		0.02		0.00		0.00
10 years		-0.40		-0.03		-0.04		-0.06		-0.05		-0.01		0.00
20 years		-1.99		-0.08		-0.04		-0.05		-0.03		-0.02		-0.01
Global PV		-0.41		-0.07		-0.03		-0.03		0.00		0.00		0.00
Model	**$A_1(3)$**		**$A_1(4)$**		**$A_1(5)$**		**$A_1(6)$**		**$A_1(10)$**		**$A_1(15)$**		**$A_1(20)$**	
σ_1	0.0077	**	0.0093	**	0.0164	**	0.0126	**	0.0149	**	0.0190	**	0.0283	**
σ_2	1		1		1		1		1		1		1	
κ_1	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0331	**
κ_2	0.1260	**	0.2498	**	0.1999	**	0.2588	**	0.6115	**	1.0219	**	1.3680	**
κ_1*	=κ_1		=κ_1		=κ_1		=κ_1		=κ_1		=κ_1		=κ_1	
κ_2*	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000	
μ_1	0		0		0		0		0		0		0	
μ_2	0.0132	**	0.0246	**	0.0402	**	0.0361		0.0741	**	0.2074	**	0.2314	**
μ_1*	0		0		0		0		0		0		0	
μ_2*	0.0082		0.0095		0.0051		0.0570		0.0000		0.0000		0.0000	
h	0.0008	**	0.0005	**	0.0004	**	0.0003	**	0.0001	**	0.0001	**	0.0000	**
lk	130,713		140,999		145,381		150,524		160,643		166,545		171,734	
Root mean squared errors (RMSE) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	0.0016	0.0019	0.0014	0.0014	0.0015	0.0013	0.0014	0.0012	0.0011	0.0009	0.0012	0.0009	0.0011	0.0008
10 years	0.0016	0.0016	0.0016	0.0014	0.0015	0.0013	0.0014	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013
20 years	0.0016	0.0028	0.0016	0.0023	0.0014	0.0014	0.0014	0.0014	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012
Avg RMSE	0.0014	0.0018	0.0014	0.0016	0.0014	0.0014	0.0014	0.0014	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012
Predictive variance (PV) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	-1.24	-4.50	-0.85	-1.90	-0.89	-1.38	-0.65	-1.15	-0.17	-0.13	-0.31	-0.11	-0.01	-0.03
10 years	-0.68	-0.67	-0.53	-0.30	-0.42	-0.06	-0.33	-0.06	-0.10	-0.02	-0.06	-0.04	-0.04	-0.01
20 years	-0.83	-4.18	-0.65	-2.73	-0.38	-0.28	-0.40	-0.26	-0.08	-0.04	0.02	-0.01	0.01	-0.02
Global PV	-0.32	-1.38	-0.25	-0.82	-0.23	-0.33	-0.19	-0.29	-0.08	-0.08	-0.08	-0.08	-0.03	-0.03
Predictive variance (PV) for one week ahead EKF predicted yields during the "great" US recession (last quarter of 2007 to second quarter of 2009).														
Maturity		Out sample		Out sample		Out sample		Out sample		Out sample		Out sample		Out sample
1 year		-0.33		0.11		0.17		0.16		0.16		0.17		0.08
10 years		-0.62		-0.22		-0.05		-0.08		-0.06		-0.06		-0.07
20 years		-7.00		-4.58		-0.23		-0.18		-0.03		0.01		0.03
Global PV		-0.85		-0.44		-0.08		-0.06		-0.01		0.02		0.00

LEGEND

The total sample is 1188 weeks for the US.

The in sample period is the first 594 weeks for the US.

The out of sample period is the last 594 weeks for the US.

The row lk reports for each model value of the maximised log of the (quasi) likelihood function from the Kalman Filter.

The parameter estimates and the likelihood lk in the top Panel are estimated using the whole US sample.

Significant parameters at the 1% level are denoted with "**".

"Avg h" is the average h_i for i=1,..,20. h_i is the estimate of the standard deviation of the Kalman filter observation error for the i-year yield.

For a given model "Avg RMSE" is the average RMSE computed across the twenty yield maturities.

Global PV is computed across all twenty yield maturities.

The column headings "In sample" and "Out of sample" respectively denote in sample and out of sample RMSE and PV for each model.

For the US out of sample RMSE and PV are computed using the models estimated in two rolling windows (weeks 1-594 and weeks 298-891).

The parameters in the yellow cells were set to 0 because their estimates were insignificant and indistinguishable from zero.

The heading "In sample" refers to the in sample period. The heading "Out of sample" refers to the out of sample period.

Table 3: Results of Extended Kalman Filter (EKF) estimation for US yields

Model	**Q(3)**		**Q(4)**		**Q(5)**		**Q(6)**		**Q(10)**		**Q(15)**		**Q(20)**	
σ_1	0.0359	**	0.0372	**	0.0537	**	0.0595	**	0.0769	**	0.0823	**	0.0911	**
κ_1	0.0125	**	0.0254	**	0.0823	**	0.0911	**	0.1095	**	0.1078	**	0.2075	**
κ_1*	1.8619	**	1.7959	**	0.5473	**	0.3645	**	0.3987	**	0.3601	**	0.2063	**
μ_1	0.0000		0.0000		0.0000		0.0000		0.0000		0.0000		0.0000	
μ_1*	0.1418	**	0.0000		0.5289	**	0.6369	**	0.9995	**	1.1540	**	1.0050	**
h	0.0008	**	0.0005	**	0.0004	**	0.0003	**	0.0001	**	0.0001	**	0.0001	**
lk	129,938		141,681		146,514		149,365		159,963		164,213		169,448	
Root mean squared errors (RMSE) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	0.0016	0.0020	0.0012	0.0010	0.0011	0.0008	0.0011	0.0008	0.0011	0.0009	0.0011	0.0008	0.0012	0.0008
10 years	0.0016	0.0014	0.0015	0.0013	0.0014	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013
20 years	0.0015	0.0019	0.0014	0.0013	0.0013	0.0012	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012	0.0012
Avg RMSE	0.0014	0.0016	0.0014	0.0013	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012
Predictive variance (PV) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	-1.42	-5.12	-0.38	-0.35	-0.07	-0.04	-0.03	-0.05	-0.13	-0.10	-0.09	0.03	-0.19	0.01
10 years	-0.66	-0.21	-0.40	-0.01	-0.18	0.00	-0.12	0.00	-0.01	-0.02	-0.01	-0.01	0.00	-0.01
20 years	-0.45	-1.46	-0.32	-0.17	-0.17	-0.04	-0.09	0.00	0.02	0.00	-0.02	0.00	-0.02	-0.03
Global PV	-0.30	-0.73	-0.19	-0.16	-0.13	-0.08	-0.08	-0.03	-0.02	-0.02	-0.02	-0.01	-0.02	-0.02
Predictive variance (PV) for one week ahead EKF predicted yields during the "great" US recession (last quarter of 2007 to second quarter of 2009).														
Maturity		Out of sample		Out of sample		Out of sample		Out of sample		Out of sample		Out of sample		Out of sample
1 year		-0.66		0.14		0.10		0.10		0.09		0.09		0.05
10 years		-0.30		-0.04		-0.05		-0.06		-0.03		-0.02		-0.01
20 years		-2.98		-0.23		0.00		-0.01		-0.02		-0.02		-0.02
Global PV		-0.65		-0.11		-0.04		-0.01		0.02		0.02		0.01

Model	**A_3(3)**		**A_4(4)**		**A_5(5)**		**A_6(6)**		**A_{10}(10)**		**A_{15}(15)**		**A_{20}(20)**	
σ_1	0.0679	**	0.0373	**	0.0355	**	0.0455	**	0.0469	**	0.0536	**	0.0531	**
κ_1	0.1024	**	0.2006	**	0.2503	**	0.3121	**	0.4999	**	0.7068	**	1.0581	**
κ_1*	0.5941	**	0.2834	**	0.4573	**	0.9125	**	1.7927	**	3.4817	**	3.7624	**
μ_1	0.0000		0.0040	**	0.0047	**	0.0072	**	0.0054	**	0.0000	**	0.0233	**
μ_1*	0.0603	**	0.1101	**	0.1150	**	0.2145	**	0.1976	**	0.2337	**	0.2452	**
h	0.0011	**	0.0010	**	0.0008	**	0.0007	**	0.0005	**	0.0004	**	0.0003	**
lk	125,811		129,087		133,240		135,660		141,888		144,350		146,718	
Root mean squared errors (RMSE) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	0.0016	0.0019	0.0014	0.0014	0.0015	0.0013	0.0014	0.0012	0.0011	0.0009	0.0012	0.0009	0.0011	0.0008
10 years	0.0016	0.0016	0.0016	0.0014	0.0015	0.0013	0.0014	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013
20 years	0.0016	0.0028	0.0016	0.0023	0.0014	0.0014	0.0014	0.0014	0.0013	0.0012	0.0012	0.0012	0.0012	0.0012
Avg RMSE	0.0014	0.0018	0.0014	0.0016	0.0014	0.0014	0.0014	0.0014	0.0013	0.0012	0.0013	0.0012	0.0013	0.0012
Predictive variance (PV) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	-1.24	-4.50	-0.85	-1.90	-0.89	-1.38	-0.65	-1.15	-0.17	-0.13	-0.31	-0.11	-0.01	-0.03
10 years	-0.68	-0.67	-0.53	-0.30	-0.42	-0.06	-0.33	-0.06	-0.10	-0.02	-0.06	-0.04	-0.04	-0.01
20 years	-0.83	-4.18	-0.65	-2.73	-0.38	-0.28	-0.40	-0.26	-0.08	-0.04	0.02	-0.01	0.01	-0.02
Global PV	-0.32	-1.38	-0.25	-0.82	-0.23	-0.33	-0.19	-0.29	-0.08	-0.08	-0.08	-0.08	-0.03	-0.03
Predictive variance (PV) for one week ahead EKF predicted yields during the "great" US recession (last quarter of 2007 to second quarter of 2009).														
Maturity		Out sample		Out sample		Out sample		Out sample		Out sample		Out sample		Out sample
1 year		-0.33		0.11		0.17		0.16		0.16		0.17		0.08
10 years		-0.62		-0.22		-0.05		-0.08		-0.06		-0.06		-0.07
20 years		-7.00		-4.58		-0.23		-0.18		-0.03		0.01		0.03
Global PV		-0.85		-0.44		-0.08		-0.06		-0.01		0.02		0.00

LEGEND

The total sample is 1188 weeks for the US.

The in sample period is the first 594 weeks for the US.

The out of sample period is the last 594 weeks for the US.

The row lk reports for each model value of the maximised log of the (quasi) likelihood function from the Kalman Filter.

The parameter estimates and the likelihood lk in the top Panel are estimated using the whole US sample.

Significant parameters at the 1% level are denoted with "**".

"Avg h" is the average h_i for i=1,..,20. h_i is the estimate of the standard deviation of the Kalman filter observation error for the i-year yield.

For a given model "Avg RMSE" is the average RMSE computed across the twenty yield maturities.

Global PV is computed across all twenty yield maturities.

The column headings "In sample" and "Out of sample" respectively denote in sample and out of sample RMSE and PV for each model.

For the US out of sample RMSE and PV are computed using the models estimated in two rolling windows (weeks 1-594 and weeks 298-891).

The heading "In sample" refers to the in sample period. The heading "Out of sample" refers to the out of sample period.

Table 4: Results of Extended Kalman Filter (EKF) estimation for Euro yields.

Model	**$A_0(3)$**		**$A_0(4)$**		**$A_0(5)$**		**$A_0(6)$**		**$A_0(10)$**		**$A_0(15)$**		**$A_0(20)$**	
σ_1	0.0003	**	0.0003	**	0.0004	**	0.0005	**	0.0009	**	0.0014	**	0.0014	**
κ_1	0.1134	**	0.1464	**	0.1821	**	0.2401	**	0.4183	**	0.5962	**	0.7508	**
κ_1^*	0.0000		0.0594	**	0.0000		0.0000		0.0000		0.0000		0.0000	
μ_1	0.0000		0.0041	**	0		0.0000		0		0.0000		0.0000	
μ_1^*	0.0019		0.0000		0		0.0000		0		0.0229	**	0.0153	**
h	0.0005	**	0.0002	**	0.0001	**	0.0001	**	0.0000	**	0.0000		0.0000	**
lk	83,294		90,183		97,012		103,055		118,526		132,234		146,490	
Root mean squared errors (RMSE) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	0.0012	0.0015	0.0009	0.0006	0.0008	0.0005	0.0008	0.0004	0.0008	0.0004	0.0008	0.0004	0.0008	0.0004
10 years	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
20 years	0.0012	0.0012	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
Avg RMSE	0.0010	0.0010	0.0010	0.0009	0.0009	0.0008	0.0009	0.0008	0.0009	0.0008	0.0009	0.0008	0.0009	0.0008
Predictive variance (PV) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	-1.37	-10.22	-0.30	-0.78	-0.03	-0.14	0.02	-0.02	0.00	0.00	0.00	0.00	0.00	0.00
10 years	-0.13	-0.16	-0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20 years	-0.57	-0.47	-0.09	-0.03	0.00	0.00	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Global PV	-0.23	-0.44	-0.08	-0.10	-0.02	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Model	**$A_1(3)$**		**$A_1(4)$**		**$A_1(5)$**		**$A_1(6)$**		**$A_1(10)$**		**$A_1(15)$**		**$A_1(20)$**	
σ_1	0.0236	**	0.0151	**	0.0125	**	0.0068	**	0.0039	**	0.0043	**	0.0109	**
σ_2	1		1		1		1		1		1		1	
κ_1	0.0470	**	0.0469	**	0.0393	**	0.0048	**	0.0000		0.0036	**	0.0121	**
κ_2	0.3627	**	0.2839	**	0.3147	**	0.3853	**	0.5699	**	0.7274	**	0.9012	**
κ_1^*	$=\kappa_1$		$=\kappa_1$		$=\kappa_1$		$=\kappa_1$		$=\kappa_1$		$=\kappa_1$		$=\kappa_1$	
κ_2^*	0.0464	**	0.1742	**	1.6192	**	0.8492	**	0.0516	**	0.0018	**	0.8936	**
μ_1	0		0		0		0		0		0		0	
μ_2	0.0232	**	0.0194	**	0.0287	**	0.0612	**	0.2912	**	0.5031	**	0.1064	**
μ_1^*	0		0		0		0		0		0		0	
μ_2^*	0.0014		0.0191		0.0633	**	0.0581	**	0.0217	**	0.0099	**	0.0612	**
h	0.0003	**	0.0001	**	0.0001	**	0.0000	**	0.0000	**	0.0000		0.0000	**
lk	85,449		97,548		104,466		115,129		133,818		151,469		159,826	
Root mean squared errors (RMSE) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	0.0011	0.0013	0.0009	0.0005	0.0010	0.0005	0.0010	0.0006	0.0008	0.0004	0.0008	0.0004	0.0008	0.0005
10 years	0.0009	0.0009	0.0009	0.0009	0.0010	0.0010	0.0010	0.0010	0.0009	0.0009	0.0009	0.0009	0.0009	0.0009
20 years	0.0010	0.0013	0.0010	0.0011	0.0010	0.0011	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010	0.0010
Avg RMSE	0.0010	0.0010	0.0010	0.0009	0.0010	0.0009	0.0010	0.0010	0.0009	0.0008	0.0009	0.0008	0.0009	0.0009
Predictive variance (PV) for one week ahead EKF predicted yields.														
Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	-0.86	-7.78	-0.13	-0.24	-0.42	-0.41	-0.42	-0.62	0.00	0.00	0.00	0.00	-0.03	-0.11
10 years	-0.07	-0.13	-0.05	-0.12	-0.15	-0.36	-0.19	-0.36	0.00	0.00	0.00	0.00	-0.03	-0.07
20 years	-0.08	-0.64	-0.03	-0.18	-0.05	-0.15	-0.05	-0.11	0.00	-0.01	0.00	-0.01	-0.02	-0.03
Global PV	-0.11	-0.45	-0.06	-0.12	-0.15	-0.27	-0.20	-0.32	0.00	-0.01	0.00	0.00	-0.03	-0.06

LEGEND

The total sample is 688 weeks for the Euro.

The in sample period is the first 344 weeks for the Euro.

The out of sample period is the last 344 weeks for the Euro.

The row lk reports for each model value of the maximised log of the (quasi) likelihood function from the Kalman Filter.

The parameter estimates and the likelihood lk in the top Panel are estimated using the whole Euro sample.

Significant parameters at the 1% level are denoted with "**".

"Avg h" is the average h_i for i=1,..,20. h_i is the estimate of the standard deviation of the Kalman filter observation error for the i-year yield.

For a given model "Avg RMSE" is the average RMSE computed across the twenty yield maturities.

Global PV is computed across all twenty yield maturities.

The column headings "In sample" and "Out of sample" respectively denote in sample and out of sample RMSE and PV for each model.
For the Euro RMSE and PV are computed using the models estimated in the first half of the sample (weeks 1-344).

The parameters in the yellow cells were set to 0 because their estimates were insignificant and indistinguishable from zero.

The heading "In sample" refers to the in sample period. The heading "Out of sample" refers to the out of sample period.

Table 5: Results of Extended Kalman Filter (EKF) estimation of the "classic" models for US and Euro yields.

	US		US		US		US		Euro		Euro	
Model	**$A_0(3)c$**		**$A_1(3)c$**		**$A_1(3)c$**		**$Q(3)c$**		**$A_0(3)c$**		**$A_1(3)c$**	
σ_1	0.0581		0.0452	**	0.0024	**	0.2349	**	0.0099	**	0.0572	**
σ_2	0.2360		0.0298	**	0.2168	**	0.0325	**	0.0896	**	0.0140	**
σ_3	0.2807	**	0.0000		1		0.0344	**	0.0920	**	1	
κ_1	0.0080		0.0000		0.0000		0.0000		0.0012	**	8.9151	**
κ_2	0.0964		0.0000		0.0648	**	0.0283	**	0.1614	**	0.0000	
κ_3	0.0273		0.0947	**	0.0855	**	0.1709	**	0.0379	**	0.0657	**
μ_1	0.0034		0.0000	**	0.0002	**	0.2713	**	0.0020	**	0.1576	**
μ_2			0.0000		0.0961	**					0.0000	
μ_3			0.0000									
κ_{21}			0.0000									
κ_{31}			$=\kappa_3$									
κ_{32}			$=\kappa_3$		$=\kappa_3$							
κ_1*	0.2163	**	$=\kappa_1$		0.1184	**	0.0000		0.3228	**	0.7583	**
κ_2*	0.2810		$=\kappa_2$		0.2460	**	0.0347	**	0.1708	**	0.1288	**
κ_3*	0.1226		$=\kappa_3$		0.0000		0.3376	**	0.0852	**	0.0000	
μ_1*	0.0018		0.0000	**	0.0011	**	0.1920	**	0.0042	**	0.9451	**
μ_2*			0.0000		0.0290						0.2852	**
μ_3*			0.0106									
κ_{21}*			$=\kappa_{21}$									
κ_{31}*			$=\kappa_3$									
κ_{32}*			$=\kappa_3$		$=\kappa_3$*							
ρ_{12}	0.9804	**			-0.1655	**	-0.2115	**	0.8146	**	-0.5135	**
ρ_{13}	-0.9887	**			-0.0798	**	0.0003		-0.8564	**	0.9867	**
ρ_{23}	-0.9954	**			0.9962	**	-0.0006	**	-0.9900	**	-0.4957	**
h_1	0.0053	**	0.0106	**	0.0055	**	0.0029	**	0.0057	**	0.0070	**
h_2	0.0030	**	0.0091	**	0.0031	**	0.0011	**	0.0036	**	0.0044	**
h_3	0.0016	**	0.0075	**	0.0016	**	0.0000		0.0022	**	0.0027	**
h_4	0.0006	**	0.0059	**	0.0006	**	0.0007	**	0.0013	**	0.0017	**
h_5	0.0001		0.0046	**	0.0002	**	0.0011	**	0.0007	**	0.0010	**
h_6	0.0004	**	0.0035	**	0.0004	**	0.0012	**	0.0003	**	0.0006	**
h_7	0.0006	**	0.0026	**	0.0006	**	0.0012	**	0.0001	**	0.0003	**
h_8	0.0006		0.0017	**	0.0006	**	0.0009	**	0.0000		0.0001	**
h_9	0.0006	**	0.0009	**	0.0005	**	0.0006	**	0.0001	**	0.0000	**
h_{10}	0.0007	**	0.0005	**	0.0007	**	0.0006	**	0.0001	**	0.0000	**
h_{11}	0.0007	**	0.0006	**	0.0007	**	0.0006	**	0.0001	**	0.0000	**
h_{12}	0.0006		0.0008	**	0.0006	**	0.0005	**	0.0000	**	0.0000	**
h_{13}	0.0003		0.0009	**	0.0004	**	0.0003	**	0.0000		0.0000	**
h_{14}	0.0002		0.0009	**	0.0002	**	0.0003	**	0.0000	**	0.0000	**
h_{15}	0.0003		0.0009	**	0.0002	**	0.0004	**	0.0001	**	0.0000	**
h_{16}	0.0003	**	0.0007	**	0.0002	**	0.0003	**	0.0001	**	0.0000	**
h_{17}	0.0002	**	0.0004	**	0.0000	**	0.0002	**	0.0001	**	0.0000	**
h_{18}	0.0000		0.0000		0.0003	**	0.0000		0.0001	**	0.0000	**
h_{19}	0.0003	**	0.0005	**	0.0007	**	0.0003	**	0.0000	**	0.0001	**
h_{20}	0.0007	**	0.0010	**	0.0011	**	0.0006	**	0.0000		0.0001	**
Avg h	0.0009		0.0027		0.0009		0.0007		0.0007		0.0009	
$x_{1,1}$ or $z_{1,1}$	-0.0000		0.0000		0.0000		0.0000		-0.0000		1.2690	**
$x_{2,1}$ or $z_{2,1}$	-0.0000		0.0446		0.0058		0.0002		0.0303	**	2.2617	**
$x_{3,1}$ or $z_{3,1}$	0.0000		0.0716		0.0068		0.2776	**	0.0000		0.0112	**
lk	138,770		118,930		137,639		139,735		96,962		100,048	

Root mean squared errors (RMSE) for one week ahead EKF predicted yields.

Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	0.0030	0.0061	0.0018	0.0023	0.0030	0.0068	0.0021	0.0060	0.0033	0.0105	0.0035	0.0112
10 years	0.0015	0.0014	0.0020	0.0019	0.0014	0.0013	0.0018	0.0018	0.0010	0.0009	0.0009	0.0009
20 years	0.0013	0.0013	0.0019	0.0024	0.0014	0.0017	0.0015	0.0015	0.0011	0.0010	0.0010	0.0010
Avg RMSE	0.0016	0.0018	0.0016	0.0018	0.0014	0.0018	0.0015	0.0018	0.0012	0.0019	0.0012	0.0021

Predictive variance (PV) for one week ahead EKF predicted yields.

Maturity	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample	In sample	Out sample
1 year	-6.83	-53.29	-1.88	-6.52	-7.06	-65.94	-3.03	-51.56	-15.71	-566.32	-17.32	-647.98
10 years	-0.47	-0.18	-1.58	-1.37	-0.33	-0.15	-0.94	-1.00	-0.23	-0.05	0.01	-0.01
20 years	-0.07	-0.08	-1.56	-2.97	-0.32	-0.93	-0.46	-0.48	-0.26	-0.01	0.03	-0.08
Global PV	-0.63	-2.01	-0.69	-1.21	-0.42	-2.20	-0.38	-1.88	-1.15	-12.35	-1.10	-14.61

Predictive variance (PV) for one week ahead predicted yields during the "great" US recession (last quarter 2007 to second quarter 2009).

Maturity	Out sample	Out sample	Out sample	Out sample
1 year	-9.88	-1.08	-13.94	-4.59
10 years	-0.32	-1.84	-0.17	-2.07
20 years	-0.02	-3.01	-0.15	-1.52
Global PV	-1.39	-0.90	-1.82	-1.24

LEGEND

The total sample is 1188 weeks for US and 688 weeks for the Euro.

The in sample period is the first 594 weeks for the US and the first 344 weeks for the Euro.

The out of sample period is the last 594 weeks for the US and the last 344 weeks for the Euro.

The row lk reports for each model value of the maximised log of the (quasi) likelihood function from the Kalman Filter.

The parameter estimates and the likelihood lk in the top Panel are estimated using the whole US and Euro samples.

Significant parameters at the 1% level are denoted with "**".

"Avg h" is the average h_i for i=1,..,20. h_i is the estimate of the standard deviation of the Kalman filter observation error for the i-year yield.

For a given model "Avg RMSE" is the average RMSE computed across the twenty yield maturities.

Global PV is computed across all twenty yield maturities.

The column headings "In sample" and "Out of sample" respectively denote in sample and out of sample RMSE and PV for each model.

For the US out of sample RMSE and PV are computed using the models estimated in two rolling windows (weeks 1-594 and weeks 298-891)

For the Euro RMSE and PV are computed using the models estimated in the first half of the sample (weeks 1-344).

The heading "In sample" refers to the in sample period. The heading "Out of sample" refers to the out of sample period.

TABLE 6: Likelihood, information criterion and Vuong likelihood ratio tests for US and Euro curves and for all models.

	A: Values of maximised log-likelihood functions lk						B: SBIC (Schwartz Bayesian information criterion)					
	US	US	Euro	US	Euro	US	US	US	Euro	US	Euro	US
MFFP Models	**$A_n(n)$**	**Q(n)**	**$A_0(n)$**	**$A_0(n)$**	**$A_1(n)$**	**$A_1(n)$**	**$A_n(n)$**	**Q(n)**	**$A_0(n)$**	**$A_0(n)$**	**$A_1(n)$**	**$A_1(n)$**
n=1	92251	93278	48172	79717			-184459	-186513	-96302	-159392		
n=2	115427	123278	73118	117465	73803	117775	-230811	-246513	-146194	-234887	-147556	-235500
n=3	125811	129938	83294	129837	85449	130713	-251579	-259834	-166545	-259631	-170848	-261376
n=4	129087	141681	90183	140367	97548	140999	-258132	-283319	-180324	-280691	-195046	-281949
n=5	133240	146514	97012	146118	104466	145381	-266438	-292986	-193982	-292193	-208882	-290713
n=6	135660	149365	103055	149658	115129	150524	-271278	-298688	-206068	-299273	-230208	-300998
n=7	138820	151680	107521	151921	119143	155163	-277597	-303317	-215000	-303799	-238236	-310277
n=8	139271	156422	112058	154075	125809	157728	-278500	-312802	-224074	-308107	-251569	-315407
n=9	140451	158540	114764	156170	129473	159369	-280860	-317038	-229485	-312297	-258896	-318689
n=10	141888	159963	118526	157737	133818	160643	-283734	-319883	-237009	-315431	-267586	-321237
n=11	143070	161260	121164	159167	137899	161816	-286097	-322477	-242285	-318291	-275748	-323582
n=12	143508	162162	123903	160718	141721	163182	-286974	-324282	-247763	-321393	-283393	-326315
n=13	143791	162361	126636	162065	145721	164466	-287540	-324680	-253230	-324087	-291392	-328883
n=14	144082	163773	129219	163189	148316	165456	-288122	-327504	-258396	-326336	-296582	-330863
n=15	144350	164213	132234	164491	151469	166545	-288658	-328383	-264426	-328939	-302889	-333041
n=16	144607	166532	134715	165557	153594	167765	-289171	-333021	-269388	-331071	-307137	-335480
n=17	144863	167446	137894	166354	155778	168883	-289683	-334849	-275745	-332666	-311506	-337716
n=18	145176	168629	140438	167009	157411	169923	-290310	-337215	-280833	-333975	-314772	-339796
n=19	145768	169406	144056	167611	158845	170885	-291493	-338770	-288070	-335180	-317640	-341720
n=20	146718	169448	146490	168207	159826	171734	-293393	-338854	-292937	-336371	-319603	-343419
	A3(3)c	Q(3)c	A0(3)c	A0(3)c	A1(3)c	A1(3)c	A3(3)c	Q(3)c	A0(3)c	A0(3)c	A1(3)c	A1(3)c
Classic models	118930	139735	96962	138770	100048	137635	-237606	-279200	-193654	-277272	-199826	-275000

The last row in Panels A and B shows lk and SBIC for the tested "classic" models.
Panel A reports the values of the maximised log-likelihood function lk as the number of factors n increases.
Panel B reports the values of the Schwartz information criterion (SBIC) for all models as the number of factors n increases.
The Akaike information criterion gave almost the same values as SBIC and therefore is not reported.

	C: p value of Vuong tests; MFFP models with n factors vs n-1 factors						D: p value of Vuong tests; MFFP models with n factors vs classic models					
	US	US	Euro	US	Euro	US	US	US	Euro	US	Euro	US
Null Models	**$A_n(n)$**	**Q(n)**	**$A_0(n)$**	**$A_0(n)$**	**$A_1(n)$**	**$A_1(n)$**	**$A_n(n)$**	**Q(n)**	**$A_0(n)$**	**$A_0(n)$**	**$A_1(n)$**	**$A_1(n)$**
n=1	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	0.00	0.00	0.00	0.00	n.a.	n.a.
n=2	1.00	1.00	1.00	1.00	n.a.	n.a.	0.00	0.00	0.00	0.00	0.00	0.00
n=3	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00
n=4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.00	1.00
n=5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=11	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=12	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=13	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=15	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
n=20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Alternative models	**$A_{n-1}(n-1)$**	**Q(n-1)**	**$A_0(n-1)$**	**$A_0(n-1)$**	**$A_1(n-1)$**	**$A_1(n-1)$**	**$A_3(3)c$**	**Q(3)c**	**$A_0(3)c$**	**$A_0(3)c$**	**$A_1(3)c$**	**$A_1(3)c$**

E: p values of Vuong tests of pairs of different models, where each model in the pair has the same number of factors n, with n=5,10,15,20.

MFFP models with five stochastic factors, i.e. n=5.

Alternative models \ Null Models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$
$A_n(n)$	n.a.	1.0000	1.0000	1.0000	n.a.	n.a.
Q(n)	0.0000	n.a.	0.0029	0.0000	n.a.	n.a.
$A_0(n)$	0.0000	0.9971	n.a.	0.0001	n.a.	1.0000
$A_1(n)$	0.0000	1.0000	0.9999	n.a.	0.0000	n.a.

MFFP models with ten stochastic factors, i.e. n=10.

Alternative models \ Null Models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$
$A_n(n)$	n.a.	1.0000	1.0000	1.0000	n.a.	n.a.
Q(n)	0.0000	n.a.	0.0000	0.9417	n.a.	n.a.
$A_0(n)$	0.0000	1.0000	n.a.	1.0000	n.a.	1.0000
$A_1(n)$	0.0000	0.0583	0.0000	n.a.	0.0000	n.a.

MFFP models with fifteen stochastic factors, i.e. n=15.

Alternative models \ Null Models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$
$A_n(n)$	n.a.	1.0000	1.0000	1.0000	n.a.	n.a.
Q(n)	0.0000	n.a.	0.8247	1.0000	n.a.	n.a.
$A_0(n)$	0.0000	0.1753	n.a.	1.0000	n.a.	1.0000
$A_1(n)$	0.0000	0.0000	0.0000	n.a.	0.0000	n.a.

MFFP models with twenty stochastic factors, i.e. n=20.

Alternative models \ Null Models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$
$A_n(n)$	n.a.	1.0000	1.0000	1.0000	n.a.	n.a.
Q(n)	0.0000	n.a.	0.0000	1.0000	n.a.	n.a.
$A_0(n)$	0.0000	0.0000	n.a.	1.0000	n.a.	1.0000
$A_1(n)$	0.0000	0.0000	0.0000	n.a.	0.0000	n.a.

LEGEND
All models in this Table are tested using the whole US and euro samples respectively.
n.a. in the above panels stands for not applicable.
Panels C, D, E in this Table report the p values of Vuong likelihood ratio statistics.
p value close to 1 supports the null model, while p value close to 0 supports the alternative model.
The Vuong likelihood ratio tests in each cell of Panel C assume that the null model is the model indicated in the column heading with n=1,..,20 factors and the alternative model is the same model, but with n-1 factors. n is the number of factors in a model.
The Vuong likelihood ratio tests in each cell of Panel D assume that the null model is the model indicated in the column heading with n=1,..,20 and the alternative model is the corresponding "classic" model indicated in bold at the bottom of each column of Panel D.
The Vuong likelihood ratio tests in each cell of Panel E assume that the null model is the model indicated in the column and the alternative model is the model indicated for the corresponding the row.
This Table presents the p values of 258 Vuong tests of pairs of models (Panel E shows two cells for each test).
This Bonferroni correction is used to ensure that the whole family of Vuong tests in Table 6 is significant at a level not greater than 1%.
The Bonferroni correction that reduces the level of significance for the single Vuong tests to 0.01/258=0.000039.
The grey cells in the Table highlight when the Vuong test is statistically significant at the 0.0039% level.
All the p values in Panels C and D are significant at the 0.0039% level.
The p values in Panel E have many decimals to show that the Vuong tests in the grey cells are still significant even after the said Bonferroni correction.
The asymptotic distribution of the Vuong statistic is the standard normal distribution.
At the 0.0039% level of significance for a single Vuong test, the null model prevails if the p value is higher than 99,99805%, while the alternative model prevails if the p value is lower than 0,00195%.

Table 7: Giacomini-White (GW) tests statistics for comparing forecast densities of pairs of models out of sample.

A: GW statistic for tests of MFFP models with different numbers of factors

Null models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$	Alternative models
n=4	-7.7	-7.6	-11.7	-6.6	-34.3	-48.6	n=3
n=5	-5.1	-13.2	-9.3	-5.6	-46.4	-57.6	n=4
n=6	-8.4	-12.9	-11.1	-10.2	-73.0	-111.6	n=5
n=10	-12.6	-19.3	-28.6	-18.6	-137.2	-112.9	n=6
n=15	-12.7	-13.1	-28.1	-31.6	-221.2	-207.2	n=10
n=20	1.9	0.0	-51.5	-32.6	-496.9	-127.6	n=15

B: GW statistic for tests of MFFP models with n factors vs respective classic models

Alternative models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$
n=3	-1.7	1.9	4.0	-0.6	9.2	7.9
n=4	2.7	4.5	8.2	2.9	14.0	18.6
n=5	4.4	5.9	11.7	6.7	17.8	21.9
n=6	5.0	6.8	13.1	11.7	20.6	27.4
n=10	6.5	8.5	15.6	14.3	27.6	34.7
n=15	7.1	9.2	17.6	15.6	33.6	42.1
n=20	7.2	10.3	18.5	16.8	40.2	45.6
Null models	**$A_3(3)c$**	**Q(3)c**	**$A_0(3)c$**	**$A_1(3)c$**	**$A_0(3)c$**	**$A_1(3)c$**

C: p values of GW tests for comparing pairs of different models with the same number of factors n, with n=5,10,15,20.

Tests of MFFP models with five stochastic factors, i.e. n=5. Tests of MFFP models with ten stochastic factors, i.e. n=10.

Null models \ Alternative models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$
$A_n(n)$	n.a.	1.0000	1.0000	0.9657	n.a.	n.a.	n.a.	1.0000	1.0000	1.0000	n.a.	n.a.
Q(n)	0.0000	n.a.	0.0000	0.0000	n.a.	n.a.	0.0000	n.a.	0.0003	0.6943	n.a.	n.a.
$A_0(n)$	0.0000	1.0000	n.a.	0.0000	n.a.	1.0000	0.0000	0.9997	n.a.	1.0000	n.a.	1.0000
$A_1(n)$	0.0343	1.0000	1.0000	n.a.	0.0000	n.a.	0.0000	0.3057	0.0000	n.a.	-	n.a.

Tests of MFFP models with fifteen stochastic factors, i.e. n=15. Tests of MFFP models with twenty stochastic factors, i.e. n=20.

Null models \ Alternative models	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$	US $A_n(n)$	US Q(n)	US $A_0(n)$	US $A_1(n)$	Euro $A_0(n)$	Euro $A_1(n)$
$A_n(n)$	n.a.	1.0000	1.0000	1.0000	n.a.	n.a.	n.a.	1.0000	1.0000	1.0000	n.a.	n.a.
Q(n)	0.0000	n.a.	0.0001	0.5315	n.a.	n.a.	0.0000	n.a.	0.8413	0.9952	n.a.	n.a.
$A_0(n)$	0.0000	0.9999	n.a.	1.0000	n.a.	1.0000	0.0000	0.1587	n.a.	1.0000	n.a.	1.0000
$A_1(n)$	0.0000	0.4685	-	n.a.	-	n.a.	0.0000	0.0048	0.0000	n.a.	-	n.a.

D: GW statistic for comparing pairs of different models (null and alternative) with the same number of factors n.

The alternative model is Q(n)	Null models	US $A_1(n)$	US $A_0(n)$	US $A_n(n)$
	n=3	-2.2	-4.4	6.6
	n=4	9.1	0.8	5.6
	n=5	9.0	4.2	10.6
	n=6	-0.2	9.5	11.9
	n=10	-0.5	3.4	22.1
	n=15	-0.1	3.8	24.7
	n=20	-2.6	-1.0	17.5

The alternative model is $A_0(n)$	Null models	US Q(n)	US $A_1(n)$	US $A_n(n)$	Euro $A_1(n)$
	n=3	4.4	5.4	6.1	11.1
	n=4	-0.8	9.3	5.6	-12.2
	n=5	-4.2	8.0	10.7	-22.8
	n=6	-9.5	-7.8	11.0	-37.3
	n=10	-3.4	-17.0	15.6	-142.1
	n=15	-3.8	-16.0	16.6	-170.3
	n=20	1.0	-17.9	16.2	-208.8

The alternative model is $A_n(n)$	Null models	US $A_1(n)$	US $A_0(n)$	US Q(n)
	n=3	-5.1	-6.1	-6.6
	n=4	-3.8	-5.6	-5.6
	n=5	-1.8	-10.7	-10.6
	n=6	-12.1	-11.0	-11.9
	n=10	-17.2	-15.6	-22.1
	n=15	-16.9	-16.6	-24.7
	n=20	-16.7	-16.2	-17.5

The alternative model is $A_1(n)$	Null models	US Q(n)	US $A_0(n)$	US $A_n(n)$	Euro $A_0(n)$
	n=3	2.2	-5.4	5.1	-11.1
	n=4	-9.1	-9.3	3.8	12.2
	n=5	-9.0	-8.0	1.8	22.8
	n=6	0.2	7.8	12.1	37.3
	n=10	0.5	17.0	17.2	142.1
	n=15	0.1	16.0	16.9	170.3
	n=20	2.6	17.9	16.7	208.8

LEGEND

n is the number of factors in a model.

n.a. in the above panels stands for not applicable.

The asymptotic distribution of the GW statistic is the standard normal distribution. The GW test is two tailed.

A GW test compares the yields forecast density of two models. This Table reports 127 GW tests (Panel D reports two cells for each test).

In order for the family-wise level of significance of all the GW in this Table is to be not more than 1%,

the level of significance for the single GW tests in this Table is not more than 0.01/127=0.000079, because of the Bonferroni correction.

The grey cells highlight GW tests whose statistic is significant at the 0.000079 level.

Large negative (positive) values of the GW statistic in this Table support the null (alternative) model.

At the 1% level of significance for the family of all GW tests in Table 7,

for a single GW test in Table 7 the alternative model prevails over the null if the GW test p value is higher than 99,99605%,

while the null model prevails if the GW test p value is lower than 0,00395%.

The GW tests in each cell of Panel A test the MFFP models indicated in the corresponding column heading: in each row

the alternative model has n=3,4,5,6,10,15 factors respectively and the null model has n=4,5,6,10,15,20 factors respectively.

The GW tests in each cell of Panel B assume that the alternative model is the model indicated in the column heading with factors n=3,4,5,6,10,15,20

as indicated in the corresponding row and the null model is the "classic" model in bold at the bottom of the corresponding column.

The GW tests in each cell of Panel C assume that the alternative model is the model indicated in the column heading

and the null model is the model indicated for the corresponding row. Panel C shows some of the p values associated with the GW statistics in Panel D.

The GW tests in each cell of Panel D assume that both the indicated alternative model and null model have the same number of factors n.

Table 8: Volatilities of and correlation between changes in yields of different maturities for the US and the Euro .

Correlations between weekly changes in one year yields and weekly changes in yields of all other maturities for the US.

Yield maturity in years	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Observed yields	0.89	0.80	0.74	0.70	0.66	0.62	0.58	0.56	0.54	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.44	0.43	0.42
Q(5)	0.94	0.85	0.78	0.72	0.68	0.64	0.61	0.58	0.56	0.54	0.53	0.51	0.50	0.49	0.48	0.47	0.46	0.45	0.44
Q(10)	0.90	0.82	0.76	0.72	0.68	0.64	0.61	0.59	0.57	0.55	0.53	0.52	0.51	0.50	0.49	0.47	0.46	0.45	0.44
Q(20)	0.90	0.81	0.76	0.72	0.67	0.64	0.61	0.58	0.56	0.55	0.53	0.52	0.51	0.50	0.48	0.47	0.46	0.44	0.43
A_0(5)	0.96	0.89	0.82	0.77	0.72	0.68	0.65	0.62	0.59	0.57	0.55	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.45
A_0(10)	0.91	0.82	0.75	0.70	0.66	0.62	0.60	0.57	0.55	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.45	0.44	0.42
A_0(20)	0.89	0.80	0.75	0.70	0.66	0.62	0.59	0.56	0.54	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.44	0.43	0.42
A_1(5)	0.96	0.89	0.83	0.77	0.72	0.68	0.65	0.62	0.59	0.57	0.55	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.44
A_1(10)	0.90	0.81	0.75	0.70	0.66	0.62	0.59	0.57	0.55	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.45	0.44	0.42
A_1(20)	0.89	0.80	0.74	0.70	0.66	0.62	0.58	0.56	0.54	0.53	0.52	0.50	0.49	0.48	0.47	0.46	0.44	0.43	0.42
A_5(5)	0.99	0.96	0.93	0.89	0.86	0.83	0.80	0.78	0.75	0.73	0.72	0.70	0.68	0.67	0.66	0.65	0.64	0.63	0.62
A_{10}(10)	0.98	0.93	0.88	0.84	0.80	0.76	0.73	0.70	0.68	0.66	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.55
A_{20}(20)	0.96	0.90	0.84	0.80	0.75	0.72	0.69	0.66	0.64	0.62	0.60	0.59	0.57	0.56	0.55	0.54	0.53	0.53	0.52
A_0(3)	1.00	0.99	0.98	0.97	0.95	0.94	0.93	0.92	0.90	0.89	0.89	0.88	0.87	0.87	0.86	0.86	0.85	0.85	0.84
A_3(3)	1.00	0.99	0.98	0.97	0.96	0.94	0.92	0.91	0.88	0.86	0.84	0.82	0.80	0.78	0.76	0.74	0.72	0.71	0.69
A_1(3)	1.00	0.98	0.96	0.94	0.91	0.88	0.85	0.82	0.79	0.76	0.73	0.70	0.68	0.66	0.63	0.61	0.59	0.57	0.55
Q3	0.99	0.96	0.92	0.88	0.84	0.80	0.76	0.73	0.69	0.67	0.64	0.62	0.60	0.58	0.57	0.55	0.54	0.53	0.52

Volatilities (i.e standard deviations) of weekly changes in yields of all maturities for the US. In this Panel yields are here measured in percentage terms.

Yield maturity in years	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Observed yields	0.08%	0.10%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
Q(5)	0.09%	0.10%	0.10%	0.11%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.11%	0.11%
Q(10)	0.08%	0.10%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
Q(20)	0.08%	0.09%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_0(5)	0.09%	0.10%	0.10%	0.11%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.11%
A_0(10)	0.08%	0.10%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_0(20)	0.08%	0.10%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_1(5)	0.09%	0.10%	0.10%	0.11%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_1(10)	0.08%	0.10%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_1(20)	0.08%	0.10%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_5(5)	0.08%	0.08%	0.08%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.09%	0.09%
A_{10}(10)	0.08%	0.09%	0.09%	0.10%	0.10%	0.11%	0.11%	0.11%	0.12%	0.12%	0.12%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.10%	0.10%	0.10%
A_{20}(20)	0.08%	0.09%	0.10%	0.11%	0.11%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.11%	0.11%	0.11%	0.11%	0.10%	0.10%
A_0(3)	0.15%	0.14%	0.13%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_3(3)	0.14%	0.13%	0.13%	0.13%	0.13%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
A_1(3)	0.14%	0.13%	0.13%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%
Q3	0.10%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%	0.12%

Correlations between weekly changes in one year yields and weekly changes in yields of all other maturities for the Euro.

Yield maturity in years	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Observed yields		0.91	0.84	0.77	0.70	0.65	0.59	0.55	0.52	0.49	0.46	0.44	0.43	0.41	0.40	0.39	0.38	0.37	0.36	0.36
A0(5)		0.96	0.89	0.81	0.74	0.68	0.63	0.58	0.54	0.51	0.48	0.45	0.43	0.41	0.40	0.38	0.37	0.37	0.36	0.36
A0(10)		0.92	0.84	0.77	0.70	0.65	0.59	0.55	0.52	0.49	0.47	0.45	0.43	0.42	0.40	0.39	0.38	0.37	0.37	0.36
A0(20)		0.91	0.84	0.77	0.71	0.65	0.60	0.55	0.52	0.49	0.46	0.44	0.43	0.41	0.40	0.39	0.38	0.37	0.36	0.36
A1(5)		0.94	0.87	0.80	0.74	0.68	0.62	0.58	0.54	0.51	0.49	0.47	0.45	0.44	0.43	0.42	0.41	0.40	0.39	0.38
A1(10)		0.91	0.84	0.77	0.71	0.65	0.60	0.55	0.52	0.49	0.46	0.44	0.43	0.41	0.40	0.39	0.38	0.37	0.37	0.36
A1(20)		0.92	0.85	0.78	0.71	0.65	0.60	0.56	0.53	0.50	0.48	0.46	0.44	0.43	0.42	0.41	0.40	0.39	0.38	0.37
A0(3)		0.99	0.95	0.89	0.81	0.71	0.62	0.53	0.45	0.38	0.33	0.28	0.25	0.22	0.19	0.18	0.16	0.15	0.15	0.14
A1(3)		0.99	0.97	0.93	0.86	0.79	0.70	0.62	0.54	0.47	0.41	0.36	0.32	0.28	0.26	0.23	0.22	0.20	0.20	0.19

Volatilities (i.e standard deviations) of weekly changes in yields of all maturities for the Euro. In this Panel yields are here measured in percentage terms.

Yield maturity in years	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Observed yields	0.07%	0.08%	0.08%	0.08%	0.08%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%
A0(5)	0.07%	0.08%	0.08%	0.08%	0.08%	0.08%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%
A0(10)	0.07%	0.08%	0.08%	0.09%	0.08%	0.08%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%
A0(20)	0.07%	0.08%	0.08%	0.08%	0.08%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%
A1(5)	0.07%	0.08%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%
A1(10)	0.07%	0.08%	0.08%	0.08%	0.08%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%
A1(20)	0.07%	0.08%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%
A0(3)	0.14%	0.12%	0.11%	0.10%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%	0.11%	0.11%	0.11%	0.11%	0.11%	0.11%
A1(3)	0.15%	0.13%	0.11%	0.10%	0.10%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.09%	0.10%	0.10%	0.10%