

Evaluation of optimal sensor placement algorithms for the Structural Health Monitoring of architectural heritage. Application to the Monastery of San Jerónimo de Buenavista (Seville, Spain).

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Abstract

In recent years, Structural Health Monitoring (SHM) based on Operational Modal Analysis (OMA) and damage detection tools has become a popular non-destructive solution to assess the real-time integrity of any kind of structure. This technique is especially well-suited for the condition-based conservation of historical structures, where minimal invasiveness must be ensured owing to their high cultural and architectural value. Optimal Sensor Placement (OSP) techniques represent a valuable tool for efficiently designing the sensor layout in a SHM system in order to achieve an effective modal identification with a reduced number of sensors and, consequently, an improved cost efficiency. In this light, this paper proposes a design methodology of sensor networks based on OSP techniques suitable for historical structures. To do so, a preliminary extensive OMA campaign is conducted in order to construct a reliable finite element (FE) model by fitting the identified modal properties. Afterwards, an optimal sensor arrangement with a limited number of sensors is obtained by applying different model-based OSP techniques. In order to improve the robustness of the solution, material uncertainties are included in the model and the optimal sensor placement is conducted within a statistical framework. This methodology is presented and evaluated with a case study of a Spanish secular building: the Monastery of San Jerónimo de Buenavista in Seville (Spain). In particular, this paper presents the results of the preliminary ambient vibration test and the modal identification of the monastery, the updating process of the FE model, as well as a critical review of the different OSP techniques within a framework of material parameter uncertainty. The presented analysis demonstrate that OSP techniques based on the rank optimization of the kinetic energy matrix of the structure yield robust sensor layout.

Keywords:

Historical building, Uncertainty analysis, Operational modal analysis, Non-destructive test, Ambient vibration test

1. Introduction

Architectural heritage in Europe consists mainly of masonry structures, such as historical bridges, castles, palaces and religious buildings. These structures are part of the national history and hold an important cultural value, as confirmed by the creation of the World Heritage List published by UNESCO. In the Spanish context, many of these still standing heritage buildings were pillaged and burned over the years, whereby their structural integrity is not always assured. The conservation and safeguarding of historical structures against material degradation and natural/human hazards represent a great concern for communities and institutions such as the International Council of Monuments and Sites (ICOMOS), which in 2003 drafted the principles for the analysis, conservation and structural restoration of architectural heritage [1].

The assessment of the health condition of historical buildings cannot usually be carried out by means of destructive tests due to the undeniable value of the historical structures. In this context, non-destructive and non-invasive methods become a fundamental technique for this type of structures. Non Destructive Tests (NDTs) such as thermography, endoscopic tests and sonic tests can be used as complementary tools to accurately determine the characteristics of masonry elements in order to validate the structural assessment [2, 3]. However, foremost among the different NDTs are OMA and model updating. These techniques allow the assessment of the structural condition through the identification and analysis of the modal properties. System identification is a research topic with decades of history, particularly in aerospace and automotive industries, where Experimental Modal Analysis (EMA) is widely used for the dynamic characterization of structural elements from input-output vibration experimental data. The implementation of this philosophy into large structures, where ambient vibration tests are easier to carry out than forced vibration tests, gave rise to output-only vibration-based system identification methods, also referred to as OMA techniques. The main objective of OMA is to identify the modal parameters (natural frequencies, vibration modes and damping ratios) through the data processing of ambient vibration records without disturbing the normal operation of the structure under study. Some basic references regarding different OMA methods and applications are [4–6]. From early approaches to now, the measurement equipment has evolved and many different algorithms have been proposed in the literature including Frequency Domain Decomposition (FDD) and Enhanced Frequency Domain Decomposition (EFDD), Fast Bayesian FFT method, Poly-Reference Least Squares Complex Frequency method (p-LSCF), Covariance-Driven Stochastic System Identification (COV-SSI), Data-Driven Stochastic System Identification (DATA-SSI), Blind Source Separation, etc. Nowadays, reliable and even automated methods are available [7], making OMA a well-established and mature technique for the structural assessment of heritage structures. The consideration of this methodology for the characterization of masonry structures is relatively recent. The PhD thesis of Ramos at the University of Minho [8] can be cited as one of the first approaches. OMA has been successfully used in [9] to identify the modal properties of a building of the Greek heritage and also in the dynamic identification of the bell tower of Trani’s Cathedral [10]. Modal updating techniques seek to minimize the mismatch between the experimentally identified modal features and the estimates of a numerical model, usually based on the Finite Element Method (FEM), through the fitting of certain modal parameters [11–14].

In the last decade, long-term vibration-based SHM is becoming increasingly popular among

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43 researchers. This non-invasive technique is mainly based on OMA techniques and makes it possible
44 to identify structural damage over time. In reference [15], SHM technology and automated OMA
45 techniques are specially applied to structures in seismically prone areas to evaluate the dynamic
46 behaviour in operational conditions and the earthquake effects. Currently, there are few SHM
47 systems installed in historical masonry structures that are faithful to the steps of the paradigm
48 defined by Farrar and Worden [16]. Initially, most monitoring systems were implemented in bell
49 towers, since in this type of structures the dynamic properties can be easily identified with moderate
50 execution costs [17–19]. In recent years, these techniques have been extended to other typologies of
51 greater geometric complexity which involve structural models with larger computational demands
52 [20, 21]. In general, SHM systems aim to control the structural behaviour of these buildings to
53 identify damage and assist decision-making for condition-based conservation management [22–24].

54 To facilitate the implementation of SHM systems in historical structures, OSP methods con-
55 stitute a useful tool. The main goal of this technique is to efficiently design the sensor layout
56 to achieve a correct identification of the vibrational properties with a reduced number of mea-
57 surement points. The reduction of the number of sensors is a very important issue in historical
58 structures because it implies less invasive monitoring, and minimal impact on the cultural and
59 artistic value of the building. Furthermore, it reduces the implementation costs and the posterior
60 data processing. The efficient design of the sensor layout is particularly relevant in geometrically
61 complex structures for which mode shapes are also complex and involve several global and local
62 modes within the frequency range of interest [25]. In these cases, a rational approach for the design
63 of SHM systems can certainly take advantage of OSP techniques in order to achieve an effective
64 modal identification. Optimal sensor placement methods have been widely employed in civil engi-
65 neering structures such as bridges, whereas the number of applications in historical structures is
66 scarce. Only a few experiences can be found in the literature [26–28]. In addition, most works on
67 the application of OSP techniques are based upon simple benchmark case studies and/or consider
68 deterministic definitions of the material properties of the monitored structure.

69 In this work, four different FEM-based OSP methodologies are considered, including two ap-
70 proaches based on the effective independence of the target mode shapes (EFI and EFIwm), and
71 two others based on the concept of energy matrix rank optimization (KEMRO and SEMRO).
72 The main objective is to evaluate the performance of the different OSP methods, especially when
73 taking into account the particular aspects involved in the numerical modelling and monitoring of
74 historical structures. The analysis of the OSP methods is performed by considering field data and
75 including material parameter uncertainties in the modelling. This research is carried out through
76 a case study of the Monastery of San Jerónimo de Buenavista (Fig. 1) in Seville, Spain. First,
77 an extensive ambient vibration test was conducted for a preliminary identification of the modal
78 features of the structure. Afterwards, a detailed three-dimensional finite element model of the
79 monastery is calibrated to fit the previously identified modal properties and, finally, the updated
80 model is used to assist different OSP techniques. The effect of material parameter uncertainties is
81 evaluated through Monte Carlo Simulations.

82 The main novelty of this work is the evaluation of the potentials of different OSP techniques for
83 the design of optimized sensor layout for the condition-based maintenance of historical buildings.
84 In the first place, the use of field data from a preliminary ambient vibration test with many
85 sensors offers a rigorous framework for the comparison of different OSP techniques and represents
86 an advanced technique for devising cost-efficient long-term monitoring systems. Secondly, the
87 incorporation of material parameter uncertainties in the three-dimensional modelling of masonry

88 structures, and the comparison of different OSP techniques through Monte Carlo Simulations also
89 represents a novel contribution in the field. The paper is organized as follows. Section 2 presents
90 the case study of the Monastery of San Jerónimo de Buenavista, including a detailed description
91 of its historical background and current condition. Section 3 shows the experimental structural
92 assessment works (ambient vibration tests, OMA) and the model updating process. The optimal
93 number of sensors for a long-term SHM system and the analysis of the four different OSP methods
94 are addressed in Section 4, and the analysis of their robustness considering material uncertainties
95 is presented in Section 5. Finally, the most relevant conclusions of this study are highlighted in
96 Section 6.



Fig. 1: View of the current state of the Monastery of San Jerónimo de Buenavista.

97 **2. Monastery of San Jerónimo de Buenavista**

98 This study focuses on the west gallery of the main cloister of the Monastery of San Jerónimo
99 de Buenavista, a historical building located in the city of Seville, Spain. The beginning of its
100 construction dates back to the early fifteenth century. Originally, the Monastery of San Jerónimo
101 consisted of an aggregate of five main buildings: the church, the eastern cloister, the main cloister,
102 the tower and the printing press (Fig. 2a). However, only part of the main cloister and the tower
103 of the church are still preserved today (Fig. 2b). The eastern cloister, the printing press and a
104 large part of the church disappeared as a result of the numerous vicissitudes that the monastery
105 has suffered over the course of centuries [29].

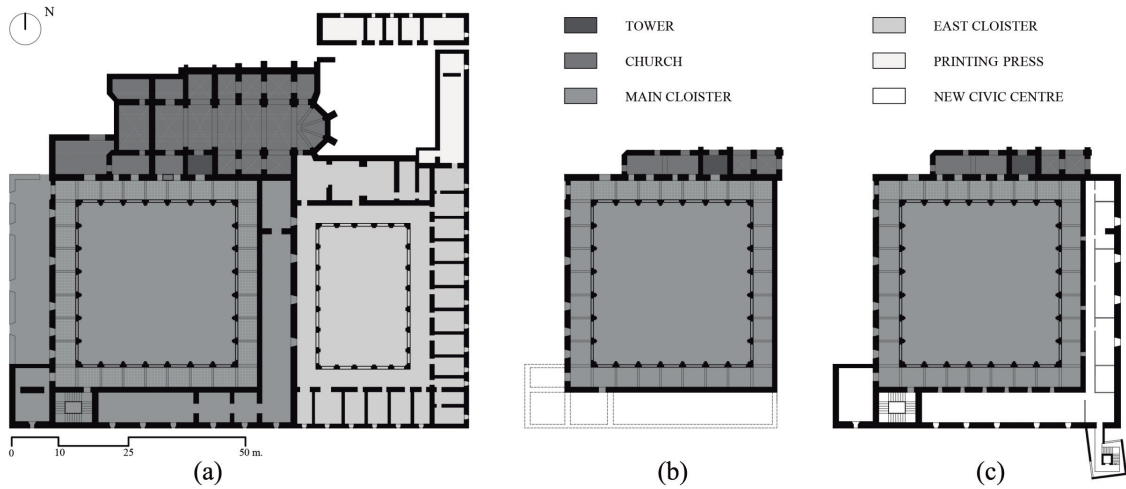


Fig. 2: Historical development of the Monastery of San Jerónimo: (a) Original configuration, 1650; (b) historical remains, 2000; and (c) current architectural configuration of the complex, 2018.

106 The remains of the main cloister constitute a Renaissance-style construction consisting of four
 107 galleries, which are made up of seven pillars on each side, attached columns and semicircular arches
 108 on the ground floor and bells on the first floor. These galleries are delimited by brick masonry
 109 walls and covered by ribbed stone vaults. Currently, in the west gallery, the entire ground floor is
 110 conserved while only the vertical structure and the arches are preserved on the upper floor (Fig.
 111 3).



Fig. 3: Views of the west gallery of the Monastery of San Jerónimo.

112 In 1964, the monastery was declared a heritage site due to the architectural importance of the
 113 whole building. Since then, it has been subjected to several restoration operations [29]. The last
 114 one was the execution of a rehabilitation and adaptation project for a civic centre in 2013 (Fig.
 115 2c). The project basically consisted of the reconstruction of the southern and eastern galleries
 116 annexed to the main cloister [30]. From a structural point of view, a mixed system of load-bearing
 117 walls and pillars supporting large-edged beams was used to construct the new building (Fig. 4).



Fig. 4: Views of the civic centre annexed to the Monastery of San Jerónimo.

118 As a result of the new protection policy for the Monastery of San Jerónimo, several damages
119 had already been detected before the construction of the new civic centre. These are related to
120 the following aspects: cracking of structural elements, cracking in auxiliary elements, excessive
121 deformations, failures in foundations, concentration of humidity, corrosion of metallic elements,
122 disintegration, cracking, factory swelling and attacks, etc. (Fig. 5). In January 2015, and due
123 to the fact that some of the damages listed above were aggravated during the works, the Town
124 Planning Department of Seville City Council commissioned a research project for the structural
125 analysis of the cloister.



Fig. 5: Detected damage in the main cloister of the Monastery of San Jerónimo.

126 The research project indicated above follows the approach ratified by the ICOMOS 14th General
127 Assembly in Victoria Falls, Zimbabwe, in 2003 [1]. The main objective was to conduct a diagnosis of
128 the building to justify different conservation proposals to ensure the integrity of all its elements. A
129 multidisciplinary work team composed of architects, engineers, archaeologists and historians carried
130 out the research project. An extensive historical study of the different interventions in the building
131 was finally presented, as well as a report about its structural health condition. To elaborate this
132 report, complex numerical analysis and several moderately destructive and non-destructive tests
133 were conducted, including ambient vibration tests, archaeological tastings, tomography and sonic
134 wave tests and georadar tests on the columns and ground. This project allowed us to obtain a vast
135 amount of experimental and numerical information from the west gallery of the cloister. The data
136 obtained from these previous works were fundamental for the development of the present study,
137 since it was possible to reduce the high level of uncertainty that usually surrounds the structural
138 characterisation of this type of buildings.

139 **3. FE modelling and analysis**

140 The reparation works in the historical building allowed us to perform several extensive exper-
 141 imental campaigns in the monastery between September and November 2013. Ambient vibration
 142 tests were performed in the west gallery with the aim of identifying the natural frequencies, mode
 143 shapes and damping ratios of the system. In order to ensure an efficient identification of the modal
 144 properties of the case study, a preliminary finite element model was previously developed with the
 145 aim of properly designing the test set-up. This numerical model was later updated based on the
 146 results obtained from the OMA of the ambient vibration tests. This model updating process is
 147 described in sections below.

148 *3.1. Preliminary FE model*

149 Based on a complex geometric survey, a sophisticated three-dimensional FE model of the whole
 150 monastery was developed in order to conduct a subsequent structural analysis that is not a subject
 151 of this work. Initially, the modal behaviour of the entire structure was analysed. **The results of**
 152 **the modal analysis of the monastery is depicted in Fig. 6.**

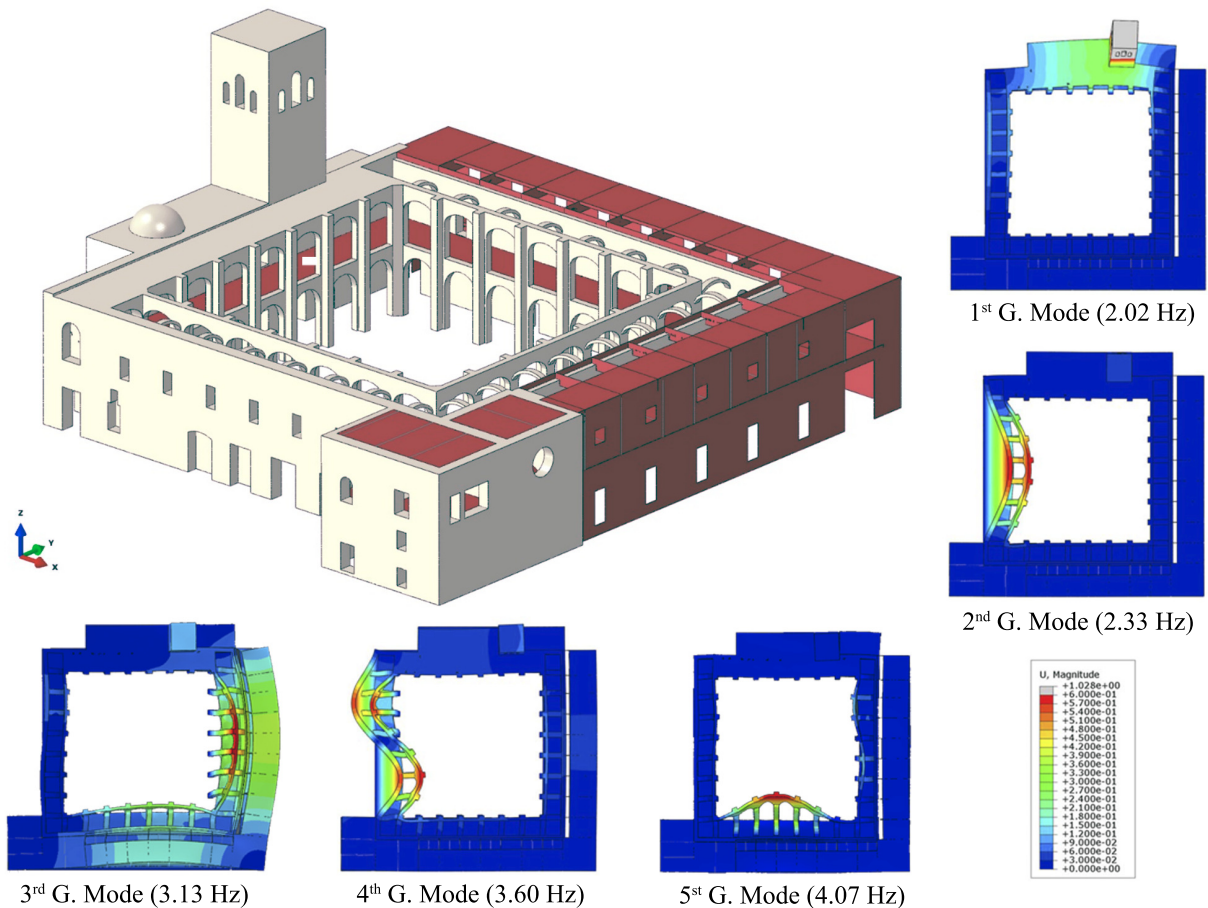


Fig. 6: **Three-dimensional FE model of the monastery and numerical identification of the first five global modes.**

153 In the light of this analysis, the dynamic independence of each gallery was observed, since each
 154 one presents its own uncoupled vibration modes. **For instance, the second and fourth global modes**

155 correspond to decoupled modes of the west gallery. Regarding that the goal of the study is the
 156 evaluation of the different OSP techniques, it was decided to perform the analysis considering only
 157 the west gallery due to the large size of the complete model. This last consideration makes it
 158 possible to reduce the computational cost of the study without losing generality in the evaluation
 159 of the different OSP techniques for their application in masonry structures. A numerical sub-model
 160 of this gallery (Fig. 7) was generated with ANSYS [31]. The influence of the other parts of the
 161 structure were considered through boundary conditions.

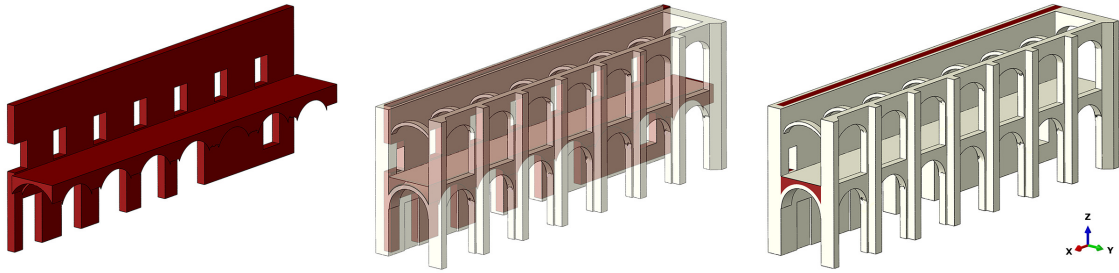


Fig. 7: Three-dimensional FE model of the west gallery.

162 With regard to the numerical definition of the preliminary FE model, the whole gallery was
 163 modelled with 4-node solid elements, with the exception of a 5-centimetre concrete compression
 164 layer located above the vaults which was modelled with 3-node triangular shell elements. In this
 165 way, the model has a total of 1462311 elements (1454047 solid and 8264 shell elements), 275475
 166 nodes and 826425 degrees of freedom. The boundary conditions are defined as constrained displacements
 167 and free rotations in the foundation, whilst the lateral boundary conditions are extracted
 168 from the general model. Table 1 summarizes the material properties used in the modelling. Note
 169 that the repair works allowed us to reliably test some elements, such as the material properties of
 170 the fillings. Dynamic tests were also carried out on the stone and brick masonries to identify the
 171 Young's moduli and densities of these elements. However, it was not possible to determine these
 172 parameters with certainty due to the great variability of the obtained results. Thus, these four
 173 parameters will be subsequently selected as the variables to be updated.

Table 1: Material properties used in the preliminary FE model.

Property	Unit	Value
Young's modulus of brick masonry (wall)	MPa	1300
Poisson's ratio of brick masonry	-	0.2
Density of brick masonry	kg/m ³	1700
Young's modulus of stone masonry (vaults and columns)	MPa	1800
Poisson's ratio of stone masonry	-	0.2
Density of stone masonry	kg/m ³	2000
Young's modulus of concrete slab	MPa	23000
Poisson's ratio of concrete slab	-	0.2
Density of concrete slab	kg/m ³	2500
Young's modulus of filling (wall)	MPa	500
Poisson's ratio of filling (wall)	-	0.2
Density of filling (wall)	kg/m ³	1500
Young's modulus of filling (vaults)	MPa	350
Poisson's ratio of filling (vaults)	-	0.2
Density of filling (vaults)	kg/m ³	900

174 As noted above, a modal analysis of this preliminary FE model was performed to identify which
 175 points experience larger modal displacements in as many vibration modes as possible. The first
 176 five mode shapes are shown in Fig. 8. It is observed that the complex geometric configuration of
 177 the arches on the first floor makes the dynamic features of the structure quite complex.

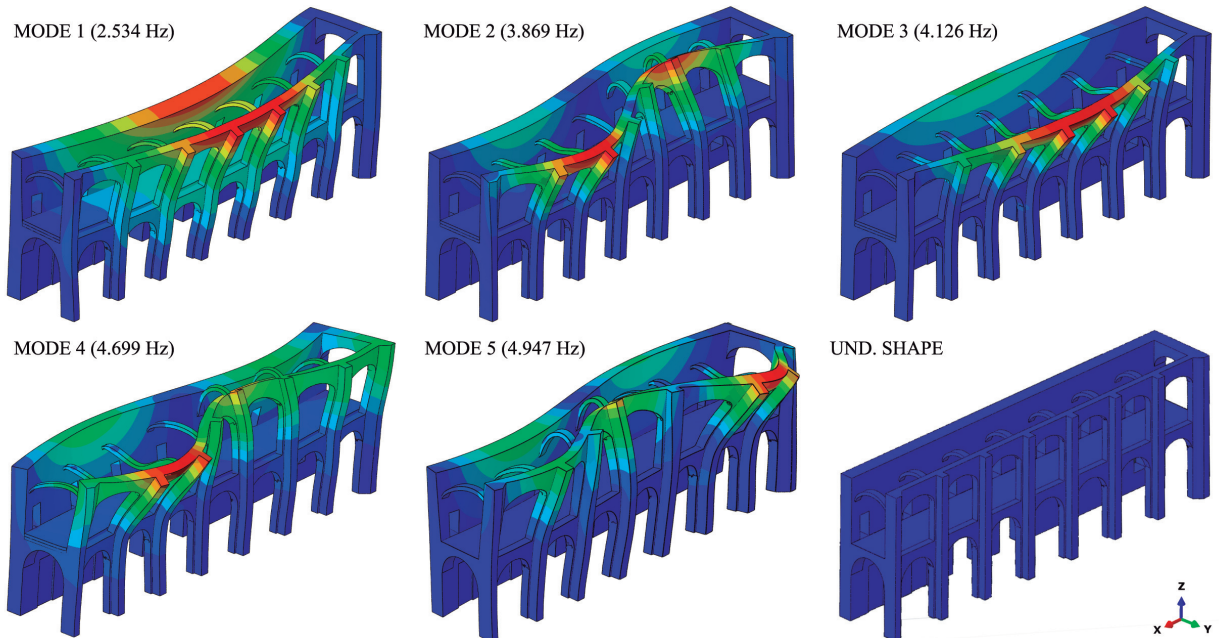


Fig. 8: First five numerical vibration modes and undeformed configuration of the FE model.

178 *3.2. Ambient vibration tests*

179 An extensive ambient vibration campaign with a large number of sensors was performed on
 180 the west gallery on November 22th, 2013. Following the results obtained from the preliminary FE
 181 model, Fig. 9 shows a representation of the considered measuring points. There are two different
 182 levels, one on the first floor of the gallery (+6.6 m.), and the other one on the start of the upper
 183 arches (+12.2 m). Accelerations were registered in X and Y directions with the aim of identifying
 184 the vibration modes in the lateral and longitudinal directions of the gallery. Eight accelerometers
 185 were used placing four of them as references (blue points in Fig. 9). The other four accelerometers
 186 were moved along the columns and the walls in the two different levels. In total, the resulting
 187 number of testing set-ups was fifteen.

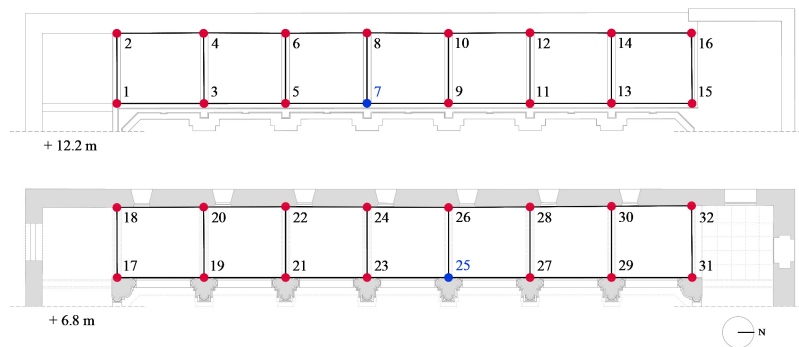


Fig. 9: Plan view of the accelerometer locations (reference accelerometers in blue).

188 The equipment used for these tests was composed of force balance accelerometers with a band-
 189 width ranging from 0.01 to 200 Hz, a dynamic range of 140 dB, a sensitivity of 10 V/g and a mass
 190 of 0.35 kg (model ES-U2). These accelerometers were connected via eight 40 m long cables to a
 191 twelve-channel data acquisition system with a 24-bit ADC, provided with anti-alias filters (model
 192 GRANITE). The equipment is manufactured by the company KINEMATRICS. The parameters
 193 set for the dynamic tests were a sampling frequency of 100 Hz and approximately 15 minutes time
 194 duration for each test. These assumptions assure that frequencies from 1 to 50 Hz would be prop-
 195 erly measured. Excitation was always associated with environmental ambient noise, and similar
 196 conditions of temperature and humidity were considered during the tests [32].

197 *3.3. OMA and model updating*

198 The data obtained in-situ were processed with the software ARTEMIS [33] using two different
 199 identification methods: Enhanced Frequency Domain Decomposition (EFDD) [34] and Stochastic
 200 Subspace Identification (SSI-UPC Merged) [35, 36] (Fig. 10).

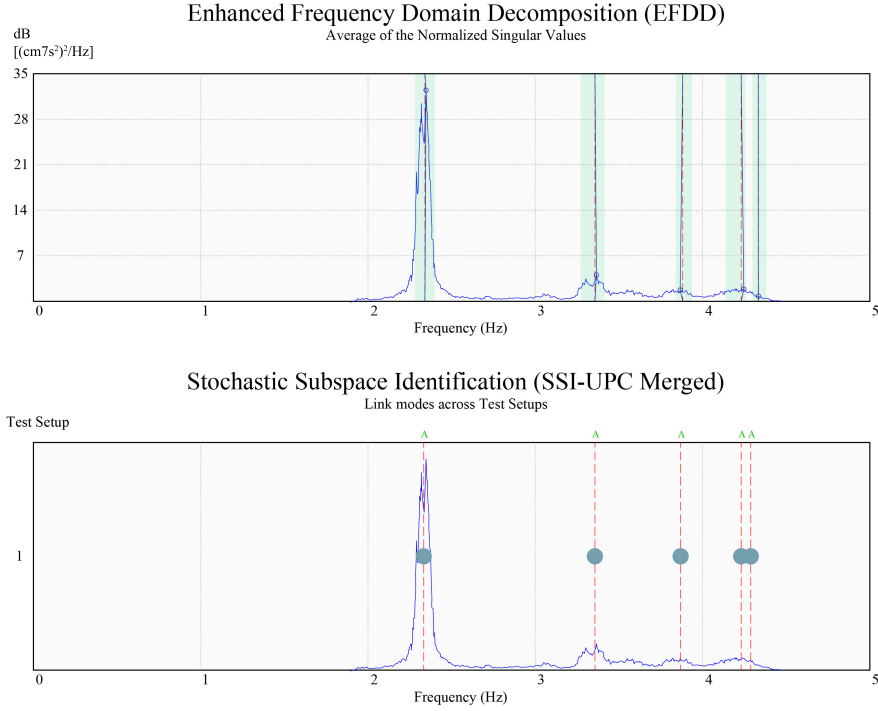


Fig. 10: Experimental identification by the EFDD and SSI-UPC-Merged methods.

201 Regarding the signal processing, a decimation factor of 5 and a spectral density resolution of
 202 1024 was used, which leads to a frequency line spacing of 0.005 Hz. In this way, the mode shapes
 203 and the modal frequencies of the gallery were obtained. The resulting modal parameters of the
 204 gallery are summarized in Table 2.

Table 2: Experimental modal parameters: natural frequencies (f), damping ratios (ξ) and standard deviation (std).

Mode N°	EFDD				SSI			
	f (Hz)	std(f)	ξ (%)	std(ξ)	f (Hz)	std(f)	ξ (%)	std(ξ)
1	2.330	0.023	0.864	0.119	2.345	0.002	2.728	0.429
2	3.359	0.021	1.292	0.468	3.357	0.002	1.411	0.104
3	3.857	0.049	0.582	0.307	3.910	0.003	2.971	0.126
4	4.311	0.056	1.565	0.799	4.286	0.001	2.019	0.054
5	4.561	0.042	1.348	0.473	4.532	0.002	1.718	0.076

205 As indicated in Table 2, the first five vibration modes of the gallery have been identified in the
 206 frequency range from 0 to 5 Hz. Frequency values are obtained with a high degree of reliability, with
 207 differences between both methods always lower than 2%. However, the values of the damping ratios
 208 present larger variability. This typical fact indicates that higher levels of excitation are usually
 209 necessary for the correct identification of the modal damping of such structures. Time domain
 210 methods have been proved to have better performance in the identification of damping ratios under
 211 operational conditions while the EFDD method, based on computation of the response spectrum

212 data, usually requires longer records to obtain an acceptable error in spectrum estimation and,
 213 therefore, to extract modal parameters in a reliable way [37]. Finally, in relation to the mode
 214 shapes, with the exception of the first mode, which is a bending mode of the full gallery, all the
 215 modes are induced by the changing shape of the transverse arches of the first floor. The great
 216 complexity of these modes can be seen in Fig. 11.

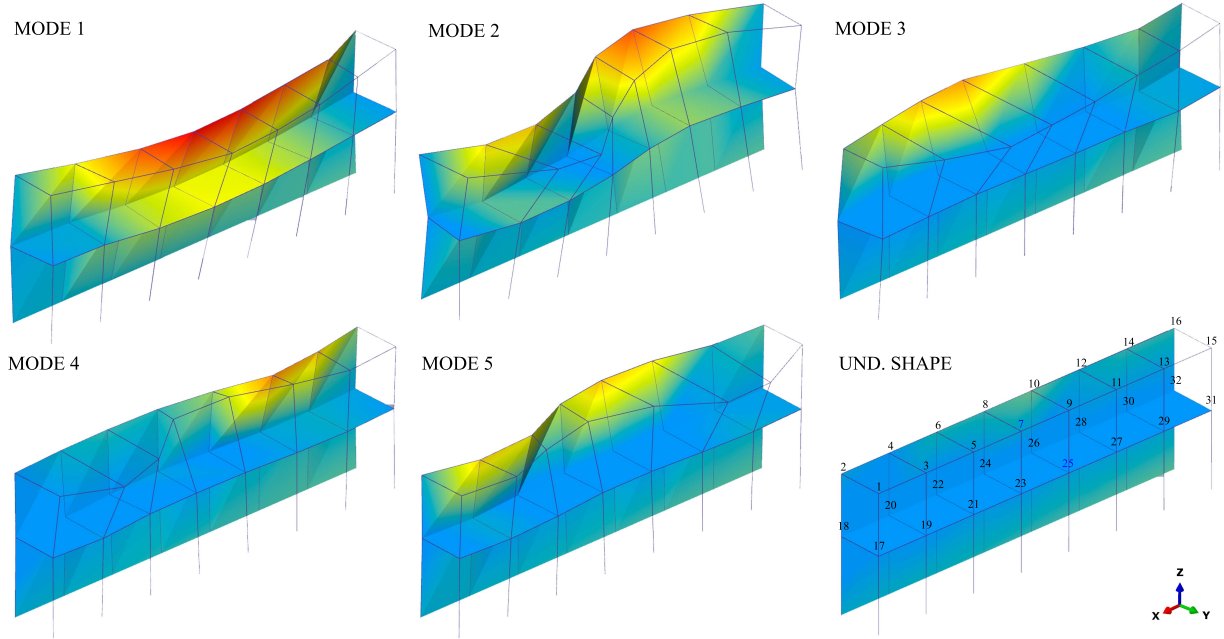


Fig. 11: First five experimental vibration modes and undeformed configuration of the experimental model.

217 Based on the experimental results, the preliminary numerical model is updated to simulate the
 218 current modal behaviour of the gallery and assist the evaluation of the OSP methods. The same
 219 procedure as in previous works by the authors was followed [38]. Hence, iterative methods are
 220 used to perform the FE model updating [39], introducing changes directly to some of the physical
 221 parameters that define the structure. As indicated above, the parameters to be updated are the
 222 Young's moduli of the stone and brick masonries due to the high degree of dispersion detected
 223 during the characterization tests. The selection of more parameters is not necessary, since their
 224 values have been reliably identified by laboratory tests.

225 Taking into account the good quality of the experimental data, the five identified modes are
 226 selected as target modes in the updating process. Only the values of the natural frequencies are
 227 selected due to the lower reliability of the identified mode shapes in comparison with the measured
 228 natural frequencies. This fact is due to the great complexity of the mode shapes (see Fig. 11).
 229 Nevertheless, the mode shapes are later validated using the Modal Assurance Criterion (MAC)
 230 [40]:

$$\text{MAC}_{j,k} = \frac{(\varphi_j^T \cdot \varphi_k)^2}{(\varphi_j^T \cdot \varphi_j) \cdot (\varphi_k^T \cdot \varphi_k)} \quad (1)$$

231 where φ_j and φ_k are two mode shapes to be compared and the superscript T designates transpo-
 232 sition.

233 Therefore, considering five identified natural frequencies, the four parameters are adjusted and
 234 the residuals are minimised during the model updating. The updating process has been performed
 235 via a genetic algorithm, as implemented in Matlab [41], according to an objective function defined
 236 by the relative differences between the experimental and the numerical modal parameters. This
 237 function is usually formulated as a least-squares problem as follows:

$$l(\theta) = \frac{1}{2} \sum_{j=1}^m w_j [z_{NUM,j}(\theta) - z_{EXP,j}]^2 = \frac{1}{2} \sum_{j=1}^m w_j r_j(\theta)^2 \quad (2)$$

238 where $z_{NUM,j}(\theta)$ are the values related to the physical parameters of the numerical model, θ (E_c and
 239 ρ_c), while the variables $z_{EXP,j}$ are the corresponding data values obtained from the experimental
 240 campaign. The differences between these variables are set as residues, $r_j(\theta)$.

241 Fig. 12 illustrates the convergence of the updating process using a genetic algorithm. Taking
 242 as a reference the parameters of the preliminary model, a controlled variability range is established
 243 for the updating parameters. Thus, the estimation of unrealistic solutions is avoided (Table 3).
 244 In each iteration, a population of 1000 vectors is created that, by using the rules of the genetic
 245 algorithm, minimize the objective function in Eq. (2). The calibration process finishes when the
 246 difference between the mean value (blue points, Fig. 12) and the best value (black points, Fig. 12)
 247 of the population is less than 1×10^{-3} .

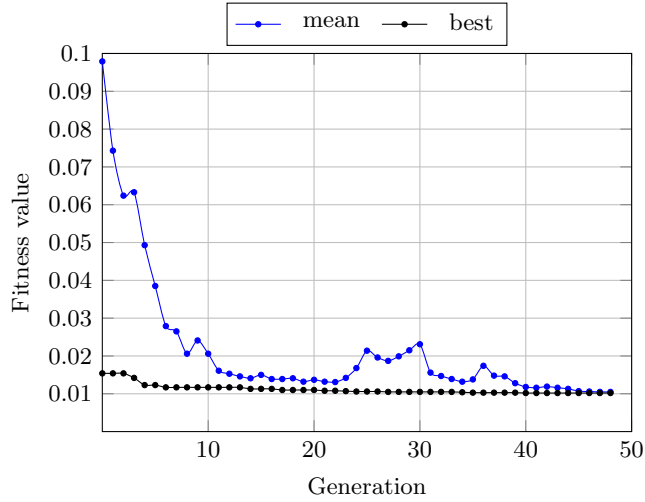


Fig. 12: Convergence plot of the genetic algorithm. Blue points: Mean values of the objective function of all the population of the corresponding generation. Black points: Best values result of an individual of the population.

248 Table 3 shows the considered lower and upper bounds for the updating parameters and their
 249 corresponding initial and updated values. The updated values differ by up to 20 % with respect to
 250 those defined in the initial model.

Table 3: Summary of updated parameters of the FE model computed by a genetic algorithm.

Parameter	Initial value	Range of variation		Updated value
		Lower	Upper	
E_b (MPa)	1300	900	1700	1000.51
ρ_b (kg/m ³)	1700	1300	2100	1805.72
E_s (MPa)	1800	1200	2200	1520.36
ρ_s (kg/m ³)	2000	1600	2400	1954.86

251 Table 4 summarizes the results following the updating process and confirms the high corre-
 252 spondence of the results between the calibrated model and those obtained from ambient vibration
 253 tests. It can be noted that the updated frequencies are close to the experimental ones, differing by
 254 less than 5% while exhibiting MAC values in a range from 0.82 (mode 3) to 0.98 (mode 1) for the
 255 five considered vibration modes.

Table 4: Comparison of numerical and experimental modal parameters, the latter identified by the SSI algorithm.

Mode	f_{exp} (Hz)	f_{num} (Hz)	$\text{MAC}_{\text{exp-num}}$	f_{upd} (Hz)	$\text{MAC}_{\text{exp-upd}}$
1	2.345	2.534 (8.05 %)	0.979	2.344 (0.04 %)	0.980
2	3.357	3.869 (15.25 %)	0.915	3.510 (4.77 %)	0.933
3	3.910	4.126 (5.52 %)	0.819	3.873 (0.94 %)	0.825
4	4.286	4.699 (9.63 %)	0.872	4.380 (2.19 %)	0.923
5	4.532	4.947 (9.15 %)	0.841	4.502 (0.66 %)	0.862

*The percentages in parenthesis correspond to the relative differences between frequencies.

256 4. Sensor placement

257 Vibration-based structural health monitoring is a suitable non-invasive technique for the long-
 258 term control of the condition of architectural heritage buildings. The placement of the sensors
 259 is a crucial design parameter of the monitoring system to efficiently identify the condition data.
 260 In this light, OSP techniques represent a very useful tool to design efficient monitoring set-ups,
 261 particularly in complex structures such as historical buildings.

262 After a concise theoretical background on OSP techniques, different OSP methods are applied
 263 to the present case study: the Monastery of San Jerónimo de Buenavista. The measuring points for
 264 the OSP problem are the same as for the ambient vibration tests (see Fig. 8). The chosen set-up
 265 has 32 possible locations for the sensors, with three different measuring directions at each point:
 266 i.e. the x , y and z directions. Therefore, a total of 96 possible sensor positions are considered in
 267 the OSP problem.

268 4.1. Theoretical background

269 Let us consider a monitoring system consisting of n sensors for the dynamic identification of a
 270 structural system. Let us also assume that the system consists of N degrees of freedom (DOFs)
 271 associated with the finite element mesh, and the numerical mode shapes φ_j are extracted by

272 performing modal analysis. Typically, only a limited number of DOFs and mode shapes of the
 273 FE model can be monitored due to accessibility or physical limitations (e.g. rotations or internal
 274 nodes). Therefore, considering N_d candidate DOFs and N_m target mode shapes, the OSP methods
 275 aim to identify the optimal positions of the n sensors among the N_d available DOFs. In this work,
 276 four different FEM-based OSP methodologies are considered, including two approaches based on
 277 the effective independence of the target mode shapes (EFI and EFIwm), and two others based
 278 on the concept of energy matrix rank optimization (KEMRO and SEMRO). In the following, the
 279 theoretical background of these techniques is concisely outlined.

280 The Effective Independence (EFI) method [42, 43] seeks the optimal location of sensors as those
 281 DOFs of the FE model that maximize the linear independence of the mode shapes. Based upon
 282 the modal analysis of the FE model, the target modal matrix $\varphi_{N_d \times N_m}$ can be computed, which
 283 contains the retained target modes as columns and the candidate DOFs as rows. Thereby, the
 284 Fisher Information Matrix, $\mathbf{FIM}_{N_m \times N_m}$, of the target modal matrix reads [44]:

$$\mathbf{FIM} = \varphi^T \varphi \quad (3)$$

285 which can be diagonalized by solving the eigenvalue/eigenvector problem:

$$(\mathbf{FIM} - \lambda_i \mathbf{I}) \psi_i = \mathbf{0} \quad (4)$$

286 as $\mathbf{FIM} = \psi \Lambda \psi^{-1}$, where $\Lambda_{N_m \times N_m}$ is the diagonal matrix of the eigenvalues λ_i of the \mathbf{FIM} , and
 287 $\psi_{N_m \times N_m}$ contains the corresponding eigenvectors ψ_i by columns. Given that the \mathbf{FIM} is symmetric
 288 and positive definite, the eigenvectors are orthogonal (i.e. $\psi^{-1} = \psi^T$) and form an orthogonal basis
 289 in an N_m -dimensional space. Therefore, the product $\varphi \psi$ yields an $N_d \times N_m$ matrix that represents
 290 the projection of the mode shapes φ_i onto the N_m -dimensional space spanned by the vectors ψ_i .
 291 Squaring each element in the $\varphi \psi$ matrix as $\mathbf{A} = \varphi \psi \otimes \varphi \psi$, with \otimes denoting term-by-term matrix
 292 multiplication, the elements A_{ij} of the resulting matrix represents the contribution of each i -th
 293 DOF to each j -th mode shape. If weighted by the inverse of the eigenvalue matrix of the \mathbf{FIM} as
 294 $\bar{\mathbf{A}} = \varphi \psi \otimes \varphi \psi \Lambda^{-1}$, each element in this matrix has equal importance. The summation by columns
 295 of the $\bar{\mathbf{A}}$ matrix leads to a $N_d \times 1$ vector \mathbf{F} whose components $F_i = \sum_{j=1}^{N_m} \bar{\mathbf{A}}_{i,j}$ represent the
 296 contribution of every i -th DOF to all the retained modes of interest. Alternatively, the orthogonal
 297 projection matrix \mathbf{E} can be defined as [42]:

$$\mathbf{E} = \varphi \psi \Lambda^{-1} (\varphi \psi)^T = \varphi \mathbf{FIM}^{-1} \varphi^T \quad (5)$$

298 The elements on the diagonal of the \mathbf{E} projection matrix equals the components of the \mathbf{F} vector
 299 and, therefore, represent the relative contribution of the candidate DOFs to the target mode shapes.
 300 It can be proved that the \mathbf{E} matrix is idempotent [42], that is $\mathbf{E}^2 = \mathbf{E}$. A well-known property of
 301 idempotent matrices is that their trace is equal to their rank. Therefore, the optimal placement of
 302 sensors can be related to the DOFs where the diagonal terms of \mathbf{E} are maximum or, in other words,
 303 the DOFs with maximum contribution to the linear independence of the target mode shapes. In
 304 this light, the EFI method is defined in an iterative way by eliminating those candidate DOFs
 305 whose contribution to the rank of the projection matrix is minimal, and the procedure finishes
 306 when the number of remaining DOFs reaches the desired number of sensors.

307 An alternative OSP method considering the mass weighting of the effective independence of
 308 the target mode shapes is also considered, usually referred to as the EFIwm method. In this case,
 309 the Fisher information matrix \mathbf{FIM} takes the form:

$$\mathbf{FIM} = \boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi} \quad (6)$$

and, consequently, the projection matrix \mathbf{E} reads [45]:

$$\mathbf{E} = \mathbf{M}^{1/2} \boldsymbol{\varphi} \mathbf{FIM}^{-1} \boldsymbol{\varphi}^T \left(\mathbf{M}^{1/2} \right)^T \quad (7)$$

where \mathbf{M} stands for the numerical mass matrix extracted from the FE model. Given that the mass matrix is diagonalizable, the square roots $\mathbf{M}^{1/2}$ in Eq. (7) can be readily obtained by eigen-decomposition. In addition, in order to eliminate the DOFs with minimum contribution to the rank of \mathbf{E} , Guyan reduction must be implemented at each iteration to reduce the mass matrix \mathbf{M} to the remaining candidate DOFs.

On the other hand, Energy Matrix Rank Optimization (EMRO) techniques are based upon the deployment of sensors at locations that maximize the energy of the monitored system, typically the strain energy or the kinetic energy [44]. Following the modal analysis of the FE model of the system, the strain energy (\mathbf{SE}) and kinetic energy (\mathbf{KE}) matrices can be written as [46, 47]:

$$\mathbf{SE} = \boldsymbol{\varphi}^T \mathbf{K} \boldsymbol{\varphi}, \quad \mathbf{KE} = \boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi} \quad (8)$$

where \mathbf{K} is the numerical stiffness matrix computed by the FEM. In this regard, the \mathbf{FIM} matrices can be assembled by using the Cholesky decomposition of the stiffness and mass matrices as follows:

$$\mathbf{K} = \mathbf{C}^T \mathbf{C}, \quad \mathbf{M} = \mathbf{D}^T \mathbf{D} \quad (9)$$

with \mathbf{C} and \mathbf{D} being upper triangular matrices. Accordingly, the \mathbf{FIM} matrices take the forms:

$$\mathbf{FIM}_{SE} = (\mathbf{C} \boldsymbol{\varphi})^T \mathbf{C} \boldsymbol{\varphi}, \quad \mathbf{FIM}_{KE} = (\mathbf{D} \boldsymbol{\varphi})^T \mathbf{D} \boldsymbol{\varphi} \quad (10)$$

and the projection matrices:

$$\mathbf{E}_{SE} = \mathbf{C} \boldsymbol{\varphi} \mathbf{FIM}_{SE}^{-1} (\mathbf{C} \boldsymbol{\varphi})^T, \quad \mathbf{E}_{KE} = \mathbf{D} \boldsymbol{\varphi} \mathbf{FIM}_{KE}^{-1} (\mathbf{D} \boldsymbol{\varphi})^T \quad (11)$$

where subscripts ‘‘SE’’ and ‘‘KE’’ relate the corresponding quantities to the strain and kinetic energies, respectively. Once the projection matrices are defined, the procedure for finding the optimal sensor localizations is identical to the previously introduced EFI algorithms. In a similar way to the EFIwm method, Guyan reduction is needed to reduce the stiffness and mass matrices at each iteration. The OSP algorithms based on the elimination of candidate DOFs with minimum contributions to the rank of \mathbf{E}_{KE} and \mathbf{E}_{SE} matrices are referred to as the KEMRO and SEMRO methods, respectively.

4.2. Identification of the optimal number of sensors

The application of the above OSP techniques to the case study is now addressed. First of all, the required number of sensors is determined. Different scenarios were explored by varying the number of accelerometers of the monitoring system from 2 to 16. The EFI method is employed to find the optimal configuration for each case. In order to analyse the goodness of the OSP solution, the data of the ambient vibration tests are used. The difference between the modal properties obtained via OMA using the complete test data (reference) and those obtained only by using

338 the data from sensors placed at the optimal configuration is analysed. In particular, the error is
 339 computed as:

$$\text{total error (\%)} = \left[\sum_{i=1}^{N_m} R_i^2 \right]^{1/2} \quad (12)$$

340 where N_m is the total number of considered modes and R_i is the relative error between the reference
 341 natural frequency of mode i ; $f_{ref,i}$, and the frequency identified using only data from sensors in
 342 the OSP solution; $f_{osp,i}$. That is:

$$R_i(\%) = 100 \cdot \left[\frac{f_{ref,i} - f_{osp,i}}{f_{osp,i}} \right] \quad (13)$$

343 In Fig. 13, the total error is shown for several cases considering different numbers of sensors
 344 in the OSP problem. Note that the error is stabilized for a number of sensors greater than 8.
 345 Given these results, it can be considered that 8 is the optimal number of accelerometers for the
 346 case of the Monastery of San Jerónimo, since a reasonable increase in the number of sensors does
 347 not significantly reduce the error in the identification of the natural frequencies of the structure.

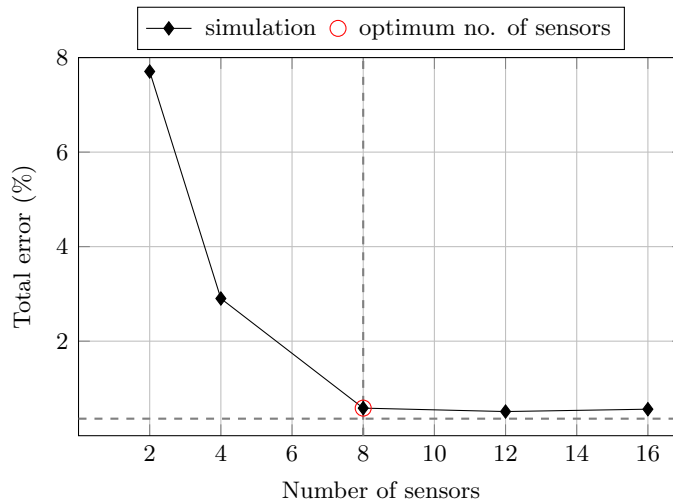


Fig. 13: Total error function for several scenarios with different numbers of sensors.

348 4.3. Evaluation of the performance of OSP methods

349 Once the required number of sensors for the long-term monitoring system has been determined,
 350 the OSP problem is performed to obtain the best positions for these 8 sensors. Four different OSP
 351 methods are evaluated, namely EFI, EFIwm, KEMRO and SEMRO.

352 In Fig. 14, the relative error in the identification of natural frequencies and the total error are
 353 shown for each method. For the EFI and EFIwm methods, the relative error is very low in all cases
 354 (below 0.5%). In the case of the EMRO methods, the error is considerably greater. Especially
 355 for the KEMRO method, for which the error in the identification of the third natural frequency
 356 amounts to almost 6%. As a partial conclusion, it can be said that the EFI methods allow the
 357 identification of the natural frequencies of the structure with a lower error.

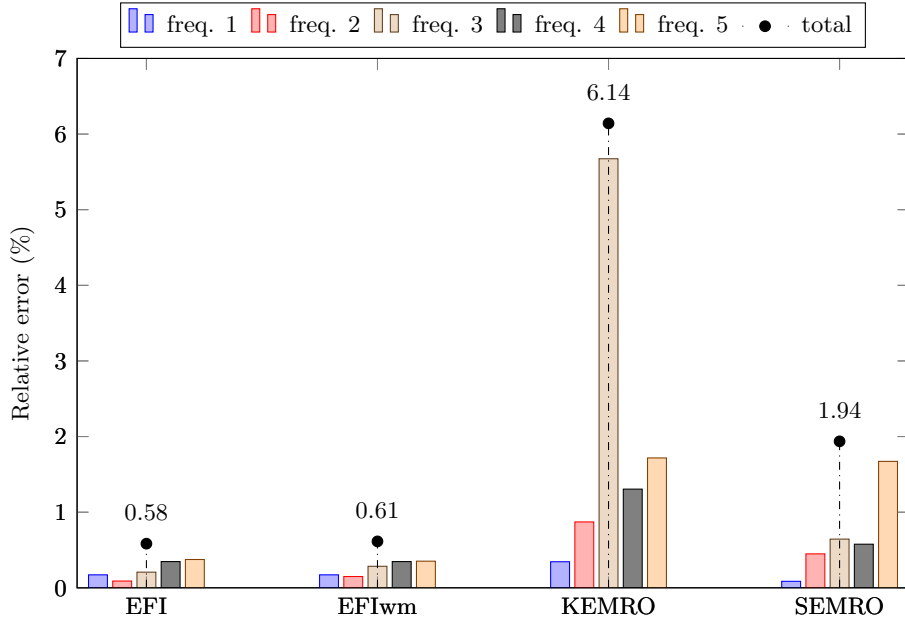


Fig. 14: Relative error function for different OSP methods.

358 Regarding the solution, the optimal locations of the 8 considered sensors obtained with the
 359 different OSP methods are shown in Fig. 15. In this bar plot, a chosen placement for a particular
 360 OSP method is represented by a bar of height 0.25. In this way, a sensor location selected by all
 361 the methods is plotted with a value of 1 on the ordinate axis. There are three common positions
 362 that are selected by all the methods: points 5, 7 and 11, all in the y direction. Another three points
 363 are selected by three of the four methods: points 6, 10 and 12, all also in the y direction. There
 364 is only one other common point chosen by both EFI methods: point 11 in the x direction. The
 365 eighth optimal position could be chosen between any of the alternatives proposed by the different
 366 methods separately.

367 Although in a particular case the optimal locations would finally depend on the choice of one of
 368 the OSP algorithms, this analysis is useful for comparing the consistency of the different methods.
 369 As a general conclusion, the solution of the EFI, EFIwm and SEMRO methods are largely similar,
 370 while that of the KEMRO method is considerably different.

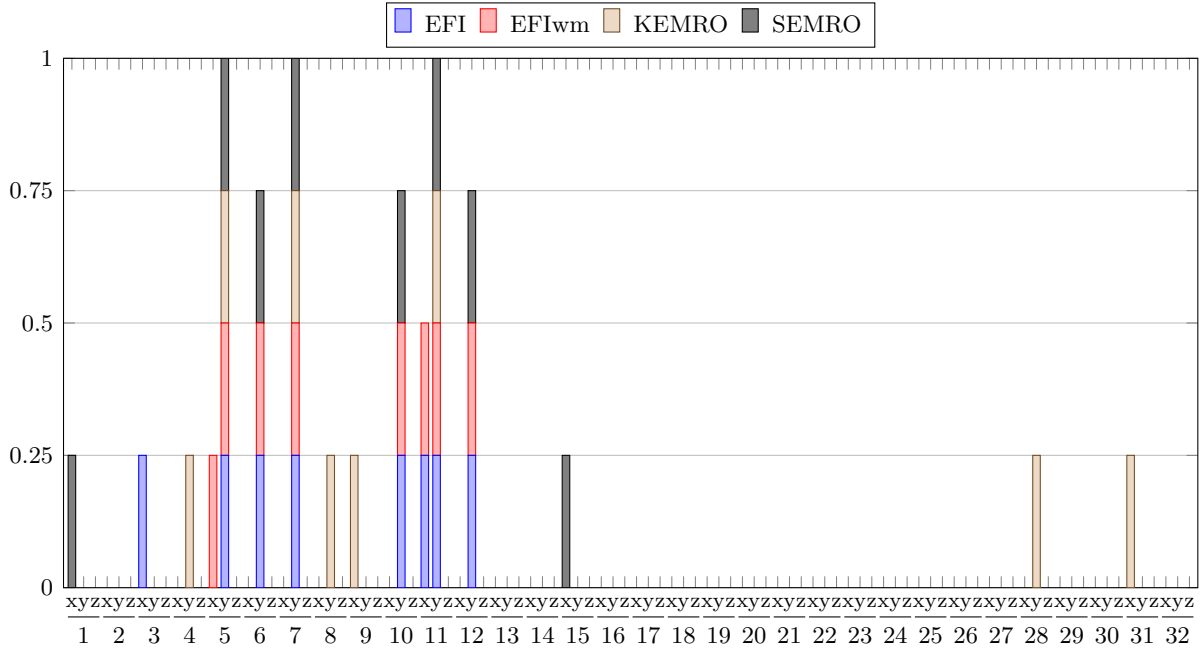


Fig. 15: Normalized sensor selection for different OSP methods considering deterministic material parameters.

371 5. An optimal sensor placement strategy considering material parameter uncertainty

372 This section assesses the robustness of the previously introduced OSP techniques considering
373 material parameter uncertainties. In particular, the elastic moduli of the brick and the stone mason-
374 ries are assumed to be stochastic with Gaussian distributions with mean values of $E_b = 1000.51$ MPa
375 and $E_s = 1520.36$ MPa, respectively, and standard deviations of 20% with respect to their mean
376 values. The optimal selection of sensors is evaluated within a stochastic framework by applying
377 Monte Carlo Simulations (MCS) as sketched in Fig. 16. Monte Carlo methods constitute a highly
378 popular and widely used solution for the uncertainty propagation analysis of structures [48]. These
379 techniques leverage the random description of certain model parameters, whereby the structural
380 analysis can be performed by a deterministic simulation framework. Albeit these techniques usually
381 require a large number of simulations and, as a consequence, considerable computational costs, the
382 uncertainty analysis can be performed using the deterministic FE model. In this work, the MCS
383 process first considers a random sampling of the elastic moduli of the brick and the stone masonries
384 using `MATLAB`. Subsequently, the FE model of the monastery is built in the finite element modelling
385 software `ANSYS`, and a modal analysis is performed to extract the mode shapes and natural frequen-
386 cies. Afterwards, the previous information is retrieved in `MATLAB`, and the optimal positions of the
387 sensors are obtained using the EFI, EFIwm, KEMRO and SEMRO algorithms. Finally, this process
388 is repeated ns times until a statistically significant population of sensor locations is achieved.

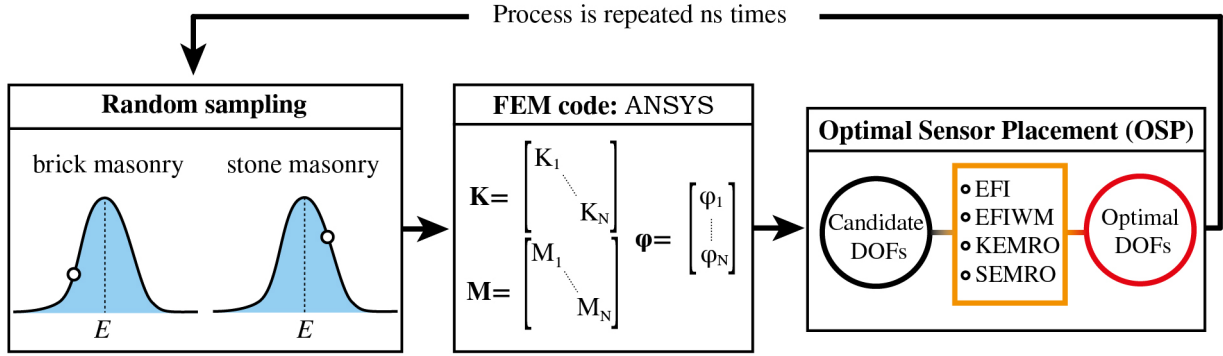


Fig. 16: The Monte Carlo simulation process.

5.1. Determination of the optimal number of samples in the MCS

An important parameter of the Monte Carlo approach is the number of samples ns required to have a representative population of the simulated stochastic process. In this case study, several simulations with different number of samples from 2 to 516 were performed. The EFI method is employed in the OSP problem. In order to determine the optimum number of samples for the MCS, the dispersion percentage is calculated for each case. The dispersion is defined as the percentage of times during the MCS that the algorithm chooses a location which is finally not part of the solution, where the solution is represented by the eight sensors that are chosen most often. It can be computed as:

$$\text{dispersion (\%)} = 100 \cdot \frac{\sum_{i=1}^{ns} \bar{p}_i}{ns \cdot n} \quad (14)$$

where ns is the number of samples in the MCS, n is the number of sensor of the monitoring system (8 in this case) and \bar{p}_i represents the number of selected points for sample i which are not present in the final solution.

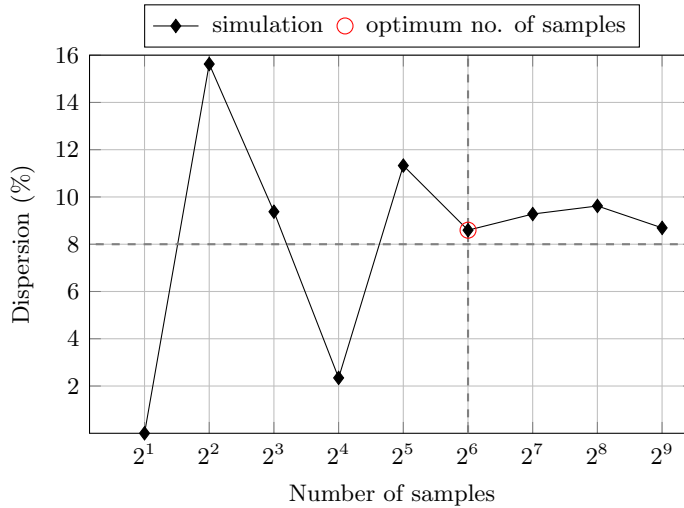


Fig. 17: Dispersion percentage for different number of samples in the MCS.

The dispersion percentage for MCS with different number of samples is presented in Fig. 17. For a low number of samples, the dispersion percentage experience strong fluctuations. However,

403 the dispersion is stabilized for a number of samples greater than 64. Consequently, 64 samples or
 404 more are considered in the following calculations.

405 *5.2. Effects of uncertainties on the OSP methods*

406 Once the minimum number of iterations for the MCS has been determined, the OSP problem
 407 is performed to evaluate the robustness of the different methods. For this purpose, first a Gaussian
 408 distribution of Young's moduli of brick and stone with standard deviations of 20% with respect
 409 to their mean values is considered. The dispersion percentage and the solution for each method is
 410 analysed.

411 In Fig. 18, the dispersion percentage computed with different numbers of samples of the
 412 MCS is shown for the four OSP methods. The EFI and KEMRO methods exhibit dispersion
 413 percentage about 10% and 22%, respectively. The SEMRO method seems to be the most stable
 414 when considering uncertainty in the value of the model parameters, with less than 3% dispersion
 415 of the results.

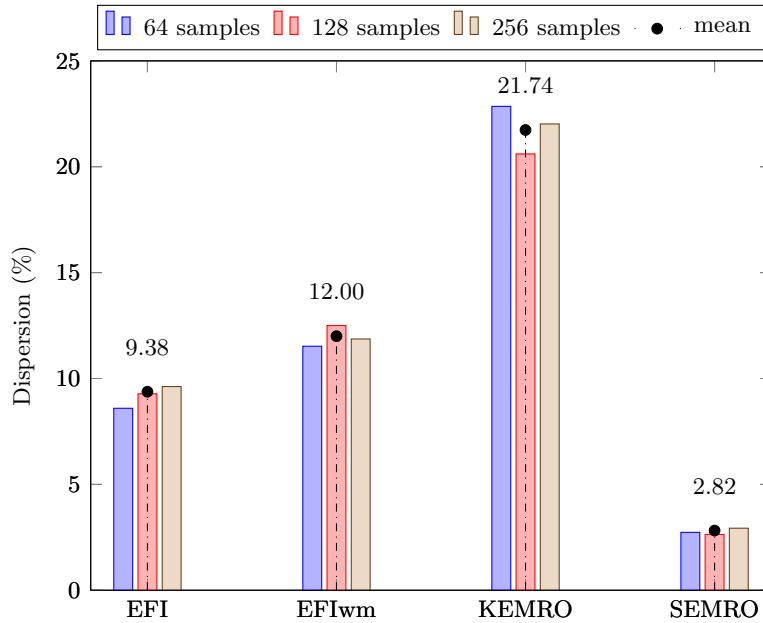


Fig. 18: Dispersion percentage for different OSP methods with 20% of parameter variability.

416 The solution considering a parametric variability of 20% is represented in Fig. 19 for different
 417 OSP methods. In this case, the probability of sensor selection is computed for each method and
 418 represented in the plot in a normalized form. There are two common positions selected by all the
 419 methods at all iterations: points 7 and 11, both in the y direction. The point $5y$ is also selected for
 420 the four methods with a combined probability greater than 95%. As in the case with no parametric
 421 uncertainties, the points 6, 10 and 12 in the y direction are selected with a high probability for
 422 the EFI, EFIwm and SEMRO methods. Another interesting result is that the point $3y$ is selected
 423 by the four methods but with a very low probability (below 25%). In view of these results, the
 424 effect of considering a parameter variability of 20% does not strongly affect the solution of the
 425 OSP problem.

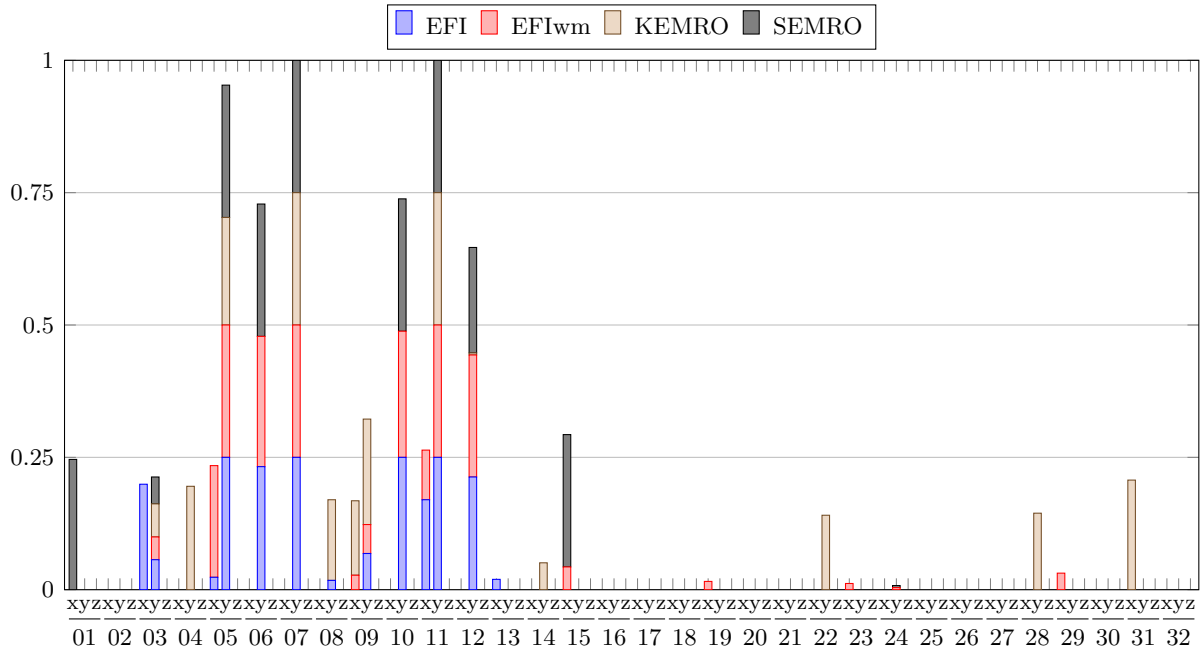


Fig. 19: Normalized probability of sensor selection for different OSP methods with 20% of parameter deviation.

426 Finally, different standard deviations for the brick and stone Young's moduli are considered.
 427 In particular, several percentages of variation between 2.5% and 40% are used to evaluate the
 428 robustness of the four considered OSP methods. The number of samples in the MCS is 64. The
 429 results are presented in Fig. 20. There is a progressive increasing trend of dispersion as the
 430 variability of the parameters grows. The SEMRO method is again the one that seems to be more
 431 stable while the KEMRO generates a greater dispersion of the results in all cases. For the two
 432 EFI methods, the dispersion percentage is similar for the different standard deviation values of the
 433 parameters.

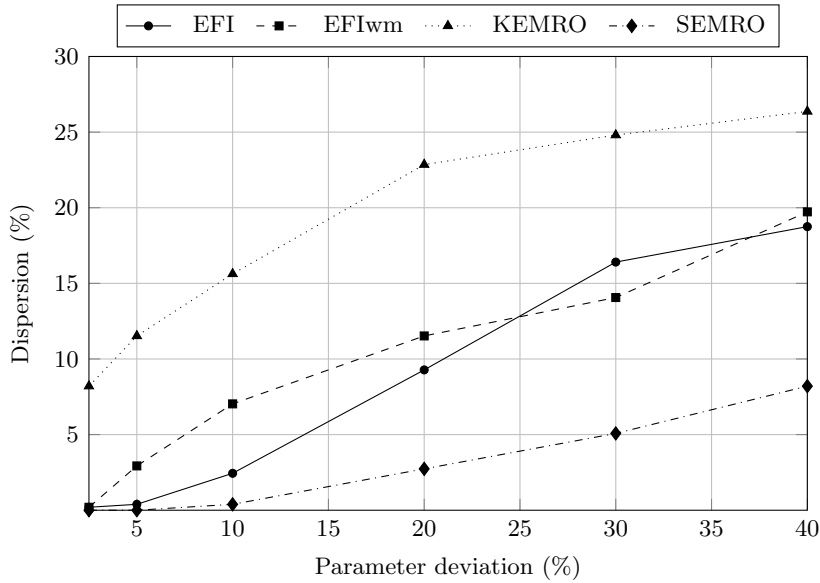


Fig. 20: Dispersion percentage versus parameter standard deviation for different OSP methods.

434 6. Conclusions

435 The paper has presented the application of the OSP methodology to historical masonry build-
 436 ings such as the Monastery of San Jerónimo de Buenavista. The use of this technique becomes
 437 relevant in this kind of structures to reduce the number of sensors in a SHM monitoring system in
 438 order to be less-invasive. In this study, four different FEM-based OSP methodologies are analysed,
 439 including two approaches based on the concept of energy matrix rank optimization (KEMRO and
 440 SEMRO), and two others based on the effective independence of the target mode shapes (EFI
 441 and EFIwm). The modelling of historical buildings is usually subjected to a high level of uncer-
 442 tainty derived from numerous aspects such as material properties. Due to this, the effect of model
 443 parametric uncertainty on the results of the different OSP algorithms is also discussed in this work.

444 First, the optimal number of sensors for the monitoring system has been determined. The
 445 results demonstrate that, with only eight sensors, the natural frequencies of the structure could
 446 be identified with a maximum error less than 1% in comparison to the extensive monitoring with
 447 ninety-six measurement points. The suitability of the OSP techniques to find a convenient set-up
 448 for a cost-efficient continuous monitoring in complex structures is thus proven. Among the four
 449 OSP algorithms analysed, the EFI methods provide a solution that allows the identification of
 450 natural frequencies with less error, whereas the solution of the KEMRO method is the one that
 451 gives the greatest error in the modal identification.

452 Including parametric uncertainty in the OSP methodology involves performing Monte Carlo
 453 simulations with a sufficient number of samples to represent the stochastic process. The SEMRO
 454 method is the one that presents the lowest dispersion in its solution, for all the different scenarios
 455 analysed. A general conclusion for the case study presented is that the choice of the optimal
 456 sensor placements do not change drastically when considering a 20% variability in value of the
 457 parameters.

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