Semi-parametric estimation of the Wilshire creep life prediction model: An application to 2.25Cr-1Mo steel.

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Abstract

The Wilshire equation is a recent addition to the literature on safe life prediction. Whilst the effect of temperature on creep life is reasonably understood, the effect of stress isn't. The Wilshire equation deals with this by partitioning over sub ranges of stress, but this approximation can lead to poor life time predictions. This paper introduces a semi-parametric procedure that allows the data itself to identify the stress relationship. When applied to 2.25Cr-1Mo steel it was found that the stress relationship is non-linear, and this semi-parametric version of the Wilshire model had better predictive performance compared to any partitioned Wilshire model. This approach contains a limit to valid extrapolation and the isothermal predictions for creep life have a more realistic pattern of behaviour.

Keywords: Creep, Arrhenius, Eyring, Wilshire equations, parametric, semi-parametric, non-parametric

Introduction

For many years now, fully parametric models have been the mainstream for obtaining predictions of safe operating life for existing high temperature materials designed for use in aero engines and power plants. These parametric models get their name from the fact that they relate life times to stress and temperature through a specific equation and that these equations contain parameters whose values are unknown and so need to be estimated from experimental data sets. In the absence of a non-thermal applied stress, the Arrhenius equation explains the effect of temperature (T) on a rate constant k

$$k = A\exp(-Q_c/RT) \tag{1a}$$

where A is a model parameter, R the universal gas constant ($8.314\ J{\cdot}mol^{-1}{\cdot}K^{-1}$) and Q_c the activation energy ($Jmol^{-1}$) for self-diffusion. This relationship was defined from empirical evidence to describe the chemical process of diffusion[1]. The modified Arrhenius equation[2] makes explicit the temperature dependence of a pre-exponential factor and is usually written as

$$k = AT^{\alpha}\exp(-Q_c/RT) \tag{1b}$$

The original Arrhenius expression above corresponds to $\alpha = 0$ but fitted rate constants typically lie in the range $-1 < \alpha < 1$, depending as it does on the specifics of the reaction dynamics. Thus, Boccaletti *et al.*[3] used $\alpha = 0$ in their study on microelectronics, Klinger[4] used $\alpha = 0.5$ in his study of plastic packaged electronic devices, whilst Nelson[5] used $\alpha = 1$ in his study of fatigue curves. Theoretical analyses on the other hand yields various predictions for α, where for example, the Eyring[6] equation, derived from transition state theory for a second order reaction, implies $\alpha = 1$ as it has the form

$$k = (k_B T/ch)\exp(-G/RT) \tag{1c}$$

where G is the Gibbs activation free enthalpy, k_B is Boltzmann's constant, c is the reference concentration of the reaction and h is Planck's constant (implying $A = k_B/ch$). Replacing the rate constant k with the minimum or secondary creep rate $\dot{\varepsilon}_m$ reveals the main building block behind parametric creep models which, to varying degrees, model the effect of temperature on the minimum creep rate reasonably well

$$\dot{\varepsilon}_m = AT^{\alpha}\exp\{-\frac{G}{RT}\} \tag{2a}$$

The main difficulty then comes with modelling creep rates in the presence of an additionally applied non-thermal stress. Eyring presented a modified version of his equation to describe this environment that took the form

$$\dot{\varepsilon}_m = AT^{\alpha}\exp\{-\frac{G}{RT} + Bf(\sigma) + D\frac{f(\sigma)}{T}\} \tag{2b}$$

where B and D are additional model parameters. The problem however is that the stress relationship is unknown and so the modified Eyring equation contains an unspecified stress function, $f(\sigma)$. An equation for the time to failure, t_f can then be obtained by making use of the empirical Monkman – Grant[7] relation that contains the constants M and ρ (where ρ is close to unity in value for many metal alloys):

$$\dot{\varepsilon}_m = (M/t_f)^{\rho} \tag{2c}$$

so that equation (2b) can be rewritten as

$$\ln(t_f) = a_0 + a_1 f(\sigma) + a_2 \ln(T) + a_3 \frac{1}{T} + a_4 \frac{f(\sigma)}{T} \quad (2d)$$

where $a_0 = \{-\ln(A)/\rho + \ln(M)\}$, $a_1 = -B/\rho$, $a_2 = -\alpha/\rho$, $a_3 = G/(\rho R)$ and $a_4 = -D/\rho$.

Many parametric creep models have equation (2d) as their basis. Thus, the so called first Soviet model[8] is very similar to equation (2d) in that $f(\sigma) = \ln(\sigma)$ for the expression in front of parameter a_1, but $f(\sigma) = \sigma$ in the expression in front of a_4. The model proposed by Larson-Miller[9] did not explicitly propose a specific stress function but applications of this model have made use of various forms including $f(\sigma) = \sigma$, $f(\sigma) = \ln(\sigma)$ or with $f(\sigma)$ being a polynomial (of varying orders) in stress or log stress (all usually with the additional restriction that $a_1 = a_2 = 0$). The Orr-Sherby-Dorn[10] model has $a_2 = a_4 = 0$ with $f(\sigma) = \ln(\sigma)$, the Wilshire[11] model has $a_2 = a_4 = 0$ with $f(\sigma) = \ln[-\ln(\sigma/\sigma_{TS})]$ and the model by Yang *et al.*[12] has $a_2 = a_4 = 0$ with $f(\sigma) = \ln(\sigma/(\sigma_{TS} - \sigma))$, where σ_{TS} is the tensile strength. The minimum commitment method[13] has $a_4 = 0$ and uses T instead of ln(T) with $f(\sigma) = \ln(\sigma) + b_1\sigma + b_2\sigma^2$, where b_1 and b_2 are additional model parameters. Finally, Manson and Haferd[14] set $a_2 = a_3 = 0$ and replace $a_4 f(\sigma)/T$ with $a_4 f(\sigma)[T-a_5]$, whilst Manson and Brown[15] further generalise using $a_4 f(\sigma)[T-a_5]^{a6}$, where a_5 and a_6 are additional model parameters (with $f(\sigma)$ being some polynomial function of stress). Table 1 summarises these competing models expressing rupture life as an algebraic function of stress and temperature.

All these models mainly differ from each other because of the uncertainty surrounding the role of stress in determining creep life and in fact all of the models contained within Table 1 suffer from an inability to adequately model the relationship between stress and creep life. That is, when creep life is adjusted for the effect of temperature in the way implied by the chosen parametric model, and when this temperature compensated creep life is plotted against the stress transformation implied by that model, the resulting plot never displays the functional relationship implied by the model. This deficiency is worse for some models, some materials and when larger ranges of stress are used.

Table 1 Summary of some popular parametric creep models for predicting time to failure

One potential solution to this problem is to remove the effect of temperature on creep life in the way suggested by a chosen creep model and then to leave the functional relationship of this temperature compensated failure time with stress unspecified. Then a non-parametric or semi-parametric procedure can be used to identify a smooth curve that adequately describes the relationship between stress and the temperature compensated failure time. Non-parametric procedures are capable of quantifying smooth curves of many varied shapes and forms and so can be applied to a great variety of creep data sets. Such a smooth curve can then be extrapolated out to all temperatures and because it does not rely on a particular equation to describe the stress relationship, it has the potential to provide more accurate predictions of creep life at stresses approaching operating conditions. The structure of the P-NID models put forward by Bolton[16] are particularly useful for the application of these non-parametric procedures. However, the non-parametric technique proposed by Bolton, namely the use of cubic spline functions, relies on the researcher having to make value judgements on what data to include and what data to exclude in order to obtain a smooth curve that describes the stress relationship that is also capable of extrapolation. On a plot of stress versus temperature compensated failure times this involves selecting either artificial coordinates or a sub set of actual data points that visually produces a smooth curve that appears to go through the middle

of all the data on such a plot. Data fitting by eye in this way is subjective and will result in different analysts obtaining different life time predictions using the same data. Provided these different predictions are not statistically significantly different from each other, then such subjectivity is not an issue (which is likely to be the case at the hands of experienced practitioners). Inexperienced users of these models may however encounter problems. An additional issue with such subjectivity is that it presents complications in producing algorithms that can be easily used by such practitioners. Objective criteria for curve fitting allows the construction of user friendly algorithms that can be used without having to have a detailed understanding of either the creep prediction model or the properties of the material being studied.

This paper therefore aims to i. propose objective non-parametric and semi-parametric procedures that remove the need for such value judgements to be made and ii. to assess the potential of these procedures for accurate creep life prediction by using data on 2.25-1Cr steel and the Wilshire model as a test bed. To this end the paper is structured as follows. The next section describes in more detail the data to be used in this paper and this is followed by a section that applies the Wilshire model to this data set as a basis for comparison to other approaches. This is followed by two sections applying the Wilshire model in partitioned form, as an illustration of how to approximate an unknown stress function using a series of segmented lines. The penultimate section outlines a new semi – parametric Wilshire model which is then applied to the creep data set. The conclusions section then identifies the better performing model and suggests some future areas of research.

The Data

The NIMS Matnavi website[17] in which Creep Data Sheet 3B resides, provides extensive rupture data for twelve batches of 2.25Cr - 1Mo steel and each batch has a different chemical composition that underwent one of four different heat treatments. The most comprehensive set of test conditions exists for the MAF batch and the chemical composition and heat treatment undergone by this batch can be found in Creep Data Sheet 3B [18]. This paper uses this batch that is made up of 102 specimens that were tested at seven different temperatures (723K, 748K, 773K, 798K, 823K, 873K and 923K) and stresses ranging from 333MPa to 14MPa. The resulting shortest failure time was 123,840 seconds and the largest was 512,694,360 seconds (or approximately sixteen and a quarter years). The data set is also made up of a number of censored times (length of an incomplete test at the time of publication of data sheet 3B). These times are shown as arrows in all subsequent figures. Figure 1 plots this data set and it is clear that the relationship between stress and time to failure is only log linear over a narrow range of stresses (this is most apparent at a temperature of 873K). For the purpose of assessing the accuracy of life time predictions made by various versions of the Wilshire model, the short-term data is restricted to 10,000 h, and failure times above this are represented in all figures as open symbols, with failure times less than 10,000 h as filled symbols. All model parameters are estimated with this short-term data only and their suitability assessed by seeing how well they predict the remaining longer-term data.

1 Plot of time to failure against stress at various temperatures for the MAF batch of 2.25Cr-1Mo steel taken from the NIMS creep data base

The Wilshire model with no partitioning (W)

The Wilshire equation for failure times has traditionally been written as

$$\sigma/\sigma_{TS} = \exp\{-k_1[t_f \exp(Q_c/RT)]^u\} \quad (3a)$$

and k_1 and u are model parameters that require estimation. This can be linearized as

$$\ln[t_f] = a_0 + a_1 \ln[-\ln(\sigma/\sigma_{TS})] + a_3[1/T] \quad (3b)$$

where $u = 1/a_1$, $k_1 = \exp(-a_0/a_1)$ and $Q_c/R = a_3$. The Wilshire model therefore implies that at some fixed datum temperature T_d, (e.g. $T_d = 773K$), there is a linear relationship between the log time to failure at that temperature, $\ln[t^d_f]$, and the transformed normalised stress $\ln[-\ln(\sigma/\sigma_{TS})]$

$$\ln\left[t_f^d\right] = \{a_0 + a_3[1/T_d]\} + a_1 \ln[-\ln(\sigma/\sigma_{TS})] \quad (3c)$$

so that the effect of temperature is to shift this linear relationship in a parallel fashion – i.e. change the intercept parameter a_0. Using equations (3b,c) it is possible to convert a failure time recorded at this datum temperature (and also some value for the normalised stress) to what that failure time would have been if the test had been conducted at a temperature different to T_d (but the same normalised stress)

$$\ln[t_f] = \ln\left[t_f^d\right] + a_3[1/T - 1/T_d] \quad (3d)$$

Put differently, $\ln\left[t_f^d\right]$ can be thought of as a temperature compensated (log) failure time that according to equation (3c) varies linearly with variations in $\ln[-\ln(\sigma/\sigma_{TS})]$ and with any additional variation being purely random in nature

$$\ln\left[t_f^d\right] = \ln[t_f] - a_3[1/T - 1/T_d] \quad (3e)$$

Estimation of the unknown parameters in equation (3b) can done using the linear least squares procedure that choses values for the parameter so that Σe^2 is minimised, where

$$e = \ln[t_f] - \{a_0 + a_1 \ln[-\ln(\sigma/\sigma_{TS})] + a_3[1/T]\} \quad (4a)$$

is the random component of creep failure times and the summation is carried out over the N test conditions leading to failure within 10,000 h.

The results of this estimation procedure, when applied to all failure times less than 10,000 h, was

$$\ln[t_f] = \underset{\{-3.14\}}{-9.798} + \underset{\{10.13\}}{4.594}\ln[-\ln(\sigma/\sigma_{TS})] + \underset{\{8.30\}}{21{,}294}[1/T] \quad (4b)$$
$$R^2 = 69.10\%$$

where student t values associated with parameters a_1 to a_3 are shown in squiggly brackets. These t values have a student t distribution with N-3 = 99 degrees of freedom under the null hypothesis that the population value for a parameter of the model equals zero. Consequently, all the parameters are statistically significantly different from zero at the 5% significance level and the value for a_3 implies an activation energy of approximately 177 $kJmol^{-1}$ (as $Q_c = Ra_3 = 8.314[21{,}294]$). The temperature compensated failure times are therefore given by $\ln\left[t_f^d\right] = \ln[t_f] - 21{,}294[1/T - 1/T_d]$ and Figure 2a plots these adjusted values against ln[-

$\ln(\sigma/\sigma_{TS})]$ using T_d = 773K. The percentage variation in $\ln[t_f]$ that can be explained by variations in 1/T and $\ln[-\ln(\sigma/\sigma_{TS})]$ is given by the R^2 value and is just under 70%.

2 Variation of the temperature compensated failure time with $\ln[-\ln(\sigma/\sigma_{TS})]$ using Qc = 177 $kJmol^{-1}$

Figure 2 reveals a problem with the Wilshire model in its simplest form, which is also true for all the parametric models summarised in Table 1. Namely, that the data on this plot should define a straight line given by equation (3c) (scatter aside), but it is clear from Figure 2 that $\ln(t^d_f)$ is not varying in a linear fashion with $\ln[-\ln(\sigma/\sigma_{TS})]$.

The Wilshire model with one partition (W_{1p})

The traditional solution to this problem is to hypothesis that over the test conditions shown, there are multiple creep mechanisms in operation, and it is this that leads to the observed non-linearity[19-21]. This solution is illustrated in Figure 3a where it is hypothesised that there is a different mechanism at work above and below a normalised stress of 0.49. This critical normalised stress was the one which gave the smallest value for Σe^2 over both creep regimes. Then equation (3b) applies above and below this critical point, but the values for a_0 to a_3 above this normalised stress will be different to those below it. Figure 3a shows the resulting kinked linear relationship based on:

If $\sigma/\sigma_{TS} > 0.49$

$$\ln[t_f] = \underset{\{-8.98\}}{-18.207} + \underset{\{7.29\}}{2.515\ln[-\ln(\sigma/\sigma_{TS})]} + \underset{\{16.59\}}{26{,}520[1/T]}$$

If $\sigma/\sigma_{TS} \leq 0.49$

$$\ln[t_f] = \underset{\{-8.08\}}{-16.457} + \underset{\{11.89\}}{7.543\ln[-\ln(\sigma/\sigma_{TS})]} + \underset{\{16.56\}}{26{,}520[1/T]} \qquad (5)$$

The estimated parameters shown in equation (5) are based only on the data points where t_f < 10,000 h. The implied activation energy for both creep mechanisms is 220 $kJmol^{-1}$ and the overall R^2 value is 89.64%, which is higher than the model without a break and the statistical significance of this non-linearity is further confirmed by the student t values associated with a_0 and a_1. It should be stressed that this abrupt break is just a simplification and in reality there is likely to be a gradual transition from one creep mechanism to another around the normalised stress of 0.49 and hence the true relationship in Figure 3a is a smooth non-linear one.

Equation (5) can be used to predict both the failure times used to estimate the parameters of this model (interpolation) and the failure times not used in such estimation (extrapolation). Figure 3b plots these predictions and the W_{1p} column of Table 2 shows the mean absolute percentage error made in interpolation and extrapolation at each temperature. Quite a complicated picture emerges. There is tendency at all temperatures (except at 748K) for the prediction errors to be the same or much larger in extrapolation compared to interpolation. This clearly suggest the model is not correctly specified in terms of stress – as the stress gets smaller the models prediction errors are getting bigger. Secondly, the model

extrapolates with smaller errors at intermediate temperatures. This implies the model is also incorrectly specified in terms of temperature.

3 (a) Variation of $\ln(t_f) - Q_c(1/RT)$ and the W_{1p} model predictions thereof with $\ln[-\ln(\sigma/\sigma_{TS})]$, (b) Predicted times to failure at different stresses and temperatures for the W_{1p} model

Table 2 The mean absolute percentage prediction error made over all stresses at different temperature using various models for creep life

The Wilshire model with two partitions (W_{2p})

Figure 3a suggests one possible reason for this poor predictive capability – namely that for this data there is a second partition, with each partition having a separate creep mechanism or regime. This idea is supported by previous work carried out by Wilshire and Whittaker[22]. For this material, these authors suggest dislocation creep processes are rate controlling at all stress levels, even though the detailed dislocation processes vary in different stress regimes. When $\sigma > \sigma_Y$, creep is controlled by the generation and movement of dislocations within the grains. In contrast, when $\sigma < \sigma_Y$, new dislocations are not generated within the grains. Instead, creep occurs within the grain boundary zones, i.e. by grain boundary sliding or diffusion along existing dislocations and grain boundaries with associated deformation in the grain regions adjacent to the boundaries. Another change in creep and creep rupture behaviour occurs when σ approximately equals $0.2\sigma_{TS}$. With this material, the original ferrite/bainite microstructure degrades to ferrite and molybdenum carbide particles in long term tests at the highest creep temperatures, with very coarse carbide particles forming along the grain boundaries. This carbide coarsening reduces creep strength in the matrix allowing diffusion to occur within the grains once again. These authors have provided similar explanations for the observed breaks in other power generating materials as well.

These critical normalised stresses given in equation (6) were the ones which gave the smallest value for Σe^2 over all creep regimes. Then equation (3b) applies either side of these critical points, but the values for a_0 to a_3 either side of these normalised stresses will be different. Figure 4a shows the resulting multi kinked linear relationship based on:

If $\sigma/\sigma_{TS} > 0.47$

$$\ln[t_f] = \underset{\{-9.99\}}{-18.658} + \underset{\{8.40\}}{2.581}\ln[-\ln(\sigma/\sigma_{TS})] + \underset{\{18.19\}}{26{,}905}[1/T]$$

If $0.47 \geq \sigma/\sigma_{TS} > 0.20$

$$\ln[t_f] = \underset{\{-9.00\}}{-16.886} + \underset{\{12.97\}}{8.642}\ln[-\ln(\sigma/\sigma_{TS})] + \underset{\{18.19\}}{26{,}905}[1/T]$$

If $\sigma/\sigma_{TS} \leq 0.20$

$$\ln[t_f] = \underset{\{-7.11\}}{-14.379} + \underset{\{1.81\}}{3.212}\ln[-\ln(\sigma/\sigma_{TS})] + \underset{\{18.19\}}{26{,}905}[1/T] \quad (6)$$

The estimated parameters shown in equation (6) are based only on the data points where $t_f < 10{,}000$ h. The implied activation energy for all creep mechanisms is 224 $kJmol^{-1}$ and the overall R^2 value is 91.50%, which is a little higher than the model with a single partition and the statistical significance of this additional partition is further confirmed by the student t values associated with a_0 and a_1 (for example, the t statistic on a_2 when $\sigma/\sigma_{TS} \leq 0.20$ suggests this parameter is not significantly different for zero, but is statistically significant over the other stress ranges). Again, it should be stressed that these abrupt breaks are just a simplification and in reality there is likely to be a gradual transition in creep mechanism around the critical normalised stresses of 0.2 and 0.47 and hence the true relationship in Figure 4a is a smooth non-linear one.

Equation (6) can be used for interpolation and extrapolation. Fig. 4b plots these predictions and Table 2 shows the mean absolute percentage error made in interpolation and extrapolation at each temperature. At temperatures of 798K or less the W_{1p} and W_{2p} have similar errors in interpolation, but at temperatures of 873K or more, the W_{2p} model has much better performance in interpolation. But the main benefit accruing from the W_{2p} model is in extrapolation where at temperatures of 773K and above, this model predicts times to failure with very much reduced errors (whilst at lower temperatures the W_{2p} is not worse than the W_{1p} model).

4 (a) Variation of $\ln(t_f) - Q_c(1/RT)$ and the W_{2p} model predictions thereof with $\ln[-\ln(\sigma/\sigma_{TS})]$, (b) Predicted times to failure at different stresses and temperatures for the W_{2p} model

Semi -Parametric Wilshire model $W_{sp\text{-}nw}$: Nadaraya & Watson

The problem with the above models is that they are approximations to the true relationship between the temperature compensated times to failure and $\ln[-\ln(\sigma/\sigma_{TS})]$ and so there could be gains in predictive accuracy in overcoming the need for such an approximation. For example, it is not clear how many critical normalized stresses (i.e. creep regimes) are needed for the approximation to correctly depict how the data actually behave. One solution to this problem is to pursue a non-parametric estimation procedure, and the technique proposed by Nadaraya[23] (and also independently by Watson) is a very early example on non-parametric estimation.

Method

In this model it is assumed that the relationship between the log time to failure and temperature is well represented by the W model described in the introduction, but that the stress relationship is not as described by this model. So instead of trying to approximate the unknown functional form of the non-linear relationship between $\ln[t^d_f]$ and $\ln[-\ln(\sigma/\sigma_{TS})]$ as seen in Figure 4a by segmented linear lines (where the correct number of required segments is not known), a smoothed curve can be used instead. When using non-parametric techniques this smoothed curve needs no equation to define its shape but instead the data itself determines the shape of the stress relationship and so should provide better interpolations/extrapolations.

To reflect the unknown relationship in Figure 4a, the right hand side of equation (3c) is set equal to f(x) which then enables equation (3e) to be written as

$$\ln\left[t_f^d\right] = \ln[t_f] - a_3[1/T - 1/T_d] = f(x) \qquad (7a)$$

where $x = \ln[-\ln(\sigma/\sigma_{TS})]$. The functional form of f(x) is unknown but one non-parametric approach to finding f(x) is to calculate a weighted average of the i = 1 to N values for $\ln[t^d_f]$,

where the weights are given by a Kernel functions. This is the basis of the Nadaraya-Watson[23] Kernel estimator

$$y_0 = f(x_0) = \frac{\sum_{i=1}^{N} K(x_i - x_0) y_i}{\sum_{i=1}^{N} K(x_i - x_0)} \quad (7b)$$

where y_0 is the value for $\ln[t^d_f]$ obtained when $x = x_0$ and where there are $i = 1$ to N values for x and y. A number of different functional forms for the Kernel K() have been proposed, but the most commonly used one is based on the Gaussian function given by

$$K(x_i - x_0) = e^{-(x_i - x_0)^2 / 2\delta^2} \quad (7c)$$

where δ is termed the band width. This band width essentially determines the degree of smoothing, such that larger values for δ lead to a smoother curve for f(x). An optimal value for δ, in the sense that it minimises the mean integrated squared error (e_0), has been proposed by Silverman[24] that is given by

$$\delta = 0.7(Q_3 - Q_1)/N^{0.2} \quad (7d)$$

where Q_3 and Q_1 are the upper and lower quartile values respectively for the N values for x (Silverman points out that this optimality is not universal and requires log failure times to follow a normal distribution). Given a value for the parameter a_3, equation (7b) is fully non-parametric in nature in that no parameters require estimation as δ is given by equation (7d). More recently, Hall and Turlach[25] have proposed a weighted version of this Kernel estimator

$$y_0 = f(x_0) + e_0 = \frac{\sum_{i=1}^{N} w_i(x_0) K(x_i - x_0) y_i}{\sum_{i=1}^{N} w_i(x_0) K(x_i - x_0)} + e_0 \quad (8)$$

where e_0 is a random error term to capture the scatter around the smooth curve $f(x_0)$ and the N weights at each value for x_0, $w_{i(x_0)}$, sum to unity. These weights are chosen so that Σe_0^2 is minimised subject to this unity constraint. Equation (8) is strictly speaking a semi-parametric regression because its application requires the joint estimation of the parameter a_3 in equation (7a) and all the weights.

One characteristic of this approach is that for a given temperature, creep life predictions can only be made within the limits of the smallest and largest values for $\ln[-\ln(\sigma/\sigma_{TS})]$. Because parameter estimation was restricted to data associated with failure times below 10,000 h these limits correspond to -1.22 to 0.75 for $\ln[-\ln(\sigma/\sigma_{TS})]$ or 0.12 to 0.74 for σ/σ_{TS}. This prediction boundary depends on temperature through the temperature dependency of the tensile strength of 2.25Cr - 1Mo steel. Over the range of temperatures within the NIMS data sheet 3B this relationship is approximately linear, so that the prediction boundary associated with the lowest stresses is given by

Lower stress prediction boundary $\approx 0.12(c_0 - c_1 T)$ where $c_0 = 1360.6$ K and $c_1 = 1.2312$

Application

Figure 5(a) shows the optimal weights for equation (8) at each and every value for $x_0 = \ln[-\ln(\sigma/\sigma_{TS})]$, where based on equation (7d), $\delta = 0.5$. It can be seen that the most important

Kernels in determining $\ln\left[t_f^d\right]$ occur at x_0 = 0.75 and x_0 = -1.14, i.e. $K(x_i - 0.75)$ and $K(x_i + 1.14)$. Figure 5b plots out these two Kernels together with the weighted average of these two Kernels based on their weights of 0.23 and 0.18 as shown in Figure 5a (this weighted average assumed all the other Kernels equal zero and the weights sum to 0.41). This weighted average Kernel is very different in shape to its constituent parts and it is this that enables very complex and flexible shapes to be derived for f(x) based on this non-parametric technique.

5 (a) Variation of the Kernel weights $w_{i(x_0)}$ with $\ln[-\ln(\sigma/\sigma_{TS})]$, (b) the Kernels associated with the largest two weights together with their weighted average

Taken into account all the Kernels associated with each value for x_0 and their corresponding weights, gave an estimated value for a_3 of 28,718 K implying an activation energy of 239 kJmol^{-1}

$$\ln\left[t_f^d\right] = \ln[t_f] - 28{,}718[1/T - 1/773] = f(x) \qquad (9a)$$

and where the resulting function f(x) is shown by the dashed curve in Figure 6a.This curve fits the failure times below 10,000 h very well (above the lower stress prediction boundary) without the need to quantify the number of, and then estimate the number of, segmented lines to approximate this curve. The dashed curve in Figure 6a (i.e. f(x)) in combination with equation (9a) enables predictions to be made for $\ln\left[t_f^d\right]$ - but only for those values of x = $\ln[-\ln(\sigma/\sigma_{TS})]$ within the prediction boundary – namely x = -1.215 to x = 0.750. Then predictions for $\ln[t_f]$ at these x values but at any temperature can be made using

$$\ln[t_f] = \ln\left[t_f^d\right] + 28{,}718[1/T - 1/773]) \quad ; \quad \text{for } x = -1.215 \text{ to } x = 0.750 \text{ only} \qquad (9b)$$

These predictions are shown in Figure 6b by the dashed curves, the end points of which define the lower stress prediction boundary. Thus, at 873K it is not possible to predict the failure times associated with specimens tested at 29, 22 and 14 MPa. That aside, the predictions look visibly superior to those obtain using the W_{2p} model in Figure 4b. This is confirmed in the last column of Table 2 where the extrapolative prediction errors from the $W_{sp\text{-}nw}$ are lower than from the W_{2p} model at all temperatures (especially so at 723K and 773K). For interpolation, the picture is more mixed with the W_{2p} model performing noticeably better at 798K and 823K, with the $W_{sp\text{-}nw}$ model performing either better than or the same as the W_{2p} model at all the other temperatures.

6 (a) Variation of $\ln(t_f) - Q_c(1/RT)$ and f(x) with $\ln[-\ln(\sigma/\sigma_{TS})]$, (b) Predicted times to failure at different stresses and temperatures for the $W_{sp\text{-}nw}$ model

Conclusions

This paper introduced a semi-parametric estimation methodology for the Wilshire model for times to failure at high temperatures. The paper also compared the predictions made using this approach to those obtained using more traditional approaches to modelling and estimation within the Wilshire methodology. When these techniques were applied to 2.25Cr-1Mo steel the following conclusions could be drawn

1. For this material it is not immediately clear from graphical presentations of the data, exactly how many partitions, and so different creep regimes, are required to adequately capture the pattern of the relationship between $\ln\left[t_f^d\right]$ and $\ln[-\ln(\sigma/\sigma_{TS})]$
2. This problem can be overcome by using a semi-parametric estimation procedure to identify the non-linear relationship between $\ln\left[t_f^d\right]$ and $\ln[-\ln(\sigma/\sigma_{TS})]$, f(x).
3. Because f(x) was used for making failure time predictions, such predictions were constrained to be within the lower stress prediction boundary. This limit can be interpreted as the limit of valid extrapolation. The boundary prevents extrapolations into stress ranges beyond that used for estimating the model's unknown parameters and so avoids the possibility of trying to make predictions where creep regimes not present in parameter estimation actually apply. Once a model has been validated by using sub sets of data, the full data that is available would be used to estimate the model and predictions made over the then enlarged prediction boundary. The presence of a prediction boundary then places more importance on the actual design of an accelerated test program – if the aim is to obtain life predictions at an operating stress for a materials, then the test program must include specimens tested at this stress with an accelerated temperature that will induce failure in a short time period (e.g. around 5,000 hours).
4. The semi-parametric version of the Wilshire model resulted in extrapolative prediction errors that are lower than from the partitioned Wilshire models at all temperatures (especially so at 723K and 773K). For interpolation, the partitioned Wilshire model performs noticeably best at 798 and 823K, with the semi-parametric version of the model performing either better than or the same as the partitioned model at all the other temperatures.
5. The shape of the predicted isothermal lines is smooth and do not have the abrupt discontinuities present in the partitioned models – a pattern of behaviour that is not a realistic description of actual creep behaviour.

Important areas for future research include applying this semi-parametric version of the Wilshire model to i. other high temperature materials and ii. to other approaches for compensating failure times for temperature. It may also be worth investigating other non-parametric and semi-parametric estimation procedures such as for example a locally weighted scatterplot smoothing (LOESS) curve, along the lines first proposed by Cleveland and Devlin[26].

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Table 1 Summary of some popular parametric creep models for predicting time to failure

Model Name	Single Comprehensive Equation
Orr-Sherby-Dorn[10] (OSD)	$\ln(t_f) = a_0 + a_1\ln(\sigma) + a_3\frac{1}{T}$
Larson-Miller[9] (LM)	$\ln(t_f) = a_0 + a_3\frac{1}{T} + a_4\frac{\ln(\sigma)}{T}$
Minimum Commitment[13] (MC)	$\ln(t_f) = a_0 + a_1\ln(\sigma) + a_2 T + a_3\frac{1}{T} + b_1\sigma + b_1\sigma^2$
Manson and Haferd[14] (MH)*	$\ln(t_f) = a_0 + a_1 f(\sigma) + a_4\frac{f(\sigma)}{T}[T - a_5]$
Manson and Brown[15] (MB)*	$\ln(t_f) = a_0 + a_1 f(\sigma) + a_4\frac{f(\sigma)}{T}[T - a_5]^{a_6}$
Soviet Model 1[8] (SM1)	$\ln(t_f) = a_0 + a_2\ln(\sigma) + a_3\frac{1}{T} + a_4\frac{\sigma}{T}$
Wilshire Model[11] (W)	$\ln(t_f) = a_0 + a_1\ln[-\ln(\sigma/\sigma_{TS})], + a_3\frac{1}{T}$
Yang *et al.*[12] Model (Y)	$\ln(t_f) = a_0 + a_1\ln[\sigma/(\sigma_{TS} - \sigma)] + a_3\frac{1}{T}$

*In these models $f(\sigma)$ is a polynomial function of stress. ln stands for natural log. t_f is the time to failure, T the test temperature, σ the test stress and σ_{TS} the tensile strength. a_0 to a_4 are model parameters that required estimation and have different meanings and values in each of the models.

Table 2 The mean absolute percentage prediction error made over all stresses at different temperature using various models for creep life

		Mean Absolute Percentage Error in prediction for various models:		
	Model:	W_{1p}	W_{2p}	$W_{sp\text{-}nw}$
723K	Interpolate	47.2%	46.6%	50.4%
	Extrapolate	**79.0%**	**79.1%**	**68.1%**
748K	Interpolate	42.8%	41.5%	22.9%
	Extrapolate	**11.3%**	**13.9%**	**11.7%**
773K	Interpolate	20.3%	24.4%	5.9%
	Extrapolate	**28.7%**	**15.4%**	**9.0%**
798K	Interpolate	80.2%	76.2%	103.0%
	Extrapolate	**-**	**-**	**-**
823K	Interpolate	49.6%	63.0%	86.4%
	Extrapolate	**48.4%**	**23.9%**	**16.2%**
873K	Interpolate	47.0%	37.1%	28.0%
	Extrapolate	**69.5%**	**24.7%**	**19.4%**
923K	Interpolate	48.2%	16.9%	18.3%
	Extrapolate	**55.4%**	**26.6%**	**26.9%**

W_{1p} is the Wilshire model with one partition, W_{2p} is the Wilshire model with two partitions and $W_{sp\text{-}nw}$ is the semi-parametric version of the Wilshire model. Interpolate refers to all test conditions leading to failure times less than 10,000 hours and extrapolate refers to all test conditions leading to failure times of more than 10,000 hours. The absolute percentage error at a particular test condition is calculated as {|predicted failure time - experimental failure time|}/experimental failure*100, where || stands for the absolute value. The Mean Absolute Percentage Error in prediction at a stated temperature is then found by adding up the absolute percentage error at all given stress conditions at that temperature and then averaging out.

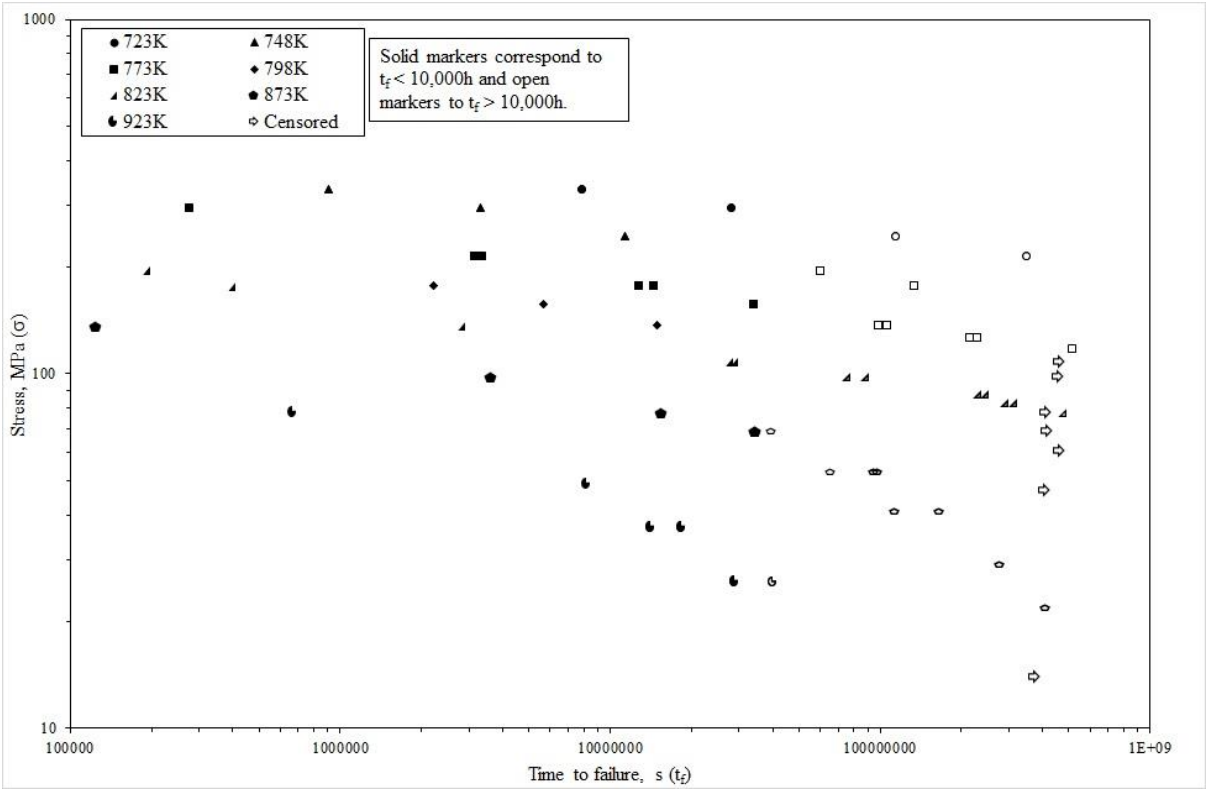

Figure 1.

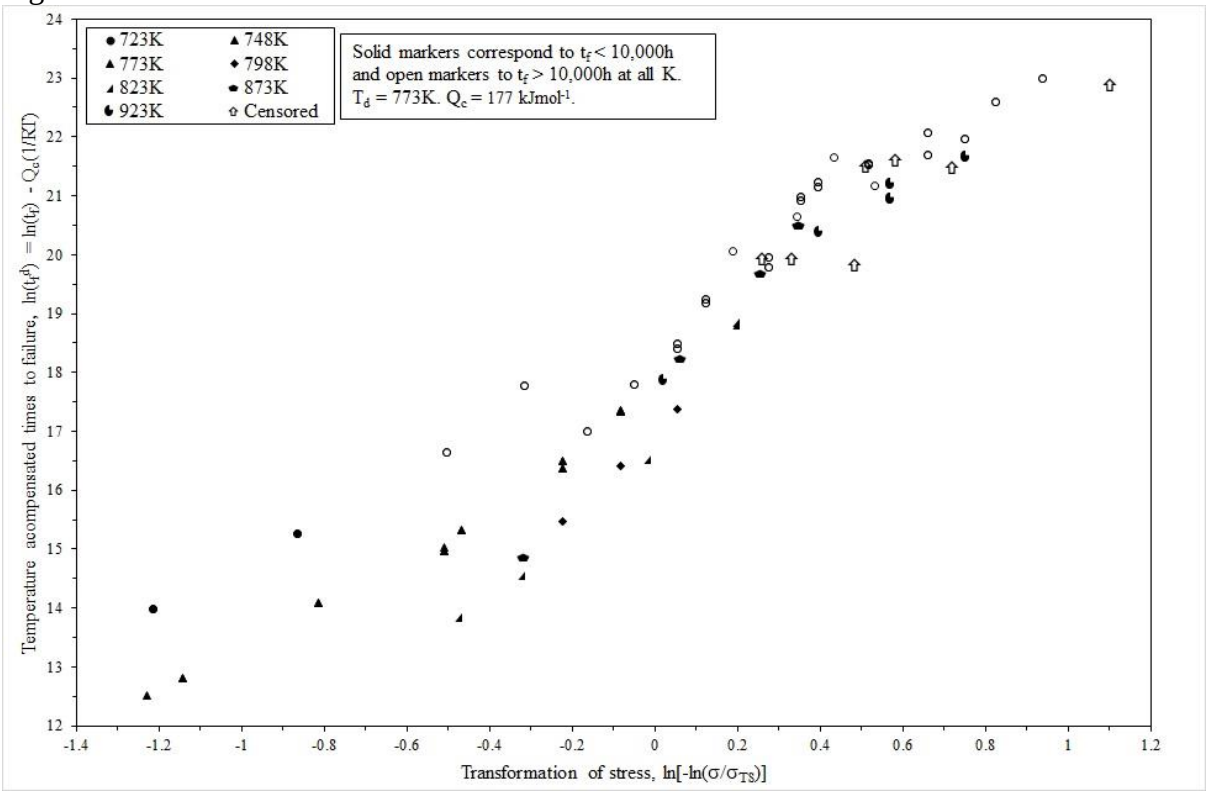

Figure 2

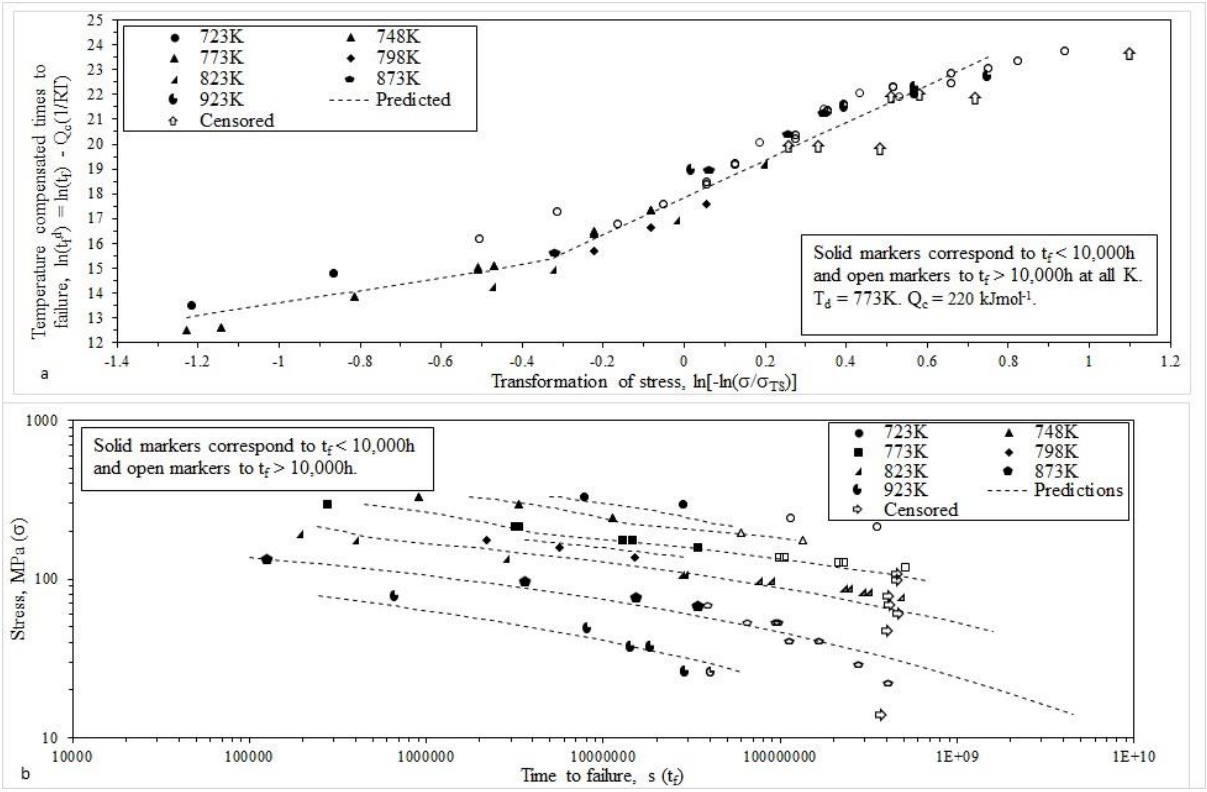

Figure 3

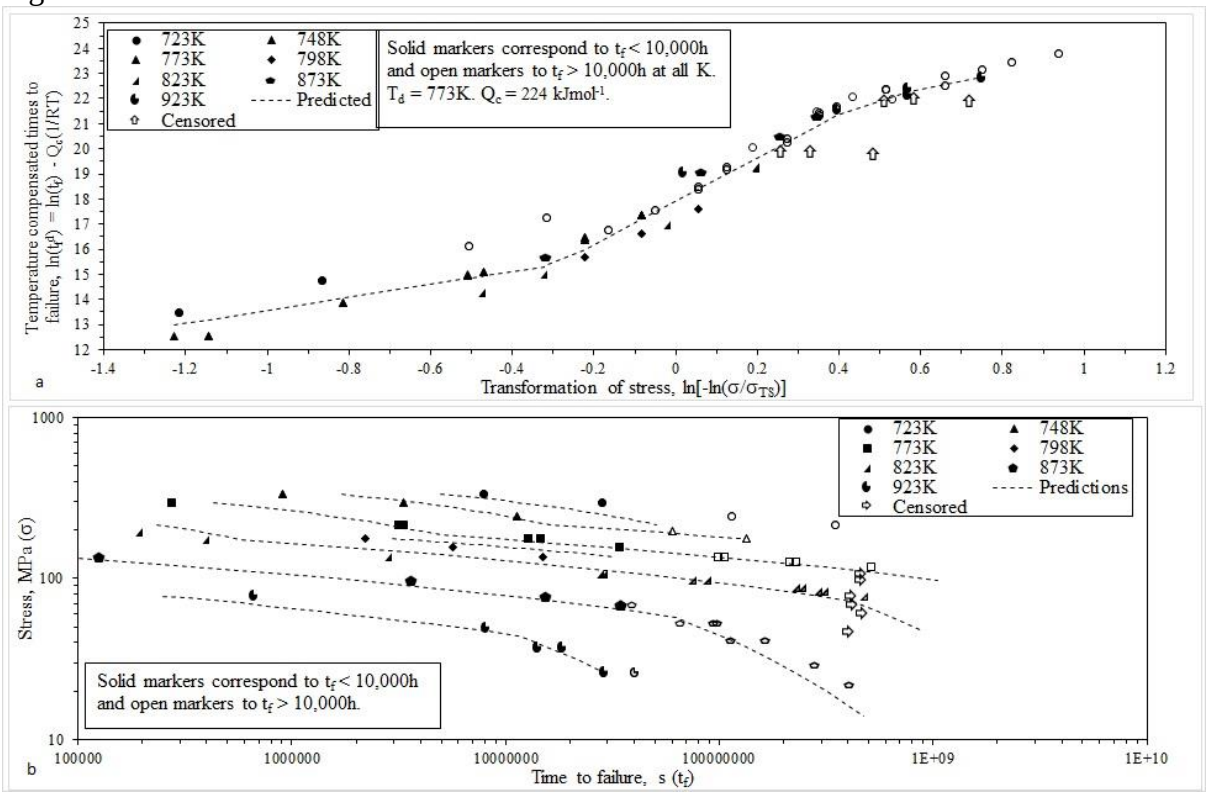

Figure 4

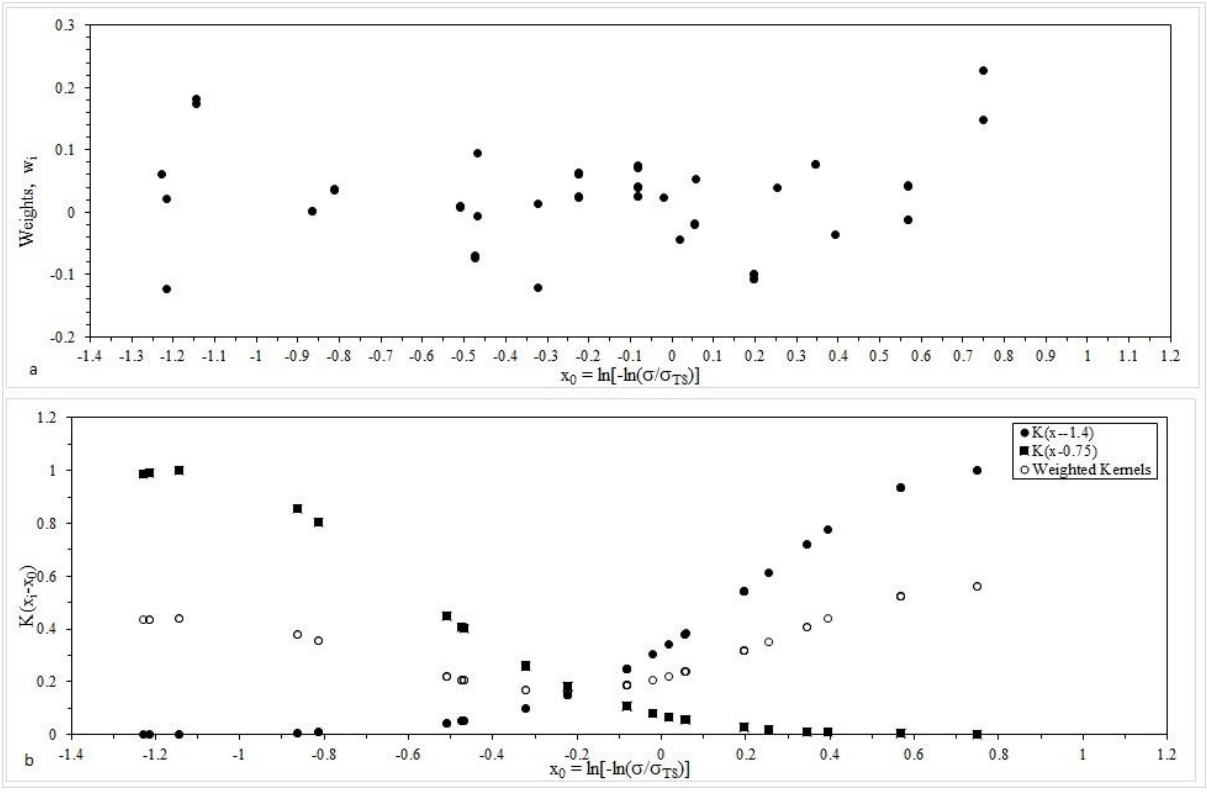

Figure 5

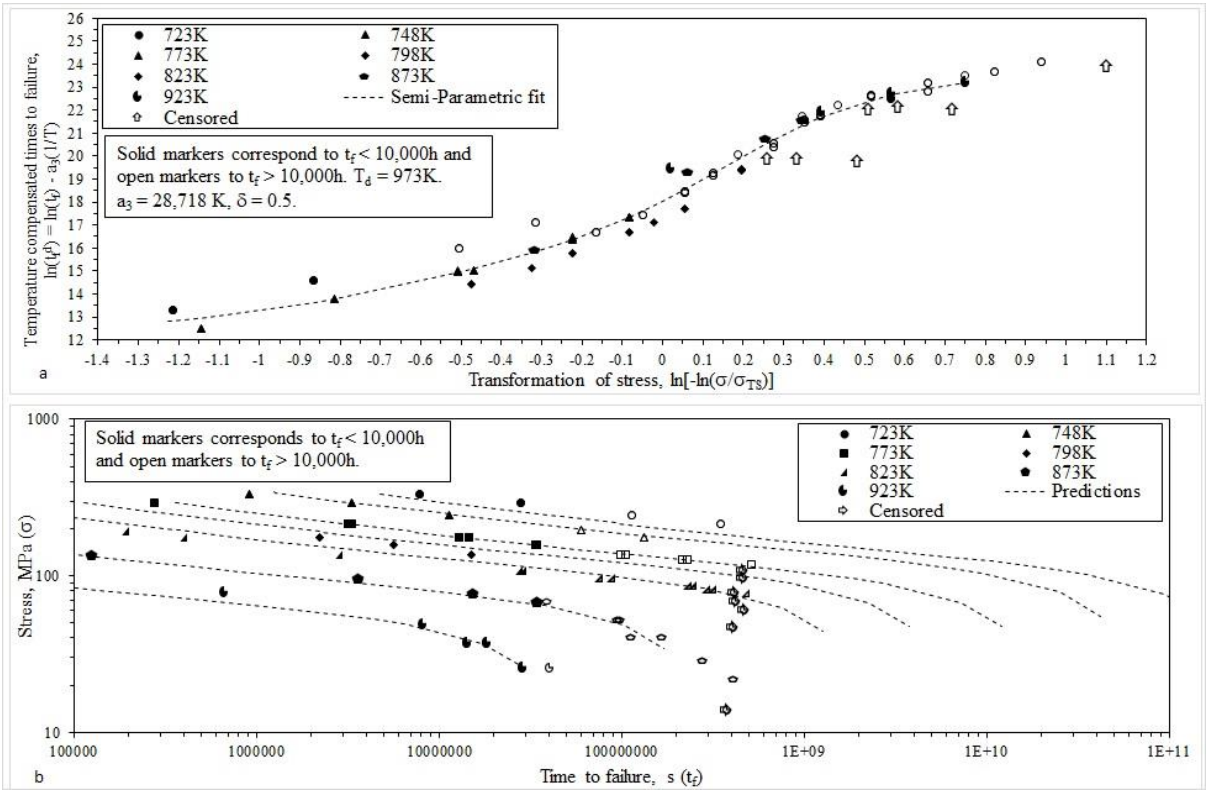

Figure 6