Union objectives, monopolistic competition and macroeconomic externalities.

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UNION OBJECTIVES, MONOPOLISTIC COMPETITION AND MACROECONOMIC EXTERNALITIES

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SUMMARY

This thesis extends a recent vein of literature which investigates the implications of wage decisions for the optimal design of the monetary regime when the goods market is monopolistically competitive. Chapter II abstracts from the existence of stochastic shocks and examines in detail the resulting macroeconomic wage-setting externality. It points out that when the monetary instrument is set after wage determination, there exists a monetary rule which can induce non-atomistic unions to set the market-clearing nominal wage, and which therefore is fully credible. This result is shown to be implicit in (and to have been overlooked by) previous contributions, and its sensitivity to the precise formulation of the union objective function is discussed. Chapter III allows for stochastic shocks, and in particular builds upon Herrendorf and Lockwood (1997) by assuming that each union receives a (common) noisy signal of a productivity shock prior to setting its individual wage. The adverse macroeconomic externality which arises in this scenario is analysed, and the circumstances under which improvements in signal quality are detrimental to union welfare are identified. It is shown that when unions are non-atomistic, the externality's strength, and hence also both employment variability and the stochastic inflation bias, are sensitive to the specification of the authorities' monetary reaction function. This implies a candidate explanation for why greater central-bank conservatism has generally not been found to be associated with greater output variability. The optimal central-bank delegation arrangements are identified, as well as the optimal amount of transparency regarding supply shocks. Chapters V and VI then analyse the externalities appertaining to indexation decisions taken in the absence of informative signals, both for the standard scenario in which the wage is indexed only to the price level, and for multiparameter indexation scenarios in which the wage is also contingent on a second aggregate variable.

DECLARATION

This work has not been previously accepted in substance for any degree and is not being concurrently submitted in candidature for any degree.

Signed ....

Date..........4 February 2007..........

STATEMENT 1

This thesis is the result of my own investigations, except where otherwise stated. No correction services whatsoever have been used in its preparation. Other sources are acknowledged by explicit references. A bibliography is appended.

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STATEMENT 2

I hereby give consent for my thesis, if accepted, to be available for photocopying and for inter-library loan, and for the title and summary to be made available to outside organisations.

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Preface

The research findings presented in Chapters III, V and VI below are the fruits of research undertaken jointly with my supervisor, Dr. Phillip Lawler. These chapters consider a model economy featuring unions, a monopolistically competitive goods market and stochastic shocks. Several working papers and, at the date of submission of this thesis for examination, two publications have sprung from this research effort. James and Lawler (2004a) is a short paper which discusses the wage-setting externality which arises when unions receive noisy signals of productivity shocks and the money supply is kept fixed, while its companion piece, James and Lawler (2004b), analyses the effect of a change in signal quality in this scenario. These papers therefore report and concisely explain the key results of Section 3 of Chapter III of this thesis. A similar task is performed in respect of the findings presented later in Chapter III for an activist monetary policy scenario by James and Lawler (2005a) and James and Lawler (2006a). The former of these two papers was presented at the 2005 MMF Research Group conference in Rethymno, Crete, and is concerned with 'economic transparency' (in other words, the disclosure by the central bank of its private information regarding supply shocks). The latter paper was presented at the 2006 ATINER conference in Athens: its focus is on the implications of the externality for the relationship between the central bank's weight parameter and employment variability. A further working paper, James and Lawler (2005b) sets out the findings of Chapter V below for scenarios in which each union chooses the degree of indexation of its wage to the price level. Finally, Chapter VI undertakes a similar investigation for the case of multiparameter indexation: its findings have found their way into print as James and Lawler (2006b).

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Key to Abbreviations

ATINER: Athens Institute for Education and Research

CES: Constant Elasticity of Substitution

i.i.d.: independent and identically distributed

iff: if and only if

MMF: Money, Macro and Finance

Chapter I

Wage-Setting Externalities and the Optimal Design of the Monetary Regime: Introduction and Literature Review

I.1 Introduction

This thesis is concerned with the relationships between three themes which are present in the macroeconomic literature. The merits and demerits of rule-based regimes considered relative to the performance of discretionary policymaking, is the first of these themes. The second is the relationship between macroeconomic outcomes and the degree of informational asymmetry between the policymaker and private-sector agents, while the third concerns the macroeconomic externalities which appertain to the decisions of such agents. Their inter-relationship is studied by means of a model economy which is subject to stochastic shocks to productivity and aggregate demand, and which features a unionised labour force employed by monopolistically competitive firms which produce differentiated products. An analysis is undertaken of the nature of the externalities arising from union wage-setting and indexation decisions, and the conditions which give rise to these externalities are identified, with particular attention being devoted to the role played in their genesis by the economy's information structure. An investigation is conducted into both the welfare repercussions of these externalities, and the implications of their existence for the conduct of monetary policy and the optimal design of the monetary regime.

One of the principal tasks of this chapter is to set the scene for our subsequent analysis by providing some background in respect of the aforementioned three major themes. This is done in sections I.2 to I.4, which describe in broad terms the development of the literature on monetary policy since the advent of rational expectations, with particular attention being paid to the important issues of (alleged) policy ineffectiveness, time inconsistency and delegation. Section I.5 provides more detailed discussion of issues which will figure prominently as aspects of the model developed in the thesis itself. Specifically, an account is given of the literature's approach to modelling monopolistic competition, as well as of the assumptions typically made in macroeconomics regarding unions' objectives in making decisions.

The survey then proceeds to discuss the role played by these issues in the evolution of the monetary policy literature. (Note that the focus here is primarily on models in which private sector agents set nominal wages, rather than the degree of wage indexation, since preparatory to Chapters V and VI, a separate survey of the macroeconomic literature on wage indexation is provided later as Chapter IV.) Considerable attention is paid to models of strategic interaction between the monetary authorities and large unions which recognise that their decisions have a non-negligible macroeconomic impact, and a fairly detailed account is given of the model of Herrendorf and Lockwood (1997), a modified version of which forms the basis of the analysis conducted in Chapter III below. A briefer penultimate section discusses the current state of the literature on transparency with regard to macroeconomic disturbances, and Section I.8 then concludes.

I.2 A Brief Outline of Rational Expectations Models of Monetary Policy

By definition, a rational expectation makes full use of every aspect of the available information set, and hence implies that expectational errors have a zero mean and are uncorrelated with any item of information available to the forecaster. The seminal contribution of Lucas (1972) showed that the empirically well-attested short-run Phillips-curve relationship between inflation and unemployment can be explained in terms of the optimising behaviour of overlapping generations of imperfectly informed private agents who form rational expectations conditional on their information about the current state of the economy. In Lucas' model, each optimising producer, in deciding how much to produce and trade, infers the aggregate price level from the prevailing market price of his particular good. Rational expectations-formation therefore involves application of the statistical technique of signal-extraction. Lucas' principal point is that when expectations are formed using a methodology of this kind which exploits all available information, a temporarily large increase in the money supply, and the induced increase in aggregate demand and hence in the particular price observed by each agent, leads imperfectly informed traders to attribute (rationally) the strength of demand in their particular market to a favourable demand shock in respect of their good, which therefore induces them to increase their supply of that good even when no such favourable shock has occurred. Consequently, the supply side of the economy exhibits a relationship of the following form between (log) output, y, and the (average) expectational error in respect of the (log) price level, $p - Ep$:[1]

$$y = y_N + a(p - Ep), \qquad a > 0 \qquad (1a)$$

where y_N is the natural level of (log) output, a is the parameter which governs the responsiveness of aggregate supply to the average expectational error in respect of p, and E is the rational-expectations operator. Equation (1a) is the Lucas surprise-supply function, and it implies an associated Phillips-curve relationship between unemployment (U), its natural rate (U_N), and expectational errors regarding inflation ($\pi - E\pi$):

$$U = U_N + b(\pi - E\pi), \qquad b < 0 \qquad (1b)$$

The Lucas model thus provides a theoretical rationale for why policymakers may, by implementing an inflationary surprise, be able to cause output temporarily to depart from the natural level.

The central implication for policy design of the Lucas model, and in particular its assumptions of rational expectations and continuously clearing markets, is summarised in the policy-ineffectiveness proposition of Sargent and Wallace (1975) and Barro (1976). This holds that the non-stochastic component of the money supply cannot systematically affect the distribution of that component of output which relates to the private sector's reactions to the information it does possess at the time prices are set. The formulation of the monetary rule only matters for output insofar as those in charge of the conduct of policy possess superior information regarding macroeconomic disturbances.[2] Furthermore, if it is assumed that the policymaker's

[1] Throughout the thesis, upper-case letters are used to denote variables in levels, while lower-case letters are generally used to denote their (natural) log counterparts.

[2] Barro concedes that were his model modified so that private agents are not informed about the (potentially arbitrary) specification of the rule (i.e. the monetary reaction function) on the date at which it is first adopted, the rule's specification would then matter for the behaviour of output in the short run during which agents gradually acquire information from which the specification may be inferred. However, as Barro emphasises, private-sector ignorance of the rule specification will be an ephemeral phenomenon, since under rational expectations the component of the variance of private-sector forecast errors regarding the price level which arises from ignorance of the rule coefficients will be rapidly eliminated. Furthermore, this learning process only becomes an issue if the policymaker's ultimate

objective is to minimise the variance of output about its full-information level,[3] an important further implication is that the policymaker need not practise activist monetary policy in order to stabilise the economy as desired, since this can alternatively be achieved by disclosing to the public, at the time output and purchase decisions are made, the authorities' superior information regarding the current values of aggregate variables.

A related elaboration of this point is the Lucas (1976) critique of the use of statistical relationships between aggregate variables to guide policy decisions. Under rational expectations, the responsiveness of aggregate supply to price-level movements depends on how individual product prices covary with the aggregate price level, and this covariance is recognised by private agents to be a function of the prevailing policy regime. Only naïve agents would expect the correlation patterns which have existed in the past between nominal variables to continue to hold after a change of policy regime. Such naivety is entirely precluded when agents form rational expectations, and hence the Lucas critique predicts that systematic attempts to exploit past statistical relationships will lead to their rapid disappearance.

While space constraints preclude a comprehensive discussion of the voluminous literature which stemmed from these important early new-classical contributions, it is fair to say that the principal focus of this research effort has been to investigate the implications of relaxing the new-classical assumption that all markets continuously clear. In other words, rational-expectations models have had incorporated into them structural features which give rise to nominal rigidities, and which therefore create the potential for policy to be non-neutral even when the policymaker lacks an informational advantage over the public. Examples include overlapping multi-period wage contracts (Fischer, 1977a; Taylor, 1979), and menu costs (Mankiw, 1985; Akerlof and Yellen, 1985). While nominal wage rigidities play an important role in the literature surveyed later in this chapter, our immediate concern must be to describe the impact of the rational-expectations concept on the debate over the wisdom of constraining the monetary authority to conduct its operations in accordance with a rule.

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objectives which determine the choice of rule are unknown to the private sector. If the policymaker's objective function is known to private agents, they will always infer the optimal choice of rule without error.

[3] In other words, it is assumed to be socially desirable that output in the representative market be at the level it would take were the participants in that market fully informed about the price level when making their demand and supply decisions.

I.3 Rules versus Discretion

I.3.1 Introductory Remarks

Prior to the nineteen-seventies such vociferous advocates of rules as Simons (1948) and Friedman (1962) failed, in general, to devise a compelling set of arguments in their favour. The only such argument of any substance identified by these authors is that a rule has the potential advantage of insulating policymakers from lobbying by politicians and private interest-groups who stand to gain, at least in the short term, from monetary policy being conducted in a particular fashion. (Note that underpinning this argument is the implicit assumption that rules, once constitutionally enacted, are inviolable.) As Barro (1986) and Fischer (1990) point out in their authoritative surveys, Friedman's other arguments for requiring the central bank to adhere to a monetary rule have little to recommend them, since they do not directly (and possibly do not even indirectly) concern the macroeconomic performance of the rule, and instead inappropriately assume that the welfare of society's members is necessarily contingent on the settings of monetary instruments, rather than on macroeconomic outcomes.[4]

I.3.2 Time Inconsistency

It is generally agreed that the most persuasive argument in favour of rules received its first clear articulation in the macroeconomic literature in the work of Kydland and Prescott (1977).[5] The key idea of their seminal contribution is that optimal plans which stipulate in advance the future setting of a choice-variable, may be time-inconsistent if the person endowed with the responsibility for implementing the plan has the discretion subsequently to alter the stipulated setting.[6] The concept of time-inconsistency is of such central importance to the remainder of this literature review, and indeed to the thesis as a whole, that a precise definition is called for. Accordingly, we provide the following, which draws on the verbal definition given by Blanchard

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[4] Another argument for rules (specifically, for constraining the authorities to increase the money supply at a constant rate) put forward by Friedman, is that discretionary policymakers who practise activist monetary policy are more likely to have a destabilising than a stabilising effect on the economy if their information regarding its current state is incomplete. However, as Barro (1986) points out, this argument inappropriately assumes that a sensible discretionary policymaker will fail to take into account the limited nature of his or her information.

[5] As pointed out by Tabellini (2005), the idea that optimal monetary policy may be time-inconsistent can be discerned in embryonic form in Simons (1948).

[6] During the course of a paper on time-inconsistency issues relating to money seigniorage, Calvo (1978) usefully points out that, in dynamic-programming terms, time-inconsistency is attributable to a change over time in the constraints facing the policymaker.

and Fischer (1989).[7] Time-inconsistency arises when it is initially optimal for a policymaker to announce a (sincere) intention to perform a certain action at a future date, only for that action to prove suboptimal, compared to some alternative action available to the policymaker, when that date arrives, and this change in the optimality status of the action in question is not attributable to any change that has occurred to the policymaker's information-set. Although time-inconsistency characterises numerous economic situations (for example, patent-protection legislation), there is a school of thought (discussed by Fischer and Summers, 1989) that macroeconomic policymaking is particularly susceptible to this problem.[8] (However, as we shall see, this view is far from universal, and some of the counterarguments which have been put forward will be discussed below.)

One of several economic examples cited by Kydland and Prescott of plans which are ex-ante optimal but time-inconsistent, is that of a policymaker faced with a variant of the Phillips-curve trade-off between inflation and unemployment represented by equation (1b) above. This scenario was analysed in somewhat greater depth by Barro and Gordon (1983a), and it is on the non-stochastic version of their model that we principally focus here. To assist the ensuing discussion we state a generalised version of (1b) which does not make specific assumptions regarding the expectation of inflation formed by private agents, so that this expectation, denoted π^e, need not be the rational expectation $E\pi$:

$$U = U_N + b(\pi - \pi^e), \qquad b < 0 \qquad (1b')$$

The policymaker aims to minimise the following social welfare function:

$$\Omega^s = (U - kU_N)^2 + c_s\pi^2 \qquad (2)$$

[7] Blanchard and Fischer (1989, p.592): "A policy is dynamically inconsistent when a future policy decision that forms part of an optimal plan formulated at an initial date is no longer optimal from the viewpoint of a later date, even though no relevant new information has appeared in the meantime." Note that throughout the thesis the term 'time inconsistency' will be used rather than its synonym 'dynamic inconsistency'.

[8] Fischer and Summers (1989) ponder whether time-inconsistency problems may be especially acute for macroeconomic policymaking, on account of those members of society who stand to lose the most from the wealth-redistribution effects of high mean inflation (e.g. recipients of non-indexed nominal incomes, creditors) being widely dispersed and poorly placed to coordinate lobbying activity against lax monetary policy.

where $c_s > 0$ and $0 \leq k \leq 1$. A crucial aspect of (2) is that it is generally assumed that $k < 1$, and hence that the natural rate of unemployment is greater than that which is socially optimal. (The two coincide only if $k = 1$.) This represents a departure from the assumption found in Barro (1976), for instance, that social welfare is appropriately measured by the variance of departures of output (or employment) from the market-clearing values which would obtain under full information, and is justified by Barro and Gordon on the grounds that individual labour-supply decisions are distorted by the presence of disincentives such as income tax and unemployment benefits.

An important implication of assuming $k < 1$ is that social welfare is increased by the reductions in unemployment below the natural rate which result when inflation is higher than expected. This in turn implies that an announced intention to deliver low inflation and not exploit any future Phillips curve, while ex-ante optimal (i.e. optimal at the date on which such an announcement is made), is nevertheless not optimal once expectations are formed and the opportunity to exploit the Phillips-curve trade-off arises. Barro and Gordon show that were this model economy also to feature some precommitment mechanism, the optimal choice of inflation rate would then be zero, and under this optimal rule private sector expectations of inflation would never be disappointed. However, in the absence of such a mechanism which ensures that announced policy intentions are binding constraints on the policymaker's subsequent conduct of policy, this optimal zero-inflation rule is not credible. Were private agents naïvely to expect adherence to such a rule, so that $\pi^e = 0$ is the case, the policymaker would have an ex-post incentive to deliver an inflation surprise and hence reduce unemployment below the natural rate. In this scenario of discretionary policymaking, it is rational for the public to expect that inflation rate to be delivered which equates the marginal benefit of an inflation surprise to its marginal cost. In other words, the inflation rate which it is rational to expect is that which is incentive-compatible, in the sense that the expectation does not provide the policymaker with an incentive to disappoint private sector agents as regards this commonly held expectation. Consequently, the rational-expectations equilibrium is characterised by $\pi^e = E\pi = \pi$, and the Phillips curve, $(1b')$, then implies that unemployment is equal to its natural rate, just as it would be under the optimal rule were the latter credible. However, equilibrium inflation under discretion is generally in excess of its social optimum (zero), as is evident from the following solution expression:

$$\pi_{NE} = \frac{b(1-k)U_N}{c_s} \tag{3}$$

This solution exhibits several key aspects of a general result which commonly arises in the literature on the time-inconsistency of discretionary monetary policy. Equation (3) indicates that the inflation bias associated with discretion would not arise were the natural rate of unemployment coincident with the socially optimal rate (i.e. if $k=1$). Furthermore, the bias is sensitive to the value of c_s, the weight placed by society on inflation stabilisation relative to employment stabilisation, as well as to the responsiveness of actual unemployment to inflation surprises (as represented by the Phillips-curve slope, b).

I.3.3 Overview of the Early Literature Subsequent to Barro and Gordon (1983a)

Research on the causes of inflation has built on the insights of the Barro and Gordon model in two principal ways. The first of these two bodies of research effort has extended the Barro and Gordon framework to allow the policymaker's reputation for disciplined control of the money supply to play a role in mitigating, and possibly entirely eliminating, the inflation bias associated with discretion. Barro and Gordon (1983a, 1983b) themselves investigated the implications of introducing reputation effects into a repeated-game version of the model presented above. Among the factors found to matter for the sustainability of a low-inflation reputational equilibrium are the non-existence of a definite termination date for the repeated game, and a sufficiently low discount factor on the part of the policymaker. Subsequent research on this theme has largely focused on incorporating asymmetric information into the repeated game, so that the policymaker's observed actions convey information about some aspect of the economy which the private sector cannot directly observe. Since this lineage of papers is largely peripheral to the subjects addressed in this thesis, it suffices here merely to give a brief description of two of its most salient examples. In Canzoneri (1985) the policymaker's forecast of the velocity shock is private information. Since the private sector cannot verify the forecast, it cannot judge the extent to which the inflation surprise delivered by the policymaker was justified by its private information. This implies that in Canzoneri's model, a reputational equilibrium is characterised by periodic episodes of high expected inflation. In Backus and Driffill (1985) on the other hand, there is 'intrinsic uncertainty'[9] in that the private sector is ignorant of the policymaker's type. In this scenario, weak policymakers who care about unemployment can, by delivering low inflation, sustain a reputation for being indifferent to unemployment for at least some repetitions of the single-period subgame.

The second main line of literature stemming from Barro and Gordon (1983a) ignores reputation issues and pursues another of these authors' hints regarding possible extensions of their analysis. This particular hint relates to the fact that, as mentioned above, the mean inflation bias, as given by (3), is dependent on the values taken by two parameters which occur in the social loss function, namely k and c_s. Barro and Gordon (1983a, p.607) suggest in passing that it may be possible to overcome the time-inconsistency problem and hence eliminate the bias if the "policymaker's preferences [are] artificially manipulated". In other words, they entertain the possibility that the central banker to whom the conduct of monetary policy is delegated may be instructed to minimise a loss function which has the same functional form as the social loss function, (2), but which differs from (2) as regards the specific values taken by the parameters. Although Barro and Gordon do not explicitly state this contrived loss function, it is clear from their verbal comments that they have in mind the following specification, where l and l_N denote employment and its natural level:[10]

$$\Omega^{cb} = (l - k_b l_N)^2 + c_b \pi^2 \tag{4}$$

where $c_b > 0$ and $0 \leq k_b \leq 1$. Ω^{cb} denotes the loss function of the central banker to whom the conduct of monetary policy is delegated. (The 'central banker' could be a committee of individuals.) There are two potential interpretations of this loss function. The first of these is to regard (4) as an objective function which is assigned to the

[9] The phrase 'intrinsic uncertainty', coined by Blackburn and Christensen (1989), refers to a lack of perfect knowledge, on the private sector's part, regarding the preferences of the policymaker, which must consequently be inferred from policy actions and/or announcements. This is a different kind of uncertainty to 'extrinsic uncertainty', which relates to the realised values of stochastic shocks.

[10] Note that since the papers which develop this idea typically specify the policymaker's loss function to be quadratic in employment or output (rather than unemployment) deviations from a target value, this more common practice is henceforth adopted. This in no way affects the results derived from the model.

relevant members of staff at the central bank, who then robotically set about minimising it by appropriate period-by-period adjustments to the monetary instrument(s). (This interpretation appears to be what Barro and Gordon themselves had in mind when making their brief comment mentioned above.) Note that this interpretation assumes that those delegated with the conduct of policy put to one side their personal views regarding which macroeconomic outcomes are desirable, and how much weight should be attached to achieving certain outcomes relative to others. In the terminology of Fischer (1995), the interpretation assumes that only 'instrument independence' is granted to the central banker to whom (4) is assigned, and that this delegate deferentially accepts that he or she does not possess 'goal independence' (in other words does not have the discretion to decide what should be the objectives of policy). The second possible interpretation of (4) instead assumes that some degree of goal independence must necessarily reside with the central banker, since the appointed person will invariably be swayed, whether consciously or not, by his or her personal views on the relative importance of stabilising inflation. Under this second interpretation, the personal preferences of candidate central bankers are observable to society (or those members of society who are responsible for taking the delegation decision), and in choosing a particular candidate society in effect accepts that policy will be directed to the task of minimising the candidate's personal objective function, (4), with c_b, and potentially k_b as well, the candidate's personal preference parameters.

I.4 Monetary Policy Delegation

I.4.1 The Rogoff (1985) Model

This alternative interpretation of the delegation issue first entered the literature by way of the important paper by Rogoff (1985), although it should be noted that Rogoff regards the two interpretations as equally valid alternatives which are not mutually exclusive. It is significant that the parameter k_b is assumed not to vary across candidates, who are all implicitly characterised by a personal k_b value which coincides with the value taken by its counterpart in the social loss function. (The significance of this assumption will become apparent below.) The candidates are distinguished from each other, rather, solely in respect of the preference parameter c_b, with those individuals who place a higher relative weight on inflation stabilisation than does society (i.e. those for whom $c_b > c_s$ is the case) being referred to as 'conservative', in the sense that they are drawn from the "conservative elements of the financial community" (Rogoff, 1985, p.1179). Following this lead, later contributions to the literature have adopted the terms 'representative' and 'liberal' to refer respectively to the cases in which $c_b = c_s$ and $c_b < c_s$. The immediate consequence of allowing c_b to differ from c_s is that the mean inflation bias which characterises the model is lower, the greater the conservatism of the appointed central banker, and is eliminated entirely in the extreme limiting case in which $c_b \to \infty$ (i.e. if the appointee is solely concerned to deliver zero inflation).

This difference in the specification of the policymaker's loss function is one of the two principal respects in which the Rogoff (1985) model differs from Barro and Gordon (1983a). The second principal difference between them concerns the assumed information structure. In Barro and Gordon both policymaker and public are ignorant of the supply shock's value when (respectively) setting the monetary instrument and forming expectations of inflation, and a stabilisation role for activist monetary policy is therefore precluded. Rogoff, by contrast, assumes that the central bank does have an informational advantage over the private sector in respect of aggregate shocks, and possesses this additional information at the time it sets its instrument. Crucially, however, the fact that the central bank's loss function may differ from that of society means that stabilisation of supply shocks will potentially be suboptimal from society's viewpoint. This fact implies that society, in making its central-banker appointment decision, is faced with a trade-off. Increasing the conservatism of the central banker lowers the mean inflation bias associated with discretion, but also causes greater emphasis than is socially optimal to be placed on offsetting the price-level impact of supply shocks, which consequently exacerbates the output impact of those shocks and results in greater variability of output about its natural level.[11] Marginal increases in conservatism (i.e. the weight c_b) therefore trade a greater distortion in the stabilisation response to supply shocks, for a lower mean inflation bias, and Rogoff demonstrates, by means of an envelope theorem argument, that the optimal c_b must be finite but

[11] As in Barro and Gordon (1983a), in the Rogoff model the natural rate itself is invariant to the specification of the policymaker's loss function.

greater than c_s. In other words, the key result of his paper is that the socially optimal delegation decision is to choose a conservative, but one who is not solely concerned with achieving price-level stability.[12]

Before moving on, we briefly discuss the appropriateness of two of Rogoff's key assumptions. The first of these is that the central bank has an informational advantage over the private sector in respect of macroeconomic shocks; the second is that monetary policy is inherently more flexible than wages. The first assumption seems eminently reasonable in the light of the finding of Romer and Romer (2000) that the macroeconomic forecasts of the US Federal Reserve greatly outperform those of private-sector professionals, and that this superiority is not attributable to the Fed's informational advantage in respect of its own imminent policy actions. The appropriateness of the second assumption remains unsettled. Like many papers, this thesis will adopt Rogoff's assumption that the monetary instrument can be adjusted after wages have been set, and that (more controversially) macroeconomic variables are rapidly responsive in the short term to such instrument adjustments. While this is a respectable view, the reader should note that there are also arguments for thinking that the transmission mechanism is sluggish: in other words, that the main macroeconomic impact of monetary policy actions occurs only after a considerable lapse of time. In papers which consider this alternative view, such as Goodhart and Huang (1998), Jerger (2002) and Geraats (2004a), the policy game is therefore modelled as one in which the central bank sets the money supply either simultaneously with, or prior to, the setting of wages by the private sector.

I.4.2 Alternative Delegation Arrangements

This seems an appropriate point at which to mention that the economic (as opposed to logical) legitimacy of the Barro-Gordon-Rogoff framework has been questioned in

[12] Rogoff's paper also examines the consequences of assigning the central bank a loss function which consists of the social loss function itself, supplemented by an additional quadratic term in the deviation of the money supply (or nominal income, or the interest rate) from a target value. In the money-supply targeting case, for example, (4) is modified to $\Omega^{cb} = (l - kl_N)^2 + c_s\pi^2 + \mu(m - \tilde{m})^2$ where μ is a weight parameter and $\tilde{m}$ the money-supply target. The relative merits of these regimes are assessed in relation to a variety of exogenous factors, including the relative importance of supply shocks versus aggregate-demand shocks, and the information available to the central bank when setting its instrument. Since these issues are only distantly related to the themes addressed in this thesis, we do not discuss them further here.

some quarters. The earliest expression of scepticism as to whether the implicit assumptions of this framework faithfully capture the attitudes of real-world policymakers appears to have been Taylor (1983), who gave instances of other economic situations in which the authorities abstain from reneging on preannounced policy intentions, even though a precommitment technology does not exist and short-run welfare gains may accrue from such behaviour. Elaborating on Taylor's point, McCallum (1995, 1997, 1999) has argued that central bankers are typically wise enough to appreciate that securing short-run output gains by means of inflation surprises is ultimately futile, since myopic conduct of this kind leads to a worse outcome, on average, in the long run. Furthermore, the public quickly comes to recognise (correctly) that this view prevails at the central bank, thus making a low-inflation equilibrium possible, despite the discretionary central bank's rule-like intentions being technically time-inconsistent. It would appear that McCallum has in mind the scenario which Blinder, during his professional stint as a central banker, found to be the actual situation at the US Federal Reserve:[13] namely one in which, contrary to the assumption made by Rogoff, the parameter k_b in the central banker's objective function is equal to unity. In other words, it is argued that the professional ethos of central bankers is such as to lead them to view the stabilisation of output at its natural level as desirable, regardless of how this rate compares with the social optimum. While this argument is undoubtedly cogent, its relevance appears to be restricted to those countries in which central bank staff have long established a reputation for disciplined conduct of monetary policy.

Rogoff's seminal paper initiated a very large body of research with arguably two or three major preoccupations. The first of these principal concerns has been to devise alternative proposals for institutional design which will improve upon Rogoff's conservative central banker. Contributions in this vein include Lohmann (1992), who argues that the appointment of a Rogoffian conservative ought to be supplemented with a constitutional provision which allows the government to over-ride this central banker and assume control over monetary policy when large supply shocks occur. Although the inflation bias is worsened by such a provision, society is made better off since the very large swings in output which would occasionally arise in the provision's absence are avoided. The most important papers to identify institutional

[13] As described in Blinder (1997).

arrangements which can potentially outperform Rogoffian delegation in macroeconomic terms are Persson and Tabellini (1993), Walsh (1995) and Svensson (1997). These contributions build on Rogoff's own observation that the policymaker's objective function in his model can be decomposed into the sum of the social loss function and an additional quadratic term in inflation, with the latter to be interpreted as a penalty (perhaps taking the form of a loss of prestige) incurred by the central banker for failing to achieve the socially optimal inflation rate. As pointed out by Walsh (1995), Rogoff's implicit restriction that this term be quadratic is arbitrary. Relaxing this restriction, Walsh finds that imposing instead a contractual obligation on the central bank to minimise a loss function which is a weighted sum of the social loss and a *linear* term in inflation, can, if the additional weight parameter is set appropriately, induce the central banker to both deliver zero mean inflation (thus eliminating the bias) and stabilise the economy in accordance with society's preferences. This loss function has the following specification:

$$\Omega^{cb} = (l - kl_N)^2 + c_s\pi^2 + 2\tau\pi \tag{5}$$

where τ is the linear contract parameter. Persson and Tabellini (1993) obtain a more general set of results which encompasses that of Walsh, but which also covers cases in which the central bank's informational advantage over the private sector cannot ultimately be verified, and hence there is a need to induce, via the penalty term in the contract, a truthful announcement by the bank of its current (i.e. state-contingent) inflation target. Svensson (1997), meanwhile, showed that an alternative modification of the policymaker's loss function could also eliminate the bias without distorting stabilisation responses to shocks: his proposed objective function only differs from that of society as regards the specified target inflation rate.

The papers discussed in the last few paragraphs impinge on a second major concern of the post-Rogoff literature, namely the identification of the optimal degree of conservatism (and perhaps other aspects of regime design) when Rogoff's assumptions regarding the economy's structure are modified. The most significant such modification from the point of view of this thesis is the introduction of non-atomistic agents, and Section I.6 below is devoted to discussion of this portion of the literature. It would be inappropriate to provide a detailed survey of those contributions which build on Barro-Gordon and Rogoff whilst maintaining their assumption that private-sector agents are atomistic, and we therefore confine our remarks here to the principal structural modifications which have been attempted. One such approach, exemplified by Waller (1992), has been to endogenise the degree of conservatism by linking it to the electoral cycle. Another important series of papers considers the implications for optimal regime design of persistence in unemployment over time. Hysteresis implies that current monetary policy decisions will affect real outcomes in future periods, and this consideration prompts policymakers to utilise Bellman's principle to determine the optimal setting of the monetary instrument (Lockwood and Phillipopoulos, 1994, Svensson, 1997, Lockwood et al. 1998). Beetsma and Jensen (1998), Muscatelli (1998) and Walsh (1999), meanwhile, study how optimal delegation arrangements are affected by the presence of intrinsic uncertainty, while another set of papers focuses on the implications for optimal regime design of the functional relationship between the policymaker's intended instrument setting and either its actual realisation or the realisation of inflation itself, being subject to multiplicative uncertainty (Schellekens, 2002, Lawler, 2004).

I.4.3 Empirical Evidence and Interpretation

The literature's third major preoccupation since Rogoff (1985) overlaps, to a certain extent, the theme of structural modifications that has just been discussed. This is the concern to provide explanations for an alleged empirical failure of the Rogoff model, namely that empirical studies generally do not confirm its prediction that more conservative central banks ought to be associated with lower mean inflation and greater output variability than their less conservative counterparts. To name but three such studies, Grilli et al. (1991), Alesina and Summers (1993) and Crosby (1998) all find that inflation is significantly negatively related to measures of central bank independence, but that there is no systematic relationship between these measures and output variability. One response to these findings is to question their validity on methodological grounds. Such studies typically assume that the conservatism of the central banker is measurable by an index of central bank independence, but as Forder (1996) and Romer (1996) point out, this approach potentially confuses the issue of how well-shielded the central bank is from political pressures, with Rogoff's idea that the optimal central banker's objective function ought to differ from that of society.[14] This criticism would appear to apply to some of the theoretical explanations which have been devised for the Rogoff model's empirical failure. Alesina and Gatti (1995), for instance, construct a model in which greater central bank independence (rather than, properly speaking, Rogoffian conservatism) reduces output variability on account of the reduced susceptibility of the economy to boom-bust episodes induced by politicians. Svensson (1997) briefly discusses some rather more mundane explanations, and points out that his proposed low-inflation targeting regime (which results in socially optimal stabilisation) may already have evolved into an institutional feature of many central banks. Crosby (1998), meanwhile, suggests an explanation in terms of country-specific supply-shock variances: those countries with a lower such variance will tend to favour greater central bank conservatism, since for them the additional output-variability cost of securing lower mean inflation is comparatively modest.

I.5 Unions and Monopolistic Competition in Macroeconomics

I.5.1 Introductory Remarks

For the purposes of this thesis, the most important papers to have modified the Rogoff model's structural underpinnings do so by allowing for unions and/or monopolistically competitive markets. These features are natural extensions of the original framework, since they provide additional justifications for assuming the level of output to be socially suboptimal. This section describes how these features are generally modelled in macroeconomics, and thus prepares the ground for the subsequent survey of the literature on their implications for regime design.

I.5.2 A Macroeconomic Model with Monopolistically Competitive Markets

Blanchard and Kiyotaki (1987) is a general-equilibrium model featuring monopolistically competitive firms, each of which produces a specialised good using

[14] Also pertinent in this respect is the Posen (1995) critique, which argues that studies which allegedly find a negative relationship between inflation and central bank independence are potentially biased, in that they inappropriately treat the latter as an exogenous variable, and do not allow for the possibility that it may be endogenously determined by social attitudes and/or interest groups which differ across countries.

a CES (constant elasticity of substitution) production technology. Each household uniquely specialises in a particular variety of labour, which is a potential input into every firm's production process. (Households therefore may alternatively be regarded as craft syndicates representing workers endowed with a particular skill.) The demand for each good, Y_i^D, is derived from the aggregation of the individual consumption decisions of optimising households. Subject to its budget constraint, the individual household maximises a utility function which is increasing in real money balances, and in a CES function of consumption-good varieties, but is decreasing in hours of work. As a consequence of these household optimisation decisions, the fraction of aggregate demand, Y^D, accounted for by the individual firm's product demand, is found to be given by the following equation:[15]

$$\frac{Y_i^D}{Y^D} = K_C\left(\frac{P_i}{P}\right)^{-\varepsilon} \tag{6a}$$

where K_C is a constant, P_i and P are respectively the price of the individual firm's good and the aggregate price level, and ε is an elasticity parameter which, for an equilibrium to exist, is constrained to exceed unity. Each household, meanwhile, similarly faces a derived demand for its labour type, denoted N_j^D, which depends on the aggregate demand for labour, N^D, and on the ratio of its individual wage, W_j, to the aggregate wage, W:

$$\frac{N_j^D}{N^D} = K_N\left(\frac{W_j}{W}\right)^{-\sigma} \tag{6b}$$

where K_N is a constant, and σ is the elasticity of substitution of labour varieties in production. Each household takes into account the implications of this relationship for its labour demand when setting its wage, in order to bring about its individually optimal combination of leisure and labour income, where the latter is devoted to the

[15] This equation corresponds to equation (7) of Blanchard and Kiyotaki (1987, p.650), with the notation altered in the interests of closer conformity with the practice adopted in subsequent chapters of this thesis. Equation (6b) is essentially the same as equation (10) of Blanchard and Kiyotaki (1987, p.664).

purchase of its optimal basket of consumption goods (and perhaps to an optimal adjustment in money holdings as well). Blanchard and Kiyotaki assume continuous clearing of both goods and labour markets, and focus on the symmetric equilibrium in which decisions relating to wages, prices and consumption expenditures are taken simultaneously by atomistic participants. Hence each household, in choosing its individual wage regards this decision's impact on firm profits and on aggregate variables as negligible. Similarly, even though each firm's profit depends upon aggregate demand, the repercussions of its pricing decision for aggregate demand are disregarded by the individual firm. One of the paper's key findings follows directly from this fact: namely that the symmetric Nash equilibrium is associated with an adverse externality. Were every firm to lower its price and every household to lower its wage, the resulting expansion of aggregate demand and hence output would make every household better off, despite the reduction in leisure hours. In equilibrium, however, each firm and household lacks an incentive to lower its price or wage, since the benefits from the resulting marginal increase in aggregate demand accrue to other firms and households, rather than to itself. Although the introduction of a small menu cost to each price adjustment does create a channel for monetary non-neutrality, Blanchard and Kiyotaki concede that the version of their model with menu costs is not fully satisfactory.[16] The details need not concern us, since the essential point for our purposes is that the subsequent literature has combined Blanchard and Kiyotaki's monopolistically competitive markets with other structural features which are not microfounded (in the sense that they are consistent with the specified utility functions of the agents within the model), but are argued to characterise real-world economies. For instance, several important contributions discussed below locate monopolistic competition solely in the goods market, while assuming the labour market features nominal wage rigidities which can impede continuous market clearing. Other papers, meanwhile, retain Blanchard and Kiyotaki's continuum of imperfectly substitutable labour varieties, and study instead the implications of relaxing the assumption that wage-setters do not set wages strategically.

[16] The reason is that if money supply expansions are to cause substantial movements in output in this version, the elasticity of labour supply with respect to the real wage must take an implausibly high value.

I.5.3 The Monopoly Union Model

Economic theorists have devised a number of hypotheses regarding the objectives which impel trade unions during wage negotiations. A plausible assumption common to these hypotheses is that a trade union which enjoys monopoly power over either a particular type of skilled labour, or, via the 'closed shop', over the potential pool of labour available to a firm or industry, will typically be motivated to take steps to increase the real wage received by its employed members above the market-clearing real wage. While this idea underlies every proposal for the form of the trade-union objective function to be used in theoretical work, differences of opinion exist as to whether its specification should capture the welfare of those union members who become unemployed as a result of the obstruction of market clearing. Oswald (1982, 1985), for example, discusses an objective function which is a weighted sum of the utility levels received respectively by employed and unemployed members. The utility function is identical across members, and is concave in the wage (or goods-consumption) and leisure, with unemployed members presumed to receive a reservation level of utility, while the weights depend simply on the proportions of the membership in employment.[17] (One potential objection to this approach is that if the real wage is persistently in excess of its market-clearing level, those among the classically unemployed who were once union members will, sooner or later, lose their insider status and enter the body of outsiders in respect of whose welfare the union is indifferent.[18])

An alternative approach to formulating union objectives is also discussed in Oswald (1985). This does not attempt to derive these objectives using the representative member's postulated utility function. Instead the objective function's specification is designed to capture the intuitively appealing idea that the union views increases in the real wage as desirable, and that the reduction in labour demand that results (given a negatively sloped labour demand curve) from a higher real wage is viewed as undesirable. The particular specification cited by Oswald as illustrative of this notion is the Stone-Geary utility function. However, since the bulk of the macroeconomic

[17] In other words, the union's objective is to maximise the expected utility of the representative member. In the closely related 'utilitarian' formulation, the union objective function is simply the sum of all members' utility, whether they are employed or not.

[18] This idea is central to the Blanchard and Summers (1986) hysteresis model, which endeavoured to explain secular trends in unemployment in terms of the interaction of exogenous supply-side shocks with endogenous changes in the proportion of the workforce who are insiders, and the concomitant evolution of trade-union objectives.

literature which adopts this approach assumes union objectives take the form of a loss rather than utility function, we couch the present discussion instead in terms of the two principal loss-function formulations. The first of these assumes a quadratic term in (log) employment deviations from a specified value (here denoted l_u), together with a linear term (with negative coefficient) in the (log) real wage:

$$\Omega_j^u = (l_j - l_u)^2 - 2c_u'(w_j - p) \tag{7a}$$

where $c_u' > 0$ is the relative-weight parameter. The alternative union loss function ascribes a particular desired real wage to the union in addition to the ostensible preferred employment level of l_u:

$$\Omega_j^u = (l_j - l_u)^2 + c_u(w_j - p - w_u^{real})^2 \tag{7b}$$

With this specification, if the loss function is to exhibit the desirable property that a real wage in excess of the market-clearing wage represents a better outcome for the union than the market-clearing outcome, restrictions must be imposed on how l_u and the 'target' real wage, w_u^{real}, are related to one another. Chapter II provides a detailed discussion of this loss function, and consequently we do not pursue this matter further here. However, two important points regarding (7b) ought immediately to be mentioned. Firstly, with appropriate values assigned to the two 'targets' l_u and w_u^{real}, (7b) can be argued to capture the welfare loss experienced by union members as a result of being exposed to the outcome-uncertainty associated with stochastic productivity shocks. Such shocks necessarily cause at least one of (and potentially both) the real wage and employment to depart from their respective unconditional expected values. It is intuitively plausible that union members, and consequently the union that represents them as well, would prefer both their hours of employment and their real wage to be constant, rather than subject to random fluctuations caused by productivity shocks. Equation (7b) therefore seems a suitable union objective function to adopt in macroeconomic models featuring stochastic productivity shocks, and indeed this is the very specification used in Herrendorf and Lockwood (1997).

The second important point regarding (7a) and (7b) is that both implicitly assume that the union's objectives exist independently of the labour-demand conditions which it faces, in much the same way as the specification of a representative consumer's utility function is independent of that consumer's budget constraint. The specification given by (7b) has greater generality than (7a) in this respect, however, since it is possible to restrict the two objectives l_u and w_u^{real} to represent a point on the expected labour demand curve. This issue is also the subject of detailed investigation in Chapter II: for the present, it suffices to mention that this point is central to the Cubitt (1997) critique of models of monetary policy featuring non-atomistic unions, namely that the conclusions to be drawn regarding the efficacy of delegation arrangements in unionised economies may be sensitive to the assumption that non-atomistic unions ignore the constraints facing them when formulating their objectives.

This completes our discussion of monopoly-union objective functions. Papers deploying this approach very generally assume that each union is empowered to set the contract nominal wage, and abstract from the possibility of wage bargaining,[19] while the firm is assumed to have the freedom to decide upon the level of employment.[20]

I.6 Monetary Policy Games with Non-Atomistic Unions

I.6.1 Introductory Remarks

This section reviews the macroeconomic literature featuring non-atomistic wage-setters (which are generally, although not universally, interpreted to be trade unions).[21] Each such union perceives the influence of its wage on aggregate variables

[19] The profession's consensus view, as stated by Oswald (1985, p.169) for instance, appears to be that few, if any, additional insights are to be gained by incorporating a bargaining stage into the model. Note also that in the literature relating to trade-union objectives much attention has been devoted to the question of whether contracts which stipulate unilateral determination of employment by the firm are efficient (i.e. are Pareto-optimal agreements between the firm and the union), McDonald and Solow (1981) being a well-known instance. The details of this debate are, however, of no concern to this thesis.

[20] This assumption that employment under wage contracts is demand-determined is widely considered to be consistent with the findings of econometric studies. For example, Blanchard (1979, p.802) describes the evidence for this as "overwhelming", while Card (1990) is a much-cited paper which provides supporting empirical evidence.

[21] In the interests of brevity, no discussion is provided here of contributions to this literature which concern macroeconomic topics only remotely related to the theme of the thesis. This includes papers which investigate the implications of strategic wage-setting for monetary-union and open-economy

to be non-negligible, and the strategic considerations which follow from this have important consequences for the individually optimal choice of wage. This literature is divisible into two major strands, one of which assumes that union objectives are represented by either (7a) or (7b), and hence that inflation in itself is not viewed as inherently undesirable: for such inflation-indifferent unions, price-level movements are only potentially detrimental to union welfare insofar as they affect the real wage or employment. The second strand allows for union inflation-aversion by incorporating a quadratic term in inflation into the union objective function, so that (7a) and (7b) respectively become:

$$\Omega_j^u = (l_j - l_u)^2 - 2c_u'(w_j - p) + \mu(\pi - \pi_u)^2 \tag{8a}$$

$$\Omega_j^u = (l_j - l_u)^2 + c_u(w_j - p - w_u^{real})^2 + \mu(\pi - \pi_u)^2 \tag{8b}$$

where $\mu > 0$ is an additional relative-weight parameter, $\pi \equiv p - p_{-1}$ is inflation, and π_u is the union's preferred inflation rate. Although it is to this second strand concerning inflation-averse unions that we first direct our attention, our discussion of it will be relatively brief, since throughout the thesis unions are assumed to be inflation-indifferent.

I.6.2 The Macroeconomics of Inflation-Averse Non-Atomistic Unions

The four closely related papers by Cubitt (1992, 1995, 1997) and Gylfason and Lindbeck (1994) figure prominently among the contributions which initiated the study of the macroeconomic consequences of inflation-averse unions.[22] These papers investigated the implications of strategic interaction between the monetary authority and a single economy-wide inflation-averse union in the absence of stochastic

issues (for example: Forteza, 1998; Gruner and Hefeker, 1999; Cukierman and Lippi, 2001; Holden, 2003; Coricelli et al., 2004b).

[22] Agell and Ysander (1993) is another of the earliest papers to consider a single economy-wide inflation-averse union. Its focus is on how the union may be induced to be more moderate in wage-setting (with potentially beneficial effects on inflation) when its after-tax real wage is doubly vulnerable to erosion by inflation, as a consequence of the progressive income-tax system featuring non-indexed nominal tax brackets.

shocks.[23] While space constraints preclude a detailed discussion of every aspect of these papers, it is worth mentioning the key finding that, under discretion, the resulting moderation in wage-setting induced by the union's inflation-aversion can render a regime in which the money supply is set in a discretionary fashion after wages, superior to one of monetary precommitment. The major focus of the literature subsequent to Cubitt has been on working out the implications of this finding for optimal regime design. Skott (1997), for instance, allows for multiple non-atomistic unions and derives the socially optimal output target that should be assigned to a discretionary central bank, while Lawler (2001) identifies the optimal combination of state-contingent inflation target and central bank 'liberalism' when Cubitt's single-union economy is subject to stochastic shocks, confirming in so doing Cubitt's result regarding the superiority of (optimally delegated) discretion over precommitment.[24]

There are two aspects of this literature which are of interest to this thesis. The first is the 'Cubitt critique' briefly mentioned earlier. Cubitt (1997) presents a model which subsumes, as distinct special cases, the models of Cubitt (1992, 1995) and Gylfason and Lindbeck (1994). The latter model suggests that it may be possible for the policy game in which the money supply and wages are set simultaneously to exhibit both an inflation bias and an employment outcome below either of the two players' desired employment levels, whereas such a result cannot arise in the Cubitt (1992, 1995) model. In his 1997 paper, Cubitt showed that the difference in results is attributable to Gylfason and Lindbeck's use of a specification equivalent to (8b), and that a necessary condition for their stagflation result to be possible, is that the combination of union employment and real wage targets (l_u and w_u^{real}) be 'over-ambitious', in the sense that it is not a feasible outcome given the economy's assumed supply-side structure. (Cubitt in his earlier papers had omitted a quadratic real-wage term, and hence had implicitly assumed that the union's objectives were feasible.) These facts prompted the critique of policy-game models featuring non-atomistic unions outlined in the concluding section of Cubitt (1997), the gist of which has already been summarised in Section 5.3 above.

[23] In a response to Gylfason and Lindbeck (1994), Acocella and Ciccarone (1997) established that similar results arise when unions are inflation-indifferent but share some other macroeconomic objective with the authorities, such as the budget deficit.

[24] Lawler (2000) conducts a similar exercise with regard to the optimal linear-in-inflation contract, and finds that, for optimality, this alternative delegation arrangement should involve the appointment of a 'liberal' central banker. In Jerger (2002) and Lawler (2005) an inflation contract is found to be beneficial, when there are multiple non-atomistic unions, even in the absence of stochastic shocks.

The second aspect of this strand of literature which needs to be mentioned is that its later contributions have wedded the monopolistically competitive structure of Blanchard and Kiyotaki (1987) to the inflation-averse unions framework of such papers as Skott (1997), in the hope of thus being able to provide theoretical explanations for the stylised fact to which Calmfors and Driffill (1988) drew attention. The empirical regularity in question is that the mean unemployment rate and the degree of wage-bargaining centralisation (which, roughly speaking, can be equated to the number of unions) exhibit a non-monotonic 'hump-shaped' relationship. (In other words, countries with atomistic or highly centralised wage-bargaining have tended to experience lower unemployment than countries with intermediate levels of wage-bargaining.) The model of Cukierman and Lippi (1999), which assumes each union controls a monopolistically competitive labour variety, is partly successful in this regard, since it does exhibit the non-monotonic Calmfors-Driffill relationship, provided unions are inflation-averse. However, another important paper, Coricelli et al. (2004a, 2006), which instead assumes homogenous labour and locates the monopolistic competition in the goods market, contrastingly finds that mean unemployment is increasing in the number of unions, regardless of the strength of their inflation aversion.[25]

I.6.3 The Macroeconomics of Inflation-Indifferent Non-Atomistic Unions

I.6.3 (i) Initial Remarks

We now turn to a discussion of the related strand of literature in which non-atomistic unions are indifferent to inflation. The earliest such papers need not concern us, since they either ignore monetary issues entirely, analysing instead an economy in which a single union sets the real wage while the government's policy variable is real public-sector expenditure (Driffill, 1984; Calmfors and Horn, 1985; Söderström, 1985), or alternatively consider a scenario in which a single union can, via an

[25] Guzzo and Velasco (1999) also investigated the implications for optimal delegation of monopolistic competition and strategic wage-setting by inflation-averse unions. Detailed discussion of this paper is unnecessary, however, since it contains an error pointed out by Lippi (2002): specifically, it is inappropriately assumed that the individual union treats the real (rather than the nominal) wages of other unions as beyond its influence. As is apparent from the exchange between Lippi (2002) and Guzzo and Velasco (2002), when the equilibrium expressions for this model with inflation-averse unions are correctly derived, the conclusions which follow are not dissimilar to those obtained by Cukierman and Lippi (1999).

appropriate setting of the nominal wage, dispel the intrinsic uncertainty it faces regarding the monetary policymaker's type (Tabellini, 1988; Andersen, 1989). Our principal interest lies, rather, with those contributions which feature monopolistically competitive markets, and these are conveniently divisible according to whether or not they assume monetary policy is able to influence real outcomes by affecting aggregate demand.

I.6.3 (ii) Monopolistically Competitive Models without an Aggregate Demand Channel

The earliest paper to demonstrate that institutional design is non-neutral as regards real outcomes when wage-setters are non-atomistic and inflation-indifferent, was the little-noticed paper by Akhand (1992). This paper is closely related to the version of Cukierman and Lippi (1999) in which unions are not inflation-averse, in that the starting-point for the analysis in both papers is an expression for the (non-stochastic) aggregate demand for labour which is solely a function of the aggregate real wage. The assumption that monopolistic competition is located only in the labour market ensures that the aggregate demand for goods, and hence for labour also, is invariant to changes in real money balances, so that monetary policy can only influence real outcomes via the real wage. The results which follow from this assumption differ considerably between the two papers, and this is attributable to the difference in specification of the wage-setters' objective function. Since the specification adopted by Akhand is not easily motivated on intuitive grounds, we will largely confine attention here to the findings of Cukierman and Lippi, who assume unions minimise (7a).[26] In both papers, the extent to which the real wage exceeds its full-employment level depends on the degree of monopolistic competition between labour varieties, with the real wage being lower, the higher the degree of substitutability of labour types in production. A coordination failure in wage-setting causes the equilibrium real wage to differ not only from its market-clearing value, but also from its ideal value from the representative union's viewpoint. In Cukierman and Lippi, the real wage is lower, and indeed is further below the representative union's desired real wage, the larger is the number of unions, since the stronger competition between labour varieties

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[26] In Akhand (1992), the wage-setter's utility function is (apparently arbitrarily) assumed to be:
$Utility = [1 + \log(real\ wage)] \times Share\ of\ aggregate\ employment$.

induces each union exclusively representing a particular variety to exercise greater restraint in wage-setting.[27] So far as delegation is concerned, greater central-bank conservatism reduces equilibrium employment, and this has to do with its exacerbation of the externality present in individual union wage-setting choices (although the authors themselves do not use this terminology, couching the discussion instead in terms of a "competition-induced strategic non-neutrality" (Cukierman and Lippi, 1999, p.1411)). Rephrased somewhat, their explanation is essentially as follows. When labour types are monopolistically competitive, an individual union can, given the nominal wage of other unions, secure an increase in its real wage by raising its own nominal wage, at the acceptable cost to it of lower employment as a result of its reduced wage-competitiveness. The greater is central-bank conservatism, and hence the less accommodating of wages is monetary policy, the stronger is the real wage impact of a given increase in the individual union's nominal wage relative to other unions' wages. Hence the incentive to raise the nominal wage is stronger, and the equilibrium real wage premium (i.e. over the competitive real wage) is greater, the more conservative is the central bank.[28]

I.6.3 (iii) Some General Remarks on Monopolistically Competitive Models which Incorporate an Aggregate Demand Channel

By introducing monopolistic labour-market competition into a model featuring non-atomistic unions, Cukierman and Lippi (as well as Akhand) showed that design-aspects of the monetary regime can matter for equilibrium real outcomes even when unions are not inflation-averse. A limitation of these papers, however, is their assumption that the strategic behaviour of unions can only affect the aggregate demand for labour via the equilibrium real wage. No allowance is made for the possibility that each union may also perceive its nominal wage to have an indirect influence on the derived demand for its labour, as a consequence of the contributory effect of its wage in inducing changes in monetary response, the price level and aggregate demand. Contributions contemporaneous with, and subsequent to

[27] Akhand's objective function generates a result contrary to that obtained by Cukierman and Lippi, namely that failure to coordinate wage-setting causes the equilibrium real wage to exceed the desired real wage, and the extent of this departure is worse the stronger the competition between unions (i.e. the more of them there are).

[28] Again, Akhand's assumed objective function causes the relationship between central-bank conservatism and employment to be the very reverse of that found by Cukierman and Lippi.

Cukierman and Lippi have therefore sought to allow for alternative channels through which the strategic interaction of unions' wage decisions with monetary policy can affect employment outcomes. Two such channels have been modelled. The 'output channel' has its sole exemplar in Lippi (2003), who uses a model that has similarities to Cukierman and Lippi (1999), since it features monopolistic competition in the labour market and perfect competition in goods. It differs from the earlier paper, however, by building on microfoundations for optimising firms and workers, and hence does not rely on the assumption implicit in Cukierman and Lippi that aggregate output is taken as given by each union.[29] Secondly, several papers alternatively introduce an aggregate demand channel by modelling the goods market as monopolistically competitive, while labour is homogenous but immobile between monopoly unions, each of which has exclusive control over the workforce available to a particular firm. In what follows, we first of all discuss the three papers in this category which feature policy rules, and which therefore implicitly assume that the real wage cannot be affected by monetary surprises. We follow this with a comparison of Coricelli et al. with Lippi (2003), since although these papers differ in that they locate monopolistic competition in different markets, they share in common the assumption that the monetary regime is one of discretion.

Before proceeding to these tasks, however, it is appropriate briefly to discuss Bleaney (1996), which features monopolistic competition among both firms and unions. While Bleaney's non-stochastic model does exhibit the Calmfors-Driffill hump-shaped relationship between unemployment and the number of unions, equilibrium real outcomes are found by him to be independent of the central bank's weight parameter. This result is attributable to his assumption that wages and the monetary instrument are set simultaneously. As previously noted by Akhand (1992), strategic non-neutralities cannot arise in a simultaneous-move game of this kind, since every wage-setter treats the price level as beyond its influence.

I.6.3 (iv) Models with an Aggregate Demand Channel and a Monetary Rule

Bratsiotis and Martin (1999) and Soskice and Iversen (2000) differ from Bleaney in that, like Cukierman and Lippi (1999), they assume monetary policy is implemented

[29] Another way of making this point is to say that Lippi (2003) is a general equilibrium model, whereas Cukierman and Lippi (1999) is not.

after unions have set wages. Furthermore, both papers assume there is credible precommitment to a rule which involves the money supply being adjusted, in a mechanical fashion, according to the realised value of the price level. The two papers principally differ in that Bratsiotis and Martin allow for stochastic shocks, whereas Soskice and Iversen do not. A second difference is that whereas in Bratsiotis and Martin each of the set of differentiated products is uniquely produced by a particular firm, in Soskice and Iversen this role is allotted to industry sectors, with the firms in each sector instead assumed to be oligopolistic Bertrand competitors. Consequently, in Soskice and Iversen's model each sector's product price is always equal to the nominal wage: hence the equilibrium real wage is zero, and the impotence of monetary policy to influence employment by altering the real wage is secured in a second way quite distinct from the assumption that monetary policy is conducted in accordance with a rule. While the two papers also differ in their assumptions regarding the objective functions of unions (Bratsiotis and Martin assume (7b)[30], while Soskice and Iversen adopt the utility function of Blanchard and Kiyotaki (1987)), both specifications ensure the individual union, in choosing its wage, faces a trade-off between inducing the real wage to move closer towards, and employment further away from, their respective desired values. Given these specifications, the key result common to both models then follows, namely that mean employment is higher, the less accommodating of prices is the monetary rule. This is because the lower the degree of accommodation, the greater the impact on aggregate demand, and hence on the derived demand for the individual union's labour, of an incremental increase in its wage.[31] Less accommodation therefore induces greater wage restraint on the part of each non-atomistic union.

These two models differ somewhat, however, as regards the relationship between the number of unions and mean employment. Soskice and Iversen find mean employment to be monotonically falling in the number of unions. This monotonicity appears to be a consequence of their assumption of Bertrand competition between sectors, each of which is associated with a single monopoly union. Each such union

[30] Bratsiotis and Martin depart from other models which use (7b) by specifying the union's desired real wage, w_u^{real}, to vary according to the realised value of the stochastic productivity shock. However, since each union in their model possesses no information regarding these shocks when choosing its individual wage, this departure from the standard specification is of no consequence for their results.
[31] Such an increase in its wage also reduces the union's derived demand for labour by a more direct route, namely by causing the employing firm (or sector's) product price to rise relative to the average price, and hence reducing that product's share of aggregate demand.

therefore realises that the product real wage of its employer firms is zero, and that therefore its wage choice can only affect its employment via the contribution made by that wage to the determination of aggregate demand. Things are rather more complicated in Bratsiotis and Martin since with monopolistic competition between firms, rather than sectors, each non-atomistic union recognises that its wage choice not only affects its employment outcomes indirectly via aggregate demand, but also via the product real wage. Hence in their model the sign of the impact of an increase in the number of unions on mean employment is ambiguous, and depends on how accommodating of prices is the monetary rule.

A limitation of both these papers pointed out by Holden (2005) is that their models confine attention to a one-shot game, and neglect to consider whether different wage-setting behaviour might arise in a repeated-game context. Using a (non-stochastic) hybrid of these two models, Holden shows that with infinite-horizon repeated play, coordinated wage-setting can be maintained in an equilibrium featuring trigger strategies, provided the unemployment costs of a breakdown in coordination are sufficiently high. As discussed above, equilibrium unemployment in the one-shot game is higher, the less accommodating is the monetary rule. The upshot is that when Soskice and Iversen's and Bratsiotis and Martin's models are extended to allow for repeated play, the conclusion of these papers that a stricter (i.e. less accommodating) rule is beneficial in terms of unemployment outcomes is shown to be questionable, since a stricter rule effectively reduces the unemployment costs of a breakdown, and hence increases the minimum sustainable unemployment level under coordination.

Bratsiotis and Martin (1999) is one of the minority of papers in this field which allow for stochastic shocks. The remaining aspects of it which require discussion are its conclusions regarding the relative merits of alternative targeting regimes when the economy is subject to such shocks. One finding is that while a regime which places a higher weight on stabilising inflation at its socially optimal value results in both lower mean inflation and lower mean unemployment, this double benefit is offset by higher unemployment variability. This trade-off is reminiscent of that present in Rogoff (1985), save that, as we saw earlier, in the Rogoff model the assumption of an atomistic private sector causes mean unemployment to be invariant to the regime's design. (Although the regime in question here involves a rule rather than discretion, it nevertheless corresponds to the appointment of a more conservative central banker in the Rogoff model. The reason for the correspondence is that the assignment of a loss function to the central bank is equivalent to assigning it a particular monetary reaction function or rule.[32]) Another of Bratsiotis and Martin's findings is that in their framework, a regime akin to that proposed by Svensson (1997), in which the central bank announces an inflation target, does not affect equilibrium mean employment. Hence the Svensson model's prediction that such a regime can lower mean inflation without affecting real outcomes is found to be robust to the introduction of monopolistic competition and strategic wage-setting. Bratsiotis and Martin also compare the relative merits of rules according to which the money supply is adjusted in response to observed movements in output or nominal income, and generally find that mean unemployment varies systematically with the resulting degree of monetary accommodation, although the accompanying amount of unemployment variability is sensitive to the nature of the rule. For example, if the central bank is instructed to solely pursue employment stability, the resulting monetary rule is fully accommodating of prices, and the mean unemployment rate is increased for the strategic-interaction reasons discussed earlier, although this drawback is offset by the complete elimination of unemployment variability about that higher mean. This real outcome is inferior to that from nominal-income targeting, which Bratsiotis and Martin also find results in the elimination of unemployment variability. This superior performance is attributable to the fact that nominal-income targeting involves a lower degree of monetary accommodation of the price level than employment targeting, so that mean unemployment is consequently lower under the former.

I.6.3 (v) Monopolistically Competitive Models with Discretion

The two key papers in this category are Lippi (2003) and the version of Coricelli et al. (2004a, 2006) in which unions are not inflation-averse.[33] Despite numerous differences in approach, both models have features which cause individual union

[32] McCallum's (1995, 1997) dictum that the delegation approach merely relocates the time-inconsistency problem, rather than definitively solving it, appears particularly compelling when this correspondence between the assignment of a loss function to the central bank (delegation), and the assignment to it of a particular monetary reaction function (a rule) is pointed out, since there is an implicit assumption underpinning the former that delegation decisions cannot be reneged upon.

[33] Acocella et al. (2005) study the policy implications of the Coricelli et al. (2004a) model when plausible numerical values are assigned to its parameters. They also conduct a similar exercise in respect of a variant model in which wages and the money supply are set simultaneously, replicating in so doing the findings of Bleaney (1996). Since this paper makes no original contribution to theory, further discussion of it here is not warranted.

wage-setting decisions to have a general-equilibrium effect. By modelling the goods market as monopolistically competitive, Coricelli et al. incorporate the aggregate-demand channel central to Bratsiotis and Martin (1999) and Soskice and Iversen (2000). It is unsurprising therefore, that Coricelli et al.'s findings have affinities to those of the latter pair of papers. In particular, greater central-bank conservatism induces greater wage restraint by inflation-indifferent unions, and hence reduces mean unemployment and inflation, while a greater number of unions always worsens macroeconomic outcomes because of the reduced internalisation by each union of the price-level impact of its wage.

In Lippi (2003) firms are perfectly competitive, and hence there is no aggregate demand channel through which the interaction of strategic wage-setting and monetary policy can affect employment. The key feature of Lippi's model is the representative firm's CES production function, which leads different labour types to be substitutable in production. Each union exclusively represents workers of a particular type, and unions are consequently monopolistic competitors in the labour market. In this respect the model has affinities to Cukierman and Lippi (1999) and Akhand (1992). It differs from them, however, in that labour demand is explicitly microfounded, and in such a way as to render different workers' labour inputs complements in production. It is the allowance made for the production-complementarity of labour inputs which underlies the divergence of the results generated by this model from those of Cukierman and Lippi (1999). It will be recalled from our earlier discussion of that paper that the individual union, in adjusting its nominal wage above the average, must weigh the resulting gain in its real wage against its loss of competitiveness vis-à-vis other unions. Higher central bank conservatism (less monetary accommodation of wages) makes this trade-off more favourable for the individual union, leading (if unions are inflation-indifferent) to a higher real wage, and hence higher unemployment and inflation. In Lippi's model there arises a second effect which works against this 'substitution effect'. The second effect is absent from Cukierman and Lippi (1999), and involves a channel running from the individual wage to labour demand via the aggregate real wage. Lippi somewhat obscures its underpinnings by describing it as an "output effect" (Lippi, 2003, p.913), and argues that the individual union, in setting its nominal wage, recognises that the resulting impact on the aggregate real wage will reduce aggregate output, and hence the derived demand for its members' labour, with this effect being stronger, the more conservative is the central bank. Doubtless Lippi's choice of descriptive phrasing here is designed to make the effect seem analogous to the aggregate demand channel of Soskice and Iversen (2000) and related papers. The analogy appears not to be a particularly close one, however, and a more straightforward explanation for the effect can be devised. The key point is that each union appreciates that the impact on the aggregate real wage of its individual nominal wage decision will affect its employer firms' demand for labour varieties other than that of its members. A contraction in demand for these other labour types must entail lower demand for the union's labour type as well, since they are complements in production. Thus a second channel, involving this output or complementarity-of-inputs effect, arises through which greater central bank conservatism can affect mean employment. The repercussions of a marginal increase in conservatism in Lippi (2003) are therefore ambiguous, since its macroeconomic impact depends on the relative strength of the substitution and output effects, with the former inducing less, and the latter more, wage restraint by each union. For plausible parameter values Lippi concludes that appointing a conservative central banker will be beneficial, a conclusion similar to that arrived at by Coricelli et al.

I.6.3 (vi) Non-neutrality of Regime Design in the Absence of Monopolistic Competition

Before concluding this section we note a recent contribution which shows that neither union inflation-aversion nor monopolistic competition is necessary for the design of the monetary regime to be non-neutral as regards mean employment. The crucial feature of the single-union economy of Lawler (2002) is that the inflation outcome which results from the central bank's setting of its instrument is subject to multiplicative uncertainty. The union seeks to minimise (7a), and recognises that while a marginally higher nominal wage leads to a higher expected price level and higher expected real wage, the latter beneficial effect has a drawback in that the consequent increase in inflation variability worsens employment variability. The more conservative is the central bank, the smaller will be the increase in mean inflation which will accompany a given increase in the nominal wage, and hence the less costly will be such a wage increase in terms of higher employment variability. Thus greater conservatism induces less moderation in wage-setting, and hence leads to a higher mean real wage and lower mean employment. This finding is obviously more reminiscent of the strand of literature featuring inflation-averse unions, than the strand which assumes more conventional union objectives and incorporates aggregate-demand or aggregate-output channels through which strategic wage decisions can influence real outcomes.[34]

I.7 The Macroeconomics of Supply-Shock Information

I.7.1 Initial Remarks

It is evident from the previous section that the strategic wage-setting literature has mainly focused on two of the three macroeconomic themes mentioned at the outset of this survey, namely the externalities theme and the differing consequences of rule-based and discretionary regimes. The third theme mentioned earlier, that of informational asymmetries between policymaker and private sector, remains largely unaddressed. In particular, no paper has investigated as yet the implications of non-atomistic unions possessing, at the time wages are set, information regarding the realised value of a supply shock. However, an important analysis of this scenario for the case of atomistic unions has been carried out by Herrendorf and Lockwood (1997). This model is described in the next subsection, following which the literature on the provision by the authorities of such information to the private sector is briefly surveyed.

I.7.2 The Herrendorf and Lockwood (1997) Model

The principal purpose of Herrendorf and Lockwood (1997) was to show that when the Rogoff model is modified to allow for a unionised labour force which possesses some information about a productivity shock's realisation at the time wages are set, a Rogoffian conservative central banker continues to be relevant to optimal regime design. Furthermore, this is shown to be the case even when the objective function assigned to the central bank features an inflation target, or a linear-in-inflation penalty, which, when appropriately set, eliminates the inflation bias associated with discretionary monetary policy.

[34] Lawler (2006) extends this framework to allow for stochastic productivity shocks, and finds the optimal delegation arrangements to be similar to those derived in Lawler (2000) for the case of an economy-wide inflation-averse union.

In Herrendorf and Lockwood's model, firms are perfectly competitive, and each concludes a wage contract with an atomistic monopoly union which represents all the (immobile) potential employees of that firm. Each union seeks to minimise, by choice of the contract wage, an objective function akin to (7b), with the values taken by the parameters l_u and w_u^{real} assumed to be such that the union's preferred mean real wage outcome (i.e. its outcome in the absence of a productivity shock) exceeds the mean market-clearing real wage. Since social welfare is assumed to be quadratic in deviations of employment from its market-clearing level, however, a discretionary policymaker concerned with social welfare engenders a mean inflation bias, for the standard time-inconsistency reasons discussed earlier.[35] Herrendorf and Lockwood introduce an additional element which complicates the delegation issue, however. Prior to setting the contract nominal wage, unions commonly observe a potentially noisy item of information regarding the productivity shock, and conditional on this 'signal' form a forecast of the shock's value. The realisation of the productivity shock in any particular period covered by the (simultaneously signed) wage contracts may therefore be decomposed into a component which is anticipated by unions at the time wages are set, and a component which they do not foresee at that juncture, but which does come to be known by the central bank at the time monetary policy is conducted.

Each union realises that the authorities would, in the absence of any nominal wage response to the signal, adjust the monetary instrument to stabilise the shock's impact optimally in accordance with the social loss function. However, this stabilisation pattern is not, in general, the pattern preferred by the unions. By forming a rational expectation of the central bank's response to the anticipated component of the shock, and adjusting its wage appropriately, each union can bring about a preferred movement in the real wage in response to this component. This adjustment is, in general, smaller in absolute terms than that which is socially optimal. The adverse consequence of this from society's viewpoint is that discretionary monetary policy is characterised by a stochastic inflation bias. In other words, the variability of inflation is higher than it would be were a policymaker concerned with social welfare able

[35] Note that Herrendorf and Lockwood do not define an objective function for society, and instead derive the monetary-delegation arrangements which are optimal from the point of view of the government's objective function. However, since the latter is essentially the same as the social objective function which appears in Rogoff (1985), and since the very title of Herrendorf and Lockwood's paper indicates that they share the same concerns as Rogoff, it seems entirely reasonable to interpret government policymakers in their model as being concerned with social welfare.

credibly to precommit to a rule under which monetary policy is devoted exclusively to shielding the price level from the impact of each shock. Such a rule is not time-consistent, since given naïve expectations that it will be adhered to, the policymaker has an incentive to deliver surprise inflation in order to stabilise employment in accordance with society's objectives rather than those of unions.[36] Herrendorf and Lockwood demonstrate that in this scenario, socially optimal delegation arrangements involve appointing a conservative central banker who places a higher (but not infinitely high) weight on inflation stabilisation, on account of the mitigating effect such an appointment has on the stochastic inflation bias.

The principal aspects of Herrendorf and Lockwood (1997) which are of interest from the perspective of this thesis have been summarised in the foregoing paragraphs. Of less interest to us is the paper's investigation of whether it is possible to implement the socially optimal state-contingent rule by making the central-banker appointment decision contingent on the realised value of the aggregate nominal wage. They find that it is indeed possible to do so provided the unions' preferences are directly observable by the designers of the monetary regime. These issues need not detain us, however, since, as Herrendorf and Lockwood themselves remark, the notion of state-contingent delegation militates somewhat against the idea that central bank appointments are long-term in nature.

I.7.3 Economic Transparency

Transparency in macroeconomics concerns the disclosure of information to the public which would otherwise be private to the policymaker. Geraats (2002) distinguishes several different categories of transparency according to whether the information in question relates to the economic environment, the actual conduct of policy (including control errors), or the policymaker's objectives. Chapter III below investigates the consequences of the policymaker divulging its private information regarding productivity shocks prior to wages and the money supply being set, and therefore constitutes an instance of what Geraats (2002, p.F547) terms "economic transparency". For this reason we confine attention here to this kind of transparency,

[36] Note that socially optimal stabilisation of the unanticipated component of the shock is not impeded: as in Barro (1976), the policymaker's informational advantage enables it to bring about its preferred outcomes in relation to the component of the shock which is not anticipated by unions.

which concerns information regarding macroeconomic disturbances possessed by the policymaker prior to setting its instrument. Most of the previous contributions on this theme are not in fact very closely related to the model developed in the thesis. Geraats (2004b) and Jensen (2000), for instance, investigate the implications of transparency regarding the central bank's macroeconomic forecasts in a two-period model featuring intrinsic uncertainty (i.e. regarding the central bank's preferences). The two papers' disagreement regarding the desirability of economic transparency is attributable to their very different assumptions regarding supply-side structure. However, since this thesis abstracts throughout from intrinsic uncertainty, a detailed discussion of these papers is not warranted. This also justifies giving only the very briefest account of Tarkka and Mayes (1999), which, in a similar manner, assumes that the private sector does not know the central bank's inflation target. Tarkka and Mayes assume output is determined according to the Lucas surprise-supply function, with supply shocks unobserved by either the public when forming expectations of inflation, or the central bank when setting the money supply. The central bank's informational advantage relates to the velocity shock, in respect of which it receives a noisy signal, while it cannot observe the private sector's expectation of inflation. In this scenario, with the private sector concerned to minimise its expectational errors regarding inflation, and the central bank concerned solely to achieve its inflation target, transparency regarding the central bank's estimate of private-sector expectations of inflation are shown to be beneficial to all parties.

This conclusion is not affirmed by the pair of papers by Cukierman (2001) and Gersbach (2002), which like Tarkka and Mayes assume output is determined by a Lucas surprise-supply function, but unlike the latter authors abstract entirely from intrinsic uncertainty, while also assuming that the central bank minimises a social loss function akin to (2). Despite superficial differences in the two models, the two papers arrive at the same principal result, namely that transparency regarding the supply shock is detrimental to welfare since, by causing private-sector agents to revise their rational expectation of inflation, it alters the marginal cost of delivering an employment-stabilising inflation surprise in such a way as to render such stabilisation measures socially undesirable. This is found to be the case regardless of the value taken by k in the social loss function.

Cukierman (2001) presents a second model which also suggests that transparency regarding supply shocks may be detrimental to social welfare. This model does not utilise the Lucas surprise-supply function, but instead assumes that deviations of output from its desirable value depend on the real interest rate expected by the private sector, as well as on a supply shock, while inflation is increasing in lagged output and is affected by a velocity shock. Cukierman shows that the central bank can, by appropriate adjustments of the nominal interest rate, achieve precisely the same stabilisation of output and inflation regardless of whether or not it discloses its information regarding both shocks prior to the formation of inflation expectations. The potential drawback to disclosure, however, is that the induced revisions of private-sector inflation expectations imply that larger (absolute value) adjustments in nominal interest rates are then required to bring about the desired stabilising movements in the (expected) real rate. The resulting increased variability of nominal interest rates may be undesirable if it increases the fragility of the financial sector.[37]

An element common to both Cukierman (2001) models and to Gersbach (2002) is the assumption that the private-sector response to the disclosure of information about the supply shock solely takes the form of a revision of inflation expectations. These papers therefore neglect to consider the possibility that transparency regarding supply shocks may, by changing a private agent's perceived opportunity set of real outcomes, induce some other change in that agent's behaviour. This issue is addressed in Chapter III below.

I.8 Concluding Remarks

This opening chapter has provided some background information on such topics as the role played by the time-inconsistency concept in the debate over rules versus discretion, proposed delegation solutions to the inflation-bias problem, and the modelling of monopolistic competition and of trade-union objectives. The later parts of the survey have focused on three partly overlapping portions of the very extensive literature on rational-expectations macroeconomic models. The first of these concerns strategic wage-setting, the second the optimal design of the monetary regime when the central bank can potentially perform stabilising actions in respect of macroeconomic shocks, while the theme of the third is economic transparency. The first two of these

[37] One difference in results between this model and the Cukierman and Gersbach models with the Lucas surprise-supply function, is that it suggests that transparency regarding velocity shocks may be undesirable.

categories overlap, albeit not to a very great extent, while the literature on economic transparency has yet to examine the implications of strategic wage-setting and monetary delegation for this issue.

In what follows, Chapter II belongs exclusively to the first of these strands of literature, and in particular to that subsidiary part of it which investigates the implications of monopolistic competition for optimal regime design. Its purpose is to address some issues which have been overlooked by previous contributions in this vein. Chapter III is rather more ambitious in its aims, in that it presents a model which falls within all three of the aforementioned areas of research. As mentioned above, the only previous paper to feature strategic wage-setting, monopolistic competition and a monetary authority that can react to shocks is Bratsiotis and Martin (1999). This paper, however, focuses on monetary rules rather than discretion, ignores the possibility that the private sector might, as in Herrendorf and Lockwood (1997), receive information about productivity shocks before setting wages, and does not address economic transparency issues. It is apparent, therefore, that a considerable gap exists in the literature, and it is this gap which Chapter III endeavours to fill.

Chapter II

Wage-Setting Externalities in the Absence of Stochastic Shocks

II.1 Introductory Remarks and Outline of the Model

As explained in the Introduction, the principal aim of this thesis is to study the influence of wage-setting and indexation decisions on macroeconomic outcomes when the economy is subject to stochastic shocks. Having said this, results relating to a purely deterministic context are of some interest in themselves, and a useful beginning can be made to the thesis by considering a basic version of our model which abstracts from stochastic disturbances. Among the advantages of initially analysing the non-stochastic model are the following. Firstly, it will allow an immediate focus to be brought to a theme which figures prominently throughout this work, namely the existence of macroeconomic externalities arising from individual unions' decisions in respect of the nominal wage, when the goods market in which their employer firms operate is monopolistically competitive. Secondly, potentially important results regarding the socially optimal monetary regime will be derived: these results are implicit in some major previous contributions to the literature, but appear to have been overlooked by the authors concerned. Thirdly, it will allow the sensitivity of such externality results to the particular specification of the individual union's objective function to be demonstrated, and a discussion of the appropriate specification of that entity to be provided.

We begin therefore by presenting non-stochastic variants of the key structural equations of the model which will feature in every chapter of the thesis. Except where stated otherwise, all variables are in logarithms, and time subscripts are almost always suppressed apart from when their use is helpful to the exposition.

There is a continuum of monopolistically competitive firms, normalised for convenience on the unit interval. The firms are identical in their production technology, each having the following log-linear production function:

$$y_i^S = \alpha\, l_i \qquad 0 < \alpha < 1 \tag{1}$$

where y_i^S and l_i are firm i's, output and labour input respectively, and α is the elasticity of output with respect to labour input. Aggregate supply is defined to be the integral over the unit interval of individual firm outputs:[1]

$$y^S = \int_0^1 y_i^S di \tag{2}$$

The individual firm's product demand, y_i^D, is a function of aggregate demand, y^D, and of the ratio of its individual price, p_i, to the aggregate price level, p, where $p = \int_{i=0}^{1} p_i di$.[2] The precise specification of this relationship is:

$$y_i^D - y^D = -\varepsilon(p_i - p), \qquad \varepsilon > 1 \tag{3}$$

The parameter ε is the elasticity of firm i's share of aggregate demand, $y_i^D - y^D$, with respect to its relative price, $p_i - p$. The interpretation assigned to ε is that it constitutes a measure of the degree of competition in the goods market, with competition being stronger the higher the value of ε, and with the limiting case in which $\varepsilon \to \infty$ corresponding to the case of perfect competition.[3]

Aggregate demand is specified to be a simple function of real money balances:

$$y^D = \gamma(m - p), \qquad \gamma > 0 \tag{4}$$

where m is the money supply (whether exogenously given, or set at the discretion of the central bank) and γ the elasticity parameter.

Throughout the thesis it is assumed that prices are fully flexible and adjust, as a result of the profit-maximising pricing decisions of firms, to ensure the goods market clears at all times. Hence $y_i^S = y_i^D = y_i$ is always the case, while at the aggregate level

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[1] Hence y^S, the arithmetic mean of individual firm log outputs, is the log of the geometric mean of firm outputs in levels.

[2] Hence p is the (log) geometric mean of firm prices in levels.

[3] The restriction $\varepsilon > 1$ arises for the standard microeconomic reason that $\varepsilon \leq 1$ implies a non-positive marginal revenue.

p adjusts to ensure the equality of y^D and y^S. The assumption of continuous market clearing allows the suppression of the superscripts 'S' and 'D', so that firm i's output and product demand may, by substituting (4) into (3), be written as follows:

$$y_i = \gamma(m - p) - \varepsilon(p_i - p) \tag{5}$$

The individual firm sets its price to maximise its profits for given values of m, p, and the predetermined contract nominal wage, w_i. The derivation of this profit-maximising price is contained in Appendix II.1. This price implies a particular demand for labour by the firm, which in the non-stochastic version of the model being considered here is given by the following expression:

$$l_i^D = \frac{\gamma(m - p) - \varepsilon(w_i - p)}{\alpha + \varepsilon(1 - \alpha)} \tag{6}$$

The supply side of the labour market is comprised of n unions, where $n \geq 1$. Each union is the exclusive representative of the immobile pool of workers available to a particular firm, and the firms to which union j is thus the monopoly supplier are arranged so as to lie contiguously on the subinterval from $(j - 1)/n$ to j/n. The demand for labour represented by union j is consequently:

$$l_j^D = \int_{(j-1)/n}^{j/n} l_i^D di \Big/ \int_{(j-1)/n}^{j/n} di = \frac{\gamma(m - p) - \varepsilon(w_j - p)}{\alpha + \varepsilon(1 - \alpha)} \tag{7}$$

where $$w_j = \int_{(j-1)/n}^{j/n} w_i di \Big/ \int_{(j-1)/n}^{j/n} di$$

The (desired) supply of labour to firm i (and represented by union j) is assumed to be perfectly inelastic, with its value normalised for convenience at zero, i.e.:

$$l_i^S = l_j^S = 0 \tag{8}$$

Note that given the specification of the union's objective function adopted below, relaxing this assumption by allowing for a positive elasticity of labour supply with respect to the real wage would not alter the conclusions to be derived from the model.

In accordance with the 'monopoly union' vein of literature, employment and the nominal wage are assumed to depend on the terms of a contract between the firm and its associated union. The union sets the nominal wage, and the firm has the right to decide upon employment unilaterally (i.e. the union agrees to supply whatever amount of labour happens to be demanded by the firm).

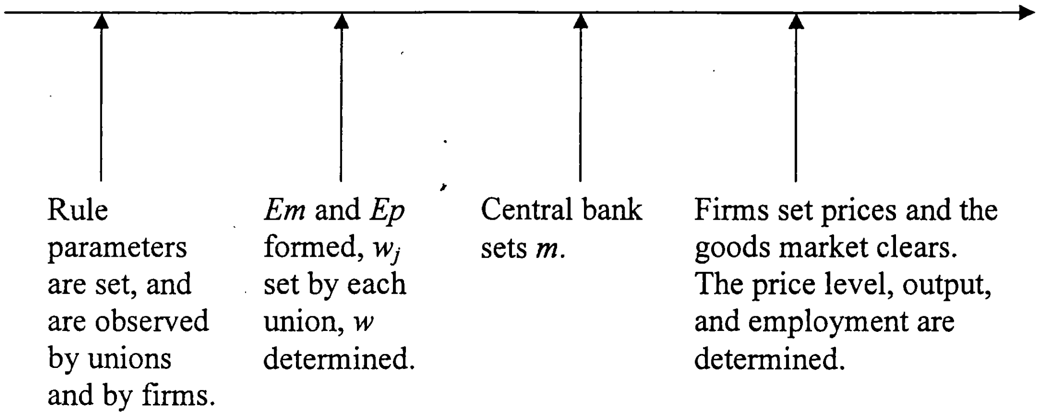
Before proceeding to analyse the principal issue of interest of this section of the thesis, namely the macroeconomic repercussions of individual unions' wage-setting decisions, it is necessary to specify the characteristics of the five monetary regimes which will be considered below (i.e. the determinants of m). The characteristics of each regime are assumed to be laid down once and for all at the very outset of the multi-stage game, and this constitutional enactment relating to the design of the monetary regime is assumed to be irreversible. Furthermore, it is assumed that every agent in the economy is fully informed about the outcome of this initial regime-design stage, and that this is common knowledge. (The model thus abstracts entirely from private-sector uncertainty regarding the nature of the regime.)

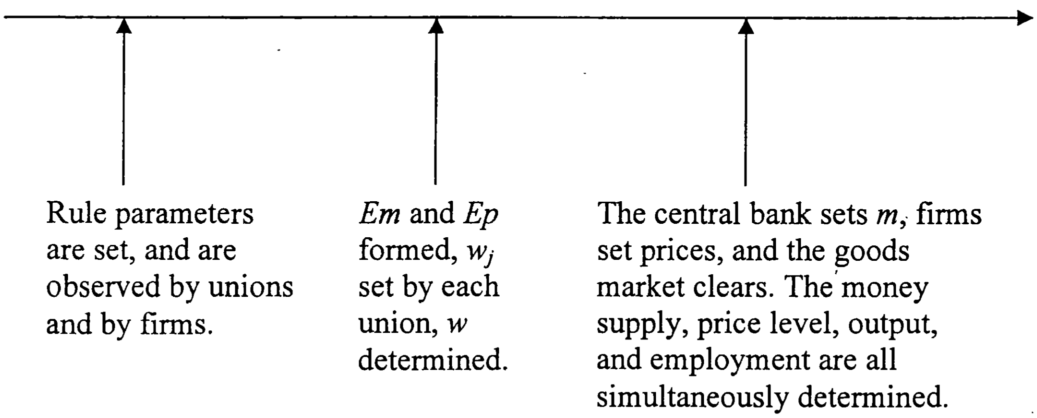
Four of the five regimes will involve the monetary authorities[4] setting their instrument, m, in accordance with a preannounced rule which is assumed to be fully credible to all agents. In the fifth regime the central bank has the discretionary power to set m as it sees fit in order minimise the objective function assigned to it under the constitution. Note that when the model features one of the rule-based regimes, the central bank is not a player in a game-theoretic sense: the part of the model subsequent to the regime-design stage simply consists of two simultaneous-move subgames, one involving wage-setting by unions, and a subsequent one involving price-setting by firms, with m being set in a purely mechanical fashion in accordance with the rule, so that in a non-stochastic context in which all agents are rational (and this is common knowledge), the precise realisation of m can be predicted without error. The four rules are a simple fixed money supply rule, a price level (or inflation) targeting rule, a nominal income targeting rule, and an employment targeting rule. For

[4] Throughout the thesis the terms 'monetary authorities' and 'central bank' are synonymous.

convenience we will refer to these in what follows as, respectively, the simple rule, the price level rule, the nominal income rule, and the employment rule.

Note that for three of the four rules, the possibility of the implementation of monetary policy under the rule taking place after wage-setting has occurred, is essential for their feasibility. The odd one out of the three is the simple rule, which differs from the other three in that the precise timing of the implementation of policy is immaterial for the resulting equilibrium: *m* may be set at its fixed value under this rule either prior to, simultaneous with, or after the determination of the aggregate nominal wage, and may either precede or occur simultaneously with the determination of the price level (i.e. with goods-market clearing). This is not true of the other three regimes, however, since each of these involves a monetary response to an aggregate variable which is necessarily determined *after* the aggregate nominal wage is determined. Consequently, for these three regimes the nature of the monetary rule is such that it necessarily envisages the actual implementation of policy under the rule (i.e. the timing of the setting of the money supply) as taking place subsequent to the determination of the aggregate nominal wage. This means that if technological constraints on the actual conduct of monetary policy are such that *m* cannot be adjusted in response to *w*, but must be set in advance of, or simultaneously with *w*, these three rules will be non-feasible, and even if adopted as rules by the monetary authorities will in fact result in the same outcome, as regards employment and output, as the simple rule. (Further comments on this will be made below when appropriate.) However, if it is possible to set *m* after *w* has been realised, so that the three non-simple rules are feasible, it is then immaterial to the results whether the money supply is set before or at the same time as the price level is determined. Assuming their feasibility, there are therefore two possible scenarios relating to these aggregate-variable-contingent rules, which differ only in the relative timing of monetary policy implementation relative to the simultaneous setting of prices by firms, and which have in common the assumption that *m* is set subsequent to *w*. The time-lines for these two scenarios are as follows:



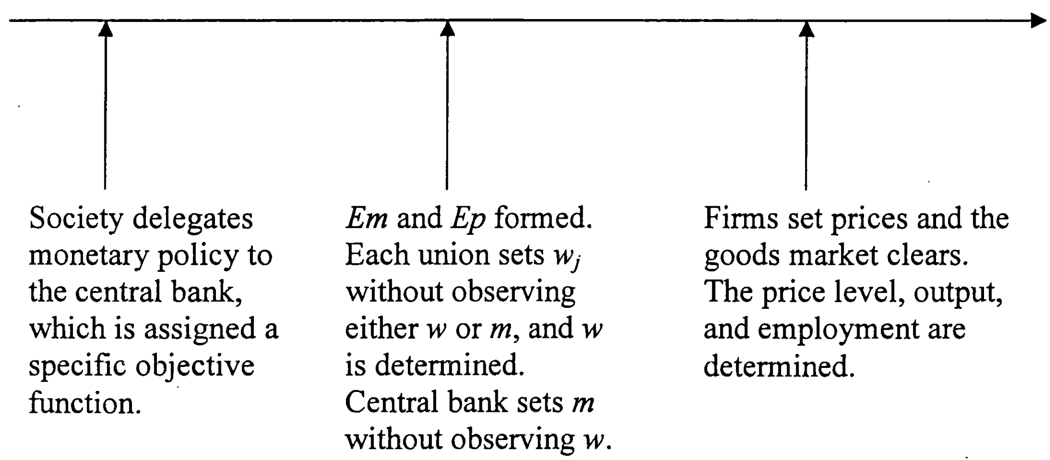


We emphasise that these two scenarios differ only in superficial respects, and in fact are equivalent in the sense that, for a given setting of the rule parameter, they will lead to the same employment and price level outcomes.

The various timing scenarios have quite different implications in the case of the discretionary regime, which involves the central bank setting m either at the same time as, or after the determination of w, and with the objective of minimising a particular loss function. There are two possibilities for the temporal sequencing of events when the regime being considered is discretionary.[5] The first has already been

[5] Arguably, there is a third possible scenario in which the central bank sets the money supply prior to unions setting wages. This scenario, however, would not in general be classified as one of discretionary

discussed by Acocella et al. (2005), and involves the central bank moving simultaneously with the determination of the aggregate nominal wage in the simultaneous-move game between unions. It is described by the following time-line:



The key point about this scenario is that each player in the game, when calculating its own optimal strategy, treats the strategies of other players as given, i.e. as beyond its influence. We will comment further on this scenario at an appropriate juncture below.

The second scenario is that investigated by Coricelli et al. (2004a, 2006), and will be further analysed later in this chapter. In this scenario the central bank's setting of its monetary instrument, m, takes place *after* the determination of the aggregate nominal wage, w, while w itself is the outcome of simultaneous wage-setting by the economy's n unions. As we shall see, a very important consequence of this temporal structure is that the individual non-atomistic union will perceive its wage to have a non-negligible influence on the subsequent realisation of the money supply. (If unions are atomistic, however, so that strategic considerations play no part in their individual wage decisions, the two scenarios turn out to be equivalent.) The time-line for this scenario is as follows:

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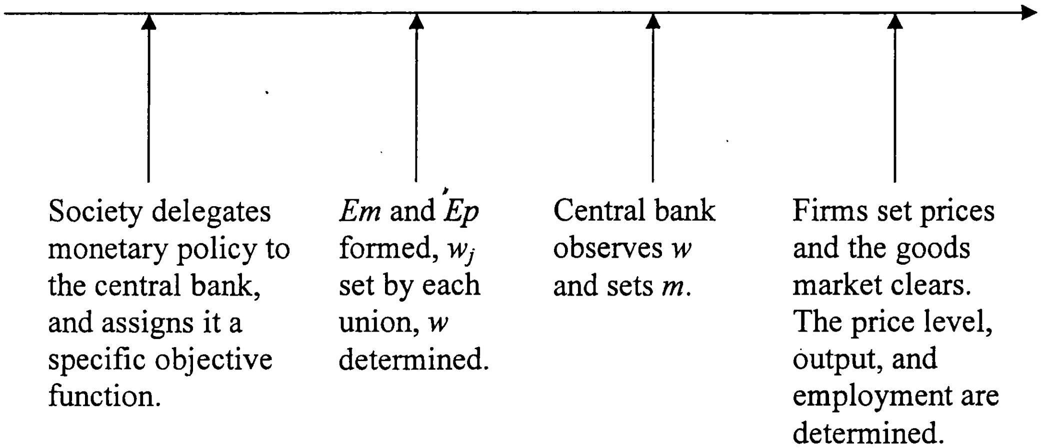
monetary policy, but rather as one of credible precommitment by the central bank to a particular setting of the money supply.



For the discretionary regime, it is important to distinguish between the loss function of the central bank and that of society itself: these two loss functions may differ. In accordance with standard practice in the literature the social loss is specified to consist of a weighted sum of quadratic terms in deviations in aggregate employment[6], $l = \int_0^1 l_i di$, and inflation from their respective socially optimal values:

$$\Omega^s = (l - l^*)^2 + c_s(\pi - \pi^*)^2 \tag{9}$$

where asterisks denote socially optimal values, $\pi \equiv p - p_{-1}$ proxies the inflation rate[7], and c_s is the relative weight placed by society on inflation deviations from the optimum. The most general specification of the loss function assigned by society to the central bank[8] is:

[6] The social loss function could alternatively feature a quadratic term in deviations of output from its socially optimal value. However, provided the socially optimal output level is defined to be consistent with the socially optimal employment level (where 'consistency' relates to the production technology as given by equation (1)), the alternative specification $\Omega^s = (y - y^*)^2 + c_s(\pi - \pi^*)^2$, $y^* = \alpha l^* + \theta$, implies that the conclusions which will be drawn from the analysis will be identical to those identified in the text.

[7] π is not the log of the inflation rate in levels, but rather is the log of $(1 + \text{inflation rate in levels})$. However, this practice is standard in the literature.

[8] The terms 'central bank' and 'central banker' will be used interchangeably throughout the thesis. This amounts to assuming that the head of the central bank has complete control over policy implementation at that institution.

$$\Omega^{cb} = (l - l_b)^2 + c_b(\pi - \pi_b)^2 + 2c_b'\pi \quad (10a)$$

This is the specification adopted by Herrendorf and Lockwood (1997) in their investigation of optimal delegation. It features assigned target values for employment (l_b) and inflation (π_b), as well as a linear penalty term in inflation, c_b' being the linear contract parameter. For the purposes of this chapter, the delegation parameters l_b, π_b and c_b' can be set to zero without any loss of insight, yielding the simpler central bank loss function in which the weight c_b is the sole delegation parameter:

$$\Omega^{cb} = l^2 + c_b\pi^2 \quad (10b)$$

Note that no restriction is placed on the sign of c_b. In line with the bulk of the literature, we will refer to cases in which $c_b > c_s$ as a 'conservative' central bank, with the limiting $c_b \to \infty$ case corresponding to that of an 'ultraconservative'. When $c_b < c_s$, the central bank is 'liberal', and the case in which $c_b = 0$ has variously been referred to as an 'ultraliberal' (Coricelli et al., 2004a), or 'populist' (Guzzo and Velasco, 1999). Underlying this nomenclature is the literature's implicit (or, in the case of many papers, explicit) assumption that negative values for c_b are inadmissible. The imposition of a lower bound of zero on the range of values which c_b can take is not a restriction compelled by logic, however. The non-negativity restriction is made on the basis of an (usually tacit) intuitive argument that the central bank ought not to be modelled as benefiting directly from inflation (or, for that matter, deflation) *per se*. In the discussion which follows, however, considerable insight will be gained into the socially optimal delegation arrangements for a discretionary regime by allowing the parameter c_b potentially to take any value on the real line.[9]

[9] Note that if $c_b < 0$ the central bank's isoloss map will lack a bliss point. For the simpler case in which $l_b = \pi_b = c_b' = c_b'' = 0$, the isoloss map will consist of hyperbolae, with the isoloss contour representing a zero realisation of the loss being a degenerate hyperbola (i.e. a pair of straight lines which intersect at the origin). Commencing from the origin, a movement along the employment axis in either direction will give rise to an increase in the loss, while movements away from the origin along the inflation axis will render the loss negative. Since for any given employment level a numerically lower value of the loss always results from a movement in inflation further away from the notional target of $\pi_b = 0$, it follows that no bliss point exists. Allowing for $l_b \neq 0$ and $\pi_b \neq 0$ does not alter the general character

On the basis of standard welfare arguments, it is customary in models of this kind to assume that the socially optimal employment level is that which would prevail under labour-market clearing. Since (8) implies (log) market-clearing employment is zero, it follows that $l^* = 0$, while for simplicity it is assumed that the socially optimal inflation rate is also zero, so that $\pi^* = 0$.[10] Following the innocuous normalisation, $p_{-1} \equiv 0$, the loss functions of society and of the central bank therefore become:

$$\Omega^s = l^2 + c_s p^2 \tag{9'}$$

$$\Omega^{cb} = l^2 + c_b p^2 \tag{10'}$$

The only structural equation in the model which remains to be discussed is the union's objective function. However, since one of the principal purposes of this section is to demonstrate the sensitivity of key results to the specification of this objective function, detailed discussion is best deferred until semi-reduced forms for the central bank's setting of *m* and the resulting realisation of the price level have been stated, and it is to the derivation of these expressions that we now turn.

Our assumption of fully flexible prices implies that individual firm prices, and consequently the aggregate price level as well, will adjust instantaneously to ensure the equality of aggregate demand with aggregate supply. In order to derive the price level as a function of *m* and *w*, we therefore initially substitute (6) into (1) and integrate over the unit interval in order to express aggregate supply as a function of *m*, *p*, and the aggregate nominal wage, $w = \int_0^1 w_i di$:

$$y^S = \frac{\alpha[\gamma(m-p) - \varepsilon(w-p)]}{[\alpha + \varepsilon(1-\alpha)]} \tag{11}$$

Equating (11) with (4) and solving for the price level yields:

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of the isoloss map: rather, it merely alters the location of the degenerate hyperbola along which the central bank's loss is zero.

[10] Although π is not the log of inflation, the adopted normalisation $p_{-1} \equiv 0$ implies that $\pi^* = 0$ corresponds to the assumption of a zero desired inflation rate in levels.

$$p = \frac{\gamma(1-\alpha)m + \alpha w}{[\alpha + \gamma(1-\alpha)]} \tag{12}$$

The simple rule prescribes that the money supply be kept fixed by the monetary authorities irrespective of the realised values of other variables. This amounts to $m_t = m_r = \overline{m}, \forall t$, where t and r denote the time period and the rule respectively, and $\overline{m}$ is a constant. Using (12) the semi-reduced form for the price level under the simple rule is therefore found to be:

$$p_{simple\ rule} = \frac{\gamma(1-\alpha)\overline{m} + \alpha w}{[\alpha + \gamma(1-\alpha)]} \tag{13}$$

The second rule which will be analysed below is the price level rule. The normalisation $p_{-1} \equiv 0$ renders the price level identical to inflation, so this rule is exactly equivalent to an inflation targeting rule. Its general form is:[11]

$$m_{price\ level\ rule} = \tau_1 p \tag{14}$$

This rule therefore prescribes a particular monetary response, the sign and magnitude of which depends on the value of the parameter τ_1, to deviations of the price level from its socially optimal value of zero. Thus τ_1 is the degree of monetary 'accommodation' of prices. With this general formulation of the price level rule, substitution of (14) into (12) and some rearrangement yields the semi-reduced form:

$$p_{price\ level\ rule} = \frac{\alpha w}{[\alpha + \gamma(1-\alpha)(1-\tau_1)]} \tag{15}$$

Substituting (15) into (14) allows the money supply to be expressed as a function of the aggregate nominal wage:

[11] Note that a still more general form of the price level targeting rule would also allow for a potentially non-zero constant term, so that $m_{price\ level\ rule} = \overline{m} + \tau_1 p$. The key result of the analysis conducted below, however, is unaffected by $\overline{m}$, and so without any loss of generality we adopt the normalisation $\overline{m} \equiv 0$ for this rule, yielding (14).

$$m_{price\ level\ rule} = \frac{\tau_1 \alpha w}{[\alpha + \gamma(1-\alpha)(1-\tau_1)]} \tag{16}$$

By a similar procedure we could also derive output as a function of w and model parameters. The expression in question, however, is not essential to the task of describing the derivation of our results for the case of the price level rule, and consequently we omit it.

Our third rule is nominal income targeting, which involves the automatic adjustment of m in response to a deviation in (log) nominal income, $y+p$, from its socially optimal value of zero[12]:

$$m_{NI\ rule} = \tau_2(y+p) \tag{17}$$

Substituting (17) into (6), and the resulting expression into (1) to obtain (using (2)) an expression for aggregate supply, and equating this with the aggregate demand expression obtained after substitution of (17) into (4), allows the semi-reduced form for the price level to be obtained for the nominal income rule scenario:

$$p_{NI\ rule} = \frac{\alpha(1-\gamma\tau_2)w}{[\alpha + \gamma(1-\alpha-\tau_2)]} \tag{18}$$

It is useful for this scenario also to state output as a function of w:

$$y_{NI\ rule} = \frac{\gamma\alpha(\tau_2 - 1)w}{[\alpha + \gamma(1-\alpha-\tau_2)]} \tag{19}$$

The degree of monetary accommodation of wages under the nominal income rule is therefore given by:

[12] As in the case of the price level rule, a more general specification of the nominal income rule would allow for a constant term $\overline{m}$. However, the explanation given in the footnote relating to equation (14) is equally applicable to (18) in that $\overline{m} \equiv 0$ can be adopted as an innocuous normalisation.

$$m_{NI\ rule} = \frac{(1-\gamma)\alpha\tau_2 w}{[\alpha + \gamma(1-\alpha-\tau_2)]} \tag{20}$$

The last of our four rules is the employment rule, according to which the money supply responds to departures of employment from the social optimum (zero).[13] A significant difference between this fourth rule and the previous three rules is that whereas the others prescribe a particular monetary response to the departure of a purely *nominal* variable from its socially optimal value, in the case of this fourth rule the target variable is real:

$$m_{employment\ rule} = \tau_3 l \tag{21}$$

By following a similar procedure to that described previously for the case of the nominal income rule, the semi-reduced forms for the price level, output, and the money supply are found to be:

$$p_{employment\ rule} = \frac{(\alpha - \gamma\tau_3)w}{[\alpha + \gamma(1-\alpha-\tau_3)]} \tag{22}$$

$$y_{employment\ rule} = \frac{-\gamma\alpha w}{[\alpha + \gamma(1-\alpha-\tau_3)]} \tag{23}$$

$$m_{employment\ rule} = \frac{-\gamma\tau_3 w}{[\alpha + \gamma(1-\alpha-\tau_3)]} \tag{24}$$

It remains to derive counterpart semi-reduced form expressions for the discretionary regime. For the purposes of this section, $(10')$, the simpler variant of the central bank loss function discussed earlier, will suffice. Substituting (12) and the aggregate counterpart to (6) into $(10')$, the central bank's loss as a function of m and w is:

[13] Note that since in this non-stochastic version of the model output is a simple monotonic function of employment, $y = \alpha l$, the employment-targeting rule given by (21) is identical to an output-targeting rule, $m_{output\ rule} = \tau_4 y = \tau_4 \alpha l$, so that $\tau_3 \equiv \tau_4 \alpha$. Note also that just as in the cases of equations (14) and (17), the implicit normalisation at zero of the constant term $\overline{m}$ does not matter in any way as regards this section's key result.

$$\Omega^{cb} = \frac{\gamma^2(m-w)^2 + c_b[\gamma(1-\alpha)m + \alpha w]^2}{[\alpha + \gamma(1-\alpha)]^2} \tag{25}$$

Differentiating with respect to m and solving the central bank's first-order condition $\partial\Omega^{cb}/\partial m = 0$, the central bank's optimal setting of m emerges as:

$$m_{discretion} = \frac{[\gamma - c_b\alpha(1-\alpha)]w}{\gamma[1 + c_b(1-\alpha)^2]} \tag{26}$$

Substituting (26) into (12) yields the price level under discretion, expressed as a function of the aggregate nominal wage:

$$p_{discretion} = \frac{w}{[1 + c_b(1-\alpha)^2]} \tag{27}$$

Having derived the semi-reduced forms for each of the five monetary regimes, we now turn to the analysis of union wage-setting behaviour. It is therefore appropriate at this juncture to set out the one remaining structural equation of the model, namely the objective function of the representative individual union. One of the two key findings of this chapter will be that many of the results obtained by previous papers which use variants of this objective function are sensitive to its precise specification. In view of this, an extensive discussion of its key features is called for.

The general specification of the individual union's loss function is:

$$\Omega_j^u = (l_j - l_u)^2 + c_u(w_j - p - w_u^{real})^2 - 2c_u'(w_j - p) \tag{28}$$

The weight parameters c_u and c_u' are restricted to be non-negative, $c_u, c_u' \geq 0$, but we impose no such sign restrictions to begin with on the union's employment objective, l_u, and real wage objective, w_u^{real}. The imposition of particular restrictions on the parameters c_u, c_u', l_u, and w_u^{real}, yield variants of (28) which have been used extensively in the literature on monetary policy games. This vein of literature has already been discussed in detail in Chapter I: it therefore suffices here to mention the two papers most closely related to the analysis of this section, namely Coricelli et al. (2004a) and Bratsiotis and Martin (1999). By imposing $c_u \equiv 0$ in (28) a specification almost identical to that used by Coricelli et al. is obtained:

$$\Omega_j^u = (l_j - l_u)^2 - 2c_u'(w_j - p) \tag{29}$$

Coricelli et al.'s union loss function differs from this in one minor and one major respect. The minor difference is that a quadratic term in unemployment deviations from zero is included, rather than the $(l_j - l_u)^2$ term of (29). Consequently, setting l_u equal to our model's market-clearing employment level of zero would correspond exactly to the assumption made by Coricelli et al. Imposing particular values on c_u' and l_u does not affect the conclusions drawn below regarding the equilibrium when the union loss function is given by (29). The major difference between (29) and Coricelli et al.'s loss function is the absence of a quadratic term in inflation from (29). Its omission is justified here since this thesis abstracts entirely from union inflation-aversion, while some of Coricelli et al.'s most interesting results are not dependent on the presence of an inflation-aversion term in their union loss function.

The key point about (29) is that this specification ensures the union loss is always directly decreasing in the real wage, and this is the case regardless of the value assigned to the employment objective l_u. One important implication of this is that given a negatively sloped labour demand curve, further increases in the real wage beyond the level which generates labour demand of l_u must entail a trade-off, so far as union welfare is concerned, between the direct beneficial effect of the real wage being higher and the detrimental indirect effect of labour demand being lowered below l_u, with the former beneficial effect outweighing the latter. The fact that the individual union which has (29) for its loss function and which faces a downward-sloping labour demand curve is always better off when employment is below l_u, and the concomitant real wage is above that which results in labour demand of l_u, implies that the union will not aim to generate a demand for its labour of l_u when setting its nominal wage. Consequently, when account is taken of the trade-off between employment and the real wage embodied in the labour demand curve, the desired employment level, l_u, in (29) may be regarded as merely notional, since its attainment conflicts with the union's concern to achieve as high a real wage as possible.

A diagrammatic representation of the union's isoloss map in real wage, employment space is helpful at this point. The linear real wage term ensures that at any particular employment level, an increase in the real wage must place the outcome on an isoloss contour which represents a (numerically) lower value of the loss, and consequently the isoloss map lacks a bliss point. The isoloss contours are parabolas with axis of symmetry given by the line $l_j = l_u$. The symmetric-wage labour demand curve has equation $l_j = -(w_j - p)/(1-\alpha)$, and therefore passes through the origin in Figure II.1, with the origin being the outcome associated with labour-market clearing.[14] The diagram assumes $l_u > 0$, although the same key point would emerge were $l_u < 0$ or even $l_u = 0$ assumed instead, namely that since the line $l_j = l_u$ is the axis of symmetry of the isoloss map, it follows that the tangency point between the map and the negatively sloped symmetric-wage labour demand curve must involve an employment level which is less than l_u. Hence l_u may be described as the union's notional employment objective which exceeds the employment level which is most desirable from the union's point of view, given the constraint it faces that the employment and real wage outcome pair must lie on the symmetric-wage labour demand curve.

To obtain the version of (28) used by Bratsiotis and Martin (1999),[15] the parameter c_u' is restricted to be zero, giving:

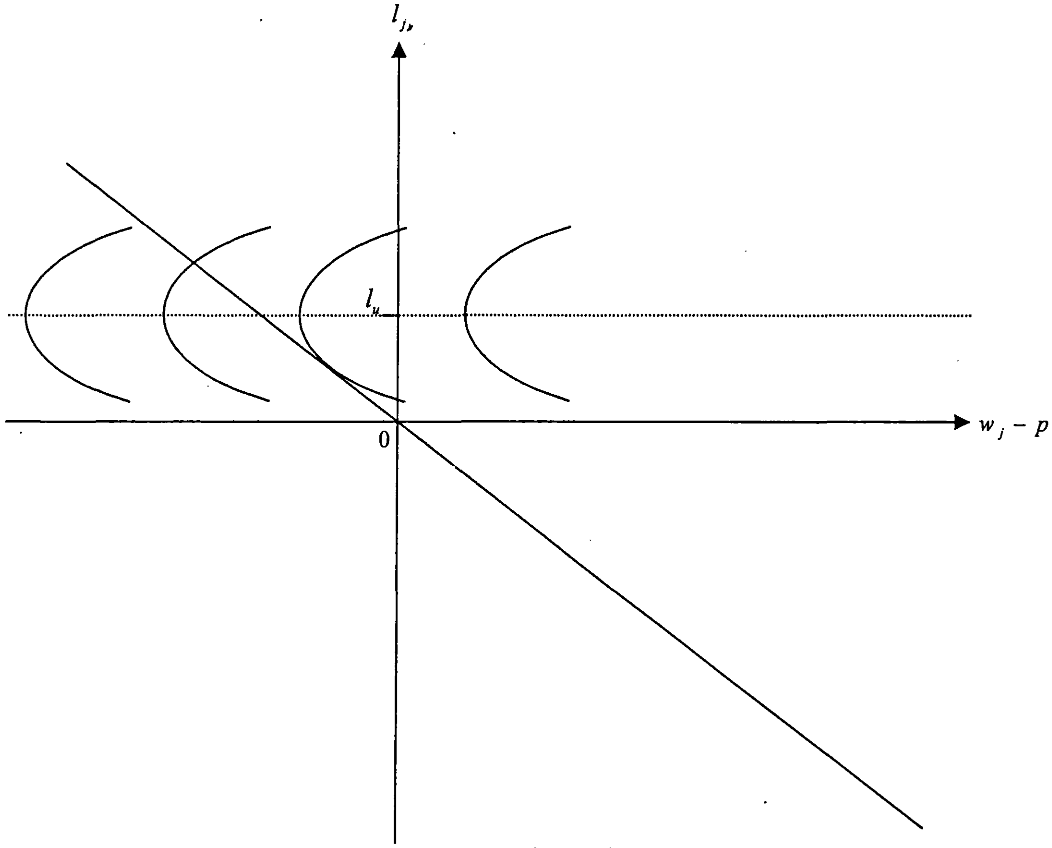
$$\Omega_j^u = (l_j - l_u)^2 + c_u(w_j - p - w_u^{real})^2 \qquad (30)$$

[14] By 'symmetric-wage labour demand curve' is meant here the labour demand curve which prevails when there is symmetric wage-setting, $w_j = w \forall j$, which in turn implies profit-maximising firms will engage in symmetric price-setting, $p_i = p \forall i$. The labour demand equation (6) is derived from the individual firm's profit-maximising price and associated output level, without making any assumptions regarding the relationship between w_j and w, i.e. without assuming that wage-setting is necessarily symmetric. (In deciding upon its own price p_i, firm i treats the aggregate price p as beyond its influence.) Hence equation (7) is not the symmetric-wage labour demand curve faced by union j. Further light will be shed on this issue by the discussion of the wage-setting equilibrium in the main body of the text, a few pages further on.

[15] More precisely, (30) is the specification of the union loss function in the non-stochastic version of Bratsiotis and Martin's model.

Figure II.1

Union *j*'s isoloss map for the loss function $\Omega_j^u = (l_j - l_u)^2 - 2c_u'(w_j - p)$



Symmetric-wage and aggregate-level labour demand curve

$$l_j = -\frac{(w_j - p)}{(1 - \alpha)}$$

Notes:

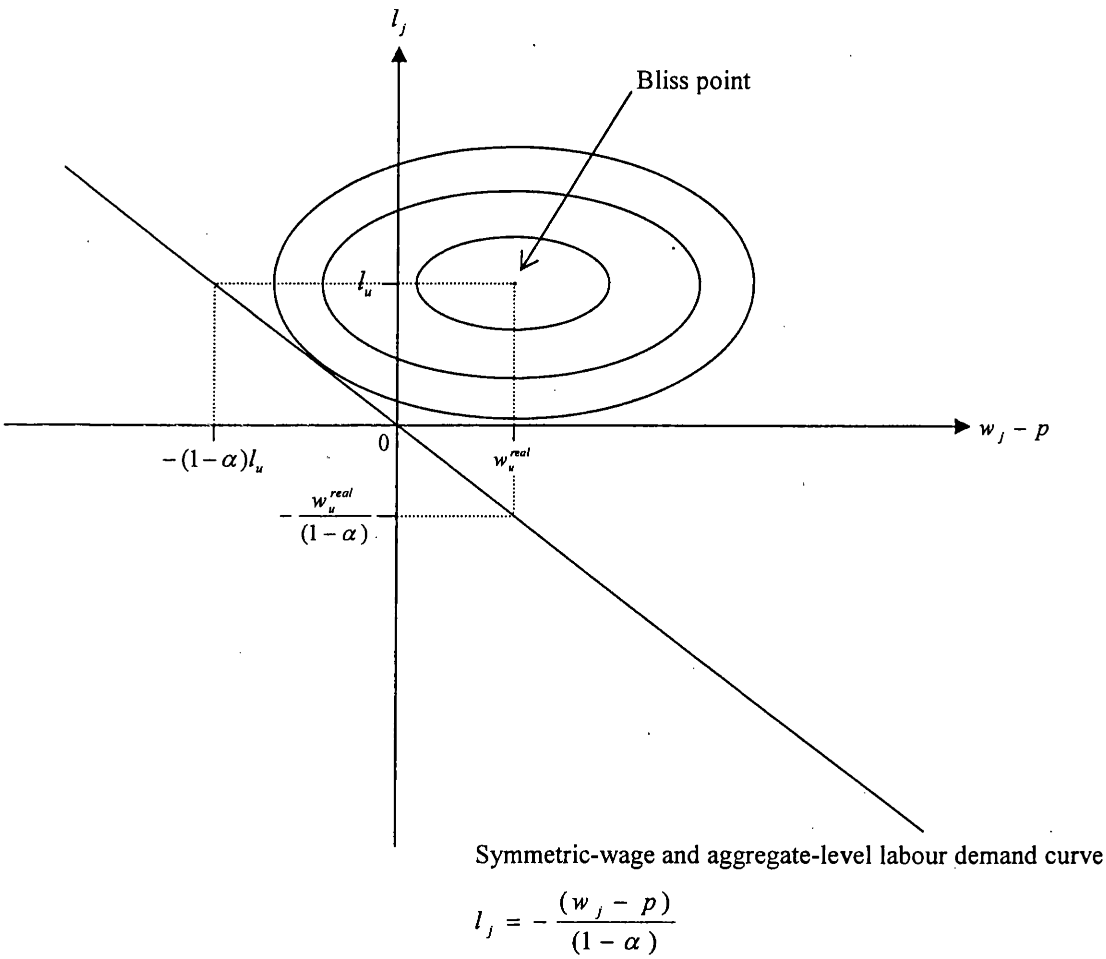
1. It is assumed for illustrative purposes that $l_u = 0$.

As is well known, the isoloss map in $l_j, w_j - p$ space generated by this specification consists of elliptical isoloss contours with axes of symmetry which pass through the map's bliss point located at (l_u, w_u^{real}).[16] It is noteworthy that whereas in the case of (29) l_u is merely notional, in the case of (30), by contrast, l_u cannot be described as notional and is indeed the union's true desired employment level. A second and closely related difference between (29) and (30) is that with (30) an increase in the real wage need not necessarily be directly beneficial to the union (where 'directly' here means that the indirect effect of an increase in the real wage on the loss via the induced change in employment is disregarded). These differences between (29) and (30) imply that, whereas with (29) the individual union will always prefer a real wage which is higher than that required to bring about the notional objective of l_u, with (30) this will only be the case if the bliss point given by (l_u, w_u^{real}) lies above the symmetric-wage labour demand curve. In algebraic terms this amounts to $l_u > -w_u^{real}/(1-\alpha)$ being the case, since the right-hand side of this inequality is the labour demand which results when w_u^{real} happens to be the real wage. This case in which the bliss point is above the labour demand curve is depicted in Figure II.2 below. If $l_u < -w_u^{real}/(1-\alpha)$, the bliss point will be below the labour demand curve and hence the tangency point between the demand curve and the isoloss map will involve a level of employment in excess of l_u (and a real wage which is less than that required to induce labour demand of l_u). If $l_u = -w_u^{real}/(1-\alpha)$, the union's bliss point will be located on the labour demand curve, with the important implication that, unlike the cases in which l_u is either greater or less than $-w_u^{real}/(1-\alpha)$, the individual union, in making its nominal wage decision, does not face a trade-off between moving its real wage closer to its notional objective of w_u^{real} and thereby incurring the cost of moving its employment further away from the notional objective of l_u. In the $l_u = -w_u^{real}/(1-\alpha)$ case an appropriate setting of the nominal wage will, given *p*, enable both objectives to be attained simultaneously. For convenience, l_u and w_u^{real} will be described as being consistent with one another if they satisfy the equation

[16] The eccentricity of the ellipses depends on the weight parameter c_u.

Figure II.2

Union *j*'s isoloss map for the loss function $\Omega^u_j = (l_j - l_u)^2 + c_u(w_j - p - w_u^{real})^2$



$l_u = -w_u^{real}/(1-\alpha)$. As we shall see, the case in which the union's objectives are consistent will figure prominently in the results reported below.

Finally, we need to discuss the nature of (28) when both of the weight parameters c_u and c'_u are positive. In this case the isoloss contours are elliptical just as they are in the $c'_u = 0$ case represented by (30). However, the presence of the linear real wage term in the general specification given by (28) does create a significant difference between its isoloss map and that associated with (30), namely that whereas the bliss point for (30) is given by (l_u, w_u^{real}), in the case of (28) the isoloss contours are centred not on (l_u, w_u^{real}) but rather on the point $\left(l_u, (c'_u/c_u) + w_u^{real}\right)$, so that whereas l_u is indeed the desired employment level of the union, its ostensible real wage objective of w_u^{real} is merely notional.[17] The true desired real wage of the union is $(c'_u/c_u) + w_u^{real}$, and this exceeds w_u^{real} if $c'_u > 0$. Despite this departure of the true bliss point from the ostensible bliss point when $c'_u > 0$, the general appearance of the union's isoloss map in this case closely resembles that of the $c'_u = 0$ case. In particular, (28), like (30), allows for the possibility that the union's preference parameters may be such as to cause the true bliss point to be located on the symmetric-wage labour demand curve. This will be the case when $l_u = -(c'_u + c_u w_u^{real})/c_u(1-\alpha)$, with $l_u > (<) -(c'_u + c_u w_u^{real})/c_u(1-\alpha)$ resulting in the true bliss point being located above (below) the symmetric-wage labour demand curve.

We are now in a position to derive the macroeconomic equilibrium for each of the five monetary regimes. We assume to begin with that each of the n unions has (29) as its objective function, and that the labour force is evenly divided between the unions (hence each union's share of the total labour force is $1/n$). The first regime we will consider is the simple rule, according to which the money supply is kept fixed regardless of the realised value of the aggregate nominal wage.

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[17] These facts are elicited by considering the slope of one of the isoloss contours of Ω_j^u, as given by (28). This slope is $dl_j/d(w_j - p) = [c'_u - c_u(w_j - p - w_u^{real})]/[(l_j - l_u)]$. Note that whereas with $c'_u = 0$ the union loss evaluated at the bliss point is zero, i.e. $\Omega_j^u\big|_{c'_u=0,\, l_j=l_u,\, w_j-p=w_u^{real}} = 0$, with $c'_u > 0$ the union loss evaluated at its true bliss point of $\left(l_j = l_u, w_j - p = (c'_u/c_u) + w_u^{real}\right)$ is negative for all $w_u^{real} > 0$: $\Omega_j^u\big|_{l_j=l_u,\, w_j-p=(c'_u/c_u)+w_u^{real}} = -(c'_u/c_u)^2 - 2c'_u w_u^{real}$. This negative minimum value of the union loss is purely a consequence of earlier normalisations.

II.2 Wage-Setting under the Simple Rule

II.2.1 Derivation of the Efficient Nominal Wage

Under this rule $m_r = \overline{m}$, where $\overline{m}$ is a constant. It is appropriate to begin by deriving the efficient nominal wage from the collective viewpoint of unions.[18] This is the nominal wage which minimises the individual union's loss, as represented by (29), given that every union in the economy abides by this particular wage, regardless of whether it is optimal for the individual union to do so. In notation, this amounts to the assumption that $w_j = w^* \forall j$, where w^* is the efficient wage. To derive w^*, we substitute equation (7) for union *j*'s demand for labour, together with (13), the price level reduced form derived earlier for the simple rule, into (29), in order to yield the individual union's loss as a function of w_j and w:[19]

$$\Omega_j^u = \left\{ \frac{\gamma[\alpha + \varepsilon(1-\alpha)]\overline{m} + (\varepsilon - \gamma)\alpha w - \varepsilon[\alpha + \gamma(1-\alpha)]w_j}{[\alpha + \varepsilon(1-\alpha)][\alpha + \gamma(1-\alpha)]} - l_u \right\}^2$$

$$-2c_u' \left\{ w_j - \frac{[\gamma(1-\alpha)\overline{m} + \alpha w]}{[\alpha + \gamma(1-\alpha)]} \right\} \qquad (31)$$

The efficient wage, w^*, is the wage w_j which minimises (31) subject to the restriction that $w_j = w \forall j$. Since Ω_j^u is quadratic in $w_j (= w)$, it follows that w^* is the solution to $d\left(\Omega_j^u \Big|_{w_j = w \forall j} \right) \Big/ dw = 0$, and this is found to be:

$$w^* = \overline{m} + \left(\frac{1}{\gamma} \right) [\alpha + \gamma(1-\alpha)][c_u'(1-\alpha) - l_u] \qquad (32a)$$

Combining this with (13) yields the price level under efficient wage-setting:

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[18] Throughout the thesis the concept of efficiency in wage-setting relates in general to its impact on the welfare of unions, rather than on the welfare of society.
[19] From this point onward subscripts denoting the type of monetary regime will be omitted in order to avoid encumbering the expressions with unnecessary notational appendages.

$$p|_{w_j=w^*\forall j} = \overline{m} + \left(\frac{\alpha}{\gamma}\right)[c_u'(1-\alpha) - l_u] \qquad (32b)$$

Combining (32a), (32b) and (7), allows us to derive the efficient real wage and employment:

$$(w-p)|_{w_j=w^*\forall j} = (1-\alpha)[c_u'(1-\alpha) - l_u] \qquad (33a)$$

$$l|_{w_j=w^*\forall j} = l_u - c_u'(1-\alpha) \qquad (33b)$$

This real wage, employment pair is located at the tangency point between union *j*'s isoloss map and the aggregate-level labour demand curve in real wage, employment space. The aggregate-level labour demand curve is simply that which relates average labour demand (across both firms and unions) to the average real wage. Its equation, derived in Appendix II.2, is:[20]

$$l = -\frac{(w-p)}{(1-\alpha)} \qquad (34a)$$

This aggregate-level curve is graphically coincident with, but not identical to, a closely related entity which will be referred to as the 'symmetric-wage labour demand curve'. The latter is the set of possible real wage, labour demand outcome pairs when wage-setting is symmetric ($w_j = w\forall j$), which in turn implies that price-setting will also be symmetric ($p_i = p\forall i$). This means that the symmetric-wage labour demand curve is a special case of the aggregate-level labour demand curve, the difference between the two being merely that in the case of the aggregate-level curve wage-setting may be either symmetric or asymmetric. The equation of the symmetric-wage labour demand curve, also derived in Appendix II.2, is:

[20] The 'D' superscript denoting 'demand' is henceforth dropped. (This notational simplification can be made because employment is demand-determined within the model.)

$$l_j\big|_{w_j=w\forall j} = -\frac{(w_j - p)}{(1-\alpha)} \tag{34b}$$

Symmetric setting of nominal wages by unions will imply particular realisations of the price level[21] and real wage, with labour demand and employment then being determined in accordance with (34a) and (34b). Note that if unions possess a means of coordinating their wage decisions, any point on the symmetric-wage labour demand curve would then be attainable by them. In particular, (13) implies that were unions to coordinate on the wage $w_j = -\gamma(1-\alpha)\overline{m}/\alpha$, the real wage and employment would be at their market-clearing values of zero. With the union loss function given by (29), however, this outcome would not be pursued by unions since it is not collectively optimal: rather, efficient wage-setting requires co-ordination on w^*, as given by (32a), since as mentioned above this wage, if adopted by every union, will ensure that the outcome in each union's individual labour market is at the tangency point with the isoloss map.

Before proceeding to discuss equilibrium wage-setting it is worthwhile pointing out that (33b) reveals efficient employment from the viewpoint of unions to be less than their notional employment objective of l_u, and that the shortfall of employment below l_u is greater, the higher is c'_u, the relative weight placed on the real wage. A second point which emerges from scrutiny of (33a) is that the efficient real wage is not necessarily greater than the market-clearing real wage of zero: this will only be the case if $c'_u(1-\alpha) > l_u$. (Coricelli at al.'s (2004a, 2006) assumption that unions care about unemployment corresponds in our notation to $l_u = 0$, and this would clearly imply an efficient real wage in excess of the market-clearing value.)

II.2.2 Derivation of the Equilibrium Nominal Wage

We now turn to equilibrium wage-setting. As shown by Coricelli at al. (2004a, 2006) the symmetric Nash equilibrium in which unions all set the same nominal wage will be unique in the model being considered here. We therefore proceed to find the

[21] This realisation of the price level would be given by (13), where w would be the symmetric wage (and hence also the average wage).

individual union's optimal wage, w_j^{**}, as a function of its rational expectation of the aggregate nominal wage and structural parameters, and then impose symmetry in wages to obtain an equation implicitly defining the symmetric Nash equilibrium wage, w_{NE}. The individual union in setting w_j regards the other unions' wage decisions as given. (Since the model assumes simultaneity of moves across the n unions, each union will have to form a rational expectation of the wages chosen by the other $n-1$ unions, and these wage decisions are beyond union j's influence.) With each union representing the same proportion of the workforce as every other union, the aggregate nominal wage can be written as follows:

$$w = \left(\frac{1}{n}\right)\sum_{j=1}^{n} w_j \tag{35}$$

(35) implies that $dw/dw_j = 1/n$, i.e. the contribution made by union j to the aggregate wage (defined to be the average wage across unions) is the fraction of the labour force constituted by union j's membership, and because of the symmetry assumption this is simply the inverse of the number of unions. Hence the total derivative with respect to w_j of union j's expected loss is:

$$\frac{dE\Omega_j^u}{dw_j} = \left.\frac{\partial E\Omega_j^u}{\partial w_j}\right|_{w\text{ fixed}} + \frac{\partial E\Omega_j^u}{\partial w}\left(\frac{dw}{dw_j}\right) \tag{36}$$

where E is the rational expectations operator and Ω_j^u is given by (31). Substituting the requisite derivatives into (36), the first-order condition for union j's optimal wage is found to be:

$$\frac{-2(\gamma\alpha + \varepsilon\Lambda)}{[\alpha + \gamma(1-\alpha)][\alpha + \varepsilon(1-\alpha)]}\left\{\frac{\gamma[\alpha + \varepsilon(1-\alpha)]\overline{m} + (\varepsilon - \gamma)\alpha Ew - \varepsilon[\alpha + \gamma(1-\alpha)]w_j}{[\alpha + \gamma(1-\alpha)][\alpha + \varepsilon(1-\alpha)]} - l_u\right\} - \frac{2c_u'\Lambda}{[\alpha + \gamma(1-\alpha)]} = 0 \tag{37}$$

where $\Lambda \equiv \alpha(n-1) + n\gamma(1-\alpha)$.

Solving for w_j yields union j's individually optimal wage, w_j^{**}:

$$w_j^{**} = \left(\frac{1}{\varepsilon}\right)\left[\frac{\gamma[\alpha+\varepsilon(1-\alpha)]\overline{m}+(\varepsilon-\gamma)\alpha Ew}{[\alpha+\gamma(1-\alpha)]} + [\alpha+\varepsilon(1-\alpha)]\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha+\varepsilon\Lambda)} - l_u\right\}\right] \tag{38}$$

Given the symmetry across unions and the assumed information structure, union j will appreciate that every other union's optimal wage will be equal to the right-hand side of (38). Hence union j's rational expectation of w will be its own optimal wage: $Ew = (1/n)(nw_j^{**}) = w_j^{**}$. Since this is true of every union, and not just of union j, $w_j^{**} = Ew = w_{NE} \forall j$ must be the case, where w_{NE} is the equilibrium nominal wage. Replacing w_j^{**} and Ew with w_{NE} in (38) therefore gives an equation which can be solved for w_{NE}:

$$w_{NE} = \overline{m} + \frac{[\alpha+\gamma(1-\alpha)]}{\gamma}\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha+\varepsilon\Lambda)} - l_u\right\} \tag{39}$$

Substituting (39) into (13), the price level under equilibrium wage-setting is:

$$p|_{w=w_{NE}} = \overline{m} + \left(\frac{\alpha}{\gamma}\right)\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha+\varepsilon\Lambda)} - l_u\right\} \tag{40}$$

It is useful for expositional purposes to state the value taken by w_{NE} in the two extreme cases of wage-bargaining structure[22], namely the case of a single economy-wide union which has the entire labour force as its membership and the atomistic case:

[22] Although the model abstracts from wage bargaining between each union and the firm(s) to which it is the monopoly supplier of labour services, it is convenient to use the phrase 'wage-bargaining structure' as a synonym for the more cumbersome 'labour-market supply-side structure'.

$$w_{NE}\big|_{n=1} = \overline{m} + \frac{[\alpha+\gamma(1-\alpha)][c_u'(1-\alpha)-l_u]}{\gamma} \tag{41}$$

$$\lim_{n\to\infty} w_{NE} = \overline{m} + \frac{[\alpha+\gamma(1-\alpha)]}{\gamma}\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]}{\varepsilon} - l_u\right\} \tag{42}$$

The effects on w_{NE} of marginal increases in the number of unions and the degree of goods market competition (as represented by ε) are given by:

$$\frac{\partial w_{NE}}{\partial n} = \frac{\alpha[\alpha+\gamma(1-\alpha)]^2 c_u'[\alpha+\varepsilon(1-\alpha)]}{(\gamma\alpha+\varepsilon\Lambda)^2} > 0 \ \forall n \geq 1 \tag{43}$$

$$\frac{\partial w_{NE}}{\partial \varepsilon} = \frac{-(n-1)\alpha[\alpha+\gamma(1-\alpha)]^2 c_u'\Lambda}{\gamma(\gamma\alpha+\varepsilon\Lambda)^2} < 0 \ \forall n > 1 \tag{44}$$

Under perfect goods market competition, the nominal wage is found to be:

$$\lim_{\varepsilon\to\infty} w_{NE} = \overline{m} + \frac{[\alpha+\gamma(1-\alpha)][c_u'(1-\alpha)-l_u]}{\gamma} \tag{45}$$

Using (39), (40) and (34b), the real wage and employment in equilibrium are:

$$(w-p)\big|_{w=w_{NE}} = (1-\alpha)\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha+\varepsilon\Lambda)} - l_u\right\} \tag{46a}$$

$$l\big|_{w=w_{NE}} = l_u - \frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha+\varepsilon\Lambda)} \tag{46b}$$

For the extremes of labour-market structure the real outcomes are:

$$(w-p)\big|_{w=w_{NE},\, n=1} = (1-\alpha)[c_u'(1-\alpha)-l_u] \tag{47a}$$

$$l\big|_{w=w_{NE},\, n=1} = l_u - c_u'(1-\alpha) \tag{47b}$$

$$\lim_{n\to\infty}(w-p)\big|_{w=w_{NE}} = (1-\alpha)\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]}{\varepsilon} - l_u\right\} \tag{48a}$$

$$\lim_{n\to\infty} l\big|_{w=w_{NE}} = l_u - \frac{c_u'[\alpha+\varepsilon(1-\alpha)]}{\varepsilon} \tag{48b}$$

The effects of marginal increases in n and ε on the real wage are:

$$\frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial n} = \frac{\gamma\alpha(1-\alpha)[\alpha+\gamma(1-\alpha)]c_u'[\alpha+\varepsilon(1-\alpha)]}{(\gamma\alpha+\varepsilon\Lambda)^2} > 0 \ \forall n \geq 1 \tag{49}$$

$$\frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial \varepsilon} = \frac{-(n-1)\alpha(1-\alpha)[\alpha+\gamma(1-\alpha)]c_u'[\alpha+\varepsilon(1-\alpha)]}{(\gamma\alpha+\varepsilon\Lambda)^2} < 0 \ \forall n > 1 \tag{50}$$

II.2.3 Discussion of the Wage-Setting Externality under the Simple Rule

Examined together, expressions (39) to (50) reveal the presence of an adverse externality arising from individual wage-setting decisions when the union loss function is given by (29) and monetary policy is conducted according to the simple rule. Comparing the equilibrium nominal wage, (39), with its efficient counterpart, (32a), it is apparent that apart from in the special cases of a single union (see (41)) or a perfectly competitive goods market (see (45)), the equilibrium nominal wage is inefficiently high. This shows up as an inefficiently high real wage (compare (46a) with (33a)), together with inefficiently low employment (compare (46b) with (33b)). The derivatives (43) and (49) reveal the extent of the inefficiency to be increasing in the number of unions, with the real wage at its highest and most inefficient under atomistic unions (see (48a)). The externality involves a failure by the individual union fully to internalise the price level repercussions of its wage decision. In setting its wage, union j will only take into account the impact of w_j on the aggregate wage, and hence its impact via (13) on the price level, to the extent that it is perceived to affect its own welfare: the impact of w_j on the welfare of other unions is disregarded. If unions are atomistic, each union's wage makes a negligibly small (formally zero) contribution to w, and so the individual union sets its wage in the (correct) belief that its individual wage will not induce any change in the price level. Consequently the adverse price level repercussions of aggregated wage decisions do not exercise any restraining influence on atomistic unions, with the result that the wage is pushed beyond its efficient value. An inefficiently high nominal wage also arises under multiple non-atomistic unions, but the recognition that the price level repercussions of union *j*'s wage are non-negligible mitigates the externality. In the non-atomistic case the fact that w_j has a non-negligible influence on p, and hence on aggregate demand also, induces wage restraint on the part of the union.

To examine this a little more deeply, it is helpful at this point to decompose the marginal impact of union *j*'s wage, w_j, on its labour demand into two effects, one involving the induced change in the real wage, and one involving the induced change in real money balances and hence aggregate demand:

$$\frac{dl_j^D}{dw_j} = \frac{1}{[\alpha + \varepsilon(1-\alpha)]}\left\{\gamma \frac{d(m-p)}{dw_j} - \varepsilon \frac{d(w_j - p)}{dw_j}\right\} \tag{51}$$

where
$$\frac{d(m-p)}{dw_j} = \left(\frac{1}{n}\right)\frac{d(m-p)}{dw} \tag{52}$$

and
$$\frac{d(w_j - p)}{dw_j} = 1 - \left(\frac{1}{n}\right)\frac{dp}{dw} \tag{53}$$

Note that (51) is the total derivative, with respect to w_j, of union *j*'s demand for labour as given by (7) rather than (34b). Equation (7) states the demand for union *j*'s labour when its employer firms have set their profit-maximising prices, taking the aggregate price level as given (i.e. symmetric price-setting by firms, $p_i = p \forall i$, has yet to occur), whereas the symmetric-wage labour demand curve, (34b), assumes symmetry of both wages and prices. The ability of union *j*'s employer firms to set a price or prices which differ from the average price is fundamental to the externality. Under monopolistic competition, there is scope for firm *i*'s price to exceed that of its competitors without the disparity triggering a complete collapse of the former's product demand, and hence also a collapse in the derived demand for union *j*'s labour. Recognising this, union *j* perceives there is scope for its own wage to exceed that set by other unions. It turns out that were every other union to abide by the efficient wage, union j would make itself better off by setting its wage in excess of the efficient wage. While this would occasion some contraction of its labour demand, and hence a larger departure of employment from its desired value of l_u, the gain in terms of a higher real wage would more than compensate for this. Alternatively put, the trade-off between employment and the real wage is advantageous for union j if all other unions set the efficient wage. In the simultaneous-move game between the unions, of course, every union perceives that its rivals will face this incentive to set a wage in excess of the efficient wage. The wage which is consistent with optimising behaviour by unions is perfectly foreseen and every union has the incentive to set this wage, w_{NE}, despite its manifest collective inefficiency. The greater the degree of goods-market competition (i.e. the higher is ε) the less scope is there for union j's employer firms to set prices in excess of the efficient wage. This explains why the equilibrium real wage is falling in ε, as revealed by (50), and why wage-setting is efficient when the goods market is perfectly competitive, as revealed by (45). Under perfect competition no firm can allow its own price to depart from the average price: hence union j cannot possibly gain from setting a wage in excess of the efficient wage.

The total derivative, (51), reveals that there are two channels via which a marginal increase in union j's wage can affect its demand for labour. One of these involves (52), the marginal impact of w_j on real money balances, and hence on aggregate demand, via the induced change in the price level. Coricelli et al. (2004a, p.3) refer to this channel, $(\gamma/[\alpha+\varepsilon(1-\alpha)])(d(m-p)/dw_j)$, as the "aggregate demand channel". Note that since $\lim_{n\to\infty} d(m-p)/dw_j = 0$, this channel is not operative in the atomistic case. It is also worth noting that under the simple rule which is the immediate context of the present discussion, the effect of a marginal increase in w_j on real money balances is negative, since m is kept fixed and an increase in w_j contributes to increase the price level. Under the alternative monetary regimes discussed below, however, the monetary response to the aggregate wage may be such as to cause $d(m-p)/dw_j \geq 0$ rather than $d(m-p)/dw_j < 0$ to be the case. The second of the two channels through which a marginal increase in w_j influences labour demand involves the real wage, and it is this real wage channel,

$-(\varepsilon/[\alpha+\varepsilon(1-\alpha)])(d(w_j-p)/dw_j)$, which is the source of the externality irrespective of the monetary regime. The reason why a smaller number of unions reduces the severity of the externality has nothing to do with the real wage channel: rather, the mitigating effect greater centralisation has on the externality is due to its influence on the channel involving aggregate demand.

In the symmetric Nash equilibrium each individual union recognises that its counterparts will face the same incentive as itself to exploit the trade-off between real wage and employment embodied in (7), and hence to set a particular nominal wage which, in the case of the simple rule, exceeds the efficient wage. Every union knows the structure of the economy, and therefore knows that symmetric pricing by firms, $p_i = p \forall i$, and symmetric wage-setting by unions, $w_j = w \forall j$, must ensue. Moreover, every union recognises that the equilibrium involves a particular nominal wage, w_{NE}, from which no union has an incentive to deviate, and that the resulting outcome in terms of employment and the real wage must lie on the symmetric-wage labour demand curve, (34b), as opposed to (7), the demand curve which holds in the absence of symmetry in prices and wages.

The efficiency of the outcome when there is a single economy-wide union has a simple explanation: the ability to set the aggregate nominal wage enables the union, by judiciously taking into account the behaviour of the price level, to set the wage which will bring about the efficient combination of real wage and employment. In other words, efficiency is achieved under a single union because the price level and aggregate demand repercussions of the wage which is set at the level of the individual firm are fully internalised.

It seems apt to conclude this discussion of the wage-setting externality under the simple rule with a diagrammatic representation of its source. To this end we introduce the notion of the individual union's perceived labour demand curve, which is to be distinguished from the symmetric-wage labour demand curve which has already been discussed. In formulating union *j*'s perceived labour demand curve it is assumed that every other union, apart from union *j*, sets the same nominal wage. Hence were union *j* also to set that particular wage, symmetric wage-setting would occur and a particular outcome on the aggregate-level labour demand curve would be brought about. It follows from this that there is an infinite set of perceived labour demand curves. Each of these curves is associated with the other $(n-1)$ unions setting a particular nominal wage, and consequently passes through the point on the aggregate-level labour demand curve associated with symmetric setting of that nominal wage.

In explaining the externality, only two members of the infinite set of perceived labour demand curves need be mentioned. The first of these is that which assumes that every other union, apart from union j, does set the efficient wage. Hence were union j also to set the efficient wage, the aggregate wage would be efficient and the efficient combination of real wage and employment would be brought about. This perceived labour demand curve of union j must therefore intersect the symmetric-wage labour demand curve at the efficient outcome, labelled point "a" in Figure II.3 below. The perceived labour demand curve's slope is the trade-off between the individual union's employment and real wage, with all other unions' nominal wages taken as given. Since this trade-off is independent of other unions' nominal wages, all of union j's possible perceived labour demand curves have the same slope. The equation of the perceived labour demand curve passing through the efficient outcome, derived in Appendix II.3, is:

$$l_j\Big|_{w_k=w^*\forall k\neq j} = \frac{(n-1)\alpha[\alpha+\gamma(1-\alpha)][l_u - c_u'(1-\alpha)] - (\gamma\alpha+\varepsilon\Lambda)(w_j - p)}{[\alpha+\varepsilon(1-\alpha)]\Lambda} \tag{54}$$

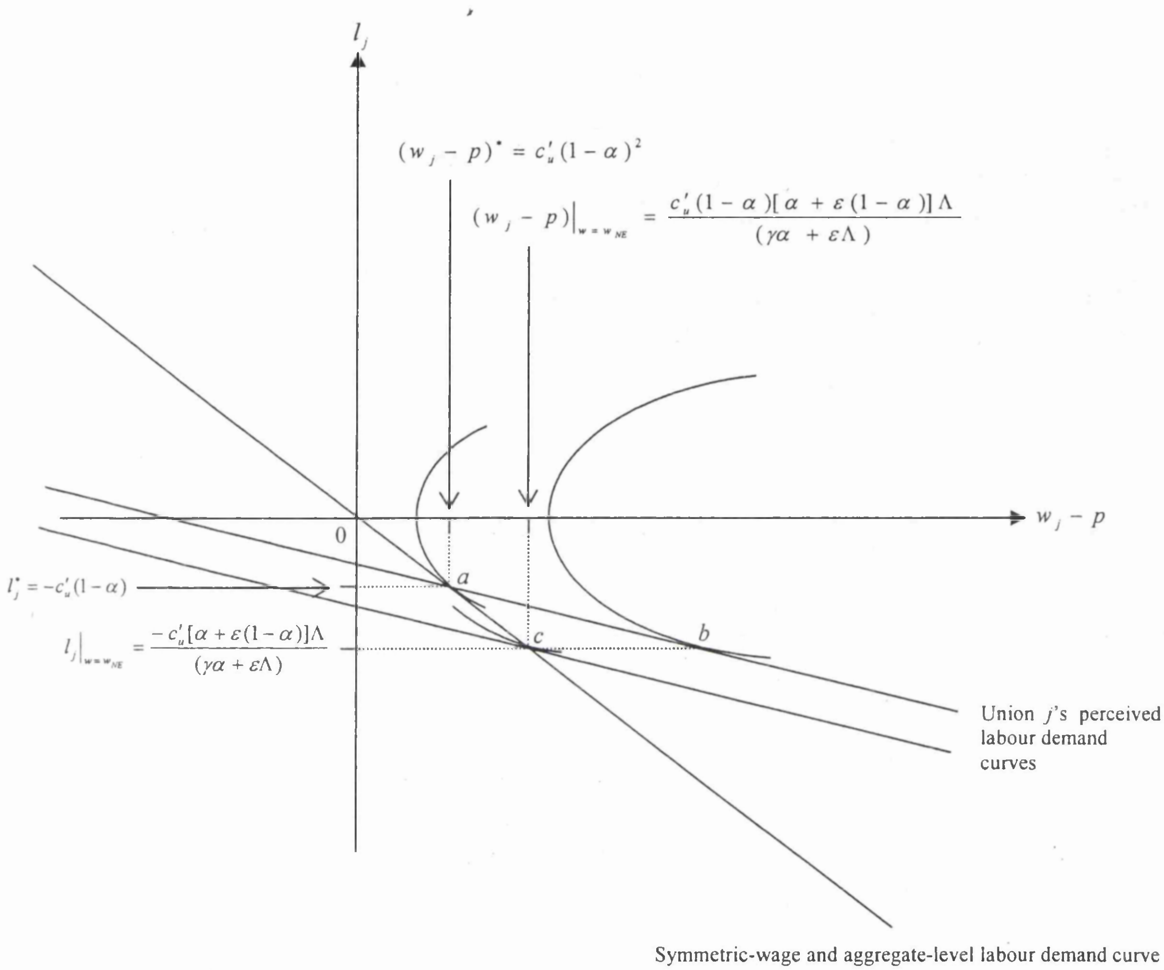
In interpreting this expression the reader should bear in mind that efficient wage-setting by all other unions save j is assumed, and that (13) therefore implies that the price level will be given by:

$$p\Big|_{w_k=w^*\forall k\neq j} = \frac{n\gamma(1-\alpha)\overline{m} + \alpha[(n-1)w^* + w_j]}{n[\alpha+\gamma(1-\alpha)]} \tag{55}$$

where w^* is given by (32a). By taking into account this expression, union j will, given $w_k = w^*\forall k \neq .j$, be able to determine a particular realisation of its real wage together with the associated employment outcome implied by (54). Thus any point on the upper perceived labour demand curve depicted in Figure II.3 will be attainable by choice of w_j, with the optimal choice that which attains the tangency point between it and the union's isoloss map. This point is labelled "b" in Figure II.3, which assumes

Figure II.3

Labour demand curves under the simple rule



for simplicity that $l_u = 0$, so that the axis of symmetry of the isoloss contours coincides with the horizontal axis.

What is true of union j is also true of every other union, of course, and thus all unions have an incentive to deviate from the efficient wage in an attempt to attain point b. Recognising this, every union realises that the symmetric Nash equilibrium wage must be that consistent with the outcome being at point c in Figure II.3, which is inefficient in that the real wage is too high and employment too low. Note that the second perceived labour demand curve of particular interest, namely the equilibrium perceived labour demand curve which passes through the equilibrium outcome at c, is tangent to the union's isoloss map at that point.[23] Were every other union to set the equilibrium wage, union j could attain any point on the equilibrium perceived labour demand curve by setting its wage appropriately. Union j's optimal strategy, of course, is to set the equilibrium nominal wage, so that it too attains the tangency point between the equilibrium perceived labour demand curve and the isoloss map. Of all the points on the aggregate-level labour demand curve, therefore, c is unique in that the individual union's perceived labour demand curve is tangent to the union's isoloss map at that point. It is this latter characteristic of c which makes it the unique symmetric Nash equilibrium outcome.

Note that the slope of the perceived labour demand curves, and hence the proximity of the equilibrium outcome, c, to the efficient outcome, a, is a function of n and ε. The slope becomes more negative (i.e. steepens) as ε increases (provided $n > 1$), but becomes less negative (i.e. gets flatter) as n increases, as is revealed by the following derivatives:[24]

$$\frac{\partial\left(dl_j/d(w_j - p)\right)}{\partial \varepsilon} = \frac{-(n-1)\alpha[\alpha + \gamma(1-\alpha)]}{[\alpha + \varepsilon(1-\alpha)]^2 \Lambda} < 0 \;\forall\; n > 1 \tag{56}$$

$$\frac{\partial\left(dl_j/d(w_j - p)\right)}{\partial n} = \frac{\gamma\alpha[\alpha + \gamma(1-\alpha)]}{[\alpha + \varepsilon(1-\alpha)]\Lambda^2} > 0 \tag{57}$$

[23] The equation of the equilibrium perceived labour demand curve is derived and stated in Appendix II.3 and differs from (54) only as regards the intercept term.

[24] The cross partial derivatives (56) and (57) relate to (54), the equation of the perceived labour demand curve passing through the efficient outcome.

The higher the degree of goods market competition, and the lower the number of unions, the weaker is the adverse externality and this is reflected diagrammatically by a clockwise rotation, as ε increases and n decreases, of the depicted perceived labour demand curves about their intersection points with the aggregate-level labour demand curve. In the limiting cases of perfect goods-market competition and a single union the perceived labour demand curve coincides with the symmetric-wage labour demand curve, ensuring equilibrium is at the efficient outcome. With l_j given by (54), we have for these extreme cases:

$$\lim_{\varepsilon \to \infty} l_j = l_j\Big|_{n=1} = -\frac{(w_j - p)}{(1-\alpha)} \tag{58}$$

The nature of the wage-setting externality has been discussed at considerable length because of its importance to the results of the remainder of this chapter and several later chapters, and because existing contributions to the literature do not provide a detailed description of its source.[25] As we shall see, the externality is essentially the same under the alternative monetary regimes which will be discussed shortly. Before moving on to these other regimes, however, we conclude our discussion of the simple rule by stating the socially optimal setting of the rule parameter $\overline{m}$, the fixed value of the money stock under the rule. It is apparent from (46a) and (46b) that real outcomes are independent of $\overline{m}$. Since there is no monetary response to the individual union's setting of its wage, regardless of how large a share of the labour force it represents, it follows that $\overline{m}$ cannot influence the trade-off faced by the union between employment and the real wage: union j need only adjust its nominal wage appropriately to take into account the perfectly foreseen contribution of $\overline{m}$ to price level determination. Hence the socially optimal setting of $\overline{m}$, denoted $\overline{m}^*$, is that which minimises inflation, i.e. the departure of p from zero. From (40) it is apparent that $\overline{m}^*$ is:

[25] Although Coricelli et al. (2004a, 2006) do not use the word 'externality' in the course of their papers, they are clearly aware of its existence, as is revealed by their use of the word "internalize" at p.19 of Section 3.2 of the longer (2004a) version of the paper, when explaining why greater centralisation of wage bargaining leads to a smaller wage premium. Bratsiotis and Martin (1999), who use a close variant of loss function (30), do not mention 'externality' or use the word 'internalise' at all.

$$\overline{m}^* = \frac{\alpha}{\gamma}\left\{l_u - \frac{c_u'[\alpha + \varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha + \varepsilon\Lambda)}\right\} \tag{59}$$

Equation (39) reveals that for any given value of $\overline{m}$, the effect of the wage-setting externality is to push up the nominal wage, which in turn, via (13), increases the price level. However, by taking this phenomenon into account, the appropriate setting of $\overline{m}$ neutralises the impact of wages on the price level completely. The social loss under the socially optimal simple rule therefore solely consists of the deviation of employment from its market-clearing value: using (9′), (40), (46b), and (59), it is found to be:

$$\Omega^s\Big|_{simple\ rule,\ \overline{m}=\overline{m}^*} = \left\{\frac{c_u'[\alpha + \varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha + \varepsilon\Lambda)} - l_u\right\}^2 \tag{60}$$

II.3 Wage-Setting under the Price Level Rule

II.3.1 Inadmissible Values of the Price Level Rule Parameter

The first of our three rules which involve the money supply being set contingent on the realised value of a macroeconomic variable is (14), the price level rule $m_r = \tau_1 p$. The relevant semi-reduced forms for this rule are equations (15) and (16). Note that this rule captures the simple rule as a special case, namely the $\tau_1 = 0$ case. A second special case encompassed by (14) is that in which monetary policy is devoted exclusively to preventing departures of p from its socially optimal value of zero: this special case is the limiting $\tau_1 \to \pm\infty$ case, and we will refer to it below as the strict price level rule. Note that Soskice and Iversen, in analysing the macroeconomic consequences of a rule of the form (14), impose an arbitrary restriction on the rule parameter τ_1, namely that it must only take a value in the unit interval, $0 \le \tau_1 \le 1$. There is, however, no economic reason to justify such a restriction (nor is one provided by Soskice and Iversen), and in this section we will assume that τ_1 may take almost any value on the real line. The very few τ_1 values which are not admissible will be discussed when appropriate during the course of the exposition. One such inadmissible value of τ_1 is immediately apparent from scrutiny of (15), however, since the price level will not be defined when $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. The reason for this is that when τ_1 has this value a non-zero realisation of the aggregate nominal wage implies there is no finite price level which can equate aggregate supply with aggregate demand.[26] Hence goods-market clearing can only occur if $w_i = 0 \forall i$ when $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. Furthermore, even if $w_i = 0 \forall i$ when τ_1 has this value, so that the goods market does clear, that market-clearing may occur at any price, since there is nothing to pin down a particular market-clearing price in this situation. Hence p is indeterminate when $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, irrespective of the value wages take.

II.3.2 Efficient Wage-Setting under the Price Level Rule

The efficient nominal wage under the price level rule is obtained by following an analogous procedure to that described earlier for the case of the simple rule, save that now account must be taken of how w influences real outcomes via the induced monetary response to the price level. This wage is found to be:

$$w^* = \frac{[\alpha + \gamma(1-\alpha)(1-\tau_1)][c_u'(1-\alpha) - l_u]}{\gamma(1-\tau_1)} \tag{61}$$

Using (61) in conjunction with (15) and (35), the efficient real wage and employment are found to be given by (33a) and (33b), the same as the efficient outcomes under the simple rule. This is not surprising: if the individual wage is set taking full account of how wages in the aggregate induce a particular monetary response, and thus contribute to determine the price level, real money balances, and aggregate demand, the best feasible outcome can almost always be brought about by means of coordinated wage-setting, irrespective of the monetary regime's characteristics.

[26] Aggregate supply and aggregate demand in this situation are given by: $y^S\big|_{\tau_1=[\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)} = [\alpha/(1-\alpha)][p - \{\varepsilon(1-\alpha)w/[\alpha+\varepsilon(1-\alpha)]\}]$, and $y^D\big|_{\tau_1=[\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)} = \alpha p/(1-\alpha)$. It is straightforward to show that $y^S = y^D$ is only possible when $\tau_1 = [\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$ if $w = 0$ is the case, and that if $w = 0$ is the case then any p will equate y^S with y^D.

The reason for the qualifying phrase "almost always" in the previous sentence is that there are two specific τ_1 values which deprive fully coordinated wage-setting of the ability to bring about the efficient wage. The first of these is $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, which, as we saw above, renders the price level indeterminate. Equation (61) therefore cannot be evaluated for this value of τ_1, since it has been derived on the assumption that $\tau_1 \neq [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$.[27] The second τ_1 value which precludes efficient wage-setting is $\tau_1 = 1$. It is apparent from (61) and (15) that the efficient nominal wage and consequently the price level also, are undefined when $\tau_1 = 1$. The explanation for this is that when $\tau_1 = 1$, coordinated wage-setting on any wage will necessarily induce a price level realisation which is equal to that wage. In other words, when $\tau_1 = 1$, and wage-setting is symmetric, $w_i = w \forall i$, profit-maximising behaviour by firms brings about a price which ensures that the real wage is zero.[28] Since attempts to bring about the efficient real wage are therefore futile when $\tau_1 = 1$, it is hardly surprising that (61) reveals w^* to be undefined when $\tau_1 = 1$.

II.3.3 Equilibrium Wage-Setting under the Price Level Rule

II.3.3.(i) Derivation of Equilibrium Expressions

Following procedures analogous to those described for the simple rule allows us to derive the equilibrium nominal wage under the price level rule:

[27] Note that while (15) implies that the real wage under the price level rule is given by $w - p = \gamma(1-\alpha)(1-\tau_1)w/[\alpha + \gamma(1-\alpha)(1-\tau_1)]$, substituting w^* as given by (61) into this expression to obtain (33a) is an invalid procedure when $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, since it involves the division of an expression by zero.

[28] Using (A.II.1.7) once again, firm i's profit-maximising price when $\tau_1 = 1$ is: $p_i^*\big|_{\tau_1=1} = \varepsilon(1-\alpha)p + \alpha w_i/[\alpha + \varepsilon(1-\alpha)]$, and consequently with symmetric wage-setting, $w_i = w \forall i$, the single market-clearing price will be $p_i = p = w$. Note also that when $\tau_1 = 1$, real money balances, and hence aggregate demand also, are always zero, regardless of the aggregate wage, w. Since aggregate supply is $y^S\big|_{\tau_1=1} = -\alpha\varepsilon(w-p)/[\alpha + \varepsilon(1-\alpha)]$, it follows that the price level must adjust to ensure that the aggregate real wage is zero.

$$w_{NE} = \frac{[\alpha + \gamma(1-\alpha)(1-\tau_1)]}{\gamma(1-\tau_1)} \left[\frac{c_u'[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]}{\{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}} - l_u \right] \tag{62}$$

The associated realisation of the price level is (using (15)):

$$p|_{w=w_{NE}} = \frac{\alpha}{\gamma(1-\tau_1)} \left[\frac{c_u'[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]}{\{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1)\}} - l_u \right] \tag{63}$$

In the extreme cases of a single union and atomistic unions, (62) becomes:

$$w_{NE}\big|_{n=1} = \frac{[\alpha + \gamma(1-\alpha)(1-\tau_1)][c_u'(1-\alpha) - l_u]}{\gamma(1-\tau_1)} \tag{64}$$

$$\lim_{n\to\infty} w_{NE} = \frac{[\alpha + \gamma(1-\alpha)(1-\tau_1)]}{\gamma(1-\tau_1)} \left[\frac{c_u'[\alpha + \varepsilon(1-\alpha)]}{\varepsilon} - l_u \right] \tag{65}$$

The equilibrium nominal wage when the goods market is perfectly competitive is:

$$\lim_{\varepsilon\to\infty} w_{NE} = \frac{[\alpha + \gamma(1-\alpha)(1-\tau_1)][c_u'(1-\alpha) - l_u]}{\gamma(1-\tau_1)} \tag{66}$$

Note that the right-hand side of (66) is identical to the right-hand sides of (64) and (61). We shall comment further on this below. Combining (62) and (63), the equilibrium real wage under the price level rule is:

$$(w-p)|_{w=w_{NE}} = (1-\alpha) \left[\frac{c_u'[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]}{\{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}} - l_u \right] \tag{67a}$$

The associated equilibrium employment level is found, using (34b) to be:

$$l|_{w=w_{NE}} = l_u - \frac{c_u'[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]}{\{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}} \tag{67b}$$

II.3.3.(ii) Discussion of Equilibrium Outcomes under the Price Level Rule

A number of interesting points can be made with regard to (67a) and (67b). The first thing to notice is that setting τ_1 equal to zero in (67a) and (67b) yields (46a) and (46b), the counterpart equilibrium expressions for the simple rule. This is unremarkable, but comparison of (67a) with (46a) for $\tau_1 \neq 0$ elicits a much more interesting point, namely that whereas equilibrium outcomes are invariant to the constant term in the rule, $\overline{m}$, regardless of the number of unions, the value taken by τ_1, the rule parameter which governs the response of the money supply to the price level, does matter for equilibrium outcomes. The sole exceptions to this are the two extremes of labour-market structure, namely a single union and the case of atomistic unions: evaluating (67a) at $n = 1$ yields (33a), the efficient real wage, while taking the limit of (67a) as $n \to \infty$ yields (48a), the real wage outcome under the simple rule when unions are atomistic. This finding that τ_1 is irrelevant to real outcomes in these extreme cases has an immediate implication: if $n = 1$ or unions are atomistic, a strict price level rule ($\tau_1 \to \pm\infty$) must be socially optimal.

The inability of the rule parameter to influence real outcomes when $n = 1$ has a straightforward explanation. A single union has the incentive to take full account of the monetary response and price level repercussions of its wage choice, and the complete internalisation of these macroeconomic repercussions ensures that the efficient real wage will always be achieved when $n = 1$, regardless of τ_1.[29] In the atomistic case, each individual union disregards the negligibly small contribution its wage makes to inducing a monetary response, and hence the response parameter τ_1 cannot influence wage-setting behaviour in any way other than through the individual union's rational expectation of the price level. Put alternatively, real outcomes are invariant to τ_1 in the atomistic case since there is no internalisation whatsoever of the monetary response repercussions of individual wage decisions. For intermediate values of *n*, such that $1 < n < \infty$, however, some (albeit incomplete) internalisation of these repercussions does occur, and the very fact that the internalisation is partial creates an opportunity for the rule parameter τ_1 to influence real outcomes.

[29] Provided, of course, that τ_1 does not take one of the two values discussed earlier which result in an indeterminate price level, namely $\tau_1 = 1$ and $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$.

Underpinning this finding is the wage-setting externality which was comprehensively discussed in our earlier section on the simple rule. Comparison of (67a) with (33a) reveals that the adverse externality also arises under the price level rule, provided of course $n > 1$. We have already established in the previous paragraph that in the atomistic case the externality has exactly the same severity under the price level rule as under the simple rule. However, for $1 < n < \infty$ the externality's strength and direction (where by 'direction' is meant whether the resulting real wage is greater than or less than the efficient wage) is subtly dependent on τ_1. This contrasts sharply with the simple rule, where the externality is independent of $\overline{m}$ and always manifests itself as an inefficiently high real wage, with the inefficiency declining in ε and increasing in n in a straightforward way. If $\tau_1 \neq 0$, however, the externality can, depending on the value of τ_1, operate in such a way as to cause the equilibrium real wage to be inefficiently low, so that equilibrium employment exceeds the efficient level.

To be more specific about this, (67a) tells us that an inefficiently low real wage, i.e. $(w-p)\big|_{w=w_{NE}} < (w-p)\big|_{w=w^*}$, will result if τ_1 is in the range $(\gamma\alpha + \varepsilon\Lambda)/\gamma[\alpha + n\varepsilon(1-\alpha)] < \tau_1 < [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. If τ_1 lies outside this interval, so that $\tau_1 < (\gamma\alpha + \varepsilon\Lambda)/\gamma[\alpha + n\varepsilon(1-\alpha)]$ or $\tau_1 > [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, a result akin to that of the simple-rule case arises, namely an inefficiently high real wage.

Scrutiny of (67a) is informative not only because it reveals which τ_1 values can result in an inefficiently low or high real wage, however, but also because it tells us that certain specific values of τ_1 give rise to particularly interesting outcomes:

(i) A strict price level rule is revealed by (67b) to outperform the simple rule as regards its employment outcome, both from the point of view of unions and from the point of view of the social loss function $(9')$. Formally, we have using (67b):

$$\lim_{\tau_1 \to \pm\infty} l\big|_{w=w_{NE}} = l_u - \frac{nc_u'(1-\alpha)[\alpha + \varepsilon(1-\alpha)]}{[\alpha + n\varepsilon(1-\alpha)]} \tag{68}$$

From comparing (68) with (46b) it is evident that equilibrium employment under the strict price level rule is closer to both its efficient level, as given by (33b), and the market-clearing level of zero, than is equilibrium employment under the simple rule. Hence the wage-setting externality under the strict price level rule is less detrimental as regards real outcomes than is the simple rule. Unsurprisingly, (63) reveals that the strict price level rule also ensures the price level attains its socially optimal value of zero. It follows that the social loss, as given by $(9')$, must be lower under the strict price level rule than under the simple rule. This finding is closely related to that of Bratsiotis and Martin (1999), who, using a variant of loss function (30) rather than (29), also find that in the absence of stochastic shocks a strict price level rule outperforms a simple rule. As we shall see, however, it remains to be investigated whether there is some less-than-strict price level rule (i.e. some finite non-zero τ_1) which can outperform the strict price level rule in the non-stochastic version of Bratsiotis and Martin's model.

(ii) It was established above that two particular τ_1 values, namely $\tau_1 = 1$ and $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, are inadmissible since they result in an undefined price level and nominal wage.[30] Strictly speaking, therefore, these two values must be excluded from the domain of the equilibrium real wage, considered as a function of τ_1, and the same is true of equilibrium employment. It is apparent from (67a), however, that the limiting value of the equilibrium real wage as τ_1 approaches unity is its value when unions are atomistic, while its limiting value as τ_1 approaches $[\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ is the efficient real wage (alternatively, its value when there is a single union):

$$\lim_{\tau_1 \to 1} (w-p)\Big|_{w=w_{NE}} = (1-\alpha)\left[\frac{c_u'[\alpha + \varepsilon(1-\alpha)]}{\varepsilon} - l_u\right] \qquad (69)$$

$$\lim_{\tau_1 \to [\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)} (w-p)\Big|_{w=w_{NE}} = (1-\alpha)\left[c_u'(1-\alpha) - l_u\right] \qquad (70)$$

[30] It is not immediately obvious from (63) that the price level under equilibrium wage-setting is undefined when $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. Recall, however, that (63) has been derived on the assumption that $\tau_1 \neq [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$.

As we shall see, there is a sound economic explanation for this pair of results: it will be demonstrated below that these limiting τ_1 values cause the trade-off between real wage and employment faced by the individual non-atomistic union to equal the trade-off which is faced under either one of the extremes of wage-bargaining structure.

(iii) (67a) and (67b) reveal the existence of a third inadmissible value for τ_1 when $1 < n < \infty$, namely $\hat{\tau}_1 \equiv (\gamma\alpha + \varepsilon\Lambda)/\gamma[\alpha + n\varepsilon(1-\alpha)]$. Note that whereas $\tau_1 = 1$ and $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ are inadmissible values for the rule parameter for all *n*, $\hat{\tau}_1$ is admissible when $n = 1$ and in the limiting $n \to \infty$ case, and is inadmissible only for $1 < n < \infty$. This is because the explanation for why $\tau_1 = \hat{\tau}_1$ results in both the equilibrium nominal wage and the price level, as given by (62) and (63), being undefined, involves the effect this τ_1 setting has on the perceived real wage-employment trade-off faced by multiple non-atomistic unions. A proper explanation of this result requires an investigation of how τ_1 influences a non-atomistic union's perceived trade-off, and this will be provided below.

(iv) The most remarkable result implied by expressions (67a) and (67b), however, is the following: there exists a value for τ_1 which, provided $1 < n < \infty$, results in the equilibrium real wage and employment attaining their market-clearing values (of zero). This τ_1 value will be denoted τ_1^*. Simply by setting (67a) or (67b) equal to zero and solving for τ_1, it is found to be:

$$\tau_1^* = \left(\frac{1}{\gamma}\right)\left\{\frac{c_u'[\alpha + \varepsilon(1-\alpha)]\Lambda - (\gamma\alpha + \varepsilon\Lambda)l_u}{nc_u'(1-\alpha)[\alpha + \varepsilon(1-\alpha)] - [\alpha + n\varepsilon(1-\alpha)]l_u}\right\} \qquad (71)$$

For the simpler case in which $l_u = 0$, which will be referred to frequently below, τ_1^* is:

$$\tau_1^*\Big|_{l_u=0} = \frac{\Lambda}{n\gamma(1-\alpha)} \qquad (71')$$

Furthermore, (62) and (63) both reveal that the equilibrium nominal wage and price level are, provided $1 < n < \infty$, both equal to zero when $\tau_1 = \tau_1^*$. With employment and the price level at their socially optimal values of zero, it follows that the social loss when $\tau_1 = \tau_1^*$ is also zero. Because it achieves the first-best outcome (society's bliss point in employment, inflation space) the rule $m_r = \tau_1^* p$ must also be time-consistent and fully credible.

Before proceeding to provide an economic explanation for this key result, there are several aspects of it which call for comment. The first thing to point out is that τ_1^* does satisfy the first-order condition for the socially optimal τ_1, namely $\partial \Omega^s\big|_{w=w_{NE}} \big/ \partial\tau_1 = 0$, it nevertheless does not satisfy the equation $\partial l\big|_{w=w_{NE}} \big/ \partial\tau_1 = 0$. Secondly, considerable insight into the result can be gained from examining the relationship between equilibrium employment and the rule parameter.

II.3.3.(iii)Equilibrium Employment as a Function of τ_1

It is straightforward to show that for all admissible values of τ_1, equilibrium employment is falling in τ_1, provided $1 < n < \infty$:

$$\frac{\partial l\big|_{w=w_{NE}}}{\partial\tau_1} = \frac{-(n-1)\gamma\alpha^2 c_u'[\alpha + \varepsilon(1-\alpha)]}{\{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}^2} < 0 \ \forall\ n > 1 \tag{72}$$

Note that for the case of a strict price level rule we have $\lim_{\tau_1 \to \pm\infty}\left(\partial l\big|_{w=w_{NE}} \big/ \partial\tau_1\right) = 0$, while from (67b) it is clear that $l\big|_{w=w_{NE}}$, is a discontinuous function of τ_1, with the discontinuity occurring at $\tau_1 = \hat{\tau}_1$.[31]

The graph of this function is depicted below as Figure II.4, which assumes that $l_u < c_u'(1-\alpha)$, so that τ_1^* lies in the interval $\hat{\tau}_1 < \tau_1^* < [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. (Note that the most obvious case to consider, that in which each union's notional desired employment level is the market-clearing level, so that $l_u = 0$, is one case which

[31] Note that $\partial l\big|_{w=w_{NE}} \big/ \partial\tau_1$, as given by (73), is also undefined at $\tau_1 = \hat{\tau}_1$.

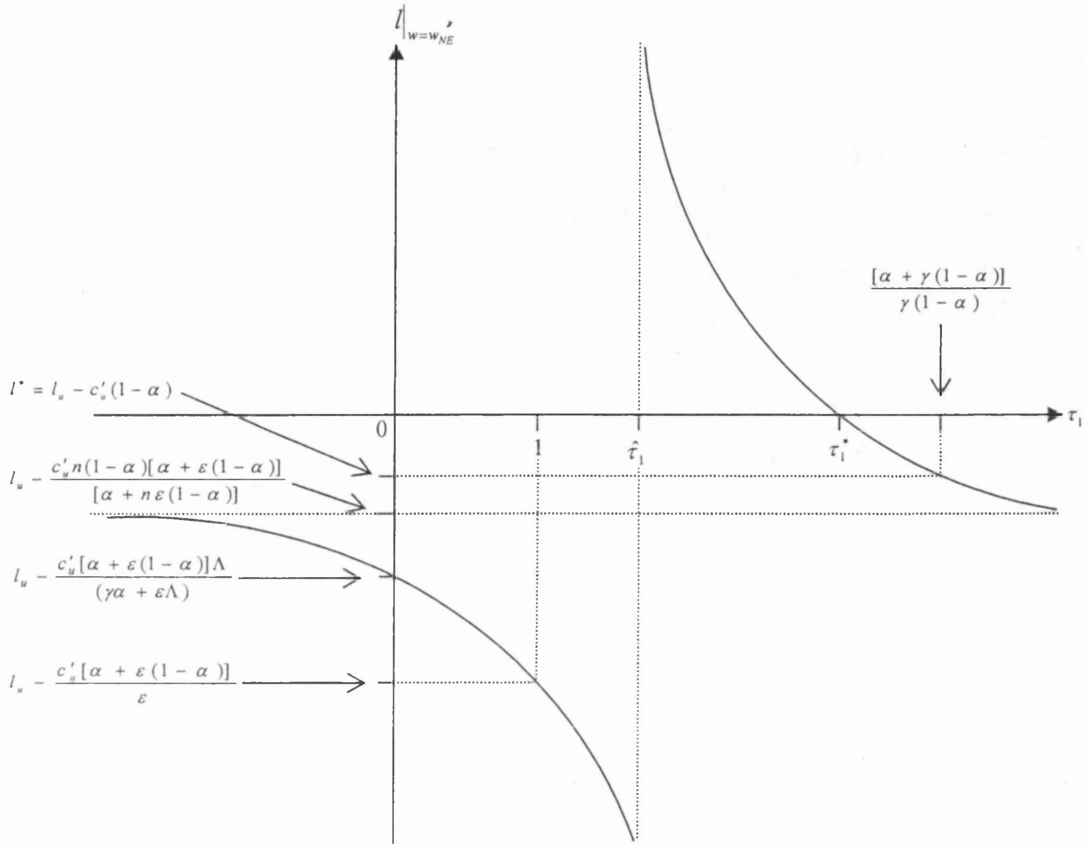
satisfies the $l_u < c_u'(1-\alpha)$ inequality, and therefore would be encompassed by Figure II.4.) Although the graph gives the appearance of equilibrium employment being a continuous function of τ_1 in the vicinity of $\tau_1 = 1$ and $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, this is not in fact the case. For the reasons given earlier, the function is discontinuous at these values, although its value in the limit as $\tau_1 \to 1$ can be seen from the graph to be the employment outcome which arises under atomistic unions, while its value in the $\tau_1 \to [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ limiting case is shown by the graph to be the efficient outcome. Figure II.4 clearly reveals that for the set of cases in which $l_u < c_u'(1-\alpha)$, if τ_1 is such that $\hat{\tau}_1 < \tau_1 < \tau_1^*$, equilibrium employment exceeds the market-clearing level (zero), while if $\hat{\tau}_1 < \tau_1 < [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ equilibrium employment is inefficiently high from the point of view of unions. Equilibrium employment is too low for both unions and society when either $\tau_1 < \hat{\tau}_1$ or $[\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha) < \tau_1$ is the case. (Although Figure II.4 relates to cases in which $l_u < c_u'(1-\alpha)$, it should be noted that the function's graph has much the same appearance for cases in which $l_u \geq c_u'(1-\alpha)$. Regardless of the value taken by l_u, the function always has a discontinuity at $\tau_1 = \hat{\tau}_1$, is always concave (from below) for $\tau_1 < \hat{\tau}_1$, and is always convex (from below) for $\tau_1 > \hat{\tau}_1$.)

II.3.3.(iv) Economic Explanation for the Key Results

The key results reported above can be explained in terms of the influence exerted by the rule parameter τ_1 on the trade-off between real wage and employment faced by the individual non-atomistic union when setting its nominal wage simultaneously with other unions. As in the case of the simple rule, the marginal effect of union *j*'s wage on its employment outcome, with every other union's wage choice treated as given, involves two channels, namely the real wage channel and the aggregate demand channel, with the latter involving an induced change in real money balances. Under the simple rule, the absence of a monetary response to the contribution made by w_j to the aggregate wage, *w*, and hence to *p* also, means that the effect of a marginal increase in w_j on the real wage is positive, while its effect on real money balances is negative. As previously discussed, union *j*'s labour demand equation, (7), therefore

Figure II.4

Equilibrium employment as a function of the price level rule parameter



Notes:

1. It is assumed that $1 < n < \infty$ and that $l_u < c_u'(1-\alpha)$. (The latter assumption is innocuous.)

2. $\hat{\tau}_1 \equiv \dfrac{(\gamma\alpha+\varepsilon\Lambda)}{\gamma[\alpha+n\varepsilon(1-\alpha)]}$

3. $\tau_1^* \equiv \left(\dfrac{1}{\gamma}\right)\left\{\dfrac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda-(\gamma\alpha+\varepsilon\Lambda)l_u}{nc_u'(1-\alpha)[\alpha+\varepsilon(1-\alpha)]-[\alpha+n\varepsilon(1-\alpha)]l_u}\right\}$

4. $l|_{w=w_{NE}}$ is undefined when $\tau_1 = 1$ and $\tau_1 = [\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$.

implies that under the simple rule union *j*, by increasing its nominal wage, can obtain a higher real wage in return for lower employment occasioned both by the higher real wage itself, and (if $n < \infty$) by the reduction in aggregate demand caused by the increase in w_j. The trade-off between real wage and employment associated with a marginal increase in w_j is therefore unambiguously inverse in nature under the simple rule, and this trade-off is more favourable to union *j* the larger is *n* and the lower is ε. As will become apparent below, when $\tau_1 \neq 0$, so that the money supply is not kept fixed but is responsive to *p* (and hence to *w*), union *j*'s trade-off may, depending on τ_1, be such that a marginal adjustment in w_j can increase both its real wage and its employment. Thus whereas under the simple rule the individual union's perceived labour demand curve is negatively sloped, this is not necessarily the case under the price level rule: certain settings of τ_1 can cause this curve to be positively sloped. These considerations will play a central role in the economic interpretations provided below of our key results.

The marginal effect of w_j on non-atomistic union *j*'s real wage, $w_j - p$, can itself be decomposed into the direct effect of an increase in w_j and an indirect effect working via the induced change in *p*. This indirect effect involves a causal chain whereby w_j contributes to determine *w*, and *w* in turn contributes to the simultaneous determination of *m* and *p*. Denoting the full marginal effect of w_j on $w_j - p$ by $d(w_j - p)/dw_j$, we therefore have:

$$\frac{d(w_j - p)}{dw_j} = \underbrace{1}_{\text{Direct Effect}} \underbrace{- \frac{\partial p}{\partial w}\left(\frac{\partial w}{\partial w_j}\right)}_{\text{Indirect Effect}} \tag{73}$$

where $\partial w / \partial w_j = 1/n$.

Clearly, the direct effect is always equal to unity regardless of τ_1. The indirect effect, by contrast, can be positive, or negative (or zero), and depends on both τ_1 and *n*. τ_1 is central to the strength and sign of the indirect effect, since it is this degree of monetary accommodation of prices which determines whether the price level increases or decreases in response to a marginal change in the aggregate nominal wage *w*. Using (15) it is apparent that:

$$\frac{\partial p}{\partial w} > 0 \quad \text{if} \quad \tau_1 < \frac{[\alpha + \gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{74a}$$

$$\lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \left(\frac{\partial p}{\partial w} \right) = \infty \tag{74b}$$

$$\frac{\partial p}{\partial w} < 0 \quad \text{if} \quad \tau_1 > \frac{[\alpha + \gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{74c}$$

$$\lim_{\tau_1 \to \pm\infty} \left(\frac{\partial p}{\partial w} \right) = 0 \tag{74d}$$

Equations (15) and (73) together imply:

$$\frac{d(w_j - p)}{dw_j} = \underbrace{1}_{\text{Direct Effect}} \underbrace{- \frac{\alpha}{n[\alpha + \gamma(1-\alpha)(1-\tau_1)]}}_{\text{Indirect Effect}} \tag{75}$$

Whether the direct effect outweighs, or is outweighed by, or works in the same direction as the indirect effect, and hence whether a marginal increase in w_j occasions an increase or a decrease in $w_j - p$, depends on τ_1 and *n*. Specifically, we have:

$$\frac{d(w_j - p)}{dw_j} > 0 \quad \text{if} \quad \tau_1 < \frac{\Lambda}{n\gamma(1-\alpha)} \quad \text{or} \quad \tau_1 > \frac{[\alpha + \gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{76a}$$

$$\frac{d(w_j - p)}{dw_j} = 0 \quad \text{if} \quad \tau_1 = \frac{\Lambda}{n\gamma(1-\alpha)} \tag{76b}$$

$$\frac{d(w_j - p)}{dw_j} < 0 \quad \text{if} \quad \frac{\Lambda}{n\gamma(1-\alpha)} < \tau_1 < \frac{[\alpha + \gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{76c}$$

$$\lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \frac{d(w_j - p)}{dw_j} = -\infty \tag{76d}$$

(76c) indicates there is a range of τ_1 values which cause the indirect effect of a marginal change in w_j working through *w* and *p* to outweigh the direct effect, so that a fall in union *j*'s real wage results when w_j is marginally increased. We know from (74a) that when τ_1 is in the range stated above in (76c) a marginal increase in w_j will, with other unions' wages taken as given, cause an increase in the price level. Furthermore, when τ_1 is in this range the eroding effect of this price level increase on union *j*'s real wage exceeds the augmenting impact of the marginal increase in w_j itself. In other words, the direct effect is outweighed by the indirect effect working against it. When $\tau_1 < \Lambda/n\gamma(1-\alpha)$, the indirect effect is the weaker of the two, so that a marginal increase in w_j causes an increase in the real wage, while $\tau_1 = \Lambda/n\gamma(1-\alpha)$ is the special case in which the opposing direct and indirect effects precisely cancel. From (74c) we know that when the monetary rule is sufficiently accommodating (specifically, when $\tau_1 > [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$), a marginal increase in w_j will occasion a fall in the price level, so that the indirect effect is not counteracting the direct effect but on the contrary is augmenting it. Unsurprisingly, (76a) indicates that the marginal effect of an increase in w_j on $w_j - p$ is positive when $\tau_1 > [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$.[32]

A final noteworthy aspect of this analysis of the marginal effect of w_j on union *j*'s real wage is that the τ_1 value which happens to ensure that $d(w_j - p)/dw_j = 0$, namely $\tau_1 = \Lambda/n\gamma(1-\alpha)$, turns out, when $l_u = 0$, to be τ_1^*, the socially optimal τ_1 value which brings about labour-market clearing, as can be seen from equation (71′). We shall have occasion to refer to this result again shortly.

We now turn to analyse similarly the marginal effect of w_j on real money balances, which will be denoted here by $d(m-p)/dw_j$, where m is given by (14), i.e. $m = \tau_1 p$. Using (15) and (16), we have:

$$\frac{d(m-p)}{dw_j} = -\frac{\alpha(1-\tau_1)}{n[\alpha + \gamma(1-\alpha)(1-\tau_1)]} \tag{77}$$

Clearly, the marginal effect of w_j on real money balances is smaller, in absolute terms, the larger is *n*, and is zero in the case of atomistic unions. Scrutiny of (77) reveals that for a non-atomistic union:

32. Result (76d) is attributable to the fact that the price level is undefined when $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$.

$$\frac{d(m-p)}{dw_j} < 0 \quad \text{if} \quad \tau_1 < 1 \quad \text{or} \quad \tau_1 > \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{78a}$$

$$\lim_{\tau_1 \to 1} \frac{d(m-p)}{dw_j} = 0 \tag{78b}$$

$$\frac{d(m-p)}{dw_j} > 0 \quad \text{if} \quad 1 < \tau_1 < \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{78c}$$

$$\lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \frac{d(m-p)}{dw_j} = \infty \tag{78d}$$

The explanation for these results is straightforward, since the marginal effect of w_j on $(m-p)$ can be expressed as follows:

$$\frac{d(m-p)}{dw_j} = \frac{(\tau_1 - 1)}{n}\frac{\partial p}{\partial w} \tag{79}$$

where the sign of $\partial p/\partial w$ depends on τ_1, as previously analysed in expressions (74a) to (74d). This expression shows clearly that the influence of a marginal increase in w_j on real money balances, with all other unions' wages taken as given, depends on the interaction of the resulting change in the price level (which depends of course on τ_1) with the induced movement in the money supply in response to that price level change. When $\tau_1 < 1$ the rule is such that a wage increase leads to an increase in the price level, and the monetary response to the price level increase is insufficient to prevent a reduction in real money balances.[33] In the limiting $\tau_1 \to 1$ case, the rule operates to keep real money balances equal to a fixed value regardless of price level movements, and hence the marginal effect (78b) is also zero. In the case represented by (78c) the rule parameter is still within the range of values which result in $\partial p/\partial w > 0$, but the monetary response to the increase in p caused by an increase in w is positive and sufficiently elastic to ensure that the price level increase is accompanied by an increase in, rather than a reduction of, real money balances. If $\tau_1 > [\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$, a marginal increase in w_j leads to a fall in the price level,

[33] In order to provide an intuitive explanation, m has been envisaged here as responding to p. Within the model, of course, m and p are simultaneously determined.

while the rule here prescribes a monetary response of the same sign as the price level movement. The price level fall occasioned by a marginal increase in w_j therefore induces a contraction of the money supply, and the latter is strong enough to ensure that real money balances fall, despite the fall in the price level.

Next, we consider how the marginal effect of w_j on $w_j - p$ and $m - p$ combine to determine the impact of a marginal increase in w_j on union j's employment. Using (7) together with our assumption of demand-determined employment, we have:

$$\frac{dl_j}{dw_j} = \frac{1}{[\alpha + \varepsilon(1-\alpha)]}\left[\gamma \frac{d(m-p)}{dw_j} - \varepsilon \frac{d(w_j - p)}{dw_j}\right] \tag{80}$$

It is apparent that the sign of dl_j/dw_j will depend upon how the aggregate demand channel, $(\gamma/[\alpha + \varepsilon(1-\alpha)])(d(m-p)/dw_j)$, compares with the real wage channel, $-(\varepsilon/[\alpha + \varepsilon(1-\alpha)])(d(w_j - p)/dw_j)$. Using (75), (77) and (80) we have:

$$\frac{dl_j}{dw_j} = \frac{\{-(\gamma\alpha + \varepsilon\Lambda) + \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}}{n[\alpha + \varepsilon(1-\alpha)][\alpha + \gamma(1-\alpha)(1-\tau_1)]} \tag{81}$$

This expression implies that:[34]

$$\frac{dl_j}{dw_j} < 0 \quad \text{if} \quad \tau_1 < \hat{\tau}_1 \quad \text{or} \quad \tau_1 > \frac{[\alpha + \gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{82a}$$

$$\frac{dl_j}{dw_j} = 0 \quad \text{if} \quad \tau_1 = \hat{\tau}_1 \tag{82b}$$

$$\frac{dl_j}{dw_j} > 0 \quad \text{if} \quad \hat{\tau}_1 < \tau_1 < \frac{[\alpha + \gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{82c}$$

$$\lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \frac{dl_j}{dw_j} = \infty \tag{82d}$$

[34] Note that $\hat{\tau}_1 \equiv (\gamma\alpha + \varepsilon\Lambda)/\gamma[\alpha + n\varepsilon(1-\alpha)]$.

Result (82a) is attributable to the fact that when τ_1 satisfies one of these two inequalities, it is either the case that the aggregate demand and real wage channels are both negative, or if the former is non-negative, the latter is sufficiently negative to outweigh it. In other words, if $\tau_1 < \hat{\tau}_1$ or $\tau_1 > [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, a marginal increase in w_j both increases the real wage and leads to a fall in labour demand, irrespective of whether the w_j increase causes aggregate demand also to fall (which is the case when $\tau_1 < 1$ or $\tau_1 > [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$), or to increase (which is the case when $1 < \tau_1 < \hat{\tau}_1$). When $\tau_1 = \hat{\tau}_1$ the aggregate demand channel is positive and has exactly the same absolute value as the negative real wage channel, so that the net effect on union *j*'s employment of a marginal increase in its wage is zero: hence result (82b). The result stated in (82c) is attributable to the aggregate demand channel being positive for the stated range of τ_1 values, with the real wage channel either negative but insufficiently strong to outweigh it (which is the case when $\hat{\tau}_1 < \tau_1 < \Lambda/n\gamma(1-\alpha)$), or positive and hence working in the same direction as the aggregate demand channel (the case when $\Lambda/n\gamma(1-\alpha) < \tau_1 < [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$). The fact that *p* is undefined when $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ underlies (82d). Table II.1 summarises how the relative strength and sign of the aggregate demand and real wage channels determine the sign of dl_j/dw_j, and also indicates the sign of $dl_j/d(w_j - p)$, to the discussion of which we now turn.

Union *j*'s perceived trade-off between its employment and real wage as a result of marginal increases in its nominal wage is given by $dl_j/d(w_j - p)$, and this is also the slope of union *j*'s perceived labour demand curve. Using (75) and (80), together with the fact that $dl_j/d(w_j - p) = (dl_j/dw_j)/(d(w_j - p)/dw_j)$, this trade-off is found to be:

$$\frac{dl_j}{d(w_j - p)} = \frac{\{-(\gamma\alpha + \varepsilon\Lambda) + \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}}{[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]} \tag{83}$$

It is apparent from (83) that:

$$\frac{dl_j}{d(w_j - p)} < 0 \quad \text{if} \quad \tau_1 < \hat{\tau}_1 \quad \text{or} \quad \tau_1 > \frac{\Lambda}{n\gamma(1-\alpha)} \tag{84a}$$

$$\frac{dl_j}{d(w_j - p)} = 0 \quad \text{if} \quad \tau_1 = \hat{\tau}_1 \tag{84b}$$

$$\frac{dl_j}{d(w_j - p)} > 0 \quad \text{if} \quad \hat{\tau}_1 < \tau_1 < \frac{\Lambda}{n\gamma(1-\alpha)} \tag{84c}$$

$$\frac{dl_j}{d(w_j - p)} = \infty \quad \text{if} \quad \tau_1 = \frac{\Lambda}{n\gamma(1-\alpha)} \tag{84d}$$

Results (84a) to (84d) relating to the sign of union *j*'s perceived trade-off are unaffected by the value of l_u. Because the economic explanation for why τ_1^* induces non-atomistic unions to set the market-clearing nominal wage is straightforward when $l_u = 0$, we will focus here on this case, and the not particularly interesting complications which arise when $l_u \neq 0$ will be dealt with briefly later. Expressions (84a) to (84d) reveal that when $n > 1$, the slope of the perceived labour demand curve of a non-atomistic union can, by appropriate choice of τ_1 be assigned almost any value, whether positive, negative, or zero. The sole exceptions are the slopes associated with the two τ_1 values which render the price level indeterminate, and which are therefore inadmissible, namely $\tau_1 = 1$ and $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. To appreciate the influence of τ_1 on the perceived trade-off it is instructive to note that, firstly, the trade-off which arises under strict price level targeting (the limiting $\tau_1 \to \pm\infty$ cases) is steeper (more negative) than that faced by an atomistic union, and secondly, an increase in τ_1 causes the perceived labour demand curve to rotate in an anticlockwise direction:

$$\frac{\partial[dl_j/d(w_j - p)]}{\partial \tau_1} = \frac{(n-1)\gamma\alpha^2}{[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]^2} \tag{85}$$

The right-hand side of (85) is strictly positive for all finite $n > 1$. Thus as τ_1 increases from negative values the perceived labour demand curve rotates anticlockwise,

Table II.1

	$\tau_1 < 1$	$\lim \tau_1 \to 1$	$1 < \tau_1 < \hat{\tau}_1$	$\tau_1 = \hat{\tau}_1$	$\hat{\tau}_1 < \tau_1 < \frac{\Lambda}{n\gamma(1-\alpha)}$
Aggregate Demand Channel $\frac{\gamma}{[\alpha+\varepsilon(1-\alpha)]} \frac{d(m-p)}{dw_j}$	< 0	$= 0$	> 0	> 0	> 0
Real Wage Channel $\frac{-\varepsilon}{[\alpha+\varepsilon(1-\alpha)]} \frac{d(w_j-p)}{dw_j}$	< 0	< 0	< 0	< 0	< 0
Remarks			$\left\lvert \gamma \frac{d(m-p)}{dw_j} \right\rvert < \left\lvert -\varepsilon \frac{d(w_j-p)}{dw_j} \right\rvert$	$\left\lvert \gamma \frac{d(m-p)}{dw_j} \right\rvert = \left\lvert -\varepsilon \frac{d(w_j-p)}{dw_j} \right\rvert$	$\left\lvert \gamma \frac{d(m-p)}{dw_j} \right\rvert > \left\lvert -\varepsilon \frac{d(w_j-p)}{dw_j} \right\rvert$
$\frac{dl_j}{dw_j}$	< 0	< 0	< 0	$= 0$	> 0
$\frac{dl_j}{d(w_j-p)}$	< 0	$= \frac{-\varepsilon}{[\alpha+\varepsilon(1-\alpha)]}$	< 0	$= 0$	> 0

Table II.1 (Continued)

	$\tau_1 = \frac{\Lambda}{n\gamma(1-\alpha)}$	$\frac{\Lambda}{n\gamma(1-\alpha)} < \tau_1 < \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}$	$\lim \tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}$	$\frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} < \tau_1$
Aggregate Demand Channel $\frac{\gamma}{[\alpha+\varepsilon(1-\alpha)]} \frac{d(m-p)}{dw_j}$	> 0	> 0	$= \infty$	< 0
Real Wage Channel $\frac{-\varepsilon}{[\alpha+\varepsilon(1-\alpha)]} \frac{d(w_j-p)}{dw_j}$	$= 0$	> 0	$= \infty$	< 0
Remarks				
$\frac{dl_j}{dw_j}$	> 0	> 0	$= \infty$	< 0
$\frac{dl_j}{d(w_j-p)}$	$= \infty$	< 0	$= -\frac{1}{(1-\alpha)}$	< 0

assumes the slope associated with the simple rule when $\tau_1 = 0$, and approaches the atomistic union's perceived labour demand curve as τ_1 approaches unity from below. Formally we find that:

$$\lim_{\tau_1 \to 1} \left(\frac{dl_j}{d(w_j - p)} \right) = \frac{-\varepsilon}{[\alpha + \varepsilon(1-\alpha)]} \tag{86}$$

The economic explanation for this result is that in the limit as $\tau_1 \to 1$, real money balances are constant and independent of w. Hence the aggregate demand channel becomes irrelevant to the non-atomistic union's wage decision. Although the price level itself remains responsive, via w, to union j's choice of w_j, and this responsiveness is taken into account by union j when choosing w_j, the fact that the aggregate demand channel is completely nullified deprives the union of any incentive to exercise wage restraint. Since w_j cannot adversely affect labour demand via aggregate demand, each non-atomistic union when setting its wage acts in the same way as would an atomistic union.

As explained earlier, when τ_1 is such that $1 < \tau_1 < \hat{\tau}_1$, the monetary response to p is sufficiently elastic to cause an increase in w_j to induce an increase in real money balances, so that the aggregate demand channel whereby a marginal increase in w_j influences union j's labour demand is positive and counteracts the negative real wage channel. As τ_1 increases gradually from unity the aggregate demand channel strengthens relative to the real wage channel, and union j's perceived labour demand curve, while still negative in slope, becomes flatter, and indeed is horizontal when $\tau_1 = \hat{\tau}_1$. As previously explained, as $\tau_1 \to \hat{\tau}_1$ the two channels come ever closer to counterbalancing each other, and the marginal effect of w_j on j's labour demand approaches zero. The marginal effect of w_j on j's real wage remains positive however. Consequently as $\tau_1 \to \hat{\tau}_1$ from below the restraining influence on union j of a marginal increase in its wage having a negative impact on its employment diminishes, and the pursuit of a higher real wage plays a greater part in union j's wage choice. This circumstance renders the externality particularly acute as $\tau_1 \to \hat{\tau}_1$, with the equilibrium real wage higher and less efficient, the closer is τ_1 to $\hat{\tau}_1$. When $\tau_1 = \hat{\tau}_1$ every union has an incentive to pursue as high a real wage as possible, with the result that w, p, and real outcomes are all undefined.

A positive perceived labour demand curve results when τ_1 satisfies $\hat{\tau}_1 < \tau_1 < \Lambda/n\gamma(1-\alpha)$, and the perceived labour demand curve is vertical when $\tau_1 = \Lambda/n\gamma(1-\alpha)$. In the $l_u = 0$ case this setting of τ_1 turns out to be socially optimal: $\tau_1^*\big|_{l_u=0} = \Lambda/n\gamma(1-\alpha)$. The reason has to do with the fact that when $l_u = 0$ the axis of symmetry of union *j*'s isoloss contours coincides with the horizontal axis, and hence a vertical perceived labour demand curve will cause union *j* to set its wage so as to attempt to achieve a tangency point on the horizontal axis, which implies an individual employment outcome equal to the market-clearing outcome. Of course, the union's wage choice also involves an attempt to secure a real wage in excess of the market-clearing wage[35], but the working of the externality is such that the real wage outcome is inefficiently low from unions' collective viewpoint, but is equal to the socially optimal real wage which clears the labour market. In economic terms $\tau_1 = \Lambda/n\gamma(1-\alpha)$ causes the marginal effect of w_j on union *j*'s real wage to be zero. Powerless to influence its real wage, the individual union's optimal strategy is then to set its wage so as to bring about, via the induced change in aggregate demand, its desired employment level of l_u. If it so happens that l_u is the market-clearing level of employment, it then follows directly that this value of τ_1 must be socially optimal: hence $\tau_1^*\big|_{l_u=0} = \Lambda/n\gamma(1-\alpha)$.

If $\tau_1 > \Lambda/n\gamma(1-\alpha)$, the perceived labour demand curve of union *j* will once again be negatively sloped, and while $[\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$ is an inadmissible value for τ_1, it is nevertheless the case that as τ_1 approaches $[\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$ the slope of the perceived labour demand curve approaches that of the symmetric-wage labour demand curve, i.e. union *j*'s perceived trade-off between its real wage and employment approaches that prevailing at the aggregate level:

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[35] When $l_u = 0$ and $\tau_1 = \tau_1^*\big|_{l_u=0}$, the real wage union *j* would achieve, by its optimal w_j, were every other union to abide by the efficient wage, is the efficient real wage itself, together with employment of zero, which for union *j* is preferable to the (negative) efficient employment outcome.

$$\lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \left(\frac{dl_j}{d(w_j - p)} \right) = \frac{-1}{(1-\alpha)} \tag{87}$$

Hence in the limiting $\tau_1 \to [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ case the rule induces efficient wage-setting by non-atomistic unions. It can be shown that were $[\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ an admissible value for τ_1, it would be the only τ_1 value which equates *both* the marginal effect of w_j on real money balances *and* the marginal effect of w_j on the real wage to the marginal effects of their aggregate counterparts. In other words, only the limiting $\tau_1 \to [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ case ensures both $d(m-p)/dw_j = d(m-p)/dw$ and $d(w_j - p)/dw_j = d(w-p)/dw$.[36]

II.3.3.(v) Influence of Goods-Market Competition and Wage-Bargaining Structure on the Wage-Setting Externality

We have yet to discuss the influence of the parameters ε and n on the wage-setting externality, and hence on equilibrium outcomes, under the general price level rule given by (14). We saw earlier that under the simple rule (i.e. when $\tau_1 = 0$) a higher ε mitigated the externality by reducing the scope for union *j*'s employer firms' prices to depart from the average price, while a higher *n*, by reducing union *j*'s incentive to internalise the full price level repercussions of its wage decision, exacerbated it. Things are not as straightforward when τ_1 is not restricted to be zero: for certain values of τ_1, a marginal increase in ε can actually worsen the adverse externality, while a marginal increase in *n* can reduce its severity, the very reverse of our findings for the case of the simple rule. To demonstrate this, we examine the derivatives of the equilibrium real wage, as given by (67a), with respect to *n* and ε:

[36] The limiting case in which $\tau_1 \to 1$ is the only other 'setting' of τ_1 which ensures $d(m-p)/dw_j = d(m-p)/dw$. However, $\lim_{\tau_1 \to 1} d(w_j - p)/dw_j = (n-1)/n > \lim_{\tau_1 \to 1} d(w-p)/dw = 0$, and therefore the limiting $\tau_1 \to 1$ case cannot induce efficient wage setting. Although the limiting $\tau_1 \to [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ case is the only instance of a finite τ_1 which can ensure $d(w_j - p)/dw_j = d(w-p)/dw$, strict price level targeting can also ensure this, since $\lim_{\tau_1 \to \pm\infty} d(w_j - p)/dw_j = \lim_{\tau_1 \to \pm\infty} d(w-p)/dw = 1$. However, if $n > 1$, strict price level targeting cannot ensure the equality of individual and aggregate marginal wage effects on real money balances: $\lim_{\tau_1 \to \pm\infty} d(m-p)/dw_j = -\alpha/n\gamma(1-\alpha) > \lim_{\tau_1 \to \pm\infty} d(m-p)/dw = -\alpha/\gamma(1-\alpha)$.

$$\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}=\frac{c_u'\gamma\alpha(1-\alpha)[\alpha+\varepsilon(1-\alpha)](1-\tau_1)[\alpha+\gamma(1-\alpha)(1-\tau_1)]}{\{\gamma\alpha+\varepsilon\Lambda-\gamma[\alpha+n\varepsilon(1-\alpha)]\tau_1\}^2} \qquad (88)$$

It is apparent that:

$$\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}>0 \text{ if } \tau_1<1 \text{ or } \tau_1>\frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \qquad (89a)$$

$$\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}<0 \text{ if } 1<\tau_1<\hat{\tau}_1 \text{ or } \hat{\tau}_1<\tau_1<\frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \qquad (89b)$$

$$\lim_{\tau_1\to 1}\left(\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}\right)=\lim_{\tau_1\to\frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}}\left(\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}\right)=0 \qquad (89c)$$

$$\lim_{\tau_1\to\hat{\tau}_1}\left(\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}\right)=\infty \qquad (89d)$$

To gain intuition regarding these results, it is helpful to examine the derivative of the perceived labour demand curve's slope with respect to n. Using (83) we have:

$$\frac{\partial[dl_j/d(w_j-p)]}{\partial n}=\frac{\gamma\alpha(1-\tau_1)[\alpha+\gamma(1-\alpha)(1-\tau_1)]}{[\alpha+\varepsilon(1-\alpha)][\Lambda-n\gamma(1-\alpha)\tau_1]^2} \qquad (90)$$

It is apparent that:

$$\frac{\partial[dl_j/d(w_j-p)]}{\partial n}>0 \text{ if } \tau_1<1 \text{ or } \tau_1>\frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \qquad (91a)$$

$$\frac{\partial[dl_j/d(w_j-p)]}{\partial n}<0 \text{ if } 1<\tau_1<\frac{\Lambda}{n\gamma(1-\alpha)} \text{ or } \frac{\Lambda}{n\gamma(1-\alpha)}<\tau_1<\frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \qquad (91b)$$

$$\lim_{\tau_1 \to 1} \left(\frac{\partial [dl_j / d(w_j - p)]}{\partial n} \right) = \lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \left(\frac{\partial [dl_j / d(w_j - p)]}{\partial n} \right) = 0 \qquad (91c)$$

$$\lim_{\tau_1 \to \frac{\Lambda}{n\gamma(1-\alpha)}} \left(\frac{\partial [dl_j / d(w_j - p)]}{\partial n} \right) = \infty \qquad (91d)$$

The two pairs of results {(89c), (91c)} and {(89d), (91d)} are the most straightforward to explain. It has already been established that as $\tau_1 \to 1$ and $\tau_1 \to [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ in the limit, each non-atomistic union's perceived real wage-employment trade-off tends to one or the other of the trade-offs which arise under the two extremes of wage-bargaining structure. It is therefore unsurprising that the equilibrium real wage and the perceived trade-off should be invariant to n in these cases (results (89c) and (91c)). In the limiting $\tau_1 \to \hat{\tau}_1$ case the perceived labour demand curve becomes horizontal and the real wage is undefined, regardless of n: it is this indeterminacy of the real wage which explains (89d). When $\tau_1 = \Lambda/n\gamma(1-\alpha)$, by contrast, the real wage is defined, but is beyond each non-atomistic union's influence, regardless of the value of n (provided $n > 1$ of course). This is a consequence of the perceived labour demand curve being vertical when $\tau_1 = \Lambda/n\gamma(1-\alpha)$, and the fact that its slope is infinite explains result (91d).

To appreciate the full significance of result (89a) it is useful to bear in mind that when $\tau_1 < 1$ or $\tau_1 > [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, the equilibrium real wage is inefficiently high: result (89a) therefore tells us that a marginal increase in n here worsens the externality, just as it does under the simple rule.[37] To understand why, recall that when $\tau_1 < 1$ or $\tau_1 > [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ the perceived labour demand curve is negatively sloped, flatter than the symmetric-wage labour demand curve, but steeper (more negative in slope) than the perceived labour demand curve of an atomistic union. Result (91a) therefore reveals that, in these cases, as n increases the perceived labour demand curve of a non-atomistic union becomes flatter, and its perceived trade-off more favourable, for the straightforward reason that the individual union's wage makes a smaller contribution to w and hence to p. This worsens the externality, and increases the inefficiently high real wage.

[37] Note that the simple rule, i.e. the $\tau_1 = 0$ case, is subsumed by result (89a).

Result (89b) is the least straightforward of the four. It is in fact divisible into four subsets of cases: for three of these subsets result (91b) indicates that the perceived labour demand curve rotates in an anticlockwise direction as n increases.

(i) The first subset of τ_1 cases encompassed by result (89b) is that for which $\Lambda/n\gamma(1-\alpha) < \tau_1 < [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. When τ_1 lies in this interval and $l_u = 0$ the equilibrium real wage is inefficiently low, although it does exceed its market-clearing value of zero. Result (89b) therefore reveals that a higher n here worsens the externality: the inefficiently low real wage becomes lower still as n increases. The explanation lies in terms of the fact that the perceived labour demand curve is here more negatively sloped than the symmetric-wage labour demand curve, while (91b) tells us that as n increases its slope departs further from that of the symmetric-wage labour demand curve. Hence for this subset of cases the aggravation of the externality caused by an increase in n is attributable to an increase in the departure of the individual union's perceived trade-off from that prevailing at the aggregate level.

(ii) The second subset of cases contains only a single τ_1 value, namely $\tau_1 = \Lambda/n\gamma(1-\alpha)$, the setting of τ_1 which renders the perceived labour demand curve of each non-atomistic union vertical, and which induces unions to set the market-clearing nominal wage in the $l_u = 0$ case. (Note that if $l_u = 0$ the equilibrium real wage, when $\tau_1 = \Lambda/n\gamma(1-\alpha)$, is inefficiently low from unions' perspective so that the adverse wage-setting externality is present.) It turns out that, regardless of l_u, a marginal increase in n has no impact on the slope of the perceived labour demand curve, nor on the equilibrium real wage, nor on the strength of the externality, when $\tau_1 = \Lambda/n\gamma(1-\alpha)$. This is not immediately obvious from (88) and (90) since those derivatives implicitly treat τ_1 as fixed. In this special case, allowance must be made for the indirect impact of n, working via the induced change in $\Lambda/n\gamma(1-\alpha)$, on the equilibrium real wage when $\tau_1 = \Lambda/n\gamma(1-\alpha)$. The following total derivative is therefore relevant:

$$\frac{d(w-p)|_{w=w_{NE}}}{dn} = \left.\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}\right|_{\tau_1\ fixed} + \left(\frac{\partial(w-p)|_{w=w_{NE}}}{\partial \tau_1}\right)\left(\frac{\partial \tau_1}{\partial n}\right) \qquad (92)$$

Evaluating the two terms on the right-hand side of (92) for $\tau_1 = \Lambda/n\gamma(1-\alpha)$, they are found to be equal in magnitude but of opposite sign:

$$\left.\frac{\partial(w-p)|_{w=w_{NE}}}{\partial n}\right|_{\tau_1=\Lambda/n\gamma(1-\alpha)} = \frac{-c_u'(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]}{(n-1)\alpha} \tag{92'}$$

$$\left(\left.\frac{\partial(w-p)|_{w=w_{NE}}}{\partial \tau_1}\right|_{\tau_1=\Lambda/n\gamma(1-\alpha)}\right)\left(\frac{\partial(\Lambda/n\gamma(1-\alpha))}{\partial n}\right) = \frac{c_u'(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]}{(n-1)\alpha} \tag{92''}$$

It follows that $\left.\left(d(w-p)|_{w=w_{NE}}/dn\right)\right|_{\tau_1=\Lambda/n\gamma(1-\alpha)} = 0$.

(iii) The third subset of cases encompassed by result (89b) involves τ_1 values such that $\hat{\tau}_1 < \tau_1 < \Lambda/n\gamma(1-\alpha)$. For this subset, when $l_u = 0$ the equilibrium real wage is inefficiently low, so low in fact that it lies below its market-clearing value of zero. Thus our finding that $\partial(w-p)|_{w=w_{NE}}/\partial n < 0$ when τ_1 is in this range of values indicates that a marginal increase in *n* exacerbates the externality. This is because the slope of the perceived labour demand curve (which is positive for this subset of cases) is falling in *n*. Hence the greater the number of unions, the more favourable is the individual non-atomistic union's perceived trade-off between real wage and employment when τ_1 belongs to this subset.

(iv) Finally, we come to the sole subset of cases for which a marginal increase in *n* actually has the effect of mitigating the externality. This subset involves τ_1 being in the interval $1 < \tau_1 < \hat{\tau}_1$, so that the equilibrium real wage is inefficiently high, and the perceived labour demand curve of each non-atomistic union is negatively sloped but flatter than that of an atomistic union. The fact that $\partial(w-p)|_{w=w_{NE}}/\partial n < 0$ for this subset of cases indicates that a marginal increase in *n* here brings the equilibrium real wage closer to its efficient value. The derivative (91b) indicates that an increase in *n* here steepens the perceived labour demand curve and hence reduces the extent of its departure from the symmetric-wage labour demand curve. However, a deeper economic explanation for the result can be provided by decomposing the marginal impact of union *j*'s wage on its employment into effects working via the real wage and aggregate demand channels. For this subset of cases, $d(w_j - p)/dw_j > 0$ and $d(m-p)/dw_j > 0$, but, as revealed by Table II.1, the negative real wage channel outweighs the positive aggregate demand channel. The real wage channel is stronger the larger is *n* since this reduces the contribution made by w_j to *w* and *p*, and hence increases the extent to which the direct effect of an increase in w_j on the real wage outweighs its indirect effect working via the induced change in *p*. However, for this subset of cases the aggregate demand channel is weaker the larger is *n*. Thus the effect of a marginal increase in w_j on union *j*'s employment must be stronger, the larger is *n*. It follows that a marginal increase in *n* renders the individual (non-atomistic) union's perceived trade-off less favourable, and hence mitigates the externality.

Proceeding now to analyse the effect of a marginal increase in ε on the equilibrium real wage, we have, using (67a):

$$\frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial\varepsilon} = \frac{-(n-1)c_u'\alpha(1-\alpha)[\alpha+\gamma(1-\alpha)(1-\tau_1)][\Lambda - n\gamma(1-\alpha)\tau_1]}{\{\gamma\alpha+\varepsilon\Lambda-\gamma[\alpha+n\varepsilon(1-\alpha)]\tau_1\}^2} \tag{93}$$

Provided $1 < n < \infty$, it is apparent that:

$$\frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial\varepsilon} < 0 \quad \text{if} \quad \tau_1 < \frac{\Lambda}{n\gamma(1-\alpha)} \quad \text{or} \quad \tau_1 > \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{94a}$$

$$\frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial\varepsilon} > 0 \quad \text{if} \quad \frac{\Lambda}{n\gamma(1-\alpha)} < \tau_1 < \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{94b}$$

$$\left.\frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial\varepsilon}\right|_{\tau_1=\Lambda/n\gamma(1-\alpha)} = \lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial\varepsilon} = 0 \tag{94c}$$

$$\lim_{\tau_1 \to \hat{\tau}_1} \frac{\partial(w-p)\big|_{w=w_{NE}}}{\partial\varepsilon} = \infty \tag{94d}$$

The marginal effect of ε on the slope of the perceived labour demand curve of a non-atomistic union is:

$$\frac{\partial[dl_j/d(w_j-p)]}{\partial\varepsilon} = \frac{-(n-1)\alpha[\alpha+\gamma(1-\alpha)(1-\tau_1)]}{[\alpha+\varepsilon(1-\alpha)]^2[\Lambda-n\gamma(1-\alpha)\tau_1]} \tag{95}$$

It is apparent that, provided $1<n<\infty$:

$$\frac{\partial[dl_j/d(w_j-p)]}{\partial\varepsilon} < 0 \ \text{ if } \ \tau_1 < \frac{\Lambda}{n\gamma(1-\alpha)} \ \text{ or } \ \tau_1 > \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{96a}$$

$$\frac{\partial[dl_j/d(w_j-p)]}{\partial\varepsilon} > 0 \ \text{ if } \ \frac{\Lambda}{n\gamma(1-\alpha)} < \tau_1 < \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)} \tag{96b}$$

$$\lim_{\tau_1 \to \frac{[\alpha+\gamma(1-\alpha)]}{\gamma(1-\alpha)}} \frac{\partial[dl_j/d(w_j-p)]}{\partial\varepsilon} = 0 \tag{96c}$$

$$\lim_{\tau_1 \to \frac{\Lambda}{n\gamma(1-\alpha)}} \frac{\partial[dl_j/d(w_j-p)]}{\partial\varepsilon} = \infty \tag{96d}$$

When τ_1 is in the range of values mentioned in (94b) the equilibrium real wage exceeds the market-clearing value, but is nevertheless inefficiently low from the point of view of unions. Result (94b) therefore indicates that when τ_1 is in this interval, an increase in ε has the effect of mitigating the externality just as it does in the case of the simple rule. The reason is the straightforward one that greater goods-market competition here renders the individual perceived trade-off less favourable: for this subset of τ_1 values, the perceived labour demand curve of a non-atomistic union is more negative in slope than the symmetric-wage labour demand curve. (96b) reveals that as ε increases marginally the perceived labour demand curve becomes flatter and rotates towards the symmetric-wage labour demand curve, so that the perceived trade-off between real wage and employment more closely approximates the aggregate-level trade-off.

The results in (94c) are also easily explained: in these cases τ_1 causes the perceived labour demand curve to be independent of ε, either by depriving the individual union of influence over its real wage (the $\tau_1 = \Lambda/n\gamma(1-\alpha)$ case), or by inducing efficient wage-setting by every union (the limiting $\tau_1 \to [\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$ case). As for (94d), this result is due to the real wage not being defined when $\tau_1 = \hat{\tau}_1$.

Result (94a) covers three subsets of cases:

(i) When $\tau_1 > [\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$, the equilibrium real wage is inefficiently high and result (94a) therefore indicates that an increase in ε mitigates the externality. In this subset of cases the perceived labour demand curve of a non-atomistic union is negatively sloped, and while steeper than the perceived labour demand curve of an atomistic union, it is flatter than the symmetric-wage labour demand curve: result (96a) therefore reveals that a marginal increase in ε here causes the departure of the perceived labour demand curve's slope from that of the symmetric-wage labour demand curve to diminish.

(ii) The second subset of cases is that for which $\tau_1 < \hat{\tau}_1$, and this subset is also characterised by an inefficiently high real wage, so that as for subset (i) the externality is mitigated by greater goods-market competition. The perceived labour demand curve of each non-atomistic union is negatively sloped but flatter than the symmetric-wage labour demand curve. Result (96a) indicates that the perceived labour demand curve steepens (becomes more negative in slope) as ε rises marginally when $\tau_1 < \hat{\tau}_1$, so that the disparity between its slope and that of the symmetric-wage labour demand curve is falling in ε.

(iii) The third subset is that for which $\hat{\tau}_1 < \tau_1 < \Lambda/n\gamma(1-\alpha)$, and this subset is characterised by an equilibrium real wage below the market-clearing value, and which consequently is inefficiently low. Result (94a) therefore tells us that, unlike all the other cases discussed previously, for this subset of cases an increase in ε worsens the externality. The perceived labour demand curve of each non-atomistic union is positively sloped, and (96a) indicates that the slope is falling in ε, which here amounts to an improvement in the union's perceived trade-off. Economic intuition for this result can be provided in terms of the relative strength of the aggregate demand and real wage channels via which a marginal increase in the individual nominal wage, w_j, influences employment. For this subset of τ_1 cases, a marginal increase in w_j does increase the real wage, and consequently the real wage channel is negative, and operates to counteract the aggregate demand channel, which for this subset of cases is positive. The positive aggregate demand channel outweighs the negative real wage channel, so that a marginal increase in w_j has the effect of increasing both the real wage and employment: this is why the perceived labour demand curve is positively sloped. The key point is that whereas a marginal increase in ε weakens the positive aggregate demand channel, $\gamma(m-p)/[\alpha+\varepsilon(1-\alpha)]$, it strengthens the negative real wage channel, $-\varepsilon(w_j-p)/[\alpha+\varepsilon(1-\alpha)]$. Consequently, the higher is ε, the smaller the perceived cost, in terms of the increased departure of employment from its desired value of l_u, of the increase in the real wage which results from a marginal increase in w_j. In other words the nature of the aggregate demand and real wage channels in this subset of cases is such that a marginal increase in ε renders the perceived trade-off faced by each non-atomistic union more favourable, which therefore exacerbates the externality.

II.3.3.(vi) Final Comments relating to Equilibrium Wage-Setting under the Price Level Rule

The most interesting aspects of the equilibrium outcomes relating to the general price level rule $m_r=\tau_1 p$ have now been discussed. For the sake of completeness, however, we briefly consider the implications of relaxing our assumption, made in much of the above, that the union's notional employment objective of l_u coincides with market-clearing employment. The first thing to note is that l_u plays no part in determining the slope of the perceived labour demand curve, and hence all the previous arguments relating to the influence of τ_1, n, and ε on the perceived trade-off continue to apply. The principal consequence of l_u being non-zero is that the efficient outcome may be located anywhere on the symmetric-wage labour demand curve, and does not necessarily involve (as it does in the $l_u = 0$ case) an efficient real wage in excess of the market-clearing real wage of zero. The value taken by l_u determines the position of the efficient outcome on the symmetric-wage labour demand curve, and consequently also determines which of union j's infinite set of possible perceived labour demand curves is the one which passes through the efficient outcome. However, these diagrammatic changes merely complicate the economic interpretation to be assigned to the equilibrium expressions, and do not really lead to any great advance in insight. The most important point relating to $l_u \neq 0$ has already been made when first discussing (71), namely that regardless of the value taken by l_u, there exists a setting for τ_1 which can induce non-atomistic unions to set their wages in such a way that the market-clearing real wage is (inadvertently, in the eyes of unions) brought about. In the $l_u = 0$ case this socially optimal setting, $\tau_1^*\big|_{l_u = 0} = \Lambda/n\gamma(1-\alpha)$, works by rendering the perceived labour demand curve vertical, so that each union perceives its real wage to be beyond its influence and consequently sets w_j in order to ensure that $l_j = l_u$. However, if $l_u \neq 0$ setting $\tau_1 = \Lambda/n\gamma(1-\alpha)$ will induce unions to set their wages so as to bring about an employment outcome which is socially suboptimal. Clearly, when $l_u \neq 0$ a non-vertical perceived labour demand curve is desirable, and the precise value of its slope will depend on l_u as well as the structural parameters. The point is that by taking into account the incentives facing each non-atomistic union, together with the workings of the externality, the design of the policy rule can bring about socially optimal employment and price level outcomes regardless of the unions' notional objective l_u.

Before ending our discussion of the price level rule, two comments regarding our principal results are in order. The first concerns the practical policy relevance of our key result that appropriate setting of the rule parameter can induce non-atomistic unions to set the market-clearing nominal wage. The practicality of this may be questioned because of the potentially close proximity of τ_1^*, the socially optimal setting of the rule parameter, to $\hat{\tau}_1$, the value of τ_1 at which employment and the price level are undefined. The greater this proximity, the greater the risk that inaccurate estimation of the values of the structural parameters γ, ε, and α (and perhaps n too) will lead to the true value of τ_1^* being inaccurately estimated. In the $l_u = 0$ case, for example, underestimation of the true τ_1^* could result in τ_1 being set undesirably close to $\hat{\tau}_1$, leading to equilibrium employment being substantially below its socially optimal level. The danger that an inaccurate estimate of τ_1^* may lead to a disastrous choice of τ_1 seems to be more acute the larger is n and/or the larger is ε.[38] For an economy with only a handful of unions and a low degree of goods-market competition, however, the gap between $\hat{\tau}_1$ and τ_1^* appears to be sufficiently large in the $l_u = 0$ case, to suggest that the key findings of this chapter may have some policy relevance. For example, if we assume, in line with widespread opinion, that $\gamma = 1$ and $\alpha = 0 \cdot 7$ are reasonably accurate estimates for these two parameters, then with $\varepsilon = 2$ and $n = 2$, we have $\tau_1^* = 2 \cdot 167$ versus $\hat{\tau}_1 = 1 \cdot 737$. This seems a significant difference, since τ_1 is, after all, an elasticity.[39]

The second comment relates to the model's assumptions regarding the timing of moves. As mentioned earlier, if the rule prescribes a monetary response to the price level, or any other aggregate variable which is necessarily realised after *w* has been determined, then it is only practicable in an economy in which *m* can indeed be set after the relevant aggregate variable has been realised. This means that if the monetary transmission mechanism or technological constraints are such that *m* must be set simultaneously with, or earlier than *w*, formulating the rule as a response to *p* (or to nominal income or employment realisations) is futile, since regardless of *n* the individual union will treat *m* as beyond its influence when setting its wage, and the employment outcome will be independent of the rule parameter and identical to that which arises in the case of the simple rule, namely (46b).

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[38] Note that for the $l_u = 0$ case, $\hat{\tau}_1$ is less than τ_1^* provided $1 < n < \infty$, and that $\partial(\tau_1^* - \hat{\tau}_1)\big|_{l_u=0} \big/ \partial n < 0 \; \forall \; n > 1$. Furthermore, $\partial(\tau_1^* - \hat{\tau}_1)\big|_{l_u=0} \big/ \partial \varepsilon < 0$ as well.

[39] A great many purely theoretical papers implicitly assume $\gamma = 1$ in specifying an equation for aggregate demand (for example: Gray, 1976; Waller and VanHoose, 1992; Duca and VanHoose, 2001), while papers which undertake simulation studies of monetary policy also generally assume a value of unity for this parameter: examples include Henderson and McKibbin (1993) and Drudi and Giordano (2000). In the latter two papers, as well as in Jerger (2002), α is assigned a value of 0·7. Papers which assign a specific value to ε are rather thinner on the ground. However, Jerger (2002, p.770) assumes its value to be 2, while Acocella et al. (2005), drawing in turn on Gordi (1995), argue that ε "is generally unlikely to exceed $1 \cdot 5$, and only infrequently to exceed $2 \cdot 0$" (Acocella et al., 2005, p.16).

II.4 Wage-Setting under the Nominal Income and Employment Rules

Having completed our discussion of the price level rule, we now proceed to give a relatively brief account of the outcomes associated with equilibrium wage-setting under the two remaining rules, namely the nominal income rule, $m_r = \tau_2(y+p)$, and the employment rule, $m_r = \tau_3 l$. In both cases the qualitative results are very similar to those for the price level rule, so that a brief treatment is justified.

For the nominal income rule, as given by (17), the relevant semi-reduced forms are given by equations (18), (19) and (20). It is immediately apparent from these expressions that $[\alpha+\gamma(1-\alpha)]/\gamma$ is an inadmissible value for the nominal income rule parameter τ_2, just as we earlier found $[\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$ to be an inadmissible value for the price level rule parameter τ_1. The reason is the same: namely that if $\tau_2 = [\alpha+\gamma(1-\alpha)]/\gamma$, $w=0$ is then a necessary condition for the price level to be able to adjust to bring aggregate supply into equality with aggregate demand and hence ensure goods-market clearing. However if $w=0$ does happen to be the case, any realisation of p will then ensure aggregate demand is equal to aggregate supply. Hence p is indeterminate when $\tau_2 = [\alpha+\gamma(1-\alpha)]/\gamma$.[40]

If we proceed, on the assumption that $\tau_2 \neq [\alpha+\gamma(1-\alpha)]/\gamma$, to derive the equilibrium nominal wage for the nominal income rule (by following a similar procedure to that described earlier for the price level rule), it is found to be:

$$w_{NE} = \frac{[\alpha+\gamma(1-\alpha-\tau_2)]}{\gamma(1-\tau_2)}\left[\frac{c_u'[\alpha+\varepsilon(1-\alpha)][\Lambda-\gamma(n-\alpha)\tau_2]}{\{\gamma\alpha+\varepsilon\Lambda-\gamma[\alpha+\varepsilon(n-\alpha)]\tau_2\}} - l_u\right] \tag{97}$$

It is apparent from this expression that a setting of unity is inadmissible for τ_2, just as we earlier found it to be inadmissible for τ_1, and indeed for the same reason: in both cases a rule parameter of unity implies that prices will adjust to ensure the aggregate real wage is zero[41], which in turn implies that no individual union will find it optimal

[40] Aggregate supply and aggregate demand when $\tau_2 = [\alpha+\gamma(1-\alpha)]/\gamma$ are given by:
$y^S\big|_{\tau_2=[\alpha=\gamma(1-\alpha)]/\gamma} = [\alpha/(1-\alpha)]\{p-\varepsilon w/[\varepsilon+\alpha(1-\gamma)]\}$ and $y^D\big|_{\tau_2=[\alpha=\gamma(1-\alpha)]/\gamma} = \alpha p/(1-\alpha)$.

[41] Aggregate supply and aggregate demand when $\tau_2 = 1$ are given by:
$y^S\big|_{\tau_2=1} = -\varepsilon\alpha(w-p)/[\varepsilon(1-\alpha)+\alpha(1-\gamma)]$ and $y^D\big|_{\tau_2=1} = 0$.

to set its wage equal to the aggregate nominal wage. This prevents symmetric wage-setting and causes the equilibrium nominal wage (and consequently the price level also) to be undefined when $\tau_2 = 1$.

Assuming that $\tau_2 \neq [\alpha + \gamma(1-\alpha)]/\gamma$ and that $\tau_2 \neq 1$, the equilibrium price level and employment outcomes are then found to be:

$$p\big|_{w=w_{NE}} = \frac{\alpha(1-\gamma\tau_2)}{\gamma(1-\tau_2)}\left[\frac{c_u'[\alpha+\varepsilon(1-\alpha)][\Lambda-\gamma(n-\alpha)\tau_2]}{\{\gamma\alpha+\varepsilon\Lambda-\gamma[\alpha+\varepsilon(n-\alpha)]\tau_2\}} - l_u\right] \tag{98}$$

$$l\big|_{w=w_{NE}} = l_u - \frac{c_u'[\alpha+\varepsilon(1-\alpha)][\Lambda-\gamma(n-\alpha)\tau_2]}{\{\gamma\alpha+\varepsilon\Lambda-\gamma[\alpha+\varepsilon(n-\alpha)]\tau_2\}} \tag{99}$$

It is evident from (98) and (99) that when $\gamma = 1$ both the equilibrium price level and employment are independent of τ_2. This is because when $\gamma = 1$ the aggregate demand equation (4) necessarily implies that nominal income is equal to the money supply, so that the responsiveness of the money supply to nominal income is irrelevant. The implication is that when $\gamma = 1$ equilibrium outcomes under the nominal income rule are exactly the same as under the simple rule, and the socially optimal design of the rule involves setting its constant value, $\overline{m}$, so as to ensure inflation is zero.[42]

The nominal income rule does differ in one subtle respect from the price level rule, therefore, since the conclusions previously arrived at regarding the latter are not dependent at all on $\gamma \neq 1$ being the case. Nevertheless, comparing the equilibrium price level and employment outcomes for the nominal income rule, (98) and (99), with their counterparts for the price level rule, (63) and (67b), we find that all the key features of the wage-setting equilibrium under the price level rule also arise in the case of the nominal income rule. In particular, it is noteworthy that neither rule can influence equilibrium employment when there is either a single union or unions are atomistic: setting $n = 1$ in (99), yields the efficient outcome, while taking the limit of (99) as $n \to \infty$ yields the outcome for the case of atomistic unions. In these extreme cases the socially optimal setting of τ_2 is that which ensures the price level outcome is zero, namely $\tau_2 = 1/\gamma$.

[42] The constant $\overline{m}$ has been omitted from (17) purely for expositional convenience.

The strong affinity between the two rules' equilibrium outcomes also extends to the case of multiple non-atomistic unions. Provided $1 < n < \infty$, equilibrium employment considered as a function of τ_2, is discontinuous at one particular value of τ_2, namely $\tau_2 = (\gamma\alpha + \varepsilon\Lambda)/\gamma[\alpha + \varepsilon(n-\alpha)]$, while as τ_2 approaches in the limit either of its inadmissible values equilibrium employment tends to either its efficient value or the atomistic-unions outcome. Most significantly of all, there is one particular setting of τ_2 which induces each union to set the market-clearing nominal wage, and therefore ensures both equilibrium employment and the price level attain their socially optimal values of zero. We denote this optimal setting of τ_2 by τ_2^*:

$$\tau_2^* = \left(\frac{1}{\gamma}\right)\left\{\frac{c_u'[\alpha + \varepsilon(1-\alpha)]\Lambda - (\gamma\alpha + \varepsilon\Lambda)l_u}{c_u'(n-\alpha)[\alpha + \varepsilon(1-\alpha)] - [\alpha + \varepsilon(n-\alpha)]l_u}\right\} \tag{100}$$

As in the case of the price level rule, evaluating τ_2^* for the extremes of wage-bargaining structure yields inadmissible values for this parameter: we find that $\tau_2^*\big|_{n=1} = 1$, while $\lim_{n\to\infty} \tau_2^* = [\alpha + \gamma(1-\alpha)]/\gamma$.

One difference between the equilibrium outcomes under the two rules, when $1 < n < \infty$, is that whereas equilibrium employment is a decreasing function of τ_1 in the case of the price level rule, under the nominal income rule it is decreasing in τ_2 if $\gamma < 1$, but is increasing in τ_2 if $\gamma > 1$:

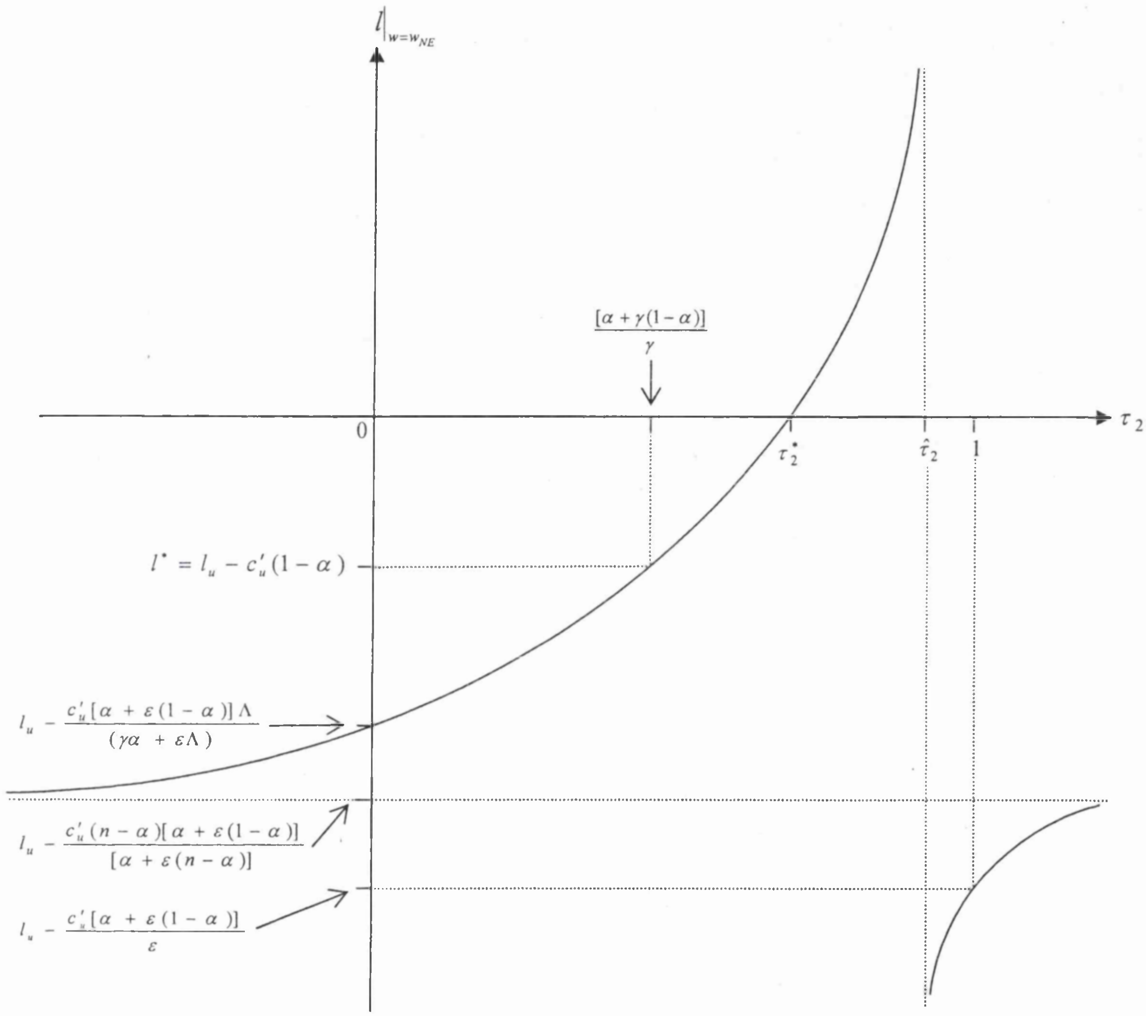
$$\frac{\partial(l\big|_{w=w_{NE}})}{\partial\tau_2} = \frac{(n-1)c_u'\gamma(\gamma-1)\alpha^2[\alpha + \varepsilon(1-\alpha)]}{\{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_2\}^2} \tag{101}$$

The graph of equilibrium employment as a function of τ_2 therefore resembles Figure II.4 only for cases in which $\gamma < 1$. This functional relationship for cases in which $\gamma > 1$ is depicted below as Figure II.5.

Before moving on to discuss the employment targeting rule, we will compare equilibrium employment under strict nominal income targeting, as given by equation (102) below, with its counterparts under the simple rule (equation (46b)) and strict

Figure II.5

Equilibrium employment as a function of the nominal income rule parameter



Notes:

1. It is assumed that $\gamma > 1$, that $1 < n < \infty$ and that $l_u < c_u'(1-\alpha)$.

2. $\hat{\tau}_2 \equiv \dfrac{(\gamma\alpha+\varepsilon\Lambda)}{\gamma[\alpha+\varepsilon(n-\alpha)]}$

3. $\tau_2^* \equiv \left(\dfrac{1}{\gamma}\right)\left\{\dfrac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda-(\gamma\alpha+\varepsilon\Lambda)l_u}{c_u'(n-\alpha)[\alpha+\varepsilon(1-\alpha)]-[\alpha+\varepsilon(n-\alpha)]l_u}\right\}$

4. $l|_{w=w_{NE}}$ is undefined when $\tau_2 = 1$ and $\tau_2 = [\alpha+\gamma(1-\alpha)]/\gamma$.

price level targeting (equation (68)).[43]

$$\lim_{\tau_2 \to \pm\infty} l\big|_{w=w_{NE}} = l_u - \frac{c_u'(n-\alpha)[\alpha+\varepsilon(1-\alpha)]}{[\alpha+\varepsilon(n-\alpha)]} \tag{102}$$

The comparison with (46b) and (68) is most straightforward for the $l_u = 0$ case, and we will focus on this case here. It is apparent that the simple (i.e. $\tau_2 = 0$) rule results in lower employment than strict nominal income targeting if $\gamma < 1$, whereas if $\gamma > 1$ the simple rule yields a superior outcome which is less negative (i.e. closer to the social optimum of zero).[44] This indicates that the wage-setting externality is more severe under the simple rule than under strict nominal income targeting when $\gamma < 1$, and less severe when $\gamma > 1$.[45] The first step to understanding why this is so is to note that when $\gamma = 1$ any rule which involves a response to nominal income merely replicates the simple rule: hence the externality must have the same strength in the simple $\tau_2 = 0$ case as in the strict $\tau_2 \to \pm\infty$ case. The second step is to note that (18) implies that $\lim_{\tau_2 \to \pm\infty} p_{NI\ rule} = \alpha w$, while (18) and (20) together imply that $\lim_{\tau_2 \to \pm\infty} \gamma(m_r - p)_{NI\ rule} = -\alpha w$. It follows that the impact of a marginal increase in union *j*'s nominal wage on its real wage and its impact on aggregate demand must both be independent of γ under the strict nominal income rule. This is not the case under the simple rule, however, since the lower is γ, the weaker is the aggregate demand channel whereby marginal increases in union *j*'s wage reduce the derived demand for its labour. Hence a marginal decrease (increase) in γ from unity must render union *j*'s perceived trade-off more (less) favourable under the simple rule, which consequently worsens (ameliorates) the externality.

Comparing the strict nominal income rule with the strict price level rule, it is apparent that, provided $1 < n < \infty$, equilibrium employment is lower under the former.

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[43] Comparison of the price level outcomes for these regimes is not of great interest since we have assumed the constant $\overline{m}$ is set to zero under the state-contingent rules.

[44] In fact, this statement that, when $\gamma > 1$, the departure of equilibrium employment from the market-clearing level of zero is smaller under the simple rule than under the strict nominal income rule, is true not only for the $l_u = 0$ case but for all $l_u \leq 0$, while it is also true for sufficiently low positive values of l_u. If l_u is sufficiently high and positive, however, the departure of equilibrium employment from zero is, when $\gamma > 1$, larger under the simple rule than under the strict nominal income rule.

[45] This statement is true regardless of the value of l_u (provided $1 < n < \infty$, of course).

Therefore, for the $l_u = 0$ case the strict price level rule unambiguously outperforms the strict nominal income rule as regards employment.[46] The explanation for this is that union *j*'s perceived trade-off between employment and the real wage is more favourable under the strict nominal income rule than under the strict price level rule.[47] The relevant expressions are:

$$\lim_{\tau_1 \to \pm\infty} \left(\frac{dl_j}{d(w_j - p)} \right) = \frac{-[\alpha + n\varepsilon(1-\alpha)]}{n(1-\alpha)} \tag{103a}$$

$$\lim_{\tau_2 \to \pm\infty} \left(\frac{dl_j}{d(w_j - p)} \right) = \frac{-[\alpha + \varepsilon(n-\alpha)]}{(n-\alpha)[\alpha + \varepsilon(1-\alpha)]} \tag{103b}$$

It is straightforward to show that (103b) is strictly greater than (103a), implying that union *j*'s perceived labour demand curve is flatter (less negative in slope) under the strict nominal income rule than under the strict price level rule, which leads to a more severe externality under the former.

The fact that when $1 < n < \infty$ the functional relationship between equilibrium employment and the nominal income rule parameter τ_2 has many similarities to the relationship between equilibrium employment and the price level rule parameter τ_1 is not entirely surprising. The reason for the similarity is that a particular setting of either τ_1 or τ_2 implies a particular monetary response to the aggregate nominal wage. Both the price level and nominal income rules are respectively equivalent to rules of the form $m_r = \tau_1 \kappa_1 w$ and $m_r = \tau_2 \kappa_2 w$, where κ_1 and κ_2 are composite parameters. Indeed, in Appendix II.4 it is shown that for the rule $m_r = \rho w$ there is a particular setting of the response parameter ρ which induces non-atomistic unions to set the market-clearing nominal wage. The socially optimal such rule is:

$$m_r^* = \frac{\{c_u'[\alpha + \varepsilon(1-\alpha)]\Lambda - (\gamma\alpha + \varepsilon\Lambda)l_u\}w}{\gamma[\alpha + \varepsilon(1-\alpha)][c_u'(1-\alpha) - l_u]} \tag{104}$$

[46] This is the case for $l_u < 0$ also, as well as for some low positive values of l_u.

[47] This is the case regardless of l_u.

This optimal rule can be reformulated to yield the optimal price level rule, (71), or the optimal nominal income rule, (100). Another alternative formulation would involve the optimal setting of τ_3 in the employment rule, (21), and this is stated below as (108), together with expressions for the equilibrium nominal wage, price level and employment which can be derived by applying to the semi-reduced forms (22), (23) and (24), an analogous procedure to that which has been described for the other rules:

$$w_{NE} = \frac{[\alpha+\gamma(1-\alpha-\tau_3)]}{\gamma}\left[\frac{c'_u[\alpha+\varepsilon(1-\alpha)][\Lambda-(n-1)\gamma\tau_3]}{\{(n-1)\varepsilon[\alpha+\gamma(1-\alpha-\tau_3)]+\gamma[\alpha+\varepsilon(1-\alpha)]\}}-l_u\right] \tag{105}$$

$$p\big|_{w=w_{NE}} = \frac{(\alpha-\gamma\tau_3)]}{\gamma}\left[\frac{c'_u[\alpha+\varepsilon(1-\alpha)][\Lambda-(n-1)\gamma\tau_3]}{\{(n-1)\varepsilon[\alpha+\gamma(1-\alpha-\tau_3)]+\gamma[\alpha+\varepsilon(1-\alpha)]\}}-l_u\right] \tag{106}$$

$$l\big|_{w=w_{NE}} = l_u - \frac{c'_u[\alpha+\varepsilon(1-\alpha)][\Lambda-(n-1)\gamma\tau_3]}{\{(n-1)\varepsilon[\alpha+\gamma(1-\alpha-\tau_3)]+\gamma[\alpha+\varepsilon(1-\alpha)]\}} \tag{107}$$

$$\tau_3^* = \frac{c'_u[\alpha+\varepsilon(1-\alpha)]\Lambda-(\gamma\alpha+\varepsilon\Lambda)l_u}{(n-1)\gamma\{c'_u[\alpha+\varepsilon(1-\alpha)]-\varepsilon l_u\}} \tag{108}$$

We shall refrain from discussing in detail the correspondence between these expressions and their counterparts for the price level and nominal income rules. The graph of equilibrium employment as a function of τ_3 is very similar, in the $l_u = 0$ case, to the equivalent graph for the nominal income rule when $\gamma > 1$, namely Figure II.5, since provided $1 < n < \infty$ equilibrium employment is an increasing function of τ_3:

$$\frac{\partial(l|_{w=w_{NE}})}{\partial\tau_3} = \frac{(n-1)c'_u\gamma^2\alpha[\alpha+\varepsilon(1-\alpha)]}{\{(n-1)\varepsilon[\alpha+\gamma(1-\alpha-\tau_3)]+\gamma[\alpha+\varepsilon(1-\alpha)]\}^2} > 0\,\forall n>1 \tag{109}$$

One point worth making, however, is that (105) indicates strict employment targeting is inadmissible in this model, since $\lim_{\tau_3\to\pm\infty} w_{NE} = \infty$. (Note also that (107) implies that

$\lim_{\tau_3 \to \pm\infty} l\big|_{w=w_{NE}} = l_u - \{c_u'[\alpha + \varepsilon(1-\alpha)]/\varepsilon\}$. This is significant, since for the price level and nominal income rules when $1 < n < \infty$ is the case, only the substitution of an inadmissible value for the rule parameter, specifically $\tau_1 = 1$ or $\tau_2 = 1$, into the expression for equilibrium employment can cause the latter to take the value it has under atomistic unions.) The inadmissibility of strict employment targeting in the present model raises questions about Bratsiotis and Martin's (1999, p.252) assumption that strict employment targeting is a feasible policy rule in their model.[48]

II.5 Wage-Setting under the Discretionary Monetary Regime

We have found that for each of the regimes analysed above which implicitly prescribe an indirect monetary response to the aggregate nominal wage, there exists a setting of the rule parameter which, when there are multiple non-atomistic unions, brings about the social optimum of full employment (i.e. labour-market clearing) and zero inflation. Such rules are therefore fully credible and render irrelevant the time-inconsistency problem alleged to be inherent to discretionary monetary regimes. These findings obviously need to be reconciled with those of Coricelli et al. (2004a, 2006), who investigate discretionary monetary policy and optimal delegation in the context of the model considered here. As already mentioned these authors also allow for inflation-aversion on the part of unions: however, for the version of their model in which the union loss function lacks a term in inflation, so that the loss function is given by our equation (29) with $l_u = 0$, Coricelli et al. find that an ultra-conservative central banker is socially optimal, so that inflation is eliminated but at the cost of some unemployment in equilibrium. Our finding that an appropriately specified rule can improve upon this outcome suggests there may be some specification of the monetary reaction function implicitly assigned to the discretionary central banker (i.e. some setting of the key delegation parameter, the relative weight on the inflation term in the central bank's loss function (10a)) which is superior to that identified as optimal by Coricelli et al. It will emerge below that an important assumption implicitly made by these authors in arriving at this recommendation is that this weight, c_b, must be

[48] It is noteworthy that Bratsiotis and Martin's expression for the equilibrium price level (equation (19) on page 251 of their paper) does appear to be undefined as the degree of monetary accommodation of price level movements tends in the limit to unity, which corresponds in their model to a strict employment-targeting regime.

positive. As hinted in our earlier discussion of (10a) and (10′), our analysis departs from Coricelli et al. by relaxing this restriction on the sign of this delegation parameter.

The extent to which this restriction can be relaxed turns out to depend upon the precise nature of the delegation. It is useful to remind the reader at this point of the two alternative interpretations of monetary-policy delegation introduced to the literature by Rogoff (1985), and which have already been briefly discussed in Chapter I. The first interpretation implicitly assumes that only instrument independence is conferred on the central bank. It views delegation as the assignment of a particular objective function to the central bank's staff, whose professionalism is such that their personal views on policy objectives exercise no influence over their discretionary settings of the policy instrument, and who obediently conduct monetary policy purely in respect of the objective function that has been assigned to them. Under this interpretation, the assignment of an objective function to the central bank is equivalent to the choice of a value for the response-to-wages parameter ρ in the monetary reaction function $m = \rho w$.[49] Note that the foregoing sentences are not specific as to whether the quadratic 'objective function' is a loss or utility function. This is because when delegation solely involves instrument independence, it is irrelevant whether the resulting objective, considered as a function of the instrument *m*, involves a unique minimum (and therefore may be regarded as a loss function), or a unique maximum (a utility function). The essential point is that with this type of delegation, central bank staff routinely set *m* equal to the value which solves the first-order condition $\partial\Omega^{cb}/\partial m = 0$.

The alternative interpretation of the delegation decision assumes that the relative weight placed on inflation stabilisation by each of the candidates to head the central bank is observable, and that society appoints that candidate whose preferences will result in the best attainable outcomes from society's viewpoint when that person is endowed with the discretionary power to conduct monetary policy entirely in

[49] An equivalence consequently arises between choosing the delegation parameter c_b in an assigned loss function, and choosing the response parameter in a monetary rule to which the central bank is required to adhere, such as the price level rule, $m_r = \tau_1 p$. This becomes readily apparent from comparison of the expressions for $m_{discretion}$ and $m_{price\ level\ rule}$, as given by (26) and (16) respectively. Equating the right hand sides of these equations, and solving for τ_1, we have $\tau_1 = 1 - c_b\alpha(1-\alpha)\gamma^{-1}$. Hence if monetary policy can be delegated without the concession of goal independence to the central bank, a particular c_b choice is equivalent to the choice of a particular value for the rule parameter τ_1.

accordance with his or her personal predilections. This second interpretation implicitly assumes that appointing someone to head the central bank involves conferring not only instrument independence, but also goal independence, on the appointee. Since the objective function (10′) has a unique minimum not only for all positive values of c_b, but also for all values of this parameter which are sufficiently large in numerical terms (specifically, for all $c_b > -1/(1-\alpha)^2$), it clearly makes sense to regard all potential central-banker candidates as having a loss function, rather than a utility function, including those for which $c_b < -1/(1-\alpha)^2$. An immediate implication of doing so, however, is that only candidates for which $c_b > -1/(1-\alpha)^2$ will set m in accordance with equation (26), since only for this subset of candidates is the second-order condition for a minimum satisfied by this setting of m. For those persons whose weight parameter c_b is more negative than $-1/(1-\alpha)^2$, setting m equal to the right-hand side of (26) will maximise their personal loss, and indeed there is no choice of m which minimises the loss for a central banker who belongs to this category. Hence in equilibrium m is undefined for this subset of candidates, and we therefore henceforth treat c_b values which are less than $-1/(1-\alpha)^2$ as inadmissible when delegation involves goal independence.[50] Consequently, for this kind of delegation, the equivalence between choosing a central banker and choosing the response coefficient ρ in $m = \rho w$ is somewhat less general than when central bankers lack the power to formulate policy goals.[51]

Since our key result only involves the weight c_b, and is independent of the other delegation parameters in (10a), it suffices here to use the simpler central bank loss function (10′) instead. The relevant semi-reduced forms for the money supply and price level are therefore given by (26) and (27). These expressions immediately reveal

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[50] The second derivative of the central bank's objective function with respect to m is $\partial^2\Omega^{cb}/\partial m^2 = 2\gamma^2[1+c_b(1-\alpha)^2]/[\alpha+\gamma(1-\alpha)]^2$. One way around the difficult identified in this passage would be to define the objective function (10′) to be a loss function if $c_b > -1/(1-\alpha)^2$, and to be a utility function if $c_b < -1/(1-\alpha)^2$, so that every potential central banker would set m in accordance with equation (26) if appointed. However, proceeding in this way would introduce an element of arbitrariness to our definitions which is not easily justifiable.

[51] For the price level rule $m_r = \tau_1 p$, the equivalence between appointing a particular goal-independent central banker, and setting τ_1 equal to a particular value, only extends to τ_1 values which are less than $[\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$. (In other words, there is no choice of goal-independent central banker which can replicate the outcomes associated with a value for τ_1 such that $\tau_1 > [\alpha+\gamma(1-\alpha)]/\gamma(1-\alpha)$.)

that, regardless of n, $c_b = -1/(1-\alpha)^2$ is an inadmissible value for the weight parameter, and it turns out, for the same reason that $\tau_1 = [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$ is inadmissible under the price level rule, namely that it renders the price level indeterminate. The parallel between discretion and the rules also extends to a second setting of c_b which is inadmissible regardless of n, namely $c_b = 0$, which, like $\tau_1 = 1$ in the case of the price level rule, necessarily results in real money balances being maintained constant regardless of w,[52] and which consequently implies that were symmetric wage-setting to occur, prices would then adjust in such a way as to cause the market-clearing real wage to prevail at every firm, thus depriving each union of an incentive to set any particular nominal wage.

Assuming $c_b > -1/(1-\alpha)^2$ and $c_b \neq 0$, the individual union's optimisation problem can be solved in an analogous fashion to that described for the rule-based regimes, which in turn leads to the following expressions for the equilibrium nominal wage, price level, real wage, and employment:[53]

$$w_{NE} = \frac{[1+c_b(1-\alpha)^2]}{c_b(1-\alpha)}\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Phi}{[c_b\alpha(1-\alpha)+\varepsilon\Phi]} - l_u\right\} \tag{110}$$

where $\Phi \equiv n[1+c_b(1-\alpha)^2]-1$

$$p\big|_{w=w_{NE}} = \frac{1}{c_b(1-\alpha)}\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Phi}{[c_b\alpha(1-\alpha)+\varepsilon\Phi]} - l_u\right\} \tag{111}$$

$$(w-p)\big|_{w=w_{NE}} = (1-\alpha)\left\{\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Phi}{[c_b\alpha(1-\alpha)+\varepsilon\Phi]} - l_u\right\} \tag{112a}$$

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[52] Because of the adopted normalisations, the fixed value of real money balances when $c_b = 0$ is zero, as is apparent from equations (26) and (27).

[53] Once again, it is convenient to simplify the notation by dropping the 'discretion' appendage used in (26) and (27). Note that had we conducted the analysis using the more general loss function given by (10a), the equilibrium expressions for w and p would then be functions not only of c_b but also of the additional delegation parameters l_b, π_b and c_b'. The important point, however, is that the counterpart expressions for the real wage and employment would be exactly the same as those reported here, namely (112a) and (112b).

$$l\big|_{w=w_{NE}} = l_u - \frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Phi}{[c_b\alpha(1-\alpha)+\varepsilon\Phi]} \qquad (112b)$$

As one might expect, evaluating (112a) and (112b) for $n=1$ yields the efficient real outcomes, as does taking the limit of (112a) and (112b) as $\varepsilon \to \infty$, while the limits of (112a) and (112b) as $n \to \infty$ are found to be the inefficient real outcomes under atomistic unions. In other words, real outcomes in these extreme cases of goods and labour-market structure are independent of the parameter c_b, confirming once again that the characteristics of the monetary regime, whether discretionary or rule-based, cannot influence equilibrium employment in these extreme cases. For the more interesting intermediate cases for which $1<n<\infty$, however, (112b) indicates that the discretionary regime parameter c_b closely parallels the rule parameters in its ability to influence equilibrium employment. Unsurprisingly, an ultraconservative central banker is found to yield the same employment and price level outcomes as a strict price level rule:

$$\lim_{c_b \to \infty} l\big|_{w=w_{NE}} = l_u - \frac{nc_u'(1-\alpha)[\alpha+\varepsilon(1-\alpha)]}{[\alpha+n\varepsilon(1-\alpha)]} \qquad (113)$$

This is (with $l_u=0$ of course) the result found by Coricelli et al., who emphasise the following derivative when discussing the policy implications of their model:

$$\frac{\partial(l\big|_{w=w_{NE}})}{\partial c_b} = \frac{(n-1)c_u'\alpha(1-\alpha)[\alpha+\varepsilon(1-\alpha)]}{[c_b\alpha(1-\alpha)+\varepsilon\Phi]^2} > 0\,\forall n>1 \qquad (114)$$

If c_b is restricted to be positive, it follows directly from (114) that an ultra-conservative central banker not only eliminates inflation but also brings about the best possible employment outcome. Relaxing this sign restriction on c_b, however, reveals numerous affinities with the rules analysed earlier. It is noteworthy, for example, that if delegation does not involve goal independence, the extreme case in which $c_b \to -\infty$ yields precisely the same employment and price level outcomes as that in which $c_b \to \infty$: $\lim_{c_b \to -\infty} l\big|_{w=w_{NE}} = \lim_{c_b \to \infty} l\big|_{w=w_{NE}}$. Another parallel with the rules is that, provided

$1 < n < \infty$, equilibrium employment is a discontinuous function of the regime parameter: the discontinuity is independent of l_u and occurs at $c_b = \hat{c}_b \equiv -(n-1)\varepsilon/(1-\alpha)[\alpha + n\varepsilon(1-\alpha)]$ and is analogous to the discontinuity at $\tau_1 = \hat{\tau}_1$ in the case of the price level rule. (Note that $\hat{c}_b > -1/(1-\alpha)^2$, so that the discontinuity exists regardless of the nature of monetary-policy delegation.) Furthermore, as $c_b \to -1/(1-\alpha)^2$ and $c_b \to 0$, equilibrium employment approaches its value under efficient wage-setting and atomistic unions respectively, which obviously parallels what happens under the price level rule as τ_1 approaches its two inadmissible values of unity and $[\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$. The important point, however, is that (112b) reveals that there is one particular setting of c_b which induces non-atomistic unions to set the market-clearing nominal wage, and which therefore ensures that society's bliss point is achieved. We will use c_b^* to denote this c_b value:[54]

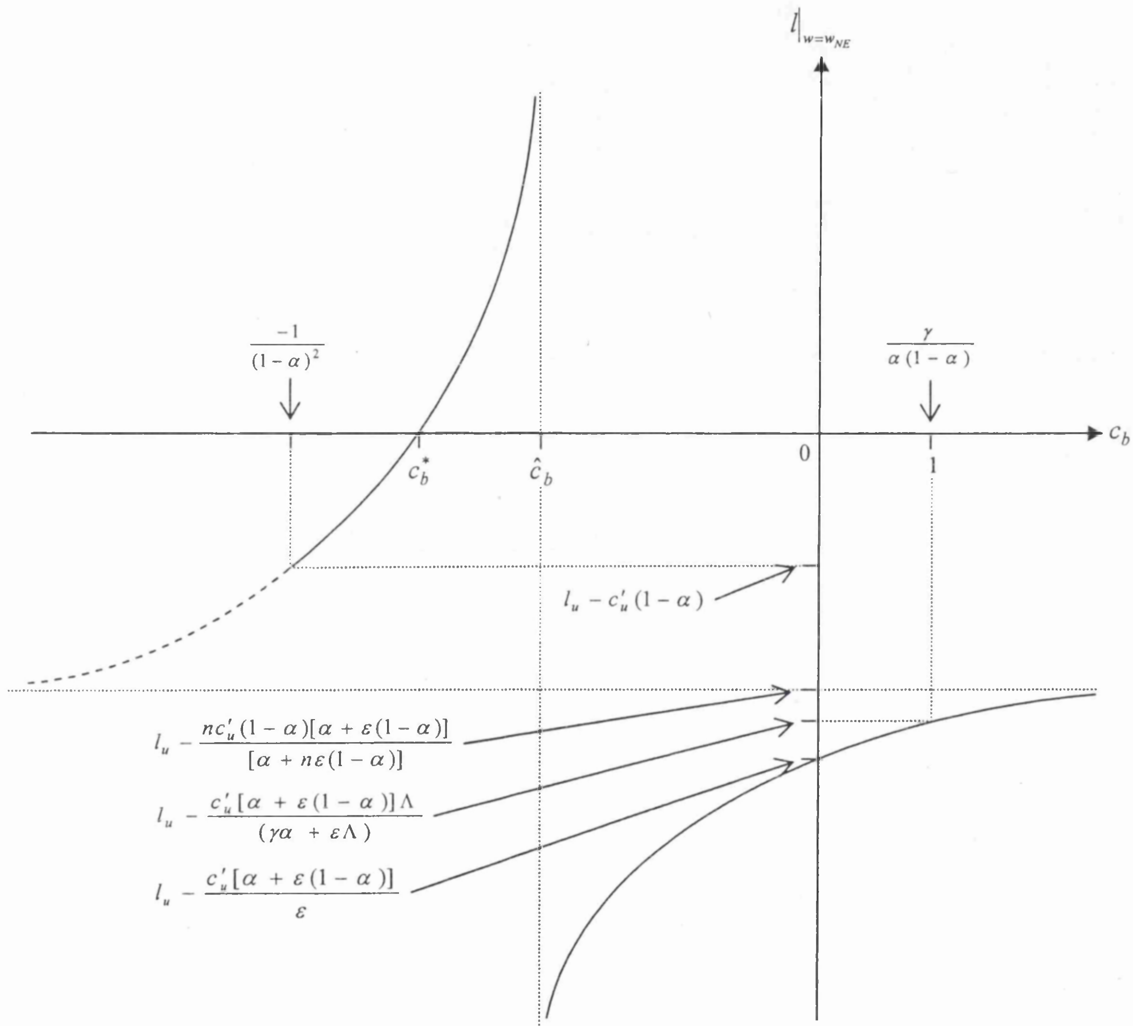
$$c_b^* = -\frac{(n-1)}{(1-\alpha)}\left\{\frac{c_u'[\alpha + \varepsilon(1-\alpha)] - \varepsilon l_u}{nc_u'(1-\alpha)[\alpha + \varepsilon(1-\alpha)] - [\alpha + n\varepsilon(1-\alpha)]l_u}\right\} \tag{115}$$

If delegation merely involves assigning an objective function to an instrument-independent central bank, equation (115) tells us the optimal weight for that objective function. However, if delegation also involves goal independence, and thus necessitates the choice of a central banker with particular preferences, the social optimum will not be attainable if $c_b^* < -1/(1-\alpha)^2$. Substituting the right-hand side of (115) into this inequality condition reveals that if the representative union's preference parameters l_u and c_u' are such that both the inequalities $nc_u'(1-\alpha)[\alpha + \varepsilon(1-\alpha)]/[\alpha + n\varepsilon(1-\alpha)] > l_u > c_u'(1-\alpha)$ hold, it will not be possible to choose an inflation-loving central banker whose fondness for inflation is sufficiently strong to induce each union to set the market-clearing nominal wage. However, if either $l_u > nc_u'(1-\alpha)[\alpha + \varepsilon(1-\alpha)]/[\alpha + n\varepsilon(1-\alpha)]$ or $l_u < c_u'(1-\alpha)$ holds, then $c_b^* > -1/(1-\alpha)^2$ is found to be the case, and the optimum is attainable even when the

[54] The fact that $c_b = c_b^*$ achieves socially optimal price level stabilization when $1 < n < \infty$ implies that the additional delegation parameters in (10a) are redundant (i.e. given $c_b = c_b^*$, when unions are non-atomistic the socially optimal value of each of them is zero).

Figure II.6

Equilibrium employment as a function of the central bank's weight parameter



Notes:

1. It is assumed that $1 < n < \infty$ and that $l_u < c_u'(1-\alpha)$.

2. $\hat{c}_b \equiv \dfrac{-(n-1)\varepsilon}{(1-\alpha)[\alpha + n\varepsilon(1-\alpha)]}$

3. $c_b^* \equiv -\dfrac{(n-1)}{(1-\alpha)}\left\{\dfrac{c_u'[\alpha+\varepsilon(1-\alpha)]-\varepsilon l_u}{nc_u'(1-\alpha)[\alpha+\varepsilon(1-\alpha)]-[\alpha+n\varepsilon(1-\alpha)]l_u}\right\}$

4. $l|_{w=w_{NE}}$ is undefined when $c_b = -1/(1-\alpha)^2$ and when $c_b = 0$.

5. The graph of $l|_{w=w_{NE}}$ is shown as a dashed line for $c_b < -1/(1-\alpha)^2$, since for these c_b values equilibrium employment is not defined if delegation involves appointing a goal-independent central banker.

central banker is goal independent. The scenario in which $l_u < c_u'(1-\alpha)$ is depicted in Figure II.6 above. Note that this encompasses the most plausible case of union preferences which might be considered, namely that in which l_u coincides with the market-clearing level of employment. For this $l_u = 0$ case, we find that:

$$c_b^*\Big|_{l_u=0} = -\frac{(n-1)}{n(1-\alpha)^2} \tag{115'}$$

Just as in the case of the price level rule, these results for the discretionary regime can be explained in terms of the influence of the regime parameter on the trade-off between real wage and employment perceived by a non-atomistic union. Using (7), (26) and (27), the marginal effects of union j's wage on its real wage and employment, and the slope of its perceived labour demand curve are as follows:

$$\frac{d(w_j - p)}{dw_j} = \frac{\Phi}{(1+\Phi)} \tag{116}$$

$$\frac{dl_j}{dw_j} = -\frac{[c_b\alpha(1-\alpha)+\varepsilon\Phi]}{[\alpha+\varepsilon(1-\alpha)](1+\Phi)} \tag{117}$$

$$\frac{dl_j}{d(w_j - p)} = -\frac{[c_b\alpha(1-\alpha)+\varepsilon\Phi]}{[\alpha+\varepsilon(1-\alpha)]\Phi} \tag{118}$$

Evaluating (117) for $c_b = \hat{c}_b$ reveals that this setting of the weight parameter causes the real wage and aggregate demand channels to neutralise each other exactly, so that $\left(dl_j/d(w_j - p)\right)\Big|_{c_b=\hat{c}_b} = 0$, and each union perceives the employment cost of its wage decision to be zero, and hence pursues as high a real wage as possible. When $c_b = \hat{c}_b$, therefore, the union's circumstances are the same as under the price level rule when $\tau_1 = \hat{\tau}_1$. Using (118) it is similarly straightforward to show that non-atomistic union j's perceived trade-off approaches, respectively, that of a single union as $c_b \to -1/(1-\alpha)^2$, and that of an atomistic union as $c_b \to 0$. Finally,

$c_b = -(n-1)/n(1-\alpha)^2$ is revealed by (116) to have the effect of rendering the real wage channel inoperative (the perceived labour demand curve is vertical), so that each union then sets w_j so as to bring about its employment objective of l_u. If $l_u = 0$ it then follows directly that this c_b induces non-atomistic unions to set the market-clearing nominal wage and hence is socially optimal. If $l_u \neq 0$ the explanation is more complicated, but the essential point remains that in the absence of goal independence, market-clearing wage-setting can be induced by setting the central bank's weight parameter appropriately, while if delegation does involve goal independence an appropriate choice of central banker can bring about the socially optimal outcome for many, but not all, cases of union preferences.

It seems appropriate at this point to suggest that caution be exercised in interpreting this result before arriving at a policy recommendation on the basis of it. This is because the argument put forward in the preceding paragraphs is rather more firmly grounded in the first, rather than the second, of the two aforementioned possible interpretations of society's delegation decision in respect of the discretionary monetary regime. If the appointed central banker enjoys some degree of goal independence, so that the second interpretation must be considered the more persuasive, it then follows that the real-world relevance of the result identified in this section crucially hinges upon the assumptions that inflation-loving candidate central bankers exist, that the extent to which each such person is fond of inflation is accurately discernible, and that the appointment of the most suitable such candidate will be viewed as credible by private-sector agents. All of these assumptions are questionable, of course, and for these reasons when the institutional framework involves goal independence being conferred on the head of the central bank, the key result of this section ought then to be regarded as a theoretical curiosity rather than a serious proposal for the design of the monetary regime.

Finally, it seems fitting to end our discussion of the discretionary monetary regime by emphasising the importance of our assumption that the central bank sets *m* after *w* has been determined. As mentioned in Chapter I, several papers have shown that in a game of simultaneous moves by the authorities and the unions, the employment outcome is independent of the central bank's weight parameter. The reason is the straightforward one that, since the central bank cannot observe *w* at the time it sets its instrument, each union, regardless of *n*, perceives *m* to be beyond its influence, and treats it as given in choosing its individually optimal wage. The scenario therefore corresponds very closely to that of the simple rule, and indeed the equilibrium employment outcome under discretionary monetary policy when the central bank sets m at the same time as w is determined is given by (46b), the outcome under the simple rule.[55]

II.6 Wage-Setting when the Union Loss Features a Quadratic Real-Wage Term

This final section compares the results described above for the case in which the union loss function is given by (29) (i.e. features a linear term, and no quadratic term in the real wage), with the results which arise when the quadratic real wage term is present, as in union loss functions (28) and (30). Since all the key points of the argument can be made by referring to the simpler specification given by (30), we will concentrate on this specification, and only briefly comment on the implications of (28). Only the price level rule will be examined here: needless to say, very similar conclusions arise in respect of the nominal income and employment rules.

When the analytical procedures described in our previous section on the price level rule are applied to union loss function (30), the efficient nominal wage is found to be:

$$w^* = \frac{[\alpha + \gamma(1-\alpha)(1-\tau_1)][c_u(1-\alpha)w_u^{real} - l_u]}{\gamma(1-\tau_1)[1 + c_u(1-\alpha)^2]} \tag{119}$$

Combining (119) with (15) yields the efficient real wage, which in turn can be substituted into the symmetric-wage labour demand curve, (34b), in order to derive the efficient employment outcome:

$$(w_j - p)\Big|_{w_j = w^* \forall j} = \frac{(1-\alpha)[c_u(1-\alpha)w_u^{real} - l_u]}{[1 + c_u(1-\alpha)^2]} \tag{120a}$$

[55] Acocella et al. (2005) investigate the importance of timing assumptions in the Coricelli et al. model when unions are inflation-indifferent. For the scenario in which m and w are simultaneously determined, the expression for equilibrium unemployment obtained by Acocella et al. (2005, p.11, equation (18)), can be shown, after accounting for differences in notation and for the value (unity) assigned by them to the aggregate demand elasticity, to imply equilibrium employment equal to equation (46b) of this chapter.

$$l_j\Big|_{w_j=w^*\forall j} = \frac{l_u - c_u(1-\alpha)w_u^{real}}{[1+c_u(1-\alpha)^2]} \tag{120b}$$

Note that in general these efficient outcomes differ from the individual union's notional objectives, w_u^{real} and l_u. The exception to this general finding is the special case in which w_u^{real} and l_u are consistent with one another, i.e. represent a point (union j's bliss point) on the symmetric-wage labour demand curve. This special case is that in which $w_u^{real} = -(1-\alpha)l_u$, and when w_u^{real} and l_u are restricted to satisfy this equation, (120a) and (120b) reveal that these notional objectives are indeed attained under efficient wage-setting:

$$(w_j - p)\Big|_{w_j=w^*\forall j,\ w_u^{real}=-(1-\alpha)l_u} = w_u^{real} \tag{121a}$$

$$l_j\Big|_{w_j=w^*\forall j,\ w_u^{real}=-(1-\alpha)l_u} = l_u \tag{121b}$$

The equilibrium nominal wage, real wage and employment in this scenario are as follows:

$$w_{NE} = \frac{[\alpha+\gamma(1-\alpha)(1-\tau_1)]}{\gamma(1-\tau_1)}\left(\frac{\Sigma_1}{\Sigma_2}\right) \tag{122}$$

$$(w_j - p)\Big|_{w=w_{NE}} = (1-\alpha)\left(\frac{\Sigma_1}{\Sigma_2}\right) \tag{123a}$$

$$l_j\Big|_{w=w_{NE}} = -\frac{\Sigma_1}{\Sigma_2} \tag{123b}$$

where: $\Sigma_1 \equiv c_u[\alpha+\varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]w_u^{real} - \{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}l_u$

$\Sigma_2 \equiv \gamma\alpha(1-\tau_1) + \{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}[\Lambda - n\gamma(1-\alpha)\tau_1]$.

Evaluating these expressions for $n=1$ reveals that equilibrium wage-setting is efficient when there is a single union. For $n>1$, however, the equilibrium real outcomes (123a) and (123b), differ in general from the efficient outcomes (120a) and (120b). It is evident that the loss function (30) gives rise to an adverse wage-setting externality in much the same way as (29). While it would be inappropriate to discuss in depth the externality arising from (30), given the exhaustive account of its source and nature provided earlier for the case of (29), it is worth pointing out that whereas with (29) the externality does not depend on l_u and for most values of τ_1 leads to an inefficiently high real wage, in the case of (30) the direction of the externality depends not only on τ_1 but also on how w_u^{real} and l_u relate to one another. In the simple rule ($\tau_1 = 0$) case, for example, the fact that (29) implies that the efficient employment outcome is necessarily less than l_u causes the real wage to be inefficiently high in equilibrium. When union preferences are given by (30), however, this will only be the case under the simple rule if the individual union's bliss point is located to the right of the symmetric-wage labour demand curve (in algebraic terms this will be so if $l_u > -w_u^{real}/(1-\alpha)$). If $l_u < -w_u^{real}/(1-\alpha)$ so that the bliss point is to the left of the symmetric-wage labour demand curve, the simple rule will be characterised by an inefficiently low real wage in equilibrium. Hence in the case of (30) the direction of the externality is sensitive to the union's preference parameters, as well as to the monetary rule parameter, whereas in the case of (29) it is only sensitive to the latter.[56]

It is clear from (122) that, as in the case of loss function (29), there exists a setting of τ_1 which can induce non-atomistic unions to set the market-clearing nominal wage. Setting the right-hand side of (122) equal to zero, and solving for τ_1, the socially optimal setting of this parameter is found to be:

$$\tau_1^* = \left(\frac{1}{\gamma}\right)\left\{\frac{c_u[\alpha+\varepsilon(1-\alpha)]\Lambda w_u^{real} - (\gamma\alpha+\varepsilon\Lambda)l_u}{nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]w_u^{real} - [\alpha+n\varepsilon(1-\alpha)]l_u}\right\} \tag{124}$$

The key finding of our earlier analysis therefore appears also to arise when the union loss function is given by (30) rather than (29).[57] However, there is an important

[56] Note again that imperfect competition is essential for the existence of the externality, since it is apparent from (122) that $\lim_{\varepsilon\to\infty} w_{NE} = w^*$.

[57] Note that Bratsiotis and Martin's expression for mean equilibrium unemployment (their equation (16) on p.249 of their paper) also appears to imply that in their model there is a particular value of their

difference between the cases. Whereas with loss function (29) τ_1^* exists (provided $1 < n < \infty$) regardless of the value taken by l_u, with loss function (30) τ_1^* only exists if l_u and w_u^{real} are inconsistent with one another. Setting $l_u = -w_u^{real}/(1-\alpha)$ in (122), (123a) and (123b), we find that the equilibrium outcomes are efficient and that union j's bliss point is achieved regardless of n, ε, and the rule parameter τ_1:[58]

$$w_{NE}\Big|_{w_u^{real}=-(1-\alpha)l_u} = w^* \qquad (125)$$

$$(w_j - p)\Big|_{w=w_{NE},\, w_u^{real}=-(1-\alpha)l_u} = w_u^{real} \qquad (126a)$$

$$l_j\Big|_{w=w_{NE},\, w_u^{real}=-(1-\alpha)l_u} = l_u \qquad (126b)$$

This result that the wage-setting externality only exists if the individual union's real wage and employment objectives are consistent with one another appears to be implicit in Bratsiotis and Martin's own expressions,[59] and also arises for the more general specification of the union loss function, namely (28). As explained in an earlier section of this chapter, the linear real wage term in (28) has the effect of displacing the bliss point of the isoloss map away from the point whose coordinates are the notional objectives of l_u and w_u^{real}, so that the true desired real wage accompanying the employment objective of l_u becomes $(c_u'/c_u) + w_u^{real}$. However, if these true objectives are consistent with one another, in other words if l_u and $(c_u'/c_u) + w_u^{real}$ satisfy the equation $l_u = -[c_u' + c_u w_u^{real}]/c_u(1-\alpha)$, union j's true bliss

monetary accommodation parameter (the equivalent of this chapter's τ_1) which causes equilibrium unemployment to be zero.

[58] Except, of course, for the two inadmissible values for τ_1 of unity and $[\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$, which result in indeterminacy of both nominal and real variables. Note also that while the right-hand side of (124) appears to be defined when $l_u = -w_u^{real}/(1-\alpha)$, this is illusory since τ_1^* has been derived on the assumption that $l_u \neq -w_u^{real}/(1-\alpha)$.

[59] In the non-stochastic version of Bratsiotis and Martin (1999), if the trouble is taken to work out the individual union's employment target which is consistent with its real wage target, and this employment target is then substituted into their expression for equilibrium unemployment (their equation (16) on p.249 of their paper), equilibrium unemployment is found to be zero in their model when union objectives are consistent.

point will lie on the symmetric-wage labour demand curve and equilibrium wage-setting will then be efficient and beyond the monetary regime's influence.[60]

The sensitivity of equilibrium outcomes to the assumption that each union's notional objectives are inconsistent with one another has previously been commented upon by Cubitt (1997). His remarks, however, concern models of strategic interaction between a policymaker and a single economy-wide union. This chapter is consequently the first contribution to the literature to point out that Cubitt's critique also extends to models which feature multiple unions, and which are therefore potentially characterised by wage-setting externalities. Cubitt's principal point is that when the single union's loss function has quadratic terms in both employment and the real wage, a necessary condition for stagflation to be an equilibrium outcome is that the union's objectives be formulated without account being taken of the feasibility of simultaneously attaining both these objectives given the economy's known structure, so that the resulting bliss point is not on the labour demand curve, and consequently is unfeasible.

The extension of Cubitt's critique to the multiple-union model of Bratsiotis and Martin seems straightforward, since in that model every union recognises that in the rational-expectations equilibrium the outcome must lie on the symmetric-wage labour demand curve. Given the centrality of this to the results of Bratsiotis and Martin, a compelling argument exists for constraining the individual union's objectives l_u and w_u^{real} to be consistent with the symmetric-wage labour demand curve, so that the resulting bliss point lies on that curve. Note that if we extend the model with loss function (30) by endogenising each union's choice of real wage objective, in the resulting Nash equilibrium of the simultaneous-move game each union does choose this real wage objective w_u^{real} to be consistent with l_u, so that the wage-setting externality is absent from the subsequent simultaneous-move wage-setting game. This extension to the model is described in Appendix II.5.

Although Cubitt (1997) does not discuss the case of a union objective function which features a linear rather than quadratic term in the real wage, it is nevertheless clear that a variant of the Cubitt critique can be applied to the results relating to loss function (29) reported earlier in this chapter. As mentioned earlier, the key point about

[60] Once again, this is subject to the proviso that $\tau_1 \neq 1$ and $\tau_1 \neq [\alpha + \gamma(1-\alpha)]/\gamma(1-\alpha)$.

(29) is that it does not feature a notional real wage objective and therefore lacks a bliss point.[61] This feature of (29) suggests that an argument along the following lines can be constructed in its defence: 'Union j may formulate a particular employment objective l_u (and it seems most plausible to assume at this point, as Coricelli et al. do, that $l_u = 0$, so that union j desires employment to be at its market-clearing level), but given $l_u = 0$ is achieved, union j must be better off if it so happens that its members also receive a higher real wage than the market-clearing real wage'. This corresponds closely to the notion underlying standard utility functions that if an individual agent does attain her ideal leisure outcome, her utility must then necessarily be increasing in her goods consumption. While this defence of (29) obviously has considerable intuitive appeal, there is a notable objection to it, namely that it abstracts from the possibility that the union's objective function may differ from that of its members, and that it may be necessary to distinguish between their respective objective functions even when the union's sole concern is to maximise its members' welfare.[62] Therefore, while (29) has the apparent merit of being based on plausible primitive assumptions regarding the preferences of union j's members, it nevertheless may be objected to on the grounds that it precludes the possibility that union j may formulate a specific real wage objective to accompany its notional employment objective of l_u. As discussed earlier, given the constraint of a downwardly sloped labour demand curve, the presence of the linear real-wage term in (29) means that the notional employment objective l_u cannot ever be the employment outcome under either equilibrium or (perhaps more significantly) efficient wage-setting. Thus an element of artificiality appertains to l_u in (29), which appears awkward alongside the assumption

[61] Note that when (29) is the loss function one cannot easily extend the model by endogenising the union preference parameter which is at the root of the externality, namely the weight c_u', since it is difficult not to treat c_u' as anything other than 'primitive', i.e. a fundamental structural building-block of the model. With loss function (29), for instance, the externality will disappear if $c_u' = 0$ (this would cause the isoloss map to consist of parallel straight lines, with the line at $l_j = l_u$ representing an infinite set of bliss points). However, since a key aspect of the loss function itself is in this case being adjusted, we lack a criterion for the evaluation of the impact of such a parameter change on union welfare: in effect, an adjustment in c_u' amounts to a modification of the union welfare criterion itself. Modifying w_u^{real} in (28) and (30) is not vulnerable to this criticism, since given l_u, consistency with the symmetric-wage labour demand curve implies a unique corresponding w_u^{real}.

[62] As pointed out by Acocella and Di Bartolomeo (2004, p.702), (29) also has the drawback of implausibly assuming that for any given employment level the marginal rate of substitution between employment and the real wage (i.e. the slope of the isoloss contours at that employment level) is constant and does not vary with the real wage.

that each union, given its objective function, forms its expectations rationally and sets its wage in an optimising fashion. It is then only a short step to arguing that since each union is assumed to be rational (i.e. endeavours to minimise its loss function and forms rational expectations) when setting its wage, it ought to be assumed to be rational, and hence ought to be assumed to take structural constraints into account, when formulating its objectives.

The Cubitt critique, suitably adapted, therefore appears to imply that (28), together with consistency between l_u and w_u^{real}, may be preferable to (29) as a specification of union objectives. The validity of this argument depends, of course, on whether or not those responsible for formulating union objectives can specify targets for both employment and the real wage. Clearly, if union j's leaders are 'weak', in the sense of being unable to disregard the clamour of militant members who are concerned to push up the real wage, such a circumstance may in fact make (29) a more realistic specification for the modeller to adopt.

II.6 Conclusion

This chapter has investigated various issues relating to the design of monetary institutions for a non-stochastic economy which features monopolistically competitive firms and inflation-indifferent monopoly unions. It has been found that union wage-setting decisions are characterised by a macroeconomic externality involving the price level, with this externality absent in the limiting case of a perfectly competitive goods market, or when there is a single economy-wide union. Equilibrium employment has been found to be independent of parameters relating to the monetary regime in the extreme cases of wage-bargaining structure, or when the money supply is set simultaneously with wages. The chapter's principal findings of importance, however, relate to economies with multiple non-atomistic unions, when the regime involves a monetary response to either the aggregate nominal wage or to a variable dependent upon that wage. Three monetary rules were analysed and shown to be equivalent in the sense that, with the exception of a few special cases, any particular employment outcome can be brought about by an appropriate setting of the monetary response parameter in each of these rules. An economic explanation for this was provided in terms of how the rule parameter affects the externality via its influence on the individual wage decisions of non-atomistic unions. The most interesting finding of all, however, was that appropriate design of each of these rules can induce non-atomistic unions to set the nominal wage associated with labour-market clearing. A parallel set of results was shown to arise under a discretionary regime: in particular, it was found that a central bank whose loss function features a negative relative weight on inflation can completely eliminate both unemployment and the mean inflation bias. This theoretical curiosity was not identified by Coricelli et al. (2004a, 2006) in their investigation of the optimal delegation issue on account of their plausibly restricting this relative weight to be positive.

The chapter's secondary finding of importance is that the wage-setting externality which typically arises in this framework is sensitive to the specification of the union's loss function. In particular, specifications which involve a quadratic term in the real wage, as adopted by Bratsiotis and Martin (1999), are vulnerable to the Cubitt (1997) critique, namely that such specifications ignore the possibility that unions may, in formulating their objectives, take into account the constraints facing them. Although Cubitt did not apply his critique to the specification adopted by Coricelli et al. (2004a, 2006), and which features a linear rather than quadratic term in the real wage, arguments were provided for a variant of the critique being similarly valid in respect of this linear-term specification, since this linear element entirely precludes the possibility that unions might devise a specific real wage target to accompany their notional employment objective. Ultimately, however, the chapter is compelled, like Cubitt, to conclude on a non-committal note as regards this issue, since it is far from clear whether or not each individual union in formulating its objectives, assesses their feasibility by taking into account the available information regarding the economy's structure.

Chapter III

Union Wage-Setting, Monopolistically Competitive Firms and Stochastic Shocks

III.1 Introduction

Whereas Chapter II has investigated in depth the nature of the wage-setting externalities which arise in a non-stochastic unionised economy when firms are monopolistically competitive, and has pinpointed and explained several noteworthy results which, although unnoticed, are implicit in existing published papers, this second chapter will endeavour to extend this literature by introducing stochastic shocks into the basic model outlined previously. As explained in Chapter I, an initial foray in this direction has been undertaken by Bratsiotis and Martin (1999), who study how the strategic wage-setting behaviour of non-atomistic unions affects the performance of a variety of monetary regimes, both rule-based and discretionary, when both aggregate demand and productivity are subject to stochastic shocks. This chapter differs from Bratsiotis and Martin in several respects. Firstly, whereas Bratsiotis and Martin only rather tersely describe equilibrium outcomes for a variety of regimes, including several varieties of targeting rule, this chapter provides a detailed analysis of three basic regimes, those of a fixed money supply, the optimal state-contingent rule, and a discretionary regime.[1] Secondly, we utilise a more intuitively justifiable specification of the union loss function than that adopted by Bratsiotis and Martin. Thirdly, and most importantly, the chapter departs from Bratsiotis and Martin by adopting the assumption of Herrendorf and Lockwood (1997) that each union possesses, at the time it concludes its wage agreement with its employer firm(s), some information concerning the realised value of the productivity shock relating to the period covered by the wage agreement. (Bratsiotis and Martin by contrast assume unions possess no such information, while Herrendorf and Lockwood assume atomistic unions and perfectly competitive firms.) As will be seen below, this third departure from Bratsiotis and Martin is especially significant since it represents the first attempt in the literature to investigate the implications of union wage-setting

[1] Although Bratsiotis and Martin (1999) briefly analyse the simple rule and discretion, this chapter does not reproduce their results because of differences in assumptions regarding information structure and union preferences. Note also that Bratsiotis and Martin differ from this chapter by allowing for non-atomistic firms.

behaviour for economic transparency. In other words, insofar as a central bank enjoys an informational advantage over unions as regards the realised values of macroeconomic shocks, to what extent should the central bank relinquish this advantage by disclosing some or all of its information to the unions, and what are the associated implications for optimal delegation arrangements? As mentioned in Chapter I, these issues have not hitherto been investigated in the context of a unionised economy.

The chapter is organised as follows. Section III.2 sets out the structural equations common to the three monetary regime scenarios considered subsequently, and in particular discusses the adopted version of the union loss function. Section III.3 analyses the fixed money supply regime, which, following the practice of Chapter II, will for convenience be referred to as the simple rule, while Section III.4 derives the optimal state-contingent rule for the extremes of wage bargaining structure. Section III.5 is devoted to the discretionary monetary policy regime: the section initially conducts the analysis with the quality of union information assumed to be exogenous, and then proceeds to endogenise information quality in order to study the issues of optimal economic transparency and optimal regime design.

III.2 The Model

The structural equations for this chapter's model appear below as expressions (1) to (12b). These are largely familiar from Chapter II, and will only be discussed insofar as they differ from their counterparts presented previously.

$$y_i^S = \alpha l_i + \theta, \qquad 0 < \alpha < 1 \tag{1}$$

where $\theta \sim N(0, \sigma_\theta^2)$.

$$y^S = \int_0^1 y_i^S di \tag{2}$$

$$y_i^D - y^D = -\varepsilon(p_i - p), \qquad \varepsilon > 1 \tag{3}$$

where $p = \int_{i=0}^{1} p_i di$.

$$y^D = \gamma(m - p + \phi), \qquad \gamma > 0 \tag{4}$$

where $\phi \sim N(0, \sigma_\phi^2)$.

$$l_j^S = 0 \tag{5}$$

$$l_j^D = \int_{(j-1)/n}^{j/n} l_i^D di \bigg/ \int_{(j-1)/n}^{j/n} di \tag{6}$$

$$s = \theta + u \tag{7}$$

where $u \sim N(0, \sigma_u^2)$ and $E(\theta u) = 0$.

$$\Omega^s = (l - l^*)^2 + c_s(\pi - \pi^*)^2 \tag{8}$$

$$\Omega^{cb} = l^2 + c_b \pi^2 \tag{9}$$

where $l = \int_{i=0}^{1} l_i di = \frac{1}{n}\sum_{j=1}^{n} l_j$, and $\pi = p - p_{-1}$.

$$\Omega_j^u = l_j^2 + c_u(w_j - p)^2 \tag{10}$$

The individual firm's production function, (1), now features a productivity shock, θ, while the aggregate demand equation, (4), has been modified to allow for a stochastic aggregate demand or velocity shock, ϕ.[2] Both of these shocks are assumed to be normally distributed with finite variance, and the mean value of each has been normalised for convenience at zero. Note that the θ shock is assumed to be common to all firms, and furthermore that its value is known to them when making their output, pricing, and labour demand decisions. This is entirely in accordance with the

[2] The aggregate demand shock, ϕ, can alternatively be thought of as a random additive disturbance to the money supply itself.

assumptions made by Herrendorf and Lockwood (1997), as is the implicit assumption that there are no firm-specific demand shocks.

As well as introducing stochastic shocks into the model, we also follow Herrendorf and Lockwood (1997) in assuming that unions receive a common noisy signal, s, of the productivity shock.[3] The relationship between s and θ is given by equation (7), where u is the noise term which is uncorrelated with θ, so that $E(\theta u) = 0$. It follows that the variance of the signal, σ_s^2, is given by:

$$\sigma_s^2 \equiv E(s^2) = \sigma_\theta^2 + \sigma_u^2 \tag{11}$$

Initially it is assumed that σ_u^2, like σ_θ^2, is an exogenous parameter. However, in a later section of this chapter concerned with economic transparency, σ_u^2 is made a choice variable of the central bank.

It is assumed that the variances of θ and u, σ_θ^2 and σ_u^2, are public knowledge: thus σ_s^2 is also commonly known. As in Herrendorf and Lockwood, the information structure is such that each union's rational expectation of θ, conditional on s, is formed using the ordinary least-squares population regression line slope. Denoting this slope by β, we therefore have:

$$E(\theta \mid s) = \beta s \tag{12a}$$

$$\beta = \frac{\sigma_\theta^2}{(\sigma_\theta^2 + \sigma_u^2)} \tag{12b}$$

Two extreme cases of information structure are captured by (12a) and (12b). Firstly, there is the full-information case in which every union receives a perfect signal of θ and therefore is perfectly informed of the state of productivity at the time it sets its wage: this is the case in which $\sigma_u^2 = 0$, so that $\beta|_{\sigma_u^2 = 0} = 1$ and $s = \theta$, and hence

[3] Throughout the thesis the word 'signal' is used, as in Herrendorf and Lockwood (1997), to refer to the item of information received by the individual union (or, more generally, by a private-sector agent), on the basis of which that union or agent forms a forecast of the relevant aggregate variable. Note that this use of the word differs from that of the signal-extraction literature stemming from Lucas (1972), where 'signal' denotes not the item of information but rather the unobserved variable which the individual agent attempts to forecast on the basis of its available information.

$E(\theta \mid s) = \theta$. The second extreme is the limiting case in which $\sigma_u^2 \to \infty$, so that the signals are so noisy as to be worthless to unions. In this case we have $\lim_{\sigma_u^2 \to \infty} \beta = 0$, so that $E(\theta \mid s) = 0$, and each union's best forecast (i.e. its rational expectation) of θ is this shock's unconditional mean value of zero, with the signal being entirely ignored. For intermediate degrees of signal quality between these two extremes we have $0 < \sigma_u^2 < \infty$, so that $0 < \beta < 1$, with β being closer to unity the smaller is σ_u^2.

Apart from the introduction of stochastic shocks and a common signal, the only difference between the structural equations set out above and their counterparts in Chapter II is the specification of the union loss function, equation (10). The latter is a version of Chapter II's equation (30), with the employment and real wage objectives, l_u and w_u^{real}, both set equal to zero. The adopted normalisations imply that the unconditional mean market-clearing values of the real wage and employment are both zero. Hence the quadratic union loss function, (10), embodies the idea that unions regard departures of the real wage and employment from their (unconditional) expected values as inherently undesirable. In other words, this loss function specification assumes that the representative union is averse to variability in both its real wage and employment.

It is worthwhile comparing the specification of the union loss function adopted here with the specifications used by the two papers most closely related to the model of this chapter, namely Bratsiotis and Martin (1999) and Herrendorf and Lockwood (1997). In Bratsiotis and Martin (1999), the union's objective function[4] is specified to be quadratic in deviations of the real wage from a desired or target value which varies with the productivity shock (note that this target value of the real wage is not necessarily the real wage which clears the labour market, since, as pointed out in Chapter II, Bratsiotis and Martin do not restrict the union's notional real wage and employment objectives to be consistent). Bratsiotis and Martin do not provide a rationale for this specification, which amounts to assuming, somewhat implausibly, that the union regards a certain degree of variability in the real wage as intrinsically desirable. However, the fact that Bratsiotis and Martin also assume that unions are entirely ignorant of the productivity shock's value at the time the nominal wages are set, means that their assumption that each union's desired real wage is stochastic does

[4] The relevant equation is equation (10) on p.247 of Bratsiotis and Martin (1999).

not affect their reported results. The key feature of Herrendorf and Lockwood (1997), of course, is the assumption that unions do possess noisy information about the productivity shock when setting wages. Sure enough, their union objective function[5] assumes fixed values for the notional real wage and employment objectives, and differs from this chapter's specification, equation (10), only by allowing these objectives, l_u and w_u^{real} (in our notation), respectively to fall short of and exceed their mean market-clearing values. The purpose of this assumption in Herrendorf and Lockwood's model is to endogenously generate a mean inflation bias when the central bank's delegated employment target is the socially optimal value associated with labour-market clearing. Were we also to specify $l_u < 0$ and $w_u^{real} > 0$ appropriately, the model of this chapter would, when the monetary-regime scenario is that of discretion, also exhibit a mean inflation bias. This phenomenon is of little interest to us, however, since Coricelli et al. (2004a, 2006) have already investigated the issue in a non-stochastic model featuring non-atomistic unions and a monopolistically competitive goods market. Our interest lies in combining these ingredients with the stochastic framework and information structure of Herrendorf and Lockwood, and to this end it is entirely satisfactory to adopt (10) as our specification of union objectives, since none of the results reported below would change were we to specify the notional objectives in (10) to differ from the mean market-clearing values.

Having concluded our discussion of the structural equations, we now state the profit-maximising labour demand of the individual firm implied by them. We assume that firm i in working out its profit-maximising price and associated labour demand, is fully informed about the productivity shock. The firm's profit-maximisation exercise in the presence of stochastic shocks is described in Appendix II.1, and without further ado we present firm i's optimal labour demand as a function of p and the shocks, and for given settings of the money supply, m, and of the nominal wage, w_i, specified in its contract with its associated union:[6]

$$l_i^D = \frac{\gamma(m+\phi-p)-\varepsilon(w_i-p)+(\varepsilon-1)\theta}{[\alpha+\varepsilon(1-\alpha)]} \tag{13}$$

[5] Herrendorf and Lockwood (1997) equation (2), p.480.
[6] Equation (13) is equation (A.II.1.8) from Appendix II.1, appropriately renumbered.

Although the trouble has been taken to allow for the velocity shock in deriving (13), this disturbance is of little interest in the context of this chapter, since were the individual union to posses any information regarding ϕ, an appropriate adjustment of its wage in the light of this information would enable it to stabilise both its real wage and its employment perfectly in relation to this shock's anticipated component (i.e. the union's forecast of it). The nature of velocity shocks is such that they do not give rise to a trade-off, either for an individual union or for unions collectively, between stabilising the real wage and stabilising employment: adjusting the nominal wage to neutralise a velocity shock's potential impact on the real wage necessarily also neutralises its potential impact on employment. For this reason wage-setting externalities do not arise in respect of velocity shocks, or union forecasts of them, and consequently little of substance is gained for this chapter by including them in the model. We therefore simplify matters by assuming that the variance of the ϕ shock is zero, so that this shock is always equal to its unconditional mean of zero:

$$l_i^D\Big|_{\sigma_\phi^2=0} = \frac{\gamma(m-p)-\varepsilon(w_i-p)+(\varepsilon-1)\theta}{[\alpha+\varepsilon(1-\alpha)]} \tag{13'}$$

During the course of the discussion it will be useful as well to refer to the equation of the symmetric-wage labour demand curve, that is, the labour demand curve which prevails when there is symmetric wage-setting (and hence symmetric price-setting too). This equation has already been derived for the model with stochastic shocks in Appendix II.2. It is:[7]

$$l_j\Big|_{w_j=w\forall j} = \frac{-(w_j-p)+\theta}{(1-\alpha)} \tag{14}$$

Since unions generally lack perfect information about θ (the sole exception is the $\sigma_u^2=0$ special case), a distinction needs to be drawn between the symmetric-wage labour demand curve and the expectation of it conditional on s:

[7] Equation (14) is equation (A.II.2.9) from Appendix II.2. Note that since the symmetric-wage labour demand curve is a special case of the aggregate-level labour demand curve, (14) may be rewritten with l in place of l_j, and w in place of w_j.

$$E(l_j\big|_{w_j = w\forall j} \mid s) = \frac{-E[(w_j - p) \mid s] + \beta s}{(1-\alpha)} \tag{14'}$$

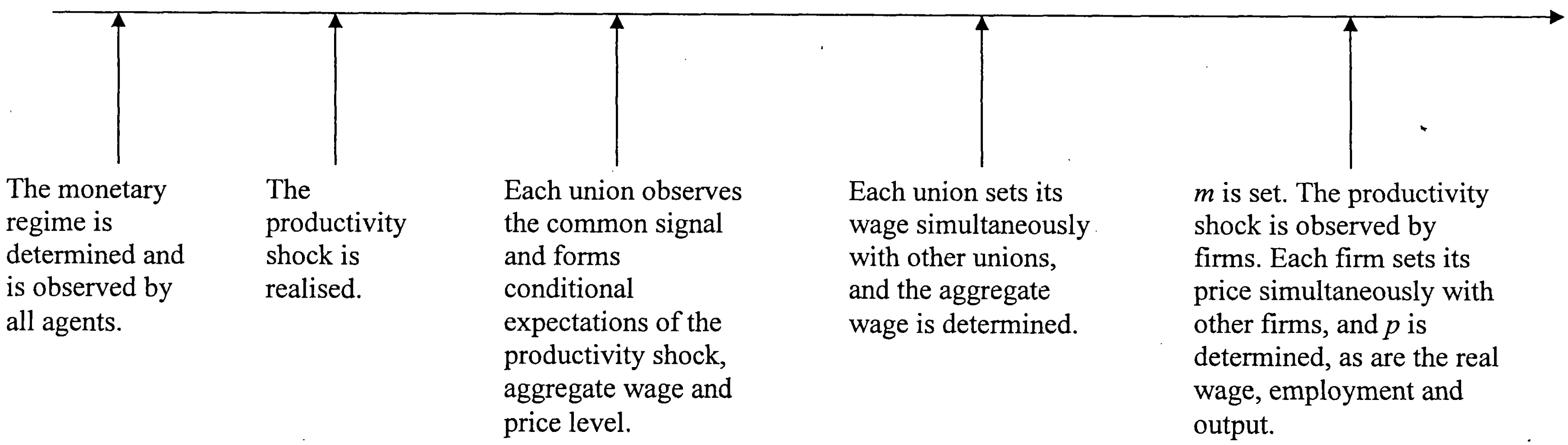
The next step in the analysis is to derive a semi-reduced form for the aggregate price level. With demand-determined employment, the substitution of l_i, as given by equation (13′), into the production function, (1), followed by aggregation across firms, and the equating of the resulting expression for aggregate supply with aggregate demand, allows p to be solved for as a function of m, $w = \int_{i=0}^{1} w_i di = \frac{1}{n}\sum_{j=1}^{n} w_j$, and θ:

$$p\big|_{\sigma_\phi^2 = 0} = \frac{\gamma(1-\alpha)m + \alpha w - \theta}{[\alpha + \gamma(1-\alpha)]} \tag{15}$$

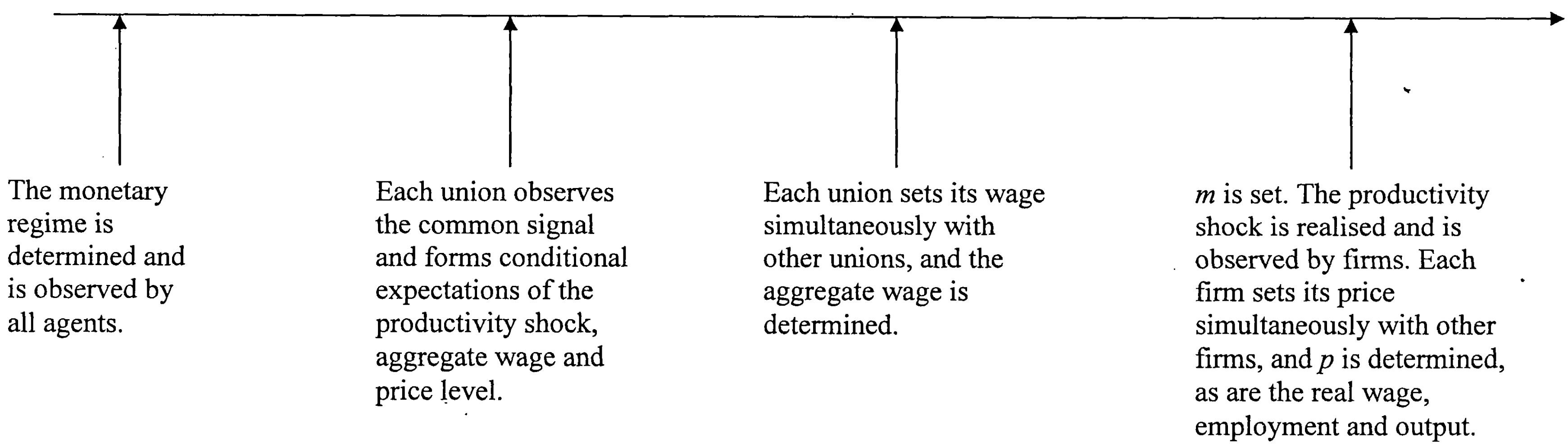
With symmetric wage-setting, $w_i = w\forall i$, every firm will have the same profit-maximising price, and a symmetric price-setting ($p_i = p\forall i$) equilibrium will result in the goods market.

In the interests of clarity, we complete our description of the model with time-line diagrams which describe two alternative scenarios, both of which are captured by our model. Although the scenarios differ as regards the timing of the realisation of θ, they are nevertheless identical as regards the model's crucial aspect, namely the structure of information regarding the productivity shock. In Time Line III.1, the realisation of θ occurs before unions observe the common signal s, whereas in Time Line III.2 it occurs after the unions' simultaneous-move wage-setting game. In fact, the realisation of θ may occur at any point in the time line, provided unions observe a noisy signal of it prior to setting their wages, and provided firms have full information about this shock when making their pricing and labour-demand decisions. Note that the time lines also assume that the money supply is set at the same time as the price level is determined. This assumption is made so as to encompass all three of the monetary regime scenarios considered below, since two of these scenarios assume the central bank has full information regarding the productivity shock and/or the aggregate wage (or price level) at the time the money supply is set. This informational assumption is irrelevant to the other monetary scenario, namely the simple rule, for which the realisation of the money supply may occur at any point in the depicted time

Time Line III.1



Time Line III.2



lines, provided unions and firms are fully informed about it at the time of making their individual moves. As in Chapter II, it is assumed that unions are always fully informed regarding the nature of the monetary regime.

III.3 Wage-Setting under the Simple Rule

III.3.1 Efficient Wage-Setting

III.3.1.(i) Derivation of the Efficient Nominal Wage

Having completed the requisite preliminary discussions, we are now in a position to study the macroeconomic repercussions of union wage-setting decisions when unions possess noisy information about the productivity shock. The first scenario we will consider is that of the simple rule according to which the money supply is kept fixed at a constant value, $\overline{m}$, regardless of the realised value of θ or of any of the aggregate variables. It is convenient to normalise $\overline{m}$ at zero, so that $m = \overline{m} \equiv 0$ can be substituted into (13′) and (15) to simplify these expressions further.[8] (Throughout the remainder of this section $m = \overline{m} \equiv 0$ is assumed, as is $\sigma_\phi^2 = 0$, and subscripts denoting these assumptions are dropped to avoid excessive notational clutter.)

Our first task must be to derive the nominal wage that is efficient from the collective viewpoint of unions. The efficient wage, w^*, is that which minimises union *j*'s expected loss, conditional on the signal, given that every union in the economy does set this particular wage. Combining (6), (10), (13′) and (15), we obtain:

$$E(\Omega_j^u \mid s) = \frac{1}{[\alpha+\gamma(1-\alpha)]^2} E\left[\left[(\gamma-1)\theta - \left\{\frac{\varepsilon[\alpha+\gamma(1-\alpha)]w_j + \alpha(\gamma-\varepsilon)w}{[\alpha+\varepsilon(1-\alpha)]}\right\}\right]^2 + c_u\{\theta + [\alpha+\gamma(1-\alpha)]w_j - \alpha w\}^2 \,\Big|\, s\right] \qquad (16)$$

The efficient wage is the solution to $dE(\Omega_j^u \mid s)\big|_{w_j = w\forall j} \big/ dw = 0$. Imposing on (16) the restriction that every union does set the same wage, i.e. $w_j = w\forall j$, obtaining the

[8] As in Chapter II, throughout this chapter the price level for the previous period is normalised at zero, $p_{-1} \equiv 0$, so that $\pi = p$ is once again the case.

derivative with respect to w of the resulting expression[9], applying the rational expectations operator (so that $E(\theta \mid s) = \beta s$), and solving the first-order condition, the efficient wage is found to be:

$$w^* = \left(\frac{1}{\gamma}\right)\frac{[\gamma - 1 - c_u(1-\alpha)]}{[1 + c_u(1-\alpha)^2]}\beta s \qquad (17)$$

III.3.1.(ii) Discussion of the Efficient Nominal Wage

A noteworthy point about the efficient wage is that the coefficient on βs is ambiguous in sign: it is not necessarily the case that the efficient response to a positive signal is to reduce the wage nominal wage. A positive signal indicates the real wage may potentially increase, as a result of the negative impact on the price level of the shock's anticipated component. While this undesirable movement in the real wage away from its unconditional mean (zero) can be dampened by a negative adjustment in the nominal wage,[10] it is nevertheless efficient in certain circumstances for the unions to respond to a positive signal by increasing the nominal wage above its unconditional mean efficient value of zero, an adjustment which exacerbates the real-wage impact of the anticipated component of the shock. The reason for this is that unions' relative aversion to employment variability may be sufficiently strong for it to be efficient for them to divert the brunt of the impact of the anticipated component on to the real wage, so that the expected impact of θ on employment is relatively mild. Sure enough, it is evident from (17) that the efficient wage response to a positive signal is positive only if the relative weight attached to real wage variability, c_u, is below a certain threshold value: specifically, $w^* > 0$ when $s > 0$ if (and only if) $c_u < (\gamma - 1)/(1-\alpha)$. (Note that since $c_u \geq 0$ this possibility can be precluded if $\gamma < 1$.)

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[9] The resulting expression will simply be (16) with every w_j replaced with w.

[10] From (15) it is evident that a negative adjustment in w will, like the positive anticipated component of the shock, cause the price level to fall. However, the effect on the expected real wage of a negative adjustment in w in response to $s > 0$ is unambiguously negative (i.e. it unambiguously dampens the anticipated component of the shock's impact on the real wage). This is readily apparent from the fact that, using (15), we have $(w-p)|_{\sigma_\phi^2 = 0} = [\gamma(1-\alpha)w + \theta]/[\alpha + \gamma(1-\alpha)]$.

The price level under efficient wage-setting is obtained by substituting (17) into (15). It is convenient to write it in terms of the union forecast, βs, and forecast error, $\theta - \beta s$, of the shock, since the covariance of these terms is zero, $E[\beta s(\theta - \beta s)] = 0$:[11]

$$p\big|_{w=w^*} = \underbrace{-\frac{[1 + c_u(1-\alpha)]\beta s}{\gamma[1 + c_u(1-\alpha)^2]}}_{E[p|_{w=w^*}|s]} - \frac{(\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{18}$$

Comparing (17) and (18), it is apparent the efficient nominal wage can be written as follows:

$$w^* = E(p\big|_{w=w^*} \mid s) + \frac{\beta s}{[1 + c_u(1-\alpha)^2]} \tag{19}$$

Using (17) and (18), the efficient real wage is:

$$(w - p)\big|_{w=w^*} = \frac{\beta s}{[1 + c_u(1-\alpha)^2]} + \frac{(\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{20a}$$

Substituting (20a) for $(w_j - p)$ in the symmetric-wage labour demand curve, (14), the efficient employment outcome is found to be:[12]

$$l\big|_{w=w^*} = \frac{c_u(1-\alpha)\beta s}{[1 + c_u(1-\alpha)^2]} + \frac{(\gamma - 1)(\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{20b}$$

It is also of interest to consider the variances of the real wage and of employment under efficient wage-setting:

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[11] Using (7) and (12b) we find that: $E[\beta s(\theta - \beta s)] = \beta E[s\theta - \beta s^2] = \beta[\sigma_\theta^2 - \beta(\sigma_\theta^2 + \sigma_u^2)]$ $= \beta(\sigma_\theta^2 - \sigma_\theta^2) = 0$, where the expectation E is unconditional.

[12] Note that l_j and l are interchangeable when considering efficient wage-setting (as also are $w_j - p$ and $w - p$), since with every union abiding by the efficient wage, individual real outcomes must be identical across all unions.

$$E((w-p)|_{w=w^*})^2 = \frac{\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]^2} + \frac{\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \tag{21a}$$

$$E(l|_{w=w^*})^2 = \frac{c_u^2(1-\alpha)^2\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]^2} + \frac{(\gamma-1)^2\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \tag{21b}$$

In these expressions E denotes the unconditional expectation, while use has also been made of the variances of the forecast, βs, and of the forecast error, $\theta - \beta s$, which are respectively given by the following expressions:[13]

$$E(\beta s)^2 = \beta^2(\sigma_\theta^2 + \sigma_u^2) = \beta\sigma_\theta^2 \tag{22a}$$

$$E(\theta - \beta s)^2 = (1-\beta)^2\sigma_\theta^2 + \beta^2\sigma_u^2 = \beta\sigma_u^2 \tag{22b}$$

Substituting (21a) and (21b) into the unconditional expectation of the union loss function, (10), yields the unconditional expected loss of each union under efficient wage-setting:

$$E(\Omega_j^u|_{w_j=w=w^*}) = \frac{c_u\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]} + \frac{[c_u+(\gamma-1)^2]\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \tag{23}$$

Expressions (20a) and (20b) decompose the realised value of the real wage and employment into a term in βs, the forecast (or anticipated component) of the shock, and a term in $\theta - \beta s$, the forecast error or unanticipated component. Under efficient wage-setting the apportionment of the anticipated component of the shock between the real wage and employment is the best that can possibly be achieved given symmetric wage-setting (i.e. $w_j = w \forall j$). Note that while the coefficients of the terms in βs in (20a) and (20b) are functions of the representative union's weight parameter c_u (which, of course, measures the union's preference for real wage stability, relative to employment stability), and of the productivity-related parameter α, they do not feature the aggregate demand elasticity γ. The reason for this is that fully coordinated

[13] Note that $E(\theta - \beta s)^2 = E[(1-\beta)\theta - \beta u]^2$.

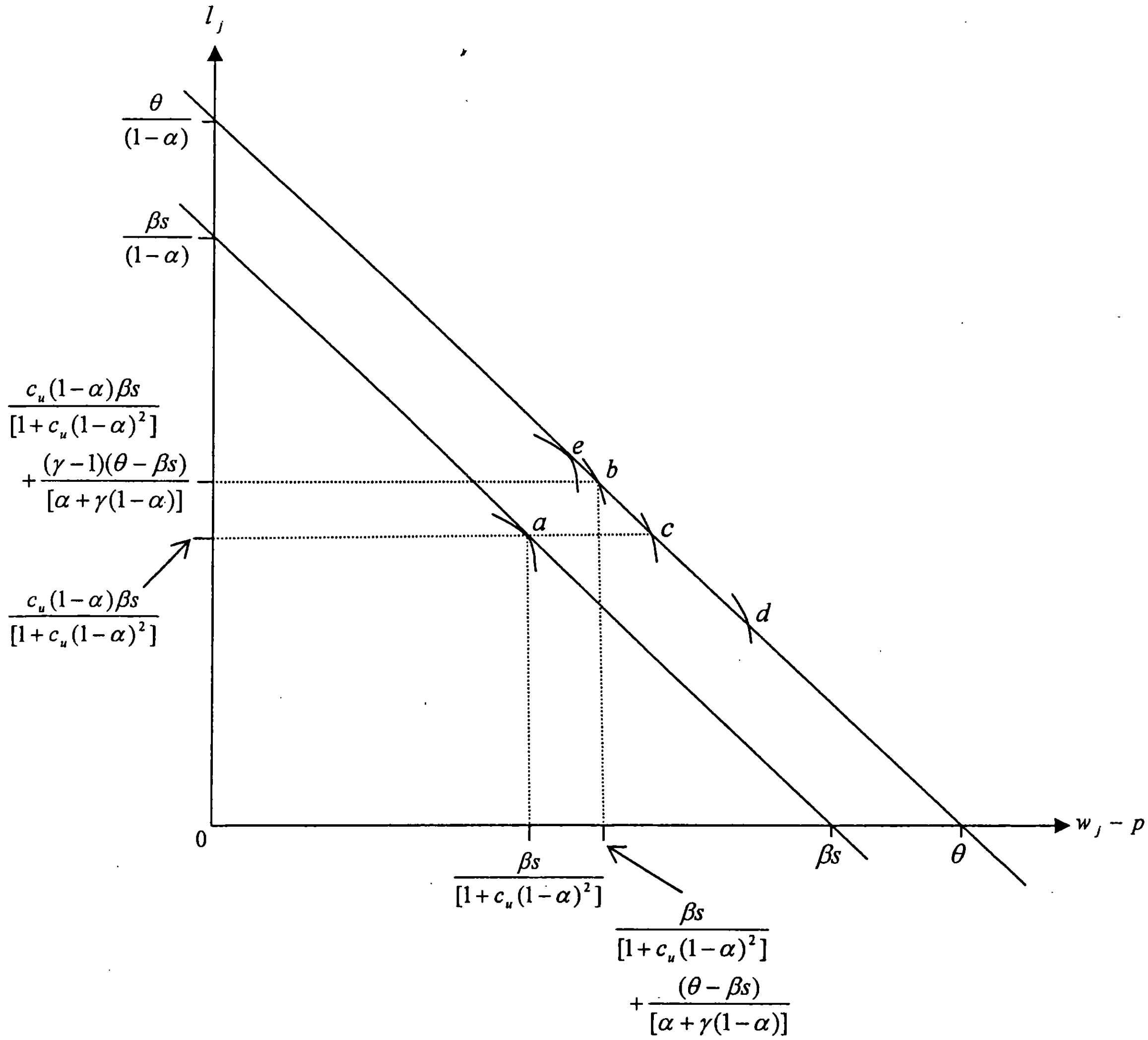
wage-setting takes full account of the price-level and aggregate-demand repercussions of the aggregate nominal wage and hence ensures that γ does not influence the impact of the shock's anticipated component on real outcomes. Since unions, by definition, possess no information about the unanticipated component of the shock at the time the efficient wage is chosen, the unions' preferences cannot influence how this component is divided between real wage and employment: hence the absence of c_u from the coefficients of the terms in $\theta - \beta s$ in (20a) and (20b). Their ignorance of $\theta - \beta s$ also means, of course, that γ influences real outcomes relating to this unanticipated component of the shock. The presence of noise in the unions' signal of θ (i.e. the fact that they are not perfectly informed about it) generally prevents efficient wage-setting from being able to bring about the ideal outcome, namely the apportionment of the entire shock, θ, between the real wage and employment in accordance with the preferences of unions. This ideal outcome is a real wage of $\theta/[1+c_u(1-\alpha)^2]$, together with employment of $c_u(1-\alpha)\theta/[1+c_u(1-\alpha)^2]$. It is readily apparent from (20a) and (20b) that under perfect information regarding θ (the $\sigma_u^2 = 0$ case, so that $\beta = 1$ and $s = \theta$ at all times) this ideal outcome is achieved by the efficient nominal wage. If information is imperfect ($\sigma_u^2 > 0$), the ideal outcome only ever arises in any particular period if the forecast happens by pure chance to be completely accurate (i.e. if $\beta s = \theta$). (Of course, when $\sigma_u^2 > 0$ is the case a lucky circumstance of this kind cannot occur systematically.) As scrutiny of (20a) and (20b) makes clear, in general when signals are noisy and wage-setting is efficient, it is only the anticipated component of the shock that is ideally apportioned between the real wage and employment. The reason for the qualifying phrase 'in general' in this explanation is that there is one special case of union preferences (a particular value which the weight c_u may take) which results in the ideal distribution of the shock itself, rather than just its anticipated component, between the real wage and employment, and this is so irrespective of the quality of unions' information (i.e. the amount of signal noise). This special case is that in which c_u has the value $\tilde{c}_u \equiv (\gamma - 1)/(1-\alpha)$.

Rather more will be said about this special case in a little while. For the moment we abstract from this case by assuming $c_u \neq \tilde{c}_u$, since we need first of all to discuss another aspect of (20a) and (20b), namely the fact that γ plays an important role in determining how the impact of the unanticipated component of the shock is divided between the real wage and employment. Note that whereas a positive forecast error regarding the shock always increases the real wage, irrespective of γ, the sign of its impact on employment is ambiguous and depends on γ. The reader at this point is referred to Figure III.1, which depicts the case in which unions set the wage efficiently following a positive signal, $s > 0$, and the forecast error turns out to be positive, $\theta - \beta s > 0$. The unions' expectation of the aggregate-level labour demand curve, conditional on the signal, is in this case above and to the right of the origin, but below and to the left of the aggregate-level labour demand curve itself. Efficient wage-setting ensures the real wage and employment outcome combination relating to the anticipated component of the shock is given by the coordinates of the tangency point (labelled a in Figure III.1) between the unions' expectation, conditional on s, of the aggregate-level labour demand curve and the isoloss map, the bliss point of which is located at the origin. For a non-zero realisation of the forecast error, of course, the aggregate-level labour demand curve differs from unions' conditional expectation of it, and the actual outcome (the diagram coordinates of which will be given by equations (20a) and (20b)) in general does not occur at a tangency point between the isoloss map and the aggregate-level labour demand curve. If $\gamma > 1$ the forecast error has a positive impact on both real wage and employment: the outcome may be at the tangency point *e*, but in general is not and instead is at a point such as b. If $\gamma = 1$ the outcome is at *c*, with the real wage solely bearing the impact of the forecast error and the employment outcome under efficient wage-setting equal to the value unions intended collectively to bring about. Finally, if $\gamma < 1$ a positive forecast error has a negative impact on employment, and the real wage increases by more than this unanticipated component of the shock[14], with the outcome being lower down the aggregate-level labour demand curve at a point such as *d*.

Thus the strength of the impact of the unions' forecast error on the real wage relative to its impact on employment depends on γ rather than on c_u. The irrelevance

[14] As is apparent from (20a), whether a positive forecast error regarding θ has a full, less-than-full, or more-than-full impact on the real wage depends on how its coefficient, $[\alpha + \gamma(1-\alpha)]^{-1}$, compares with unity. Note that $[\alpha + \gamma(1-\alpha)]^{-1} > (<) 1$ when $\gamma < (>) 1$, and that $[\alpha + \gamma(1-\alpha)]^{-1}\big|_{\gamma=1} = 1$.

Figure III.1
Actual and expected outcomes from efficient wage-setting



Notes:

1. The diagram ignores velocity shocks, and assumes for illustrative purposes that $\theta > \beta s > 0$.

2. The upper of the two diagonal lines is the symmetric-wage and aggregate-level labour demand curve. The lower diagonal line is the expectation, conditional on *s*, of the symmetric-wage labour demand curve.

3. The expected outcome conditional on *s* is at *a*, while the actual efficient outcome is at *b*. It is therefore implicitly assumed that $\gamma > 1$ and that $0 < c_u < (\gamma - 1)/(1-\alpha)$.

of the unions' weight parameter in this context ought not to surprise since unions are uninformed about $\theta - \beta s$ when setting the efficient wage and therefore cannot influence its impact on real outcomes. While the slope and position of the aggregate-level labour demand curve (and of unions' conditional expectation of this curve) are independent of γ, γ plays a key role in determining the location of the outcome on the aggregate-level labour demand curve whenever there is a non-zero forecast error. (Similarly, had we allowed for velocity shocks, and union forecasts thereof, these would be found to have no effect on the slope and position of the curves depicted in Figure III.1, and would only affect the position of the outcome on the aggregate-level labour demand curve if the unions' forecast error regarding the velocity shock happened to be non-zero.) The reason why γ exerts a major influence over the real impact of forecast errors is because it is central to the profit-maximising pricing decision of the individual firm once the nominal wage and the state of productivity have been realised. Insight into this can be gained by considering the following version of the semi-reduced form for the price level, obtained by setting $m = 0$ in (15) and decomposing θ into forecast and forecast error:

$$p\big|_{m=0} = \frac{\alpha w - \beta s - (\theta - \beta s)}{\alpha + \gamma(1-\alpha)} \qquad (15')$$

This equation reveals that when $\gamma < 1$, firm pricing behaviour is such that the price level falls by more than the value of the positive forecast error itself.[15] (In other words, when $\gamma < 1$ the price level is relatively elastic with respect to the forecast error.) Since the nominal wage does not respond at all to the forecast error, it follows that when $\gamma < 1$ the real wage must also be relatively elastic with respect to it. In the $\gamma = 1$ case the price level, and hence the real wage also, has unit elasticity with respect to the forecast error, while when $\gamma > 1$ the price level falls, and the real wage increases, by less than the full extent of a positive error. The associated impact of the error on employment involves both a direct effect, and an indirect effect working via the induced change in the real wage, as is evident from the following version of (14), the equation of the symmetric-wage labour demand curve:

[15] Note that we cannot make a similar inference from $(15')$ regarding the impact of the forecast itself (βs) on *p*, since *w* is a function of βs.

$$l_j\big|_{w_j = w\forall j} = \frac{-(w_j - p) + \beta s + (\theta - \beta s)}{(1-\alpha)} \tag{14''}$$

The impact of $\theta - \beta s$ on employment is therefore:[16]

$$\frac{dl_j}{d(\theta - \beta s)} = \frac{\partial l_j}{\partial(\theta - \beta s)} + \frac{\partial l_j}{\partial(w_j - p)}\frac{\partial(w_j - p)}{\partial(\theta - \beta s)} \tag{24}$$

where $\frac{\partial(w_j - p)}{\partial(\theta - \beta s)} = \frac{1}{\alpha + \gamma(1-\alpha)}$, and hence:

$$\frac{dl_j}{d(\theta - \beta s)} = \frac{(\gamma - 1)}{[\alpha + \gamma(1-\alpha)]} \tag{24'}$$

This last equation reveals that when $\gamma = 1$ and the forecast error is positive, the fact that the real wage increases by the full amount of the error means that employment is fully insulated from its impact, while if $\gamma < (>) 1$ the fact that the real wage increases by more (less) than the error implies that some of the impact of this unanticipated component of the shock must also be transmitted to employment.

One further aspect of Figure III.1 remains to be discussed, namely the tangency point at e between the representative union's isoloss map and the aggregate-level labour demand curve. The existence of such a tangency point implies that there is some γ value which results in the movement of the price level in response to the unanticipated component of the shock being exactly what is required to bring about the ideal distribution of the impact of this component (the forecast error) between the real wage and employment. It is clear from Figure III.1 that since point e is further up the aggregate-level labour demand curve than point c, it must involve a γ value greater than unity. This is indeed the case, since the γ value which ensures efficient wage-setting always results in the outcome being at a tangency point is $\gamma = 1 + c_u(1-\alpha)$. This equation can be written as $c_u = (\gamma - 1)/(1-\alpha)$, or as $c_u = \tilde{c}_u$, the

[16] To reduce the notational burden at this point, the '$w_j = w\forall j$' subscript is temporarily omitted.

special case mentioned earlier. To see why this γ value is of particular interest, note that equations (20a) and (20b) in this special case are as follows:

$$(w-p)\Big|_{w=w^*,\, c_u=(\gamma-1)/(1-\alpha)} = \frac{\theta}{[\alpha+\gamma(1-\alpha)]} \tag{25a}$$

$$l\Big|_{w=w^*,\, c_u=(\gamma-1)/(1-\alpha)} = \frac{(\gamma-1)\theta}{[\alpha+\gamma(1-\alpha)]} \tag{25b}$$

The coefficient of the term in βs in (20a), $1/[1+c_u(1-\alpha)^2]$, and the corresponding coefficient in (20b), $c_u(1-\alpha)/[1+c_u(1-\alpha)^2]$, comprise the optimal apportionment of the productivity shock's impact between the real wage and employment. In general, the coefficient on βs differs from that on $\theta-\beta s$ in (20a), and a similar point can be made about the two coefficients in (20b). In other words, efficient wage-setting can in general only achieve the optimal allocation of the anticipated component of the shock, and in general fails to allocate optimally the unanticipated component. However, when $\gamma=1+c_u(1-\alpha)$, the two coefficients in (20a) take the same value, implying that the adjustment in the price level occasioned by the unanticipated component of the shock is precisely what is required to bring about the most desirable adjustment in the real wage (and hence in employment too) in relation to that unanticipated component. This is a significant finding, since it implies that the signal, regardless of its quality, is of no value to unions in this special case. When $\gamma=1+c_u(1-\alpha)$, unions can achieve collective efficiency in wage-setting, irrespective of the magnitude of their forecast error, by entirely disregarding the signal and simply setting the nominal wage equal to its unconditional mean efficient value of zero, since the movement in the price level occasioned by the realisation of the shock then exactly suffices to distribute the entire shock's impact between the real wage and employment in the most desirable way from the unions' viewpoint. Sure enough, equation (17) for the efficient wage confirms this intuition:

$$w^*\Big|_{\gamma=1+c_u(1-\alpha)} = 0 \tag{26}$$

We conclude our discussion of the outcomes from efficient wage-setting under the simple rule by briefly commenting upon the unconditional expectation of each union's loss as given by equation (23). The first noteworthy point about this expression concerns the extreme $c_u = 0$ case of union relative preferences, which corresponds to the representative union being indifferent to real wage variability. For this extreme case we find that the component of the union's expected loss relating to the anticipated component of θ shocks is zero:

$$E(\Omega_j^u\big|_{w=w^*,\, c_u=0}) = \frac{(\gamma-1)^2\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \tag{27}$$

The reason for this is straightforward: unions can always, by means of an appropriate adjustment of the nominal wage, fully insulate either one of the two of real wage and employment from the anticipated component of the shock.[17] However, since productivity shocks must have an effect on either the real wage or employment, it is impossible for efficient wage-setting to prevent the anticipated component of shocks having an impact on union welfare when the union loss function features quadratic terms in both the real wage and employment (i.e. when c_u is positive and finite).

The second aspect of (23) which calls for comment is the nature of the functional dependence of its two component terms on the variance of the signal noise term, which is an obvious measure of the quality of unions' information about the θ shock. It is readily apparent from the following derivatives that whereas a marginal increase in σ_u^2 (i.e. a marginal deterioration in signal quality) has a beneficial (i.e. negative) impact on the term in the variance of the unions' forecasts in (23), namely $E(\beta^2 s^2) = \beta\sigma_\theta^2$, this is counterbalanced by its positive and therefore

[17] This reasoning suggests that, under efficient wage-setting, the part of the expected union loss relating to the anticipated component of θ shocks ought also to be zero in the converse extreme case in which each union's sole concern is to achieve real wage stability. This corresponds to the limiting $c_u \to \infty$ case of our model, and yet using (23) we find that $\lim_{c_u\to\infty} E(\Omega_j^u\big|_{w=w^*}) = \infty$. This result is solely a consequence of the way the loss function is formulated, however, and in particular arises because c_u is a *relative* weight parameter. Were we alternatively to specify the loss function as $\Omega_j^u = c_u l_j^2 + (w_j - p)^2$, then it would be found that $\lim_{c_u\to\infty} E(\Omega_j^u\big|_{w_j=w^*}) = \beta\sigma_u^2/[\alpha+\gamma(1-\alpha)]^2$. The fact that this expression lacks a term in $\beta\sigma_\theta^2$, confirms the intuition that, under efficient wage-setting, the anticipated component of a θ shock has a zero impact on the union loss when unions are indifferent to variability of either employment or the real wage.

detrimental impact on the term in the variance of the unions' forecast errors, $E[(\theta-\beta s)^2]=E[(1-\beta)^2\theta^2+\beta^2u^2]=(1-\beta)^2\sigma_\theta^2+\beta^2\sigma_u^2$.

$$\frac{\partial E(\beta^2 s^2)}{\partial\sigma_u^2}=\left(\frac{\partial\beta}{\partial\sigma_u^2}\right)\sigma_\theta^2=-\beta^2 , \tag{28a}$$

$$\frac{dE[(\theta-\beta s)^2]}{d\sigma_u^2}=\underbrace{\left.\frac{\partial E[(\theta-\beta s)^2]}{\partial\sigma_u^2}\right|_{\beta\text{ fixed}}}_{\text{Direct Effect}}+\underbrace{\frac{\partial E[(\theta-\beta s)^2]}{\partial\beta}\left(\frac{\partial\beta}{\partial\sigma_u^2}\right)}_{\text{Indirect Effect}}=\beta^2 \tag{28b}$$

Note that in (28b) the indirect effect of a marginal increase in σ_u^2 on the forecast-error variance, i.e. the effect working via the induced change in β, is zero:

$$\frac{\partial E[(\theta-\beta s)^2]}{\partial\beta}=-2(1-\beta)\sigma_\theta^2+2\beta\sigma_u^2=0 \tag{28c}$$

The reason for this is the straightforward one that the population regression line slope, β, is by definition the line of best fit through the scatterplot of points in (θ,s) space, and hence must be the best coefficient which can possibly be chosen for the forecasting equation $\theta^e\,|\,s=Coefficient\times s$, where $\theta^e\,|\,s$ denotes the expectation conditional on s.[18] In other words, considering $E[(\theta-(\theta^e\,|\,s))^2]$, where the expectation E is unconditional, as a function of the forecasting equation coefficient, this forecast-error variance is (globally) minimised when the coefficient is set equal to β. Since the first-order condition for the optimal coefficient choice is satisfied, a marginal change in σ_u^2 which induces a change in that minimand (i.e. β) necessarily has a zero impact on the minimised variable, the forecast-error variance. The indirect effect given by (28c) is therefore zero, and with the direct effect in (28b) positive, a marginal increase in the signal noise variance necessarily worsens the variance of forecast errors.

[18] $\theta^e\,|\,s$ is used here rather than $E(\theta\,|\,s)$ since the latter is the mathematical conditional expectation which exploits all available information.

Using (28a) and (28b) in conjunction with our expression for the representative union's expected loss under efficient wage-setting, (23), we find that with only a single exception, the beneficial effect of a marginal increase in σ_u^2 on the term in (23) in the variance of the forecasts is always outweighed by its detrimental effect on the term in the variance of the forecast errors:

$$\frac{dE(\Omega_j^u|_{w_j=w=w^*})}{d\sigma_u^2}=\left\{\frac{-c_u}{[1+c_u(1-\alpha)^2]}+\frac{[c_u+(\gamma-1)^2]}{[\alpha+\gamma(1-\alpha)]^2}\right\}\beta^2 \tag{29}$$

It is easily shown that the right-hand side of (29) is positive for all values of c_u, other than $c_u=\tilde{c}_u$.[19] The latter is the special case in which signals can be disregarded by unions engaged in efficient wage-setting, since price level movements associated with forecast errors regarding θ bring about the most desirable outcome from unions' collective viewpoint. Since the signals are of no intrinsic value in this special case, it is hardly surprising that it is the only one for which the marginal effect of a deterioration in signal quality on unions' expected loss under efficient wage-setting is zero rather than positive.

III.3.2 Equilibrium Wage-Setting under the Simple Rule

III.3.2.(i) Derivation of the Equilibrium Nominal Wage

We now turn to the derivation of the equilibrium nominal wage under the simple monetary rule. This involves each union setting its individually optimal nominal wage, conditional on the common signal, *s*, with the wage decisions of other unions taken as given. The relevant expression for the individual union's conditional expected loss is once again given by equation (16). The total derivative of (16) with respect to w_j takes account of the fact that for a non-atomistic union, $\partial w/\partial w_j=1/n$, so that $dE(\Omega_j^u\,|\,s)/dw_j=\partial E(\Omega_j^u\,|\,s)/\partial w_j+(1/n)(\partial E(\Omega_j^u\,|\,s)/\partial w)=0$ is union *j*'s first-

[19] The necessary and sufficient condition for the right-hand side of (29) to be positive is that $[c_u+(\gamma-1)^2][1+c_u(1-\alpha)^2]>c_u[\alpha+\gamma(1-\alpha)]^2$, or $f(c_u)>0$, where $f(c_u)\equiv(1-\alpha)^2c_u^2-2(1-\alpha)(\gamma-1)c_u+(\gamma-1)^2$. Since $\partial^2 f(c_u)/\partial c_u^2>0$, and since the equation $f(c_u)=0$ has a single repeated root (i.e. a unique solution) at $c_u=\tilde{c}_u\equiv(\gamma-1)/(1-\alpha)$, it follows that $f(c_u)>0\,\forall c_u\neq\tilde{c}_u$, and that $f(c_u)$ evaluated at $c_u=\tilde{c}_u$ is zero.

order condition for its individually optimal wage, w_j^{**}. Obtaining this total derivative, imposing the symmetric Nash equilibrium condition that $w_j^{**} = E(w \mid s) = w_{NE} \forall j$, and solving for the resulting symmetric Nash equilibrium wage, yields:

$$w_{NE} = \left(\frac{1}{\gamma}\right)\left\{\frac{(\gamma-1)(\gamma\alpha+\varepsilon\Lambda)-c_u[\alpha+\varepsilon(1-\alpha)]\Lambda}{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda}\right\}\beta s \tag{30}$$

where $\Lambda \equiv (n-1)\alpha + n\gamma(1-\alpha)$.

Substituting (30) into (15) yields the price level under equilibrium wage-setting:

$$p\big|_{w=w_{NE}} = \underbrace{-\left(\frac{1}{\gamma}\right)\left\{\frac{\gamma\alpha+\varepsilon\Lambda+c_u[\alpha+\varepsilon(1-\alpha)]\Lambda}{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda}\right\}\beta s}_{E(p|_{w=w_{NE}}|s)} - \frac{(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{31}$$

Comparing (30) and (31), it is evident that the equilibrium nominal wage can be written as follows:

$$w_{NE} = E(p\big|_{w=w_{NE}} \mid s) + \frac{(\gamma\alpha+\varepsilon\Lambda)\beta s}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}} \tag{32}$$

The equilibrium nominal wages for the extremes of wage-bargaining structure are as follows:

$$w_{NE}\big|_{n=1} = \left(\frac{1}{\gamma}\right)\frac{[\gamma-1-c_u(1-\alpha)]}{[1+c_u(1-\alpha)^2]}\beta s \tag{33}$$

$$\lim_{n\to\infty} w_{NE} = \left(\frac{1}{\gamma}\right)\left\{\frac{\varepsilon(\gamma-1)-c_u[\alpha+\varepsilon(1-\alpha)]}{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]}\right\}\beta s \tag{34}$$

By taking the limit of (30) as $\varepsilon \to \infty$, the equilibrium nominal wage when the goods market is perfectly competitive is found to be:

$$\lim_{\varepsilon\to\infty} w_{NE} = \left(\frac{1}{\gamma}\right)\frac{[\gamma-1-c_u(1-\alpha)]}{[1+c_u(1-\alpha)^2]}\beta s \tag{35}$$

The effects of marginal increases in n and ε on w_{NE} are given by:[20]

$$\frac{\partial w_{NE}}{\partial n} = \frac{-c_u\alpha[\alpha+\gamma(1-\alpha)]^2[\alpha+\varepsilon(1-\alpha)]\beta s}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} < 0 \text{ when } s>0 \tag{36}$$

$$\frac{\partial w_{NE}}{\partial \varepsilon} = \frac{(n-1)c_u\alpha[\alpha+\gamma(1-\alpha)]^2\Lambda\beta s}{\gamma\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} > 0\,\forall n>1 \text{ when } s>0 \tag{37}$$

Using (30) and (31) together with (14), the real wage and employment under equilibrium wage-setting are found to be:

$$(w-p)\big|_{w=w_{NE}} = \frac{(\gamma\alpha+\varepsilon\Lambda)\beta s}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}} + \frac{(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{38a}$$

$$l\big|_{w=w_{NE}} = \frac{c_u[\alpha+\varepsilon(1-\alpha)]\Lambda\beta s}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}} + \frac{(\gamma-1)(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{38b}$$

If there is a single union, and/or the goods market is perfectly competitive, real outcomes are as follows:

$$(w-p)\big|_{w=w_{NE},n=1} = \lim_{\varepsilon\to\infty}(w-p)\big|_{w=w_{NE}} = \frac{\beta s}{[1+c_u(1-\alpha)^2]} + \frac{(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{39a}$$

$$l\big|_{w=w_{NE},n=1} = \lim_{\varepsilon\to\infty} l\big|_{w=w_{NE}} = \frac{c_u(1-\alpha)\beta s}{[1+c_u(1-\alpha)^2]} + \frac{(\gamma-1)(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{39b}$$

For the case of atomistic unions, the corresponding pair of expressions is:

[20] In stating certain expressions which occur in this section to be positive or negative, it is generally assumed that $c_u > 0$.

$$\lim_{n\to\infty}(w-p)\Big|_{w=w_{NE}} = \frac{\varepsilon\beta s}{\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} + \frac{(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{40a}$$

$$\lim_{n\to\infty} l\Big|_{w=w_{NE}} = \frac{c_u[\alpha+\varepsilon(1-\alpha)]\beta s}{\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} + \frac{(\gamma-1)(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{40b}$$

The variances of the real wage and employment are obtained by squaring and taking the unconditional expectations of (38a) and (38b), and making use in so doing of results (22a) and (22b):

$$E[(w-p)\Big|_{w=w_{NE}}]^2 = \frac{(\gamma\alpha+\varepsilon\Lambda)^2\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} + \frac{\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \tag{41a}$$

$$E(l\Big|_{w=w_{NE}})^2 = \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Lambda^2\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} + \frac{(\gamma-1)^2\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \tag{41b}$$

Differentiating (41a) and (41b) with respect to n, we find that these derivatives can be unambiguously signed:

$$\frac{\partial E[(w-p)\Big|_{w=w_{NE}}]^2}{\partial n} = \frac{-2c_u\gamma\alpha(1-\alpha)[\alpha+\gamma(1-\alpha)][\alpha+\varepsilon(1-\alpha)](\gamma\alpha+\varepsilon\Lambda)\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^3} < 0 \tag{42a}$$

$$\frac{\partial E(l\Big|_{w=w_{NE}})^2}{\partial n} = \frac{2c_u^2\gamma\alpha[\alpha+\gamma(1-\alpha)][\alpha+\varepsilon(1-\alpha)]^2\Lambda\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^3} > 0 \tag{42b}$$

The derivatives of (41a) and (41b) with respect to ε are unambiguous in sign for all $n > 1$, but are zero for $n = 1$:

$$\frac{\partial E[w-p)\Big|_{w=w_{NE}}]^2}{\partial \varepsilon} = \frac{2(n-1)c_u\alpha(1-\alpha)[\alpha+\gamma(1-\alpha)](\gamma\alpha+\varepsilon\Lambda)\Lambda\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^3} > 0\,\forall\, n > 1 \tag{43a}$$

$$\frac{\partial E(l|_{w=w_{NE}})^2}{\partial \varepsilon} = \frac{-2(n-1)c_u^2\alpha[\alpha+\gamma(1-\alpha)][\alpha+\varepsilon(1-\alpha)]\Lambda^2\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^3} < 0 \forall n > 1 \quad (43b)$$

Before embarking on a discussion of these numerous expressions, it is useful to state as well the representative union's unconditional expected loss under equilibrium wage-setting. This is obtained by combining (41a) and (41b) with the unconditional expectation of (10):

$$E(\Omega_j^u|_{w=w_{NE}}) = \frac{c_u\{(\gamma\alpha+\varepsilon\Lambda)^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\Lambda^2\}\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} + \frac{[c_u+(\gamma-1)^2]\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \quad (44)$$

This expected loss is found to have the same value when there is a single union as when the economy has multiple unions and a perfectly competitive goods market:

$$E(\Omega_j^u|_{w=w_{NE}})\Big|_{n=1} = \lim_{\varepsilon\to\infty} E(\Omega_j^u|_{w=w_{NE}}) = \frac{c_u\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]} + \frac{[c_u+(\gamma-1)^2]\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \quad (45)$$

The expected loss of atomistic unions is:

$$\lim_{n\to\infty} E(\Omega_j^u|_{w=w_{NE}}) = \frac{c_u\{\varepsilon^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\}\beta\sigma_\theta^2}{\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} + \frac{[c_u+(\gamma-1)^2]\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \quad (46)$$

The derivatives of (44) with respect to n and ε are as follows:

$$\frac{\partial E(\Omega_j^u|_{w=w_{NE}})}{\partial n} = \frac{2(n-1)c_u^2\gamma\alpha^2[\alpha+\gamma(1-\alpha)]^2[\alpha+\varepsilon(1-\alpha)]\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^3} > 0 \forall n > 1 \quad (47)$$

$$\frac{\partial E(\Omega_j^u|_{w=w_{NE}})}{\partial \varepsilon} = \frac{-2(n-1)^2c_u^2\alpha^2[\alpha+\gamma(1-\alpha)]^2\Lambda\beta\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^3} < 0 \forall n > 1 \quad (48)$$

III.3.2.(ii) Discussion of the Wage-Setting Externality

Expressions (30) to (48) reveal that when there are multiple unions and the goods market is monopolistically competitive, the wage-setting decision of the individual union is characterised by an adverse externality which results in the departure of the symmetric Nash equilibrium nominal wage from its efficient value.[21] Comparing (30), (33) and (35) with (17), we find that save when there is a single union and/or perfect goods-market competition, the equilibrium nominal wage differs from the efficient wage. To be a little more specific: if firms are not perfect competitors then in equilibrium the wage set by unions in reaction to a positive (negative) signal is always less (more) than the efficient wage, with the latter only being set if there is a single economy-wide union.[22] As mentioned earlier, whether the efficient nominal wage response to a positive signal is positive or negative is ambiguous and depends on whether the union's weight c_u is less than or greater than a critical value, namely $\tilde{c}_u$. Despite the ambiguity regarding the sign of the efficient nominal wage, there is no ambiguity as regards the implications for the real wage of the equilibrium nominal wage response to a positive signal being numerically lower than the efficient response: the increase in the real wage associated with the shock's anticipated component must be unambiguously smaller as a result of the inefficiency in wage-setting.[23] A parallel argument exists for the case of a negative signal: the efficient nominal wage response is ambiguous in sign, but the externality always results in the equilibrium nominal wage being numerically too high, with the consequence that in relation to the shock's anticipated component the equilibrium real wage does not decrease by as much as efficiency requires. In summary, the consequence of each union setting a lower-than-efficient (higher-than-efficient) wage in response to a positive (negative) signal is, in absolute terms, a smaller-than-efficient movement in the real wage, together with a larger-than-efficient movement in employment, in relation to the shock's anticipated component. The inefficiently low absolute magnitude of movements in the real wage in response to (non-zero) signals depresses

[21] Note that the presence of γ in the coefficients on βs in (38a) and (38b) is an immediate indication of inefficiency.

[22] That $w_{NE} < w^*$ is the case when $s > 0$ (provided $n > 1$), and that $w_{NE} > w^*$ when $s < 0$ (again provided $n > 1$), follows straightforwardly from manipulation of this $w_{NE} < w^*$ inequality, where the appropriate expressions are given by (30) and (17).

[23] This follows immediately from the fact that under the simple rule and in the absence of velocity shocks, $(w - p) = [\gamma(1-\alpha)w + \theta]/[\alpha + \gamma(1-\alpha)]$.

the variance of the real wage, as given by equation (41a), below its efficient value. Concomitantly, the absolute magnitude of the movements in employment caused by equilibrium wage responses to non-zero signals is inefficiently high, and this shows up as inefficiently high employment variability, as described by equation (41b).

The source of the adverse externality is the individual union's failure to take full account of the price-level repercussions of its wage decision. If the signal happens to be non-zero, there arises a trade-off between stabilising the impact of the resulting non-zero anticipated component of the shock on the real wage and stabilising its impact on employment. Unless the union is the sole supplier of labour services in the economy, the trade-off faced by the individual union differs from that which exists at the aggregate level, and depends on the influence its wage has on the price level and thus, indirectly via real money balances and aggregate demand, on the labour demand of its employer firms, as given by equation (13). As in Chapter II, therefore, the impact of union j's nominal wage on its employment outcome can be decomposed into a real wage channel and an aggregate demand channel. The real wage channel is the ultimate source of the externality: under the simple rule the aggregate demand channel is negative and operates to reduce the externality's severity.

Intuitive explanation of the externality is facilitated by couching the discussion in terms of the individual union's optimal wage decision, given that every other union sets its wage equal to the efficient wage. (Note, however, that efficient wage-setting by other unions is not in any way essential to the engendering of the externality, which merely requires that each union takes the wage decisions of other unions as given when setting its wage.) For concreteness the discussion also focuses on the case of a positive signal: needless to say, a parallel argument exists for the case of a negative signal. The key point is that if every other union sets its wage in response to $s > 0$ equal to w^* (as given by (17)), union j can secure, in relation to the positive anticipated component of the shock, a smaller deviation of its real wage from the mean real wage outcome of zero, by setting a lower wage than w^*. This necessarily entails a larger deviation of union j's employment from its mean of zero, but this exacerbation of the impact of the shock's anticipated component on employment is worthwhile incurring in return for the dampening of its impact on union j's real wage.

It is helpful at this point to refer to a concept which will be familiar from Chapter II, namely the individual union's perceived labour demand curve (as distinct from the symmetric-wage labour demand curve, itself a special case of the aggregate-level labour demand curve). As in Chapter II, there is an infinite set of possible perceived labour demand curves, with one such perceived curve for every wage on which the $(n-1)$ unions other than union j could coordinate their wage setting. Two members of this set are of particular interest, however, namely the perceived labour demand curve which prevails when every other union apart from union j sets the efficient wage, and its counterpart for the case in which every other union sets the equilibrium wage. Since each perceived labour demand curve's slope is entirely independent of both the signal and the realised value of the shock, and furthermore does not depend in any way on the specification of the union's loss function, it must be the same, for a given monetary regime, as in the non-stochastic scenario of Chapter II. In fact, under the simple rule the equations of the two perceived labour demand curves which are of interest differ from the equations of their Chapter II counterparts only in respect of their intercepts. The perceived labour demand curve which prevails were every other union to set the efficient wage (and which therefore passes through the efficient outcome) is stated below as equation (49a): its derivation for the version of the model with stochastic shocks and the union loss function of this chapter is to be found in Appendix III.1, as is the derivation of union j's equilibrium perceived labour demand curve (i.e. that which prevails when every other union sets the equilibrium wage), the equation for which appears below as (49b):

$$E(l_j\big|_{w_k=w^*\forall k\neq j}\mid s)=\frac{1}{[\alpha+\varepsilon(1-\alpha)]\Lambda}\Bigg[\frac{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}\beta s}{[1+c_u(1-\alpha)^2]}$$

$$-(\gamma\alpha+\varepsilon\Lambda)E[(w_j-p)\big|_{w_k=w^*\forall k\neq j}\mid s]\Bigg] \tag{49a}$$

$$E(l_j\big|_{w_k=w_{NE}\forall k\neq j}\mid s)=\frac{1}{[\alpha+\varepsilon(1-\alpha)]\Lambda}\Bigg[\frac{\{(\gamma\alpha+\varepsilon\Lambda)^2+c_u[\alpha+\varepsilon(1-\alpha)]^2\Lambda^2\}\beta s}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}}$$

$$-(\gamma\alpha+\varepsilon\Lambda)E[(w_j-p)\big|_{w_k=w_{NE}\forall k\neq j}\mid s]\Bigg] \tag{49b}$$

These perceived labour demand curves for the $s > 0$ case, together with the symmetric-wage labour demand curve[24], and its expectation (conditional on s), are depicted below as Figure III.2. The actual symmetric-wage labour demand curve is located to the right of its conditional expectation: the diagram consequently implicitly assumes a positive realisation of the forecast error $\theta - \beta s > 0$. The (expected) efficient outcome is at point *a*, and in the absence of a forecast error (i.e. if $\beta s = \theta$ happens to be the case), this outcome would be achieved were every union to set the efficient wage. The (expected) outcome for union *j*, were it to set its individually optimal wage when all other unions set the efficient wage, is located at the tangency point between the lower perceived labour demand curve and the isoloss map at *b*. This can be shown for the depicted $s > 0$ case to involve (provided $n > 1$) both a lower real wage and lower employment for union *j* than the symmetric Nash equilibrium at point *c*.[25] The individual union recognises, of course, that its incentive to deviate from the efficient wage in order to attempt to attain point *b* is also faced by every other union. It follows that the only wage consistent with individually optimising behaviour and rational expectations-formation by every union is w_{NE}, as given by (30). Hence the equilibrium (expected) outcome will be at point *c*, and the actual outcome only differs from this on account of an expectational error regarding θ. Note that *c* is the unique point in the diagram which satisfies the criteria for a symmetric Nash equilibrium outcome: it is both on the expected aggregate-level labour demand curve (i.e. it is a member of the set of expected outcomes, conditional on *s*, which are attainable given symmetric wage-setting), and is at a tangency point between a perceived labour demand curve and the representative union's isoloss map.

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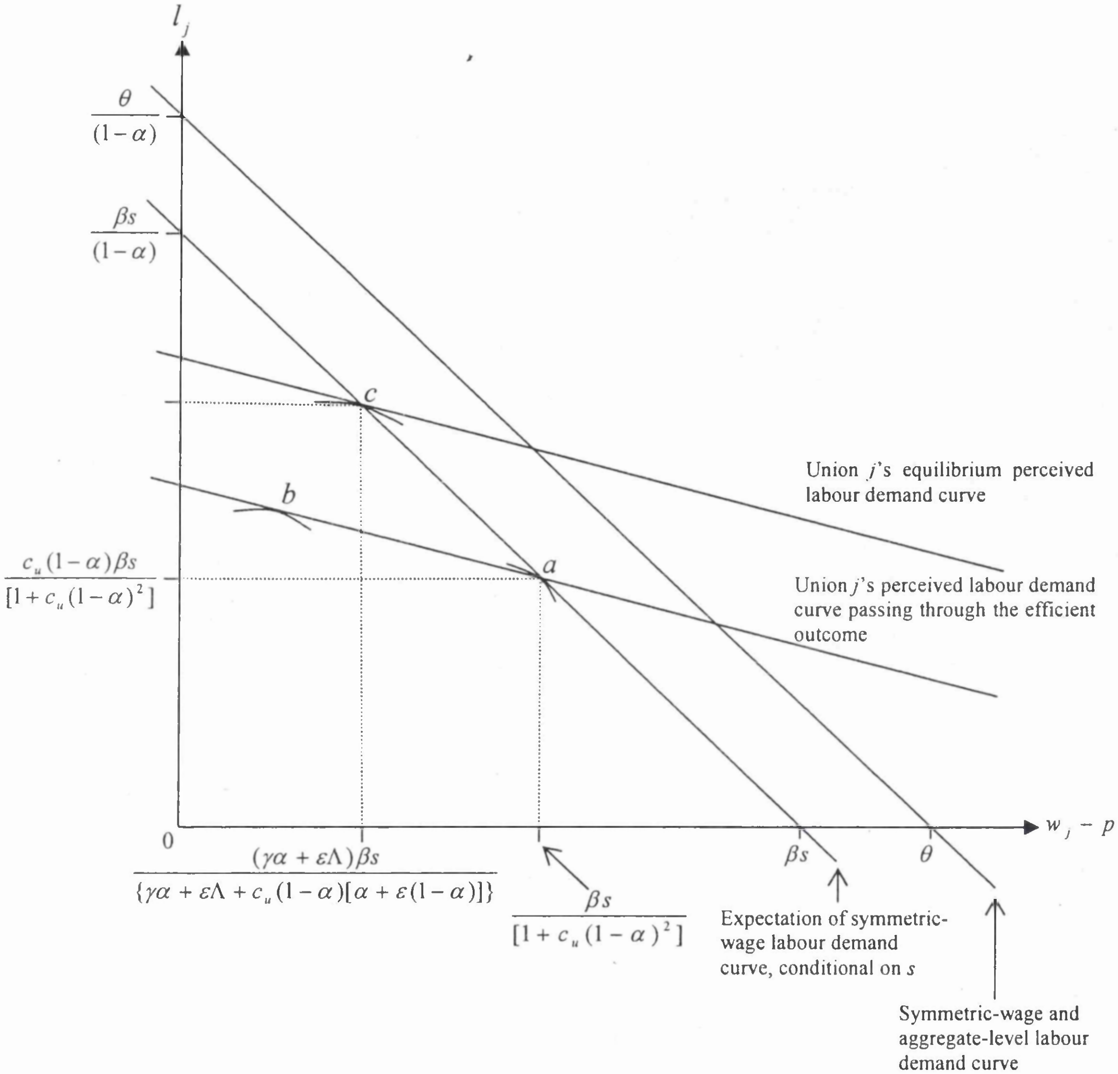
[24] Recall that the symmetric-wage labour demand curve is a special case of the aggregate-level labour demand curve, and hence in diagrammatic terms the two are coincident, as are their conditionally expected counterparts.

[25] The proof that, provided $n > 1$, point *b* is to the left of point *c* in Figure III.2 proceeds as follows. The slope of each of union *j*'s isoloss contours is $-c_u(w_j - p)/l_j$, and at *b* this is equal to the slope of union *j*'s perceived labour demand curve. Hence the coordinates of point *b* must simultaneously satisfy both the equation $c_u(w_j - p)/l_j = (\gamma\alpha + \varepsilon\Lambda)/[\alpha + \varepsilon(1-\alpha)]\Lambda$ and the equation of the perceived labour demand curve. Substituting the right-hand side of (49a) for l_j in the former equation allows point *b*'s real-wage coordinate to be solved for as:
$$w_j - p = \{\gamma\alpha + \varepsilon\Lambda + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\Lambda\}(\gamma\alpha + \varepsilon\Lambda)\beta s/[1 + c_u(1-\alpha)^2]\{(\gamma\alpha + \varepsilon\Lambda)^2 + c_u[\alpha + \varepsilon(1-\alpha)]^2\Lambda^2\}$$
Following some tedious algebra, the necessary and sufficient condition for this real wage, in the $s > 0$ case, to be less than the expected equilibrium real wage (which is given by the first term on the right-hand side of (38a)), is found to be: $\{(1-\alpha)(\gamma\alpha + \varepsilon\Lambda) - [\alpha + \varepsilon(1-\alpha)]\Lambda\}^2 > 0$, and this condition holds for all $n > 1$. The left-hand side of this inequality is zero for $n = 1$, implying that for a single union points *a*, *b*, and *c* are all coincident.

Figure III.2
Labour demand curves for the case of a positive productivity shock



Notes:

1. The diagram assumes for illustrative purposes that $\theta > \beta s > 0$.

2. It is also assumed that $1 < n < \infty$.

3. The vertical-axis coordinate of point c is equal to:
$c_u[\alpha+\varepsilon(1-\alpha)]\Lambda\beta s/\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}$.

4. The expected equilibrium outcome, conditional on s, is at c, and the expected efficient outcome is at a. The actual equilibrium outcome will lie on the aggregate-level labour demand curve, and its precise position will depend on the value of γ and on the realised value of the velocity shock. In the absence of a velocity shock it will be located on the aggregate-level curve to the right of a perpendicular line passing through c.

As in Chapter II, under the simple rule the slope of union j's perceived labour demand curve is more negative, the lower the number of unions. This slope is intimately related to the strength of the aggregate demand channel, which is stronger the smaller is n. Under the simple rule the aggregate demand channel is negative for a non-atomistic union[26], and works in the same direction as the real wage channel. Hence a strengthening of the aggregate demand channel occasioned by a marginal decrease in n will raise the employment cost to union j of deviating from the efficient wage. In diagrammatic terms, the perceived labour demand curve rotates in a clockwise direction as n decreases, and the closer is n to unity, the better does the perceived labour demand curve approximate the (conditional expectation of) the symmetric-wage labour demand curve. It is these facts which underlie (36)[27]: the departure of the equilibrium nominal wage from its efficient value is greater, the larger is n because of the weakening of the aggregate demand channel and resulting exacerbation of the externality. It is this effect which causes real wage variability to be decreasing in n, and employment variability to be increasing in n, as revealed by expressions (42a) and (42b). The externality-exacerbating effect of a marginal increase in n also explains result (47) relating to the unconditional expected union loss under equilibrium wage-setting. (Note that the derivative in (47) is zero when $n=1$ because a single union fully internalises the price-level repercussions of the wage it sets at every firm in the economy, and therefore achieves the minimum possible value of the expected loss.[28])

As in Chapter II, the slope of union j's perceived labour demand curve is shallower (less negative) under the simple rule, the lower is ε. This is because the externality which arises in the stochastic context of this chapter when the union loss function is given by (10), is dependent on the possibility of the individual firm's price exceeding the average price without such a disparity causing a total evaporation of its product demand. It is clear from (35) and from (39a) and (39b) that when the goods market is perfectly competitive, the externality is absent and each union finds it individually

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[26] In other words, controlling for the real wage channel, the effect of a marginal increase in union j's wage on employment, working via the aggregate demand channel, is negative for a non-atomistic union under the simple rule. For an atomistic union it is always zero irrespective of the monetary regime.

[27] In interpreting (36), bear in mind that for $s>0$, $w_{NE}<w^*$ is the case.

[28] Of course, a marginal increase in n when n is currently unity does not have a sensible economic meaning, since in economic terms n must take integer values. It is clear from (47), however, that a (non-marginal) increase in the number of unions from $n=1$ to $n=2$ will certainly result in an increase in the expected loss under equilibrium wage-setting.

optimal to set the collectively efficient wage. More generally, a marginal increase in ε reduces the scope for the prices of union j's employer firms to depart from the average price, and thus mitigates the externality. It is this effect which explains the sign of expressions (37), (43a), (43b) and (48), all of which are zero when $n=1$ since a single union achieves the efficient outcome regardless of goods-market conditions.

It is worthwhile stressing that it is crucial to the existence of the externality that unions are averse to both deviations of the real wage and of employment from their unconditional expected values (i.e. its existence depends on c_u being positive and finite). If the representative union's sole concern is to stabilise only the real wage or only employment, the externality does not arise since the trade-off between these entities which exists whenever the anticipated component of the shock is non-zero does not amount to a constraint on the individual union, which can achieve its sole objective (in respect of the shock's anticipated component) by setting its nominal wage appropriately. The absence of the externality when either one of the two terms on the right-hand side of (10) is zero is formally evident from the following expressions which make use of (17), (20a), (20b), (30), (38a) and (38b):

$$w^*\big|_{c_u=0} = w_{NE}\big|_{c_u=0} = \frac{(\gamma-1)\beta s}{\gamma} \tag{50a}$$

$$(w-p)\big|_{w=w^*,c_u=0} = (w-p)\big|_{w=w_{NE},c_u=0} = \beta s + \frac{(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{50b}$$

$$l\big|_{w=w^*,c_u=0} = l\big|_{w=w_{NE},c_u=0} = \frac{(\gamma-1)(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{50c}$$

$$\lim_{c_u\to\infty} w^* = \lim_{c_u\to\infty} w_{NE} = \frac{-\beta s}{\gamma(1-\alpha)} \tag{51a}$$

$$\lim_{c_u\to\infty} (w-p)\big|_{w=w^*} = \lim_{c_u\to\infty} (w-p)\big|_{w=w_{NE}} = \frac{(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{51b}$$

$$\lim_{c_u \to \infty} l\big|_{w=w^*} = \lim_{c_u \to \infty} l\big|_{w=w_{NE}} = \frac{\beta s}{(1-\alpha)} + \frac{(\gamma-1)(\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{51c}$$

Expressions (50a) to (50c) reveal that when unions are indifferent to real wage variability (the $c_u = 0$ case), equilibrium wage-setting is efficient and ensures that employment is fully insulated from the shock's anticipated component, the full impact of which consequently falls on the real wage. In the converse extreme case in which $c_u \to \infty$, which amounts to unions being indifferent to employment variability, efficiency requires that the anticipated component's impact be entirely borne by employment, with the real wage fully protected, and that this is indeed the case under equilibrium wage-setting is apparent from expressions (51a) to (51c).

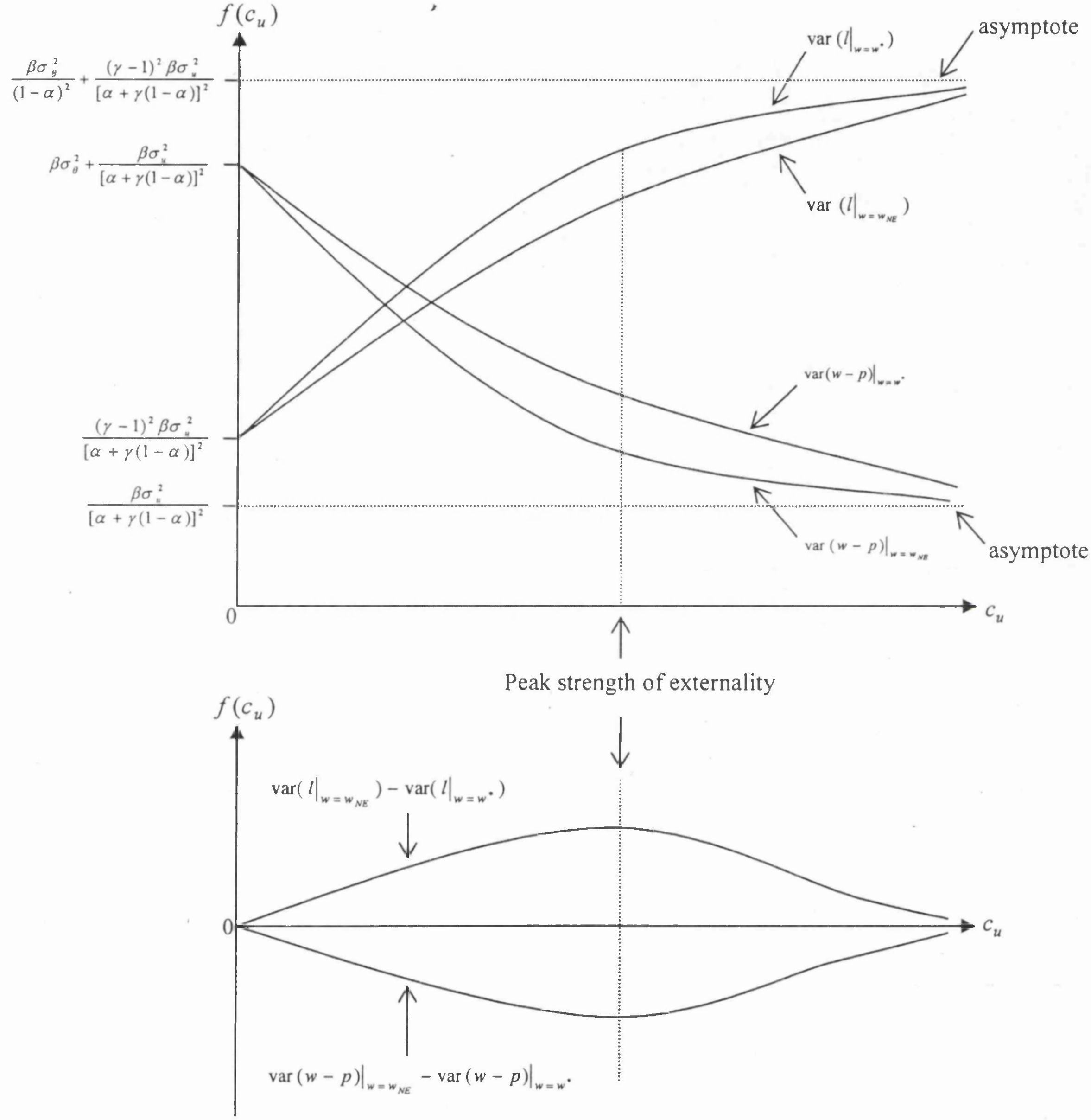
An obvious measure of the strength of the externality is the amount by which the variance of the real wage under equilibrium wage-setting falls below its efficient value. (Clearly, an alternative measure one might adopt is the excess of equilibrium employment variability over its efficient value.) Hence the following expression, obtained by combining (21a) with (41a) will prove useful to the analysis:

$$E[(w-p)\big|_{w=w_{NE}}]^2 - E[(w-p)\big|_{w=w^*}]^2 =$$

$$\left\{\frac{(\gamma\alpha+\varepsilon\Lambda)^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} - \frac{1}{[1+c_u(1-\alpha)^2]^2}\right\}\beta\sigma_\theta^2 \tag{52}$$

Note that evaluating (52) for $n=1$, or taking its limit as $\varepsilon \to \infty$ causes its right-hand side to equal zero. This is to be expected, since the externality is known to be absent in these two special cases. By tedious algebra it can be shown that, provided $n>1$, the right-hand side of (52) is unambiguously negative for all positive c_u values. We omit such a proof since this fact is implied by our earlier finding that real wage variability is inefficiently low for all $c_u > 0$, provided $n > 1$. More important for our present purposes is the nature of the functional relationship between the strength of the externality, as represented by (52), and the parameter c_u. Since it is apparent from (21a) and (41a) that under both efficient and equilibrium wage-setting the variance of the real wage is a strictly convex function of c_u for all $c_u > 0$, it is clear from (50b) and (51b) that the strength of the externality rises initially as c_u increases from zero,

Figure III.3
Variances of employment and the real wage as functions of the union weight parameter



and attains a maximum strength at some intermediate value, following which further increases in c_u must cause a diminution of the severity of the externality. This functional relationship between the externality's strength and c_u is evident from Figure III.3, which for good measure also includes graphs of the variances of the real wage and of employment under the two wage-setting scenarios in order to emphasise the important point that the externality is strongest for intermediate values of the unions preference parameter, c_u.

III.3.3.(iii) Signal Quality, Equilibrium Wage-Setting and Union Welfare

There is another interesting aspect to the equilibrium nominal wage, as given by (30), and the associated real outcomes, as given by (38a) and (38b), which calls for comment. It will be recalled that under efficient wage-setting there is a special case, involving a particular combination of values for the structural parameters, which renders the signal worthless to unions and makes it efficient to set the wage equal to its unconditional mean market-clearing value of zero, regardless of the particular realisation of the signal or the quality of the information it embodies. Significantly, this special case has a parallel under equilibrium wage-setting, in which unions are led to disregard the signal and set a wage of zero. It will be recalled from our earlier discussion of efficient wage-setting that this special case in that scenario involved the parameters α and γ taking values such that the equation $c_u = \tilde{c}_u$, where $\tilde{c}_u \equiv (\gamma - 1)/(1-\alpha)$, is satisfied. As is apparent from (30), the analogous special case under equilibrium wage-setting involves the values of α, γ, ε and n being such as to satisfy the equation $c_u = \tilde{\tilde{c}}_u$ where $\tilde{\tilde{c}}_u \equiv (\gamma-1)(\gamma\alpha + \varepsilon\Lambda)/[\alpha + \varepsilon(1-\alpha)]\Lambda$.[29]

There are two main points to note about $\tilde{c}_u$ and $\tilde{\tilde{c}}_u$. Firstly, substituting $\tilde{\tilde{c}}_u$ for c_u in (38a) and (38b), we find that the equilibrium real wage when $c_u = \tilde{\tilde{c}}_u$ is $\theta/[\alpha + \gamma(1-\alpha)]$, and that equilibrium employment is $(\gamma-1)\theta/[\alpha + \gamma(1-\alpha)]$, and that these outcomes are the same as their counterparts under efficient wage-setting when $c_u = \tilde{c}_u$. This equivalence is hardly surprising since these special c_u values in their

[29] Note that just as it was pointed out earlier that the possibility of c_u being equal to $\tilde{c}_u$ is precluded if $\gamma < 1$, so too is the possibility of c_u being equal to $\tilde{\tilde{c}}_u$ precluded if $\gamma < 1$.

respective scenarios alike cause the wage at every firm to be zero, and hence a particular realisation of θ must have the same impact on real outcomes in these two special cases. There is a significant difference between efficient wage-setting when $c_u = \tilde{c}_u$ and equilibrium wage-setting when $c_u = \tilde{\tilde{c}}_u$, however: whereas when $c_u = \tilde{c}_u$ a real wage of $\theta/[1+\tilde{c}_u(1-\alpha)^2] \equiv \theta/[\alpha+\gamma(1-\alpha)]$ is the best possible outcome for unions, in the sense that both the shock's anticipated component, βs, and its unanticipated component, $\theta - \beta s$, are efficiently distributed between the real wage and employment, under equilibrium wage-setting this particular outcome comes about not when $c_u = \tilde{c}_u$, but rather when $c_u = \tilde{\tilde{c}}_u$, and hence for the latter case of union preferences cannot be efficient. In other words, although equilibrium wage-setting when $c_u = \tilde{\tilde{c}}_u$ brings about the same real outcomes as efficient wage-setting when $c_u = \tilde{c}_u$, it does not achieve the most desirable distribution, from the point of view of unions, of either the anticipated or unanticipated component of the shock between the real wage and employment.[30]

The second main point regarding $\tilde{c}_u$ and $\tilde{\tilde{c}}_u$ is that the difference between these two values is smaller, the higher the value of ε and the lower is *n*: in other words, $\tilde{c}_u$ and $\tilde{\tilde{c}}_u$ are closer the weaker the externality in wage-setting. Indeed, we find that for the extreme cases of the model in which the externality is absent, these two c_u values coincide: $\lim_{\varepsilon\to\infty} \tilde{\tilde{c}}_u = \tilde{\tilde{c}}_u\big|_{n=1} = \tilde{c}_u$. This observation regarding the proximity of $\tilde{\tilde{c}}_u$ to $\tilde{c}_u$ when there are only a few unions in the economy and product-market competition between firms is strong will prove useful in interpreting some of the algebraic expressions which will arise below in the course of our investigation of the relationship between the expected union loss and the quality of the information available to unions regarding the productivity shock. Before turning to this major theme of the current chapter, however, it is best to explain first of all why $\tilde{c}_u$ and $\tilde{\tilde{c}}_u$

[30] Provided $n > 1$ and ε is finite, under equilibrium wage-setting the externality always prevents the achievement of the efficient distribution of the anticipated component of the shock between real wage and employment, and this is just as true of the $c_u = \tilde{\tilde{c}}_u$ case as of any other $c_u > 0$, including $c_u = \tilde{c}_u$. (This statement presupposes that $\gamma > 1$, so that $\tilde{c}_u > \tilde{\tilde{c}}_u > 0$ is the case.) As for the unanticipated component of the shock, this is only ever apportioned in the most desirable way for unions when $c_u = \tilde{c}_u$, and this is so regardless of the wage-setting scenario.

are highly pertinent to our exploration of the signal-quality issue. It will be recalled from our earlier discussion of efficient wage-setting that when $c_u = \tilde{c}_u$ happens to be the case (and $\gamma \geq 1$ is a necessary condition for this to be possible), the expected union loss is independent of signal quality (as embodied in the noise variance, σ_u^2), since unions collectively achieve the most desirable outcome by always setting a wage of zero. It follows from this that if each union sets its individually optimal wage when $c_u = \tilde{c}_u$, the most desirable outcome will only be achieved if the signal is completely uninformative and hence is ignored. If the signal is at all informative (i.e. if σ_u^2 is finite), the equilibrium wage response to a non-zero signal will not be efficient, and the apportionment of the anticipated component of the shock between the real wage and employment will consequently also be inefficient, with only the apportionment of the shock's unanticipated component being in full accordance with union preferences. It is also intuitively clear that when $c_u = \tilde{c}_u$ the inefficiency of equilibrium wage-setting must be worse, the better is signal quality (i.e. the lower is σ_u^2 and hence the closer to unity is β), since in this special case it is only the anticipated component of θ that is inefficiently divided between the real wage and employment, and this component will generally be a larger proportion of θ, the better is signal quality.[31] As for the other special case, namely that in which $c_u = \tilde{\tilde{c}}_u$, its key aspect is that the working of the externality is such that unions ignore the signal, so that the equilibrium nominal wage is zero regardless of s. It follows that when $c_u = \tilde{\tilde{c}}_u$, the expected union loss must be independent of signal quality.

Equipped with these intuitive insights regarding the relationship between the (unconditional) expected union loss and signal quality in these two special cases, we now undertake a more formal investigation of this relationship. The expression we need to consider is the unconditional expected loss of unions under efficient wage-setting, as given by (44). Bearing in mind that $\partial(\beta\sigma_\theta^2)/\partial\sigma_u^2 = -\beta^2$ and

[31] The reason why this statement contains the qualifier 'generally' is because we are considering particular realisations of signal s and noise term u, and in any particular period u may be such as to cause $\theta - \beta s$ to be greater, in absolute terms, than βs. However, the general truth of the statement is readily apparent from the fact that the variance of the ratio of βs to θ is $E[(\beta s/\theta)^2] = \beta$, which is decreasing in σ_u^2, while the variance of the ratio of $\theta - \beta s$ to θ is $E[(\theta - \beta s)^2/\theta^2] = 1 - \beta$, which is increasing in σ_u^2.

$\partial(\beta\sigma_u^2)/\partial\sigma_u^2 = \beta^2$, it follows that a marginal deterioration in signal quality (i.e. a marginal increase in σ_u^2) will decrease the expected loss if the coefficient of the first term on the right-hand side of (44), the term in the variance of the unions' forecast (i.e. the anticipated component of the shock), exceeds the coefficient of the second term, the term in the variance of the unions' forecast error (i.e. the unanticipated component of the shock).

$$\frac{dE(\Omega_j^u|_{w=w_{NE}})}{d\sigma_u^2} = \left[\frac{-c_u\{(\gamma\alpha+\varepsilon\Lambda)^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\Lambda^2\}}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} + \frac{[c_u+(\gamma-1)^2]}{[\alpha+\gamma(1-\alpha)]^2}\right]\beta^2 \qquad (53)$$

Thus under equilibrium wage-setting, the impact on unions' welfare of a marginal increase in σ_u^2 (working both directly, and indirectly, via the induced change in β), may be decomposed into two effects, one of which is beneficial and the other detrimental. The first effect is given by the first term within the square brackets on the right-hand side of (53) and may be described as the beneficial externality-mitigating effect. A deterioration in signal quality induces a decrease in β, with the result that each union's individual wage response to a non-zero signal is smaller in absolute terms: the departure of the equilibrium wage from its efficient value is therefore smaller as a result of the increase in σ_u^2. (That the externality's strength is falling in σ_u^2 can alternatively be seen from (52), where the coefficient on $\beta\sigma_\theta^2$ is unambiguously negative: since $\partial(\beta\sigma_\theta^2)/\partial\sigma_u^2 = -\beta^2$, it follows that the higher is σ_u^2, the weaker is the externality, as measured by (52).) The result is an improved distribution of the anticipated component of the shock between the real wage and employment. The second effect is the detrimental forecast-error effect given by the second term within the square brackets on the right-hand side of (53). A marginal increase in σ_u^2 necessarily increases the variance of the unanticipated component of the shock (i.e. the forecast error), which in turn increases both real wage and employment variability and hence also the expected loss. Whether a marginal deterioration in signal quality is beneficial or detrimental to unions' welfare therefore boils down to whether or not the externality-mitigating effect outweighs the forecast-error effect. Which of the two effects is the stronger depends in particular on two elements of the model. Firstly it will depend on the value of c_u, since, as we saw earlier, the externality is stronger for intermediate values of c_u, and hence the externality-mitigating effect will have a larger beneficial impact the closer is c_u to the value at which the externality is most severe. Secondly, it will depend on the value of γ, since as mentioned earlier in our discussion of efficient wage-setting, it is this parameter which is crucial for determining how the unanticipated component of θ is apportioned between the real wage and employment. Mindful of these points regarding c_u and γ, we now direct our attention once more to expression (53).

An immediate implication of (53) is the following necessary and sufficient condition for a marginal deterioration in signal quality to be beneficial (detrimental) to the welfare of non-atomistic unions:

$$\frac{dE(\Omega_j^u\big|_{w=w_{NE}})}{d\sigma_u^2} < (>) 0 \text{ iff } \Delta\Theta < (>) 0 \tag{54}$$

where $\Delta \equiv (1-\gamma)(\gamma\alpha + \varepsilon\Lambda) + c_u[\alpha + \varepsilon(1-\alpha)]\Lambda$,

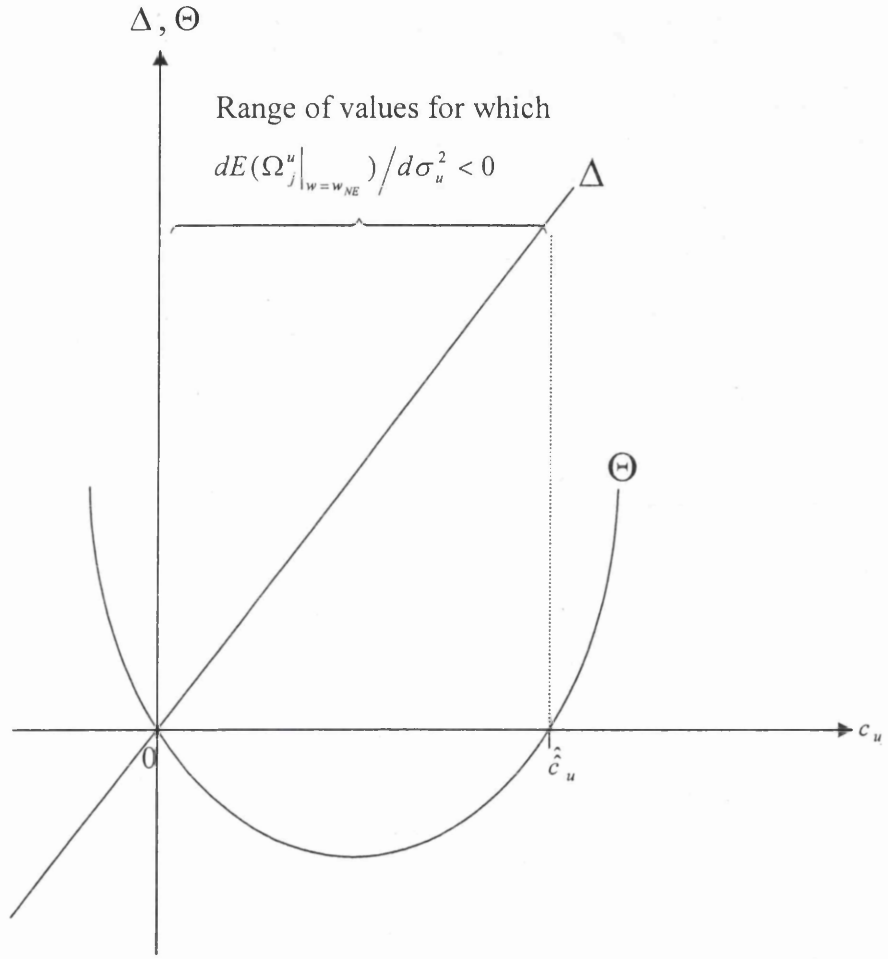
$\Theta \equiv -(n-1)c_u\alpha[\alpha + \gamma(1-\alpha)]^2 + [1-\gamma + c_u(1-\alpha)]\{\gamma\alpha + \varepsilon\Lambda + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\Lambda\}$

In interpreting (54) some insight is gained by noting that Δ is equal to minus one times the numerator of the coefficient on βs in the equilibrium nominal wage equation, (30). Interpretation of the result is also facilitated by referring to its graphical representation in Figure III.4 below, where Δ and Θ appear, respectively, as linear and quadratic functions of c_u. The graph depicts the case in which $\gamma = 1$: the counterpart diagrams for the $\gamma < 1$ and $\gamma > 1$ cases are presented and discussed in Appendix III.2. An important aspect of the $\gamma = 1$ case is that $\tilde{c}_u\big|_{\gamma=1} = \tilde{\tilde{c}}_u\big|_{\gamma=1} = 0$: thus in Figure III.4 the graph of the linear function Δ passes through the origin. One implication of the fact that $\tilde{c}_u\big|_{\gamma=1} = 0$ is that it is not necessarily the case that a set of admissible c_u values exists for which unions' welfare is falling in signal quality: with $\gamma = 1$ the possibility arises that unions with positive weight c_u are always made better off by improvements in signal quality. As for the fact that $\tilde{\tilde{c}}_u\big|_{\gamma=1} = 0$, this implies that

Figure III.4

Identification of the sign of the impact of a marginal deterioration in signal quality on the expected union loss under the simple rule

($\gamma = 1$ case)



Notes:

1. It is assumed that $n > 1$ and $\gamma = 1$. Note that when $\gamma = 1$, $\tilde{c}_u|_{\gamma=1} = \tilde{\tilde{c}}_u|_{\gamma=1} = \hat{c}_u|_{\gamma=1} = 0$.

2. It is assumed here that $\hat{\hat{c}}_u|_{\gamma=1}$, the non-zero root of the equation $\Theta = 0$, is positive. This is the case if both $\varepsilon < \alpha/(1-\alpha)$ and $n > \alpha[(\varepsilon-1)(1-\alpha)-1]/[\varepsilon(1-\alpha)-\alpha]$ (i.e. the diagram implicitly assumes that these two conditions hold).

in the extreme case in which $c_u = 0$, union welfare will be independent of signal quality.

To aid the exposition we state explicitly the expression for Θ, as given in (54), for the $\gamma = 1$ case:

$$\Theta\big|_{\gamma=1} = (n-\alpha)(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]c_u^2 + \{(1-\alpha)[\alpha+\varepsilon(n-\alpha)]-(n-1)\alpha\}c_u \quad (55)$$

It is readily apparent that, considered as an equation in c_u, the equation $\Theta\big|_{\gamma=1} = 0$ has two real solutions, one of which will be located at the origin in Figure III.4. We will denote this solution $\hat{c}_u\big|_{\gamma=1}$: we therefore have $\hat{c}_u\big|_{\gamma=1} = 0$. The non-zero solution, which we will denote $\hat{\hat{c}}_u\big|_{\gamma=1}$, may be either positive or negative. If it is negative, the product $(\Delta\big|_{\gamma=1})(\Theta\big|_{\gamma=1})$ will then be positive for all positive c_u values, implying that for all positive values of c_u, unions are made better off by improvements in signal quality. If, on the other hand, the non-zero solution is positive, $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$, it will then be the case that $(\Delta\big|_{\gamma=1})(\Theta\big|_{\gamma=1}) < (>) 0$ for all positive $c_u < (>) \hat{\hat{c}}_u\big|_{\gamma=1}$. In other words, if it so happens that $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$, there will be a range of positive c_u values (specifically, all positive c_u values less than $\hat{\hat{c}}_u\big|_{\gamma=1}$) for which improvements in signal quality make unions worse off under equilibrium wage-setting. It is this situation in which $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$ that is depicted in Figure III.4. Solving the equation $\Theta\big|_{\gamma=1} = 0$ for its non-zero solution, it is found to be:

$$\hat{\hat{c}}_u\big|_{\gamma=1} = \frac{\{[\alpha-\varepsilon(1-\alpha)]n+\alpha[(\varepsilon-1)(1-\alpha)-1]\}}{(n-\alpha)(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]} \quad (56)$$

It follows from (56) that in order for $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$ to be the case a pair of necessary conditions must be satisfied, and that their simultaneous satisfaction is sufficient to ensure $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$. This pair of conditions is as follows:[32]

$$\hat{\hat{c}}_u\Big|_{\gamma=1} > 0 \text{ iff both } \varepsilon < \frac{\alpha}{(1-\alpha)} \text{ and } n > \frac{\alpha[(\varepsilon-1)(1-\alpha)-1]}{[\varepsilon(1-\alpha)-\alpha]} \qquad (57)$$

Since the severity of the wage-setting externality is decreasing in ε and increasing in n, the clear implication of (57) is that the externality's severity must exceed a particular critical value for it to be possible for unions to be made worse off by an improvement in signal quality when $\gamma = 1$. There is no need to state this critical value explicitly: what matters is that if (and only if) both the conditions in (57) hold, then this implies goods-market conditions and wage-bargaining structure are such that the resulting externality is sufficiently severe for the beneficial externality-mitigating effect of a marginal increase in signal noise to outweigh the detrimental forecast-error effect for all positive c_u values less than $\hat{\hat{c}}_u\big|_{\gamma=1}$.

Although this finding that, when $\gamma = 1$, deterioration in signal quality can improve union welfare only if circumstances are such as to cause a sufficiently severe externality makes good intuitive sense, there is another aspect of this result which fits less comfortably with intuitive arguments advanced in earlier sections. In particular, reasons were given earlier for thinking that because the externality is more severe at intermediate values of c_u, one would expect a worsening of signal quality not to be damaging to unions when c_u takes very low positive values. This reasoning is obviously not supported by our findings for the $\gamma = 1$ case. To explain why, when $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$, we have $dE(\Omega_j^u\big|_{w=w_{NE}})\big/d\sigma_u^2 < 0$ for all positive c_u less than $\hat{\hat{c}}_u\big|_{\gamma=1}$, attention needs to be focused on a fact noted in our earlier discussion of efficient wage-setting, namely that when $\gamma = 1$, employment is fully insulated from the shock's

[32] Note with regard to (57) that if $\varepsilon < \alpha/(1-\alpha)$ then $\alpha[(\varepsilon-1)(1-\alpha)-1]/[\varepsilon(1-\alpha)-\alpha] > 1$, so that in order for $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$ to be the case there must be more than one union and goods-market competition must be sufficiently low to ensure that $\varepsilon < \alpha/(1-\alpha)$. Note also that while manipulating the numerator of $\hat{\hat{c}}_u\big|_{\gamma=1}$, as given by (56), reveals that $\hat{\hat{c}}_u\big|_{\gamma=1} > 0$ can also be the case if $\varepsilon > \alpha/(1-\alpha)$ and $n < \alpha[(\varepsilon-1)(1-\alpha)-1]/[\varepsilon(1-\alpha)-\alpha]$, this possibility can be precluded in the context of the model, since it would require n to take an inadmissible value of less than unity.

unanticipated component (i.e. unions' forecast error regarding θ), and that this component solely affects the real wage. Thus, when $\gamma = 1$ the variance of forecast errors, $\beta\sigma_u^2$, only contributes to real wage variability, and does not contribute to employment variability at all. It follows that if c_u is low, so that unions are principally concerned to avoid employment variability, and are close to being indifferent to variability of the real wage, it must then be the case that when $\gamma = 1$ the forecast error variance is a decidedly minor component of unions' expected loss. (With reference to equation (44), note that if $\gamma = 1$ and c_u is close to zero, the coefficient of the term in $\beta\sigma_u^2$ will be small compared to the coefficient of the term in $\beta\sigma_\theta^2$.) The clear implication is that when $\gamma = 1$, the detrimental forecast-error effect of a marginal increase in the signal noise variance must be weaker, the lower is c_u. Of course, the beneficial externality-mitigating effect is also weaker, the lower is c_u, but it so happens that if the conditions stated in (57) are satisfied, then for any $c_u < \hat{c}_u\big|_{\gamma=1}$, the goods market is sufficiently uncompetitive and unions are sufficiently numerous for the externality to be strong enough for the mitigating effect upon it of an increase in σ_u^2 to outweigh the forecast-error effect.

We have almost completed our investigation of the relationship between union welfare and signal quality when monetary policy is conducted according to the simple rule that the money supply be kept fixed regardless of the value of the productivity shock. The sole aspect of the simple-rule scenario which still requires comment is the case of equilibrium wage-setting when unions are atomistic. Since expressions (53) and (54) are for finite *n*, their counterparts for the atomistic case have to be stated separately:

$$\lim_{n\to\infty} \frac{dE(\Omega_j^u\big|_{w=w_{NE}})}{d\sigma_u^2} = \left[\frac{-c_u\{\varepsilon^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\}}{\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} + \frac{[c_u + (\gamma-1)^2]}{[\alpha+\gamma(1-\alpha)]^2}\right]\beta^2 \qquad (58)$$

From (58) it follows that:

$$\lim_{n\to\infty} \frac{dE(\Omega_j^u\big|_{w=w_{NE}})}{d\sigma_u^2} < (>)0 \quad \text{iff} \quad \Delta'\Theta' < (>)0 \tag{59}$$

where $\Delta' \equiv \varepsilon(1-\gamma) + c_u[\alpha + \varepsilon(1-\alpha)]$

$\Theta' \equiv -c_u\alpha[\alpha + \gamma(1-\alpha)] + [1-\gamma + c_u(1-\alpha)]\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}$

These expressions relating to the atomistic case have been stated purely for the sake of completeness, since all of the previous passages discussing whether or not there exists a range of c_u values for which the equilibrium expected loss of non-atomistic unions is falling in σ_u^2 are equally relevant for the atomistic case. The chief point to note about the atomistic-unions case in the context of the simple-rule scenario of this section is that, for given values of c_u, γ, ε and α, the wage-setting externality is more severe than when unions are non-atomistic. Consequently, where the values of the structural parameters γ, ε and α are such as to give rise to a range of c_u values for which the expected union loss under equilibrium wage-setting is falling in σ_u^2, this range of c_u values is at its broadest in the atomistic case, since the more severe the externality, the larger the set of c_u values for which the externality-mitigating effect of an increase in the signal-noise variance outweighs the forecast-error effect.

III.4 The Optimal State-Contingent Monetary Rule

This section derives the optimal rule for the extremes of wage-bargaining structure, under the assumptions that the rule is fully credible to the private sector and is indeed adhered to by the monetary authorities, and that the signal-noise variance, σ_u^2, and hence the quality of the signal received by unions, is exogenous. Our principal purpose is to ascertain the realised values of employment, the price level and the social loss under the optimal rule, in order to provide a standard against which to compare the performance of the discretionary monetary policy regime on which the analysis focuses in subsequent sections. Attention is confined here to the single-union and atomistic cases, since derivation of the optimal rule for the $1 < n < \infty$ case would not yield any additional insights.

Before embarking on this task, however, a brief digression is required to explain why, throughout the remainder of the thesis, it is assumed that the monetary authorities are perfectly informed about θ at the time monetary policy is implemented. This simplifying assumption is justified since were we to take the trouble to model the authorities as being imperfectly informed about θ at the time they set their instrument, this would merely introduce terms in the authorities' forecast error re θ into our expressions for employment, the real wage, and the price level. The coefficients of these terms would in fact be the same as the coefficients of the terms in the unions' forecast error in the relevant expressions (equations (31), (38a) and (38b)) for the simple-rule scenario analysed in the previous section. Hence, using e_{cb} to denote the central bank's forecast error, the employment expression (whether under efficient or equilibrium wage-setting) would feature the additional term $(\gamma - 1)e_{cb}/[\alpha + \gamma(1-\alpha)]$, the real wage expression the term $e_{cb}/[\alpha + \gamma(1-\alpha)]$, and the price level expression the term $-e_{cb}/[\alpha + \gamma(1-\alpha)]$. The intuitive reason for this is that the simple-rule scenario can be thought of as one in which the central bank is completely uninformed about θ when carrying out monetary policy, and therefore sets m at some fixed, rule-dictated value $\overline{m}$. If the central bank does have some, albeit imperfect, information regarding θ at the time m is set, it will, in general, adjust the money supply in response to the forecast of θ which it forms on the basis of this information. Since, however, at this date the central bank has yet to observe its forecast error, it will not adjust the money supply in reaction to it, and it is this lack of monetary response that gives rise to the parallel between the impact of this central bank forecast error on macroeconomic variables under this scenario, and the impact of the unions' forecast error on these variables under the simple rule. The key point to note about the coefficients stated just above is that the only structural parameters which occur in them are α and γ. The absence of the parameters c_b, c_u and σ_u^2 from these coefficients is attributable to both the central bank and unions having no information regarding the central bank's forecast error at the time these parties set their choice variables, while ε and n are absent since these parameters only ever influence outcomes via the wage-setting externality, which, of course, can only arise in respect of the component of the shock anticipated by unions. Since our principal interest is in studying the macroeconomic implications of the design of the monetary regime (as represented by the delegation parameter c_b), and of the quality of the signal received by unions (as represented by σ_u^2), it follows that no additional insights regarding these issues can be gained by modelling the central bank as imperfectly informed about θ. (Note that an implicit assumption underpinning this argument is that unions do not enjoy an informational advantage over the central bank as regards macroeconomic shocks. This assumption is plausible, however, if the central bank sets its monetary instrument after, or at the same time as, unions set wages.)

The general form of the state-contingent rule is:

$$m_r = \overline{m} + \lambda_1\theta + \lambda_2 s + \lambda_3 w \tag{60}$$

where $\overline{m}$ is a constant and λ_1, λ_2 and λ_3 are the rule parameters. It so happens that an alternative specification of the rule in terms of the aggregate wage (which will depend, of course, on the anticipated component of the shock, βs), and the unanticipated component of the shock, $\theta - \beta s$, leads to precisely the same results as (60), but has an advantage over (60) in being more amenable to intuitive explanation.[33] This alternative specification is:

$$m_r = \overline{m} + \rho_1 w + \rho_2(\theta - \beta s) \tag{61}$$

Since unions are entirely ignorant of $\theta - \beta s$ when setting their nominal wages, intuition suggests that the optimal setting of ρ_2 will bring about, in respect of this unanticipated component of the shock, society's preferred impact pattern for employment and the price level. It will transpire below that for the single-union and atomistic cases on which we focus here, it is not possible to set ρ_1 so as to achieve the socially optimal pattern in respect of βs.

[33] Note that (61) is less general than (60), since its formulation amounts to the imposition of a constraint on the rule parameters which appear in (60). To be specific, (61) is the version of (60) which arises when λ_1 and λ_2 are constrained to satisfy the equation $\lambda_1 = -\lambda_2/\beta$.

Combining (61) with (6), (13′) and (15), allows union j's real wage and labour demand to be expressed as functions of the shock, the forecast error, and the individual and aggregate wage:

$$w_j - p = w_j - \frac{\{\gamma(1-\alpha)[\overline{m} + \rho_2(\theta - \beta s)] + [\gamma(1-\alpha)\rho_1 + \alpha]w - \theta\}}{[\alpha + \gamma(1-\alpha)]} \tag{62a}$$

$$l_j^D = \frac{1}{[\alpha + \varepsilon(1-\alpha)][\alpha + \gamma(1-\alpha)]}\Big\{ \gamma[\alpha + \varepsilon(1-\alpha)][\overline{m} + \rho_2(\theta - \beta s)]$$

$$+ (\gamma - 1)[\alpha + \varepsilon(1-\alpha)]\theta + \{\gamma[\alpha + \varepsilon(1-\alpha)]\rho_1 + \alpha(\varepsilon - \gamma)\}w - \varepsilon[\alpha + \gamma(1-\alpha)]w_j \Big\}$$

(62b)

Union j's loss as a function of the rule parameters, its individual wage and the aggregate wage is obtained by substituting (62a) and (62b) into (10). The efficient wage for given settings of the rule parameters is derived by minimising the rational expectation, conditional on s, of this loss by choice of w_j, subject to the constraint that wage-setting be symmetric, $w_j = w \forall j$. This, of course, is also the wage that would be set by a single economy-wide union:

$$w^* = w_{NE}\big|_{n=1} = \frac{-[1 - \gamma + c_u(1-\alpha)]\beta s}{\gamma(1-\rho_1)[1 + c_u(1-\alpha)^2]} \tag{63a}$$

Proceeding similarly to minimise the same expected loss in the absence of a symmetric-wage constraint, and with the aggregate wage treated as given, allows atomistic union j's individually optimal wage to be solved for as a function of the signal, the expected value of the aggregate wage, $E(w \mid s)$, and the rule parameters. We shall not state this expression here, since our interest lies with the associated symmetric Nash equilibrium nominal wage, which may be solved for after imposing the condition that $w_j = E(w \mid s) = w_{NE} \ \forall j$:

$$w_{NE,\,atomistic\ case} = \frac{-\{\varepsilon(1-\gamma) + c_u[\alpha + \varepsilon(1-\alpha)]\}\beta s}{\gamma(1-\rho_1)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}} \tag{63b}$$

Note that ρ_2 is absent from both (63a) and (63b) for the unsurprising reason that since unions set wages in ignorance of their forecast error, the monetary response to that error cannot be foreseen and hence cannot influence wage decisions. Another noteworthy point is that these two expressions reveal unity to be an inadmissible value for the wage-response rule parameter ρ_1, a result which is familiar from our analysis of rules in the non-stochastic version of the model. (No symmetric Nash equilibrium exists if $\rho_1 = 1$ since, given symmetry in wages, the monetary response to w prescribed by the rule frustrates each union's attempts to ensure that the anticipated component of the shock is partly borne by the real wage, and thus deprives unions of an incentive to set wages symmetrically.)

The associated expressions for the equilibrium price level, real wage, and employment are obtained by substituting (63a) and (63b) into (15), (62a) and (62b) (with $w_j = w = w_{NE}$, of course):

$$p\big|_{w=w_{NE},n=1} = \frac{\overline{m}}{(1-\rho_1)} - \frac{[1-\gamma\rho_1 + c_u(1-\alpha)]\beta s}{\gamma(1-\rho_1)[1+c_u(1-\alpha)^2]} + \frac{[\gamma(1-\alpha)\rho_2 - 1](\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{64a}$$

$$p\big|_{w=w_{NE},\,atomistic\ case} = \frac{\overline{m}}{(1-\rho_1)} - \frac{\{\varepsilon(1-\gamma\rho_1) + c_u[\alpha + \varepsilon(1-\alpha)]\}\beta s}{\gamma(1-\rho_1)\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} + \frac{[\gamma(1-\alpha)\rho_2 - 1](\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{64b}$$

$$(w-p)\big|_{w=w_{NE},n=1} = \frac{\beta s}{[1+c_u(1-\alpha)^2]} + \frac{[1-\gamma(1-\alpha)\rho_2](\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{65a}$$

$$l\big|_{w=w_{NE},n=1} = \frac{c_u(1-\alpha)\beta s}{[1+c_u(1-\alpha)^2]} + \frac{[\gamma(1+\rho_2)-1](\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{65b}$$

$$(w-p)\big|_{w=w_{NE},\,atomistic\ case} = \frac{\varepsilon\beta s}{\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} + \frac{[1-\gamma(1-\alpha)\rho_2](\theta - \beta s)}{[\alpha + \gamma(1-\alpha)]} \tag{66a}$$

$$l\Big|_{w=w_{NE},\,atomistic\,case} = \frac{c_u[\alpha+\varepsilon(1-\alpha)]\beta s}{\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} + \frac{[\gamma(1+\rho_2)-1](\theta-\beta s)}{[\alpha+\gamma(1-\alpha)]} \tag{66b}$$

The expressions for the expected social loss for the single-union and atomistic cases is found by taking the unconditional expectations of the squares of (65a), (65b), (66a) and (66b) in order to obtain the variances of the price level and employment, and combining these appropriately with the unconditional expectation of (8):[34]

$$E\Omega^s\Big|_{w=w_{NE},\,n=1} = \frac{c_s\overline{m}^2}{(1-\rho_1)^2} + \frac{1}{[1+c_u(1-\alpha)^2]^2}\left\{c_u^2(1-\alpha)^2 + \frac{c_s[1-\gamma\rho_1+c_u(1-\alpha)]^2}{\gamma^2(1-\rho_1)^2}\right\}\beta\sigma_\theta^2$$
$$+\frac{\{[\gamma(1+\rho_2)-1]^2+c_s[\gamma(1-\alpha)\rho_2-1]^2\}\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2} \tag{67a}$$

$$E\Omega^s\Big|_{w=w_{NE},\,atomistic\,case} = \frac{c_s\overline{m}^2}{(1-\rho_1)^2} + \frac{\{[\gamma(1+\rho_2)-1]^2+c_s[\gamma(1-\alpha)\rho_2-1]^2\}\beta\sigma_u^2}{[\alpha+\gamma(1-\alpha)]^2}$$
$$+\frac{1}{\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2}\left\{c_u^2[\alpha+\varepsilon(1-\alpha)]^2+\frac{c_s\{\varepsilon(1-\gamma\rho_1)+c_u[\alpha+\varepsilon(1-\alpha)]\}^2}{\gamma^2(1-\rho_1)^2}\right\}\beta\sigma_\theta^2 \tag{67b}$$

The socially optimal monetary rule is given by the $(\overline{m},\rho_1,\rho_2)$ combination which solves the following three simultaneous first-order conditions:

$$\frac{\partial E\Omega^s}{\partial\overline{m}} = \frac{\partial E\Omega^s}{\partial\rho_1} = \frac{\partial E\Omega^s}{\partial\rho_2} = 0 \tag{68}$$

There is little to be gained by explicitly expressing each of the first-order conditions subsumed in (68) in terms of the structural parameters and rule coefficients, and consequently these expressions are omitted. For the $n=1$ case, the relevant loss expression is given by (67a), and after obtaining the relevant derivatives and forming the first-order conditions, the solution is found to be:

[34] As usual, we assume that society's desired inflation rate is zero, $\pi^*=0$, and that its desired level of employment is the market-clearing level, so that in equation (8) $l^*=0$ is the case. Given the normalisation $p_{-1}=0$, $\pi^*=0$ implies in turn that $p^*=0$.

$$\overline{m}^* = 0 \qquad (69a)$$

$$\rho^*_{1,n=1} = \frac{[1+c_u(1-\alpha)]}{\gamma} \qquad (69b)$$

$$\rho^*_2 = \frac{[1-\gamma+c_s(1-\alpha)]}{\gamma[1+c_s(1-\alpha)^2]} \qquad (69c)$$

Society's optimal monetary rule in the single-union case is therefore: [35]

$$m^*_{r,n=1} = \frac{[1+c_u(1-\alpha)]w}{\gamma} + \frac{[1-\gamma+c_s(1-\alpha)](\theta-\beta s)}{\gamma[1+c_s(1-\alpha)^2]} \qquad (69d)$$

Our results for $\overline{m}^*$ and ρ^*_2, namely expressions (69a) and (69c), have a straightforward explanation. The socially optimal setting of $\overline{m}$ is zero, since this ensures inflation is zero in the event that both signal and shock take their mean values of zero. The socially optimal setting of ρ_2, meanwhile, ensures that the impact of the shock's unanticipated component on the price level and employment is that which is most desirable from society's viewpoint. Substituting ρ^*_2, as given by (69c) into the coefficients on $\theta-\beta s$ in (64a) and (65b), the impact of $\theta-\beta s$ on the price level when $\rho_2=\rho^*_2$ is found to be $-1/[1+c_s(1-\alpha)^2]$, while its associated impact on employment is $c_s(1-\alpha)/[1+c_s(1-\alpha)^2]$. Another noteworthy point about expression (69d) is that it tells us that if γ and α happen to take values such that $c_s=(\gamma-1)/(1-\alpha)$, the socially optimal monetary response to θ is zero. The reason for this is that in this special case the movement in the price level occasioned by a non-zero realisation of θ would, were $w=0$ also the case, suffice by itself to bring about the optimal distribution of the shock's impact between the price level and

[35] Note that were we to allow for velocity shocks, with the central bank observing the particular realisation of this shock, ϕ, prior to setting *m*, (69d) and every other optimal-rule equation in this section would then simply feature an additional term, $-\phi$, on the right-hand side. Neutralising ϕ by an appropriate adjustment of *m* fully insulates both employment and the price level from its impact. Note also that were we to derive society's optimal state-contingent rule using the more general specification given by equation (60), the optimal values for the rule parameters in the $n=1$ case would be $\overline{m}^*=0$, $\lambda^*_1=\rho^*_2$, $\lambda^*_2=-\rho^*_2\beta$, and $\lambda^*_3=\rho^*_{1,n=1}$, where $\rho^*_{1,n=1}$ and ρ^*_2 are given by (69b) and (69c) respectively.

employment. Note that this possibility is precluded if $\gamma < 1$. This special case clearly parallels the special $c_u = \tilde{c}_u \equiv (\gamma - 1)/(1 - \alpha)$ case of union preferences discussed in Section III.3 on the simple rule. (Recall that if $c_u = \tilde{c}_u$ happens to hold, unions can then afford to disregard signals entirely.)

As for society's optimal setting of ρ_1 in the $n = 1$ case, the intuitive explanation for ρ_1^*, as given by (69b) is obvious once it is noted that ρ_1 is absent from the expression for equilibrium employment, (65b), and is present in the coefficient on βs in the expression for the equilibrium price level, (64a). A single union fully internalises the price-level repercussions of the economy-wide wage it sets in response to the signal, and hence is able to bring about the most desirable apportionment, from its point of view, of the anticipated component of the shock between employment and the real wage. Furthermore, it is able to do this regardless of the rule parameters, the only exception being when ρ_1 takes the inadmissible value of unity. Hence the socially optimal setting of ρ_1 is that which ensures that the anticipated component of the shock has no impact on the price level, so that price level variability solely arises from union forecast errors.

Proceeding similarly for the atomistic case, the $\overline{m}$ and ρ_2 solutions to (68) when (67b) is the relevant expected social loss are identical to $\overline{m}^*$ and ρ_2^* as given by (69a) and (69c) and therefore need not be repeated. The socially optimal setting of ρ_1 is:

$$\rho^*_{1,\ atomistic\ case} = \frac{\{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}}{\gamma\varepsilon} \tag{70a}$$

Society's optimal rule when unions are atomistic is therefore:[36]

$$m^*_{r,\ atomistic\ case} = \frac{\{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}w}{\gamma\varepsilon} + \frac{[1-\gamma + c_s(1-\alpha)](\theta - \beta s)}{\gamma[1 + c_s(1-\alpha)^2]} \tag{70b}$$

[36] Note that the limit as $\varepsilon \to \infty$ of (70b) is equal to (69d), confirming that in the absence of the externality the optimal rule would be given by (69d). The socially optimal settings of the rule parameters in the specification given by (60) would in the atomistic case be $\overline{m}^* = 0$, $\lambda_1^* = \rho_2^*$, $\lambda_2^* = -\rho_2^*\beta$, and $\lambda_3^* = \rho^*_{1,atomisticcase}$, where $\rho^*_{1,atomisticcase}$ is given by (70a).

Central to intuitive explanation of (70a) is the fact that ρ_1 is absent from (66b), the expression for equilibrium employment when unions are atomistic. This is attributable, of course, to each atomistic union's perception that its individual wage decision has a negligible impact on w and hence on the monetary response, regardless of this parameter's value. Hence in the atomistic case, as in the single-union case, ρ_1 can only be used to eliminate the impact of the anticipated component of the shock on the price level.

The expected social loss under the optimal rule is divisible into a term in the variance of union forecasts, $\beta\sigma_\theta^2$, and a term in the variance of union forecast errors, $\beta\sigma_u^2$. For the two extremes of wage-bargaining structure, the representative union's weight parameter c_u solely features in the term in $\beta\sigma_\theta^2$, while society's weight c_s is to be found only in the term in $\beta\sigma_u^2$:

$$E(\Omega^s|_{m=m_r^*,n=1}) = \frac{c_u^2(1-\alpha)^2\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]^2} + \frac{c_s\beta\sigma_u^2}{[1+c_s(1-\alpha)^2]} \tag{71a}$$

$$E(\Omega^s|_{m=m_r^*,\,atomistic\,case}) = \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\beta\sigma_\theta^2}{\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} + \frac{c_s\beta\sigma_u^2}{[1+c_s(1-\alpha)^2]} \tag{71b}$$

Since the wage-setting externality exacerbates employment variability, and price level variability under the optimal rule is the same for these two extreme cases, it follows that (71a) must be greater than (71b).[37]

III.5 Discretionary Monetary Policy

III.5.1 The Central Bank's Optimal Setting of its Monetary Instrument

This section modifies the basic model of this chapter by assuming that the monetary authorities are not constitutionally bound to conduct policy in accordance with a rule, and are instead endowed with the discretionary power to set the money supply as they see fit. As in previous sections, the information structure is as represented in Time

[37] It is straightforward to show, using (71a) and (71b), that this is indeed the case.

Lines III.1 and III.2, with the central bank fully informed about both the shock and the aggregate wage when setting m. The central bank's task is to minimise its assigned loss function, (9), by choice of m, and carrying out this optimisation exercise (which involves substituting (15) and the aggregate counterpart to (14) into (9)), the solution (denoted m^*) to the first-order condition $\partial\Omega^{cb}/\partial m = 0$,[38] is found to be as follows:

$$m^* = \frac{[\gamma - c_b\alpha(1-\alpha)]w + [1-\gamma + c_b(1-\alpha)]\theta}{\gamma[1+c_b(1-\alpha)^2]} \tag{72}$$

Note that for the $\sigma_\theta^2 = 0$ case, which has $\theta_t = 0 \forall t$, (72) simplifies to equation (26) of Chapter II. Hence the degree of monetary accommodation of wages in the stochastic model of this chapter is the same as in the non-stochastic version, and indeed is the same as in Coricelli et al. (2004a, 2006). It will be useful for the subsequent discussion to state the price level as a function of w and θ, given optimising central bank behaviour, and substituting (72) into (15), this p is found to be:

$$p\big|_{m=m^*} = \frac{w-\theta}{[1+c_b(1-\alpha)^2]} \tag{73}$$

III.5.2 Efficient Wage-Setting under Discretion

Appropriate substitutions involving (6), (10), (13′), (72) and (73) then yield the expected loss of the representative union conditional on s:

$$E(\Omega_j^u \mid s) = E\left[\frac{1}{[1+c_b(1-\alpha)^2]^2}\left\{\frac{\{[\varepsilon - c_b\alpha(1-\alpha)]w - \varepsilon[1+c_b(1-\alpha)^2]w_j\}}{[\alpha+\varepsilon(1-\alpha)]} + c_b(1-\alpha)\theta\right\}^2\right.$$

[38] Note that $\partial E\Omega^{cb}/\partial m = \partial\Omega^{cb}/\partial m$ here. The version of the model with discretionary monetary policy captures both the scenario in which m is set prior to the simultaneous-move price-setting game between firms (i.e. prior to the determination of p), and the scenario in which m is set simultaneously with the determination of p. The two scenarios are equivalent since even if firms do not directly observe m they can perfectly predict its value: each firm knows θ, w, and the economy's structure, including the central bank's loss function, at the time it choose its price, and hence can predict the market-clearing price and central bank's choice of m with complete accuracy. Similarly, the central bank's information is such that it has perfect foresight regarding the market-clearing price which will result from its optimal setting of m.

$$+ \ c_u\left\{w_j + \frac{(\theta - w)}{[1+c_b(1-\alpha)^2]}\right\}^2 \left| s \right] \tag{74}$$

The unions' collectively efficient wage, w^*, is the solution to the first-order condition $dE(\Omega_j^u \mid s)\big|_{w_j = w \forall j} \big/ dw = 0$:

$$w^* = \frac{(c_b - c_u)\beta s}{c_b[1+c_u(1-\alpha)^2]} \tag{75}$$

Combining (75) with (73) and the symmetric-wage labour demand curve, as given by (14), yields expressions for the price level, real wage, and employment under efficient wage-setting:

$$p\big|_{w=w^*} = -\frac{c_u\beta s}{c_b[1+c_u(1-\alpha)^2]} - \frac{(\theta - \beta s)}{[1+c_b(1-\alpha)^2]} \tag{76}$$

$$(w-p)\big|_{w=w^*} = \frac{\beta s}{[1+c_u(1-\alpha)^2]} + \frac{(\theta - \beta s)}{[1+c_b(1-\alpha)^2]} \tag{77a}$$

$$l\big|_{w=w^*} = \frac{c_u(1-\alpha)\beta s}{[1+c_u(1-\alpha)^2]} + \frac{c_b(1-\alpha)(\theta - \beta s)}{[1+c_b(1-\alpha)^2]} \tag{77b}$$

The variances of these expressions are as follows:

$$E(p\big|_{w=w^*})^2 = \frac{c_u^2\beta\sigma_\theta^2}{c_b^2[1+c_u(1-\alpha)^2]^2} + \frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{78}$$

$$E[(w-p)\big|_{w=w^*}]^2 = \frac{\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]^2} + \frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{79a}$$

$$E(l\big|_{w=w^*})^2 = \frac{c_u^2(1-\alpha)^2\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]^2} + \frac{c_b^2(1-\alpha)^2\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{79b}$$

The associated value of the unconditional expected union loss is:

$$E(\Omega_j^u\big|_{w=w^*}) = \frac{c_u \beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]} + \frac{[c_u + c_b^2(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{80}$$

Comparing (77a) and (77b) with their counterpart expressions for the case of efficient wage-setting under the simple rule, namely (20a) and (20b), reveals that the distribution of the impact of βs between the real wage and employment is the same under discretion as under the simple rule. This is not surprising: by taking into account the price-level implications of the central bank's response to the aggregate wage, unions can, via fully coordinated wage-setting, bring about their preferred distribution of βs just as they can when m is fixed. However, in general discretion does differ from the simple rule as regards how the impact of the shock's unanticipated component, $\theta - \beta s$, is divided between the real wage and employment. Comparing (77a) and (77b) with (20a) and (20b), the sole exception to this general truth is found to be the special case in which $c_b = (\gamma-1)/(1-\alpha)$. (Note that with $c_b > 0$ this case can only arise if $\gamma > 1$.) Evaluating the central bank's optimal monetary response, as given by (72), for this special case, we find that it results in the central bank disregarding the productivity shock and only responding to w:

$$m^*\big|_{c_b=(\gamma-1)/(1-\alpha)} = \frac{w}{\gamma} \tag{81}$$

The reason for this is that in this special case the movement in the price level occasioned by the unanticipated component of the shock is sufficient in itself to bring about the central bank's preferred apportionment of its impact between the price level and employment. Since the money supply is unresponsive to $\theta - \beta s$ (and, indeed, is only responsive to βs indirectly via its response to w), the impact of $\theta - \beta s$ must be the same under discretion when $c_b = (\gamma-1)/(1-\alpha)$ as under the simple rule.

A remaining noteworthy point regarding these results for the case of efficient wage-setting under discretion is that the efficient nominal wage, as given by (75), is zero and unresponsive to the signal if $c_b = c_u$ happens to be the case. The reason why efficiency requires unions to ignore signals when $c_b = c_u$ is because in this special case the central bank, in pursuing its optimal apportionment of both components of the shock between employment and the price level, necessarily brings about the most desirable apportionment of it between the real wage and employment from unions' collective viewpoint. As we shall see, this special $c_b = c_u$ case in which signals are of no value to unions which are able to coordinate their wage-setting efficiently will figure prominently in subsequent sections of this chapter.

III.5.3 Equilibrium Wage-Setting under Discretion

III.5.3.(i) Derivation of the Equilibrium Nominal Wage

We now proceed to the case of equilibrium wage-setting, in which each union chooses its individual nominal wage, w_j, to minimise its expectation, conditional on s, of its loss, as given by (74). In doing so, union j takes into account the fact that $\partial w / \partial w_j = 1/n$, and treats the wage decisions of other unions as given. This leads to the following first-order condition for union j's optimal wage:

$$\frac{dE(\Omega_j^u \mid s)}{dw_j} = \frac{2}{n[1+c_b(1-\alpha)^2]^2}\left\{ \frac{[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]}{[\alpha+\varepsilon(1-\alpha)]^2}\{[c_b\alpha(1-\alpha)-\varepsilon]E(w\mid s) \right.$$
$$+\varepsilon[1+c_b(1-\alpha)^2]w_j - c_b(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\beta s\} +$$
$$\left. + c_u\{-E(w\mid s)+[1+c_b(1-\alpha)^2]w_j+\beta s\}\Phi_b \right\} = 0 \qquad (82)$$

where $\Phi_b \equiv n[1+c_b(1-\alpha)^2]-1$.

The w_j solution to (82) is of little interest in itself, and we therefore omit this expression, proceeding directly instead to the symmetric Nash equilibrium wage which is obtained by imposing $w_j = E(w \mid s) = w_{NE}$ in (82) and solving for w_{NE}:

$$w_{NE} = \frac{1}{c_b(1-\alpha)}\left[\frac{c_b^2\alpha(1-\alpha)^2+\{c_b\varepsilon(1-\alpha)-c_u[\alpha+\varepsilon(1-\alpha)]\}\Phi_b}{c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b}\right]\beta s \qquad (83)$$

It is apparent from (83) that zero is an inadmissible value for c_b: throughout this chapter we in fact restrict c_b to be positive. Substituting (83) into (73), the price level under discretion and equilibrium wage-setting is found to be:

$$p\big|_{w=w_{NE}} = \frac{-c_u[\alpha+\varepsilon(1-\alpha)]\Phi_b\beta s}{c_b(1-\alpha)[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]} - \frac{(\theta-\beta s)}{[1+c_b(1-\alpha)^2]} \tag{84}$$

Comparing (83) and (84), it is apparent that the equilibrium wage can be written as follows:

$$w_{NE} = E[(p\big|_{w=w_{NE}}) \mid s] + \frac{[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]\beta s}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]} \tag{83'}$$

For the extremes of wage-bargaining structure we have:

$$w_{NE}\big|_{n=1} = \frac{(c_b-c_u)\beta s}{c_b[1+c_u(1-\alpha)^2]} \tag{85}$$

$$\lim_{n\to\infty} w_{NE} = \frac{\{c_b\varepsilon(1-\alpha)-c_u[\alpha+\varepsilon(1-\alpha)]\}\beta s}{c_b(1-\alpha)\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} \tag{86}$$

For the limiting case in which the goods market is perfectly competitive, we find that:

$$\lim_{\varepsilon\to\infty} w_{NE} = \frac{(c_b-c_u)\beta s}{c_b[1+c_u(1-\alpha)^2]} \tag{87}$$

The real wage and employment are found by combining (83) with (84) and the aggregate version of (14):

$$(w-p)\big|_{w=w_{NE}} = \frac{[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]\beta s}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]} + \frac{(\theta-\beta s)}{[1+c_b(1-\alpha)^2]} \tag{88a}$$

$$l\big|_{w=w_{NE}} = \frac{c_u[\alpha+\varepsilon(1-\alpha)]\Phi_b\beta s}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]} + \frac{c_b(1-\alpha)(\theta-\beta s)}{[1+c_b(1-\alpha)^2]} \tag{88b}$$

If there is a single union and/or the goods market is perfectly competitive (88a) and (88b) become:

$$(w-p)\big|_{w=w_{NE},\,n=1} = \lim_{\varepsilon\to\infty}[(w-p)\big|_{w=w_{NE}}] = \frac{\beta s}{[1+c_u(1-\alpha)^2]} + \frac{(\theta-\beta s)}{[1+c_b(1-\alpha)^2]} \tag{89a}$$

$$l\big|_{w=w_{NE},\,n=1} = \lim_{\varepsilon\to\infty}(l\big|_{w=w_{NE}}) = \frac{c_u(1-\alpha)\beta s}{[1+c_u(1-\alpha)^2]} + \frac{c_b(1-\alpha)(\theta-\beta s)}{[1+c_b(1-\alpha)^2]} \tag{89b}$$

Real outcomes in the atomistic case are:

$$\lim_{n\to\infty}(w-p)\big|_{w=w_{NE}} = \frac{\varepsilon\beta s}{\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} + \frac{(\theta-\beta s)}{[1+c_b(1-\alpha)^2]} \tag{90a}$$

$$\lim_{n\to\infty} l\big|_{w=w_{NE}} = \frac{c_u[\alpha+\varepsilon(1-\alpha)]\beta s}{\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} + \frac{c_b(1-\alpha)(\theta-\beta s)}{[1+c_b(1-\alpha)^2]} \tag{90b}$$

The variances of the real wage and employment, and their derivatives with respect to n and ε, are as follows:

$$E[(w-p)\big|_{w=w_{NE}}]^2 = \frac{[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]^2\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2} + \frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{91a}$$

$$E(l\big|_{w=w_{NE}})^2 = \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Phi_b^2\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2} + \frac{c_b^2(1-\alpha)^2\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{91b}$$

$$\frac{\partial E[(w-p)\big|_{w=w_{NE}}]^2}{\partial n} = \frac{-2c_uc_b\alpha(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)][1+c_b(1-\alpha)^2][c_b\alpha(1-\alpha)+\varepsilon\Phi_b]\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3} \tag{92a}$$

(Note that $\partial E[(w-p)\big|_{w=w_{NE}}]^2/\partial n < 0\,\forall n \geq 1$.)

$$\frac{\partial E(l|_{w=w_{NE}})^2}{\partial n}=\frac{2c_u^2c_b\alpha(1-\alpha)[\alpha+\varepsilon(1-\alpha)]^2[1+c_b(1-\alpha)^2]\Phi_b\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3}>0\,\forall\, n\geq 1 \tag{92b}$$

$$\frac{\partial E[(w-p)|_{w=w_{NE}}]^2}{\partial \varepsilon}=\frac{2(n-1)c_u\alpha(1-\alpha)[1+c_b(1-\alpha)^2][c_b\alpha(1-\alpha)+\varepsilon\Phi_b]\Phi_b\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3} \tag{93a}$$

(Note that $\partial E[(w-p)|_{w=w_{NE}}]^2/\partial\varepsilon>0\,\forall\, n>1$.)

$$\frac{\partial E(l|_{w=w_{NE}})^2}{\partial \varepsilon}=\frac{-2(n-1)c_u^2\alpha[\alpha+\varepsilon(1-\alpha)][1+c_b(1-\alpha)^2]\Phi_b^2\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3}<0\,\forall\, n>1 \tag{93b}$$

Union j's unconditional expected loss under equilibrium wage-setting is obtained by combining (91a) and (91b) with the unconditional expectation of (10):

$$E(\Omega_j^u|_{w=w_{NE}})=\frac{c_u\{c_u[\alpha+\varepsilon(1-\alpha)]^2\Phi_b^2+[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]^2\}\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2}+\frac{[c_u+c_b^2(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{94}$$

If there is a single union or the goods market is perfectly competitive, the expected loss is:

$$E(\Omega_j^u|_{w=w_{NE},\,n=1})=\lim_{\varepsilon\to\infty}[E(\Omega_j^u|_{w=w_{NE}})]=\frac{c_u\beta\sigma_\theta^2}{[1+c_u(1-\alpha)^2]}+\frac{[c_u+c_b^2(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{95}$$

If unions are atomistic, the expected loss is:

$$\lim_{n\to\infty}E(\Omega_j^u|_{w=w_{NE}})=\frac{c_u\{\varepsilon^2+c_u[\alpha+\varepsilon(1-\alpha)]^2\}\beta\sigma_\theta^2}{\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2}+\frac{[c_u+c_b^2(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{96}$$

The derivatives of (94) with respect to n and ε are as follows:

$$\frac{\partial E(\Omega_j^u|_{w=w_{NE}})}{\partial n} = \frac{2(n-1)c_u^2 c_b \alpha^2 (1-\alpha)[\alpha+\varepsilon(1-\alpha)][1+c_b(1-\alpha)^2]^2 \beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3} > 0 \forall n>1$$

(97)

$$\frac{\partial E(\Omega_j^u|_{w=w_{NE}})}{\partial \varepsilon} = \frac{-2(n-1)^2 c_u^2 \alpha^2 [1+c_b(1-\alpha)^2]^2 \Phi_b \beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3} < 0 \forall n>1 \qquad (98)$$

III.5.3.(ii) Discussion of the Wage-Setting Externality

Expressions (83) to (98) reveal that under discretion an adverse wage-setting externality arises when union wage decisions in response to a non-zero signal are uncoordinated, with the character of this externality being very similar to that which was earlier shown to be present under the simple-rule regime. Because of the closeness of the similarity, a detailed explanation of the externality's source under discretion is not warranted, and we therefore confine our remarks to the most salient points. The externality results in the term in $\beta\sigma_\theta^2$ in the variance of employment, i.e. the term relating to the component of each shock which is anticipated by unions, being inefficiently high, and also results in the corresponding term in the variance of the real wage being inefficiently low. The extent of the inefficiency is increasing in n and decreasing in ε, and the externality is absent (and hence the efficient wage is set) if there is a single union and/or perfect goods-market competition.

An important aspect of the results reported above is the presence of the central bank's weight parameter c_b in the term in βs, in the expressions for the real wage and employment under equilibrium wage-setting, namely (88a) and (88b). This indicates that, in general, a parameter which is a design feature of the monetary regime does play a role in determining how the component of productivity shocks anticipated by unions impacts upon the real wage and employment. The exceptions to this general finding are revealed by expressions (89a), (89b), and (90a), (90b): if there is a single union, or unions are atomistic, then under equilibrium wage-setting the impact of the anticipated component of the shock on real outcomes is entirely independent of c_b, a result which is reminiscent of our Chapter II findings for the non-stochastic model. This contrast between society's ability, via its delegation decision in respect of c_b, to influence the real impact of the anticipated component of the shock when unions are non-atomistic, and its complete inability to do so in the two extreme cases of $n = 1$ and $n \to \infty$, can be explained in terms of the working of the wage-setting externality. If unions are non-atomistic, the monetary response consequences of each individual union's choice of wage are not negligible and hence are taken into account when making that wage decision insofar as they matter for its individual expected loss. However, the impact of w_j on the welfare of other unions is disregarded by union j, and it is the very fact that the macroeconomic impact of the induced monetary response to w_j is only partially internalised that creates the opportunity for the design of the discretionary regime to influence how the anticipated component of the shock is divided, via equilibrium wage-setting, between the real wage and employment. In the single-union case, of course, internalisation of the wage's macroeconomic repercussions is complete, so that the regime's designers are powerless to influence the impact of βs on real outcomes. They are similarly powerless in this respect in the atomistic case, where the source of the neutrality result is the fact that each atomistic union always perceives the monetary-response and aggregate-demand repercussions of its wage decision to be negligible, regardless of the monetary regime.

This brings us to one of the key results of this chapter. The result is concerned with the relationship between the variability of employment under equilibrium wage-setting, as given by (91b), and the central bank's weight parameter c_b. The first and second derivatives of (91b) with respect to c_b are as follows:

$$\frac{\partial E(l|_{w=w_{NE}})^2}{\partial c_b} = \frac{-2(n-1)c_u^2\alpha(1-\alpha)[\alpha+\varepsilon(1-\alpha)]^2\Phi_b\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3} + \frac{2c_b(1-\alpha)^2\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^3} \tag{99a}$$

$$\frac{\partial^2 E(l|_{w=w_{NE}})^2}{\partial c_b^2} = \frac{2(n-1)c_u^2\alpha(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]^2 X_3\beta\sigma_\theta^2}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^4} + \frac{2(1-\alpha)^2[1-2c_b(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^4} \tag{99b}$$

where $X_3 \equiv \alpha(n-1+2\Phi_b)+2n(1-\alpha)\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b$.

If we proceed on the assumption that $1 < n < \infty$, it follows directly from (99a) that a necessary and sufficient condition for the variance of equilibrium employment to be a decreasing function of c_b is the following:

$$\frac{\partial E(l|_{w=w_{NE}})^2}{\partial c_b} < 0 \text{ iff } \frac{\sigma_\theta^2}{\sigma_u^2} > \Psi \tag{100}$$

where $\Psi \equiv \dfrac{c_b(1-\alpha)[c_b\alpha(1-\alpha)+\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3}{(n-1)c_u^2\alpha[\alpha+\varepsilon(1-\alpha)]^2[1+c_b(1-\alpha)^2]^3\Phi_b}$.

If it is the case that $\sigma_\theta^2/\sigma_u^2 < \Psi$, then it necessarily follows that $\partial E(l|_{w=w_{NE}})^2/\partial c_b > 0$, while the case in which $\sigma_\theta^2/\sigma_u^2 = \Psi$ is that in which a marginal increase in c_b has no impact on the variance of employment.

In interpreting (100), it is helpful to bear in mind that it has been derived on the assumption that σ_u^2, ε and n are finite and positive, and that n exceeds unity. The composite parameter Ψ has a limiting value of zero as $c_b \to 0$, is a monotonic increasing function of c_b for all admissible values of c_b (i.e. for all $c_b > 0$), and has a positive finite asymptotic value as $c_b \to \infty$.[39] The ratio $\sigma_\theta^2/\sigma_u^2$, meanwhile, is a positive decreasing convex function of σ_u^2, and therefore may take any value on the positive portion of the real line. Several important conclusions immediately follow from these facts. The principal conclusion is that when there are multiple non-atomistic unions and the goods market is monopolistically competitive, there must exist a range of c_b values for which marginal increases in c_b bring about a reduction in employment variability. This range of c_b values has a lower bound of zero and an upper bound at the c_b value which solves the equation $\sigma_\theta^2/\sigma_u^2 = \Psi$. A secondary conclusion is that if the quality of the unions' signal is sufficiently high (in other words, if the signal noise variance, σ_u^2, is small enough), the ratio $\sigma_\theta^2/\sigma_u^2$ will be large enough to ensure that $\sigma_\theta^2/\sigma_u^2 > \lim_{c_b \to \infty}\Psi$, so that increases in the weight c_b will then (provided $1 < n < \infty$) result in lower employment variability regardless of the

[39] These facts concerning Ψ are established in Appendix III.3.

current value of c_b.[40] In other words, if $\sigma_\theta^2/\sigma_u^2 > \lim_{c_b\to\infty}\Psi$ is the case, the variance of equilibrium employment is a monotonic decreasing function of c_b for all $c_b > 0$, and is therefore at its lowest under an ultraconservative central banker (i.e. in the limiting $c_b \to \infty$ case).

The significance of these results is that they provide an alternative explanation to those cited in Chapter I for why empirical studies such as Alesina and Summers (1993) have not, in general, found evidence in support of the Rogoff (1985) model's prediction that more conservative central banks will be associated with higher employment variability. The Rogoff model, it will be recalled, assumes that the economy consists of atomistic private agents who do not possess any information regarding aggregate supply shocks at the time that they form their expectations of inflation. Its structure therefore abstracts entirely from the possible existence of externalities in union wage-setting decisions made in response to signals of supply shocks.

In order to gain a little more insight into the result, and how it is underpinned by the externality and the non-atomistic status of unions, it is useful to note that (99a) decomposes the derivative of the variance of equilibrium employment into a term in $\beta\sigma_\theta^2$ (the variance of union forecasts) and a term in $\beta\sigma_u^2$ (the variance of union forecast errors). The term in $\beta\sigma_\theta^2$ is the externality-mitigating effect of a marginal increase in c_b. Note that this term is zero if $n=1$ and/or the goods market is perfectly competitive (the limiting $\varepsilon \to \infty$ case), and/or signals provide no information regarding θ (the limiting $\sigma_u^2 \to \infty$ case). In all of these special cases the externality is absent, and consequently increased central bank conservatism cannot have an externality-mitigating effect. The term in $\beta\sigma_\theta^2$ also disappears in the limiting $n \to \infty$ case, since although the externality is very much present when unions are atomistic, the fact that the money supply is beyond each union's individual influence renders the externality independent of c_b. Provided $1 < n < \infty$ is the case, however, and provided ε and σ_u^2 are finite, the term in $\beta\sigma_\theta^2$ in (99a) is negative, so that the component of

[40] Note that while (100) has been derived on the assumption that $\sigma_u^2 > 0$, it is nevertheless also the case that $\partial E[(l|_{w=w_{NE}})^2]/\partial c_b < 0$ when $\sigma_u^2 = 0$. This follows straightforwardly from (99a), the second term of which is zero when $\sigma_u^2 = 0$.

employment variability relating to unions' forecasts (the term in $\beta\sigma_\theta^2$ in (91b)), is a monotonic decreasing function of c_b (indeed, (99b) reveals it to be convex in c_b).

The reason why a higher value for c_b mitigates the externality here is essentially the same as the reason why it does so in the non-stochastic model of Chapter II (when c_b is restricted to be positive). The explanation has two parts to it. Firstly, the higher is c_b, the less accommodating of the aggregate wage is the central bank, as is evident from the following derivative, which follows directly from (72):

$$\frac{\partial^2 m^*}{\partial c_b \partial w} = \frac{-(1-\alpha)[\alpha + \gamma(1-\alpha)]}{\gamma[1+c_b(1-\alpha)^2]} < 0 \tag{101}$$

Secondly, this in turn implies that the aggregate demand channel via which the individual non-atomistic union's wage influences its labour demand is strengthened by an increase in c_b. To see this, note that the aggregate demand channel, $(\gamma/[\alpha+\varepsilon(1-\alpha)])(d(m-p)/dw_j)$, is negative for $c_b > 0$, i.e. a marginal increase in union *j*'s wage reduces real money balances and hence also aggregate demand, and causes a contraction in the demand for its labour. Furthermore, the aggregate demand channel is more negative, and hence stronger, the higher is c_b, as is evident from the following expressions obtained by combining (72) and (73):

$$m - p = \frac{-c_b\alpha(1-\alpha)w + [1+c_b(1-\alpha)]\theta}{\gamma[1+c_b(1-\alpha)^2]} \tag{102a}$$

$$\frac{d(m-p)}{dw_j} = \frac{-c_b\alpha(1-\alpha)}{n\gamma[1+c_b(1-\alpha)^2]} \tag{102b}$$

$$\frac{\partial[d(m-p)/dw_j]}{\partial c_b} = \frac{-\alpha(1-\alpha)}{n\gamma[1+c_b(1-\alpha)^2]^2} \tag{102c}$$

(102c) implies that the employment cost to union *j* of marginally increasing its wage is larger, the more conservative the central bank, and consequently greater conservatism mitigates the externality.

III.5.4 The Stochastic Inflation Bias and the Optimal Choice of Central Banker

We have yet to derive an expression for the variance of the price level under equilibrium wage-setting when the monetary regime is discretionary. Squaring the right-hand side of (84) and finding its unconditional expectation, this variance is found to be:

$$E(p|_{w=w_{NE}})^2 = \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Phi_b^2\beta\sigma_\theta^2}{c_b^2(1-\alpha)^2[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2} + \frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \qquad (103)$$

For the extreme cases of a single union and atomistic unions, (103) reduces to:

$$E(p|_{w=w_{NE},\, n=1})^2 = \frac{c_u^2\beta\sigma_\theta^2}{c_b^2[1+c_u(1-\alpha)^2]^2} + \frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \qquad (104)$$

$$\lim_{n\to\infty} E(p|_{w=w_{NE}})^2 = \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\beta\sigma_\theta^2}{c_b^2(1-\alpha)^2\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} + \frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \qquad (105)$$

We will comment initially on these two extreme cases since the case of multiple non-atomistic unions, in respect of which (103) is relevant, has several complicating aspects. The terms in $\beta\sigma_\theta^2$ in (104) and (105) are examples of the stochastic inflation bias which characterises this model. It will be recalled from the discussion of Herrendorf and Lockwood (1997) in Chapter I that the stochastic inflation bias is the bias in the variance of inflation which arises as a consequence of the assumption incorporated into the model that society's optimal rule is not credible. As we saw earlier in Section III.4, rules, no matter how designed, cannot affect the employment impact of βs when wages are set by either a single union or by atomistic unions. Society's optimal rule in these extreme cases therefore involves using m to neutralise completely the potential impact of βs on the price level. Given the model's implicit assumptions, such a rule is not credible, however. Were a single union naïvely to expect the authorities to adhere to (69d), or were atomistic unions naïvely to expect adherence to (70b), the incorporation of these naïve expectations into nominal wages would create the opportunity for the authorities to bring about, via a monetary surprise, a better distribution (from their point of view) of the impact of βs between employment and the price level, than that which would result from adherence to the optimal rule, (69d) or (70b). In other words, while it is ex-ante optimal for the authorities to declare an intention to adhere to (69d) when $n=1$, or to (70b) when unions are atomistic, once expectations have been formed, whether naïvely or rationally, the central bank's ex-post optimal policy is to set the money supply in accordance with (72) rather than (69d) or (70b). Thus the time-inconsistency of the optimal rule when there is a single union or when unions are atomistic, causes unions' rational expectation, conditional on s, of the impact of the shock on the price level to be non-zero whenever the signal happens to have a non-zero realisation. Given such a non-zero rational expectation of the price level, and the associated adjustment of nominal wage(s) by union(s), the optimising central bank cannot do better than to set m in reaction to w and βs in the way that the union(s) expect it to do. Of course, union expectations of m and p, conditional on s, may still prove incorrect, but only to the extent that their forecasts of the shock turn out to be inaccurate (i.e. if $\beta s \neq \theta$ transpires to be the case), and not because of any error in predicting the realisations of m and p which will occur in the absence of union forecast errors regarding the shock. The implication of this is that when the variance of the price level is decomposed into terms in the variances of the anticipated and unanticipated components of the shock, as has been done in (104) and (105), the term in $\beta\sigma_\theta^2$ in that decomposition is found to be positive, rather than zero, the value it would have were the rule fully credible and were the authorities always to abide by the rule. The extent by which the term in $\beta\sigma_\theta^2$ in (104) and (105) differs from zero is the stochastic inflation bias. Using these expressions, it is straightforward to demonstrate that $\lim_{n\to\infty} E(p|_{w=w_{NE}})^2 > E(p|_{w=w_{NE},\, n=1})^2$. In other words the stochastic inflation bias is larger when unions are atomistic than when there is a single union. The reason for this is that the externality which arises under atomistic unions strengthens the employment impact of a given non-zero anticipated component of the shock, and therefore increases the authorities' temptation to renege on the optimal rule. Thus the time-inconsistency problem is more severe when unions are atomistic, than when there is a single union, because the externality raises the marginal benefit of implementing a given monetary surprise without affecting that surprise's marginal cost.

That the externality exacerbates the stochastic inflation bias is readily apparent from the following expressions:

$$\frac{\partial[\lim_{n\to\infty} E(p|_{w=w_{NE}})^2]}{\partial\varepsilon} = \frac{-2c_u^2\alpha[\alpha+\varepsilon(1-\alpha)]\beta\sigma_\theta^2}{c_b^2(1-\alpha)^2\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^3} < 0 \tag{106}$$

$$\lim_{\varepsilon\to\infty}[\lim_{n\to\infty} E(p|_{w=w_{NE}})^2] = E(p|_{w=w_{NE},\ n=1})^2 \tag{107}$$

Since we already know that increases in ε mitigate the externality, (106) reveals that this mitigation effect is accompanied by a reduction in the stochastic inflation bias, while (107) tells us that under perfect goods-market competition, which is known to prevent the externality from arising, the stochastic inflation bias is the same when unions are atomistic as when there is a single union.

An important aspect of the stochastic inflation bias is that it is a monotonic decreasing (indeed, convex) function of the central bank's weight parameter c_b. A higher c_b alleviates the time-inconsistency problem underlying the bias by increasing the cost to the central bank of allowing the price level to deviate from zero, which in turn makes the bank less inclined to use monetary policy to offset the employment impact of a non-zero βs. To work out the socially optimal choice of c_b, however, we must also consider how this parameter affects the apportionment of the unanticipated component of the shock between employment and the price level. The optimal apportionment is brought about by a representative central banker (i.e. one such that $c_b = c_s$). It follows immediately that a representative central banker cannot be socially optimal since appointing someone marginally conservative must on the one hand result in only a negligibly small ('second order') increase in the part of the social loss relating to the stabilisation of the unanticipated component of the shock, but on the other hand cause a non-negligible ('first order') decrease in the stochastic inflation bias. Similar intuitive arguments rule out the possibility of a liberal central banker (i.e. one such that $c_b < c_s$) being socially optimal. If $c_b < c_s$ is the case, a marginal increase in c_b both reduces the stochastic inflation bias and brings the stabilisation of

$\theta - \beta s$ carried out by the central bank into closer alignment with society's preferences in respect of it, so that the impact of both the anticipated and unanticipated components of the shock on the social loss must be improved. It follows that for the two extremes of wage-bargaining structure society's optimal delegation choice must be such that $c_b > c_s$, and hence must involve either a conservative or an ultraconservative central banker.

To pursue this question more formally, we state society's expected loss when there is a single union. (This loss is obtained by combining (91b) evaluated for $n=1$ and (104) with the social loss function (8), with the socially optimal employment in the latter assumed to coincide with its market-clearing value of zero, while the socially optimal inflation rate is also assumed to be zero: hence $l^* = 0$ and $\pi^* = 0$.)

$$E(\Omega^s\big|_{w=w_{NE},\, n=1}) = \frac{c_u^2[c_s + c_b^2(1-\alpha)^2]\beta\sigma_\theta^2}{c_b^2[1+c_u(1-\alpha)^2]^2} + \frac{[c_s + c_b^2(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{108}$$

Differentiating (108) with respect to c_b yields:

$$\frac{\partial E(\Omega^s\big|_{w=w_{NE},\, n=1})}{\partial c_b} = \frac{-2c_s c_u^2 \beta\sigma_\theta^2}{c_b^3[1+c_u(1-\alpha)^2]^2} + \frac{2(c_b - c_s)(1-\alpha)^2\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^3} \tag{109}$$

This derivative is negative for all $c_b \le c_s$, confirming the intuitive argument provided a moment ago that society's optimal choice of central banker is neither a liberal nor one with representative preferences. It is apparent from (109) that when $c_b > c_s$ a trade-off arises between the beneficial effect on the stochastic inflation bias of an increase in conservatism, and its detrimental effect of making the distribution of $\theta - \beta s$ between employment and the price level depart still further from that preferred by society. Society's optimal choice of central banker is that which exploits this trade-off to the full. In formal terms, identifying this choice involves examining the solution(s), which we denote by c_b^*, to the first-order condition $\partial E(\Omega^s\big|_{w=w_{NE},\, n=1}) / \partial c_b = 0$. It turns out, however, that explicitly solving for c_b^* is undesirable because of the complexity of the resulting expression. Instead, following the procedure used by Herrendorf and Lockwood (1997),[41] we consider the following equation which is derived from the first-order condition, and which implicitly defines c_b^*:

$$\frac{c_s c_u^2 \sigma_\theta^2}{(c_b^*)^3[1+c_u(1-\alpha)^2]^2} = \frac{(c_b^* - c_s)(1-\alpha)^2 \sigma_u^2}{[1+c_b^*(1-\alpha)^2]^3} \tag{110}$$

The second derivative of (108) with respect to c_b is:

$$\frac{\partial^2 E(\Omega^s|_{w=w_{NE},\, n=1})}{\partial c_b^2} = \frac{6c_s c_u^2 \beta \sigma_\theta^2}{c_b^4[1+c_u(1-\alpha)^2]^2} + \frac{2(1-\alpha)^2[1+(3c_s-2c_b)(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^4} \tag{111}$$

Substituting (110) into (111) we have:

$$\left(\frac{\partial^2 E(\Omega^s|_{w=w_{NE},\, n=1})}{\partial c_b^2}\right)\Bigg|_{c_b=c_b^*} = \frac{2(1-\alpha)^2}{[1+c_b^*(1-\alpha)^2]^3}\left\{1+\frac{3(c_b^*-c_s)}{c_b^*[1+c_b^*(1-\alpha)^2]}\right\}\beta\sigma_u^2 > 0 \,\forall c_b^* > c_s \tag{112}$$

Since we have already established that there cannot be a solution to the first-order condition such that $c_b^* < c_s$, it follows from (112) that all such solutions must be minima, and that c_b^* must therefore be a unique solution. Hence it must be the case that $c_s < c_b^* < \infty$, so that the socially optimal choice of central banker when $n=1$ is a conservative who is not indifferent to employment stabilisation.

Following a similar procedure for the atomistic case reveals that for it too the socially optimal central banker is a conservative. In the atomistic case the expected social loss (obtained using (105) and the limiting $n \to \infty$ case of (91b)), and its derivative with respect to c_b, are found to be the following:

$$\lim_{n\to\infty} E(\Omega^s|_{w=w_{NE},\, n=1}) = \frac{c_u^2[c_s + c_b^2(1-\alpha)^2][\alpha+\varepsilon(1-\alpha)]^2\beta\sigma_\theta^2}{c_b^2(1-\alpha)^2\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} + \frac{[c_s + c_b^2(1-\alpha)^2]\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2} \tag{113}$$

[41] As described in the Appendix to their paper, Herrendorf and Lockwood (1997), pp.492-493.

$$\frac{\partial[\lim_{n\to\infty} E(\Omega^s|_{w=w_{NE},\, n=1})]}{\partial c_b} = \frac{-2c_s c_u^2[\alpha+\varepsilon(1-\alpha)]^2\beta\sigma_\theta^2}{c_b^3(1-\alpha)^2\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} + \frac{2(c_b-c_s)(1-\alpha)^2\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^3} \tag{114}$$

The socially optimal c_b, c_b^*, is therefore the solution to the equation $f(c_b^*,\varepsilon)=0$, where:

$$f(c_b^*,\varepsilon) \equiv \frac{-c_s c_u^2[\alpha+\varepsilon(1-\alpha)]^2\sigma_\theta^2}{(c_b^*)^3(1-\alpha)^2\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} + \frac{(c_b^*-c_s)(1-\alpha)^2\sigma_u^2}{[1+c_b^*(1-\alpha)^2]^3} \tag{115}$$

Were we to obtain the second derivative of (113) with respect to c_b, and, by making use of (115), evaluate it for $c_b = c_b^*$, we would find it to be unambiguously positive, indicating that c_b^* is such that $c_s < c_b^* < \infty$. We shall refrain from stating this second-order condition since the procedure has already been clearly set out for the $n=1$ case. One point of interest regarding society's optimal central banker for the atomistic case, however, is that since it is apparent from (115) that $\partial f(c_b^*,\varepsilon)/\partial\varepsilon > 0$ and $\partial f(c_b^*,\varepsilon)/\partial c_b^* > 0$,[42] it follows from the implicit function theorem that the socially optimal degree of central bank conservatism is lower, the stronger is competition in the goods market:

$$\frac{dc_b^*}{d\varepsilon} = -\left(\frac{\partial f/\partial\varepsilon}{\partial f/\partial c_b^*}\right) < 0 \tag{116}$$

Since in the limiting case in which $\varepsilon\to\infty$ atomistic unions set the efficient wage and bring about the same outcome as a single union, it follows that when ε is finite the optimal degree of central bank conservatism is greater when unions are atomistic than

[42] The actual expression for the derivative $\partial f(c_b^*,\varepsilon)/\partial\varepsilon$ is:
$\partial f(c_b^*,\varepsilon)/\partial\varepsilon = 2c_s c_u^2\alpha[\alpha+\varepsilon(1-\alpha)]\sigma_\theta^2/(c_b^*)^3(1-\alpha)^2\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^3 > 0$.
Since $\partial f(c_b^*,\varepsilon)/\partial c_b^*$ must have the same sign as $\left(\partial^2\{\lim_{n\to\infty}[E(\Omega^s|_{w=w_{NE}})]\}/\partial c_b^2\right)\Big|_{c_b=c_b^*}$, and the latter is known to be positive, it follows that $\partial f(c_b^*,\varepsilon)/\partial c_b^* > 0$ must be the case.

when there is a single union. Intuitively, the stronger is the externality which characterises the atomistic case, the higher is employment variability, which in turn causes a higher stochastic inflation bias. This changes the trade-off faced by society by raising the marginal benefit of an increase in c_b, without affecting its marginal cost.

Before moving on to discuss the optimal setting of c_b when there are multiple non-atomistic unions, it seems fitting to mention how the above findings regarding c_b^* would be affected were we to modify the model to allow for a non-zero mean inflation bias. Such a bias could be generated by specifying the representative union's desired mean real wage to exceed its mean market-clearing value of zero, together with appropriate specification of its desired mean employment level. The mean inflation bias would be a monotonic decreasing function of c_b, and therefore would merely increase the socially optimal setting of c_b without affecting the above conclusions qualitatively (i.e. it would remain the case that $c_s < c_b^* < \infty$). Furthermore, one of the key insights of Herrendorf and Lockwood (1997) would be applicable to our model, namely that the mean inflation bias can always be eliminated by assigning an appropriate value to an additional delegation parameter in the central bank's loss function.[43]

We now turn to discuss the relationship between the expected social loss and the central bank's weight c_b when there are multiple non-atomistic unions. The expressions relevant to the $1 < n < \infty$ case are the variance of equilibrium employment, as given by (91b), and its first and second derivatives with respect to c_b, namely (99a) and (99b), together with the variance of the price level, as given by (103). In what follows, the term in $\beta\sigma_\theta^2$ in (103) will be referred to as the stochastic inflation bias. This is slightly controversial, since it does not fully accord with the use made of the phrase by Herrendorf and Lockwood (1997), who conceive of the stochastic inflation bias as the difference between the variance of inflation under

[43] In other words, elimination of the mean inflation bias can be achieved by assigning the central bank the more general loss function mentioned in Chapter II, namely $\Omega^{cb} = (l - l_b)^2 + c_b(\pi - \pi_b)^2 + 2c_b'\pi$, with any one of the additional delegation parameters l_b, π_b or c_b' set appropriately.

discretion and the corresponding variance under society's optimal rule.[44] Although the optimal rule for the $1 < n < \infty$ case was not formally derived in Section III.4 above, it is nevertheless clear that not only the equilibrium price level, but also equilibrium employment will be a function of the rule's wage-response parameter (either λ_3 in (60) or ρ_1 in (61)). It follows from this that the optimal rule when $1 < n < \infty$ will not involve complete elimination of the impact of the anticipated component of the shock on the price level, and that price level variability under the rule will consist not only of a term in the variance of union forecast errors, $\beta\sigma_u^2$, but also of a term in the variance of the shock's anticipated component, $\beta\sigma_\theta^2$. Arguably, therefore, only a part, rather than the whole, of the term in $\beta\sigma_\theta^2$ in (103) should be referred to as the stochastic inflation bias. Despite these reservations, a persuasive case can nevertheless be made for describing this term in $\beta\sigma_\theta^2$ as a bias, provided it is borne in mind that this bias has its source in the incentives facing the discretionary central bank once it has been assigned its loss function and the weight c_b has acquired a specific fixed value. The use of the word 'bias' is justified since the representative union's perception of the central bank's incentives affects its rational expectation of the price level. Were monetary policy to be devoted to stabilising the price level perfectly with regard to the anticipated component of the shock (so that the terms in βs and $\beta\sigma_\theta^2$ in, respectively, (84) and (103) are zero), the equilibrium real wage and employment outcomes given by (88a) and (88b) could alternatively be brought about by a particular adjustment in the aggregate nominal wage which would differ from (83).[45] A declared intent on the central bank's part to follow such a policy would, in general, not be credible, however.[46] Were the central bank publicly to promise to use the money supply solely to prevent βs (when non-zero) from causing a movement in the

[44] As noted in Chapter I, Herrendorf and Lockwood do not define an objective function for society, and use the assumed objective function of the government as their reference point for deriving the optimal design of the monetary regime (i.e. the optimal set of delegation parameters in the central bank loss function). Since that government objective function is essentially the same as the social loss function of this thesis, namely (8), it follows that Herrendorf and Lockwood's definition of the stochastic inflation bias can be said to be the bias in inflation or price level variability which arises as a consequence of the entity which delegates monetary policy to the central bank not being able to precommit with full credibility to its optimal rule. In Herrendorf and Lockwood this entity is the government, whereas in our model it is society.

[45] This particular adjustment in w_{NE} would be given by the first term on the right-hand side of (88a), namely: $[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]\beta s/[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]$.

[46] It would only be credible in the limiting $c_b \to \infty$ (i.e. ultraconservative) case.

price level, it would be naïve of unions to expect the central bank to adhere to such a promise, since naïve expectations would cause the marginal benefit to the central bank of delivering a monetary surprise to exceed its marginal cost. Only the rational expectation of p, as given by (84), ensures the equality of this marginal benefit and marginal cost, and hence deprives the central bank of an incentive to deliver a monetary surprise. The presence of a term in $\beta\sigma_\theta^2$ in (103) is thus attributable to a time-inconsistency problem faced by the central bank, and for this reason it may be legitimately described as a bias.

The derivative of (103) with respect to c_b is:

$$\frac{\partial E(p|_{w=w_{NE}})^2}{\partial c_b} = \left\{ \frac{-2c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Phi_b}{c_b^3(1-\alpha)^2} \times \right.$$
$$\left. \frac{[c_b\alpha(1-\alpha)(n-1+\Phi_b)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b^2]}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3} \right\} \beta\sigma_\theta^2 - \frac{2(1-\alpha)^2}{[1+c_b(1-\alpha)^2]^3}\beta\sigma_u^2 \tag{117}$$

This expression tells us that price level variability is a monotonic decreasing function of c_b when $1<n<\infty$. Combining (117) with (99a) yields an expression for the derivative with respect to c_b of the expected social loss when $1<n<\infty$:

$$\frac{\partial E(\Omega^s|_{w=w_{NE}})}{\partial c_b} = \frac{-2c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Phi_b X_4 \beta\sigma_\theta^2}{c_b^3(1-\alpha)^2[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3} + \frac{2(c_b-c_s)(1-\alpha)^2\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^3} \tag{118}$$

where:

$$X_4 \equiv (n-1)c_b\alpha(1-\alpha)[c_s+c_b^2(1-\alpha)^2] + c_s\Phi_b[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]$$

The principal aspects of the relationship between the expected social loss and the delegation parameter c_b when $1<n<\infty$ are as follows. It is clear that (118) is negative for all admissible values of c_b such that $c_b \le c_s$. It must therefore be the case that if the monetary regime is one of discretion, appointing a conservative or ultraconservative central banker must be better for social welfare than appointing a liberal or one with 'representative' preferences. This is precisely the same result as was obtained for the two extreme cases of $n=1$ and atomistic unions. However, whereas for the latter two cases it was found that the possibility of an ultraconservative being socially optimal could be ruled out, this same conclusion does not necessarily follow for the $1<n<\infty$ case. Since employment variability can be falling in all admissible c_b provided the externality is strong enough to ensure the condition stated in (100) holds, i.e. if $\sigma_\theta^2/\sigma_u^2 > \Psi$, and since price level variability is a monotonic decreasing function of c_b regardless of signal quality, it follows that $\sigma_\theta^2/\sigma_u^2 > \Psi$ is a sufficient condition for the expected social loss (when $1<n<\infty$) to be a monotonic decreasing function of c_b, and hence for the optimal appointment to be an ultraconservative. Although marginal increases in c_b, when $c_b > c_s$, lead to worse stabilisation, from society's viewpoint, of the unanticipated component of shocks, if the externality is severe enough the externality-mitigating effect of greater conservatism, and its associated beneficial effect in reducing the stochastic inflation bias, outweighs the former detrimental effect relating to stabilisation of union forecast errors, and hence leads to an improvement in social welfare. If the quality of unions' signals is sufficiently poor, however, so that the externality is relatively weak and the unanticipated component of each shock contributes substantially to employment variability, the optimal choice of central banker is a conservative, as was found to be the case for the two extremes of wage-bargaining structure. A necessary condition for this to be the case is that $\sigma_\theta^2/\sigma_u^2 < \Psi$, since this condition must hold if the variance of employment, considered as a function of c_b, is to have a (unique) interior minimum.

III.5.5 Signal Quality and Union Welfare under Discretion

An important aspect of the equilibrium outcomes under discretion remains to be discussed, namely the relationship between unions' equilibrium expected loss and the quality of the signal, which depends of course on the variance of signal noise, σ_u^2. Bearing in mind results (28a) and (28b) regarding how a marginal increase in σ_u^2

affects the variances of the anticipated and unanticipated components of θ, it is apparent from (94) that a marginal deterioration in signal quality will have both a beneficial externality-mitigating effect, and a detrimental exacerbating effect on union forecast errors. (The former effect concerns the term in $\beta\sigma_\theta^2$ in (94), the latter effect the term in $\beta\sigma_u^2$):

$$\frac{dE(\Omega_j^u|_{w=w_{NE}})}{d\sigma_u^2} = \left\{ \frac{-c_u\{c_u[\alpha+\varepsilon(1-\alpha)]^2\Phi_b^2+[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]^2\}}{[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2} + \frac{[c_u+c_b^2(1-\alpha)^2]}{[1+c_b(1-\alpha)^2]^2} \right\}\beta^2 \tag{119}$$

It follows from (119) that when unions are non-atomistic, a necessary and sufficient condition for the externality-mitigating effect to outweigh (be outweighed by) the forecast-error effect, and hence for a marginal deterioration in signal quality to have a net beneficial (detrimental) effect on union welfare is the following:

$$\frac{dE(\Omega_j^u|_{w=w_{NE}})}{d\sigma_u^2} < (>) 0 \quad \text{iff} \quad \Delta\Theta < (>) 0 \tag{120}$$

where: $\Delta \equiv -c_b^2\alpha(1-\alpha)^2 + \{c_u[\alpha+\varepsilon(1-\alpha)] - c_b\varepsilon(1-\alpha)\}\Phi_b$

$\Theta \equiv [1+c_u(1-\alpha)^2]\Delta - 2(n-1)c_u\alpha[1+c_b(1-\alpha)^2]^2$

For convenience, and because there seems very little danger of confusing the reader, the same notation has been used here as was used to describe the equivalent result for the simple rule scenario, namely (54). The affinity of (120) to (54) is plain: under discretion, whether a marginal deterioration in signal quality is beneficial or not to union welfare depends on the sign of the product of two terms, one of which, Δ, is linear in c_u, and the other of which, Θ, is quadratic in c_u. Note that as in the simple rule scenario, Δ under discretion is equal to minus one times the numerator of the coefficient on βs in the expression for the equilibrium nominal wage (as can be seen from comparing Δ in (120) with (83)).

The graphs of Δ and Θ are depicted below in Figure III.5. These two functions of c_u have a common vertical intercept. For conformity with our notational practice for the simple rule scenario, the points at which the parabola of Θ intersects the horizontal axis (i.e. the c_u solutions to the equation $\Theta = 0$) are denoted $\hat{c}_u$ and $\hat{\hat{c}}_u$. The parallel between results (120) and (54) extends further in that just as under the simple rule we saw that there was a particular c_u value which led unions to disregard the signal, and hence rendered their equilibrium expected loss independent of σ_u^2, so too is this the case under discretion. From scrutiny of (83) it is apparent that when $c_u = c_b(1-\alpha)[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]/[\alpha+\varepsilon(1-\alpha)]\Phi_b$, the externality works in such a way as to lead unions to set a zero nominal wage regardless of s. It seems fitting to use $\tilde{\tilde{c}}_u$ to denote this particular value, since it is the counterpart under discretion of the simple-rule scenario special case discussed earlier, namely $c_u = \tilde{\tilde{c}}_{u,\,Simple\ Rule} \equiv (\gamma-1)(\gamma\alpha+\varepsilon\Lambda)/[\alpha+\varepsilon(1-\alpha)]\Lambda$. Note that $\Delta\big|_{c_u=\tilde{\tilde{c}}_u} = 0$, and hence that:

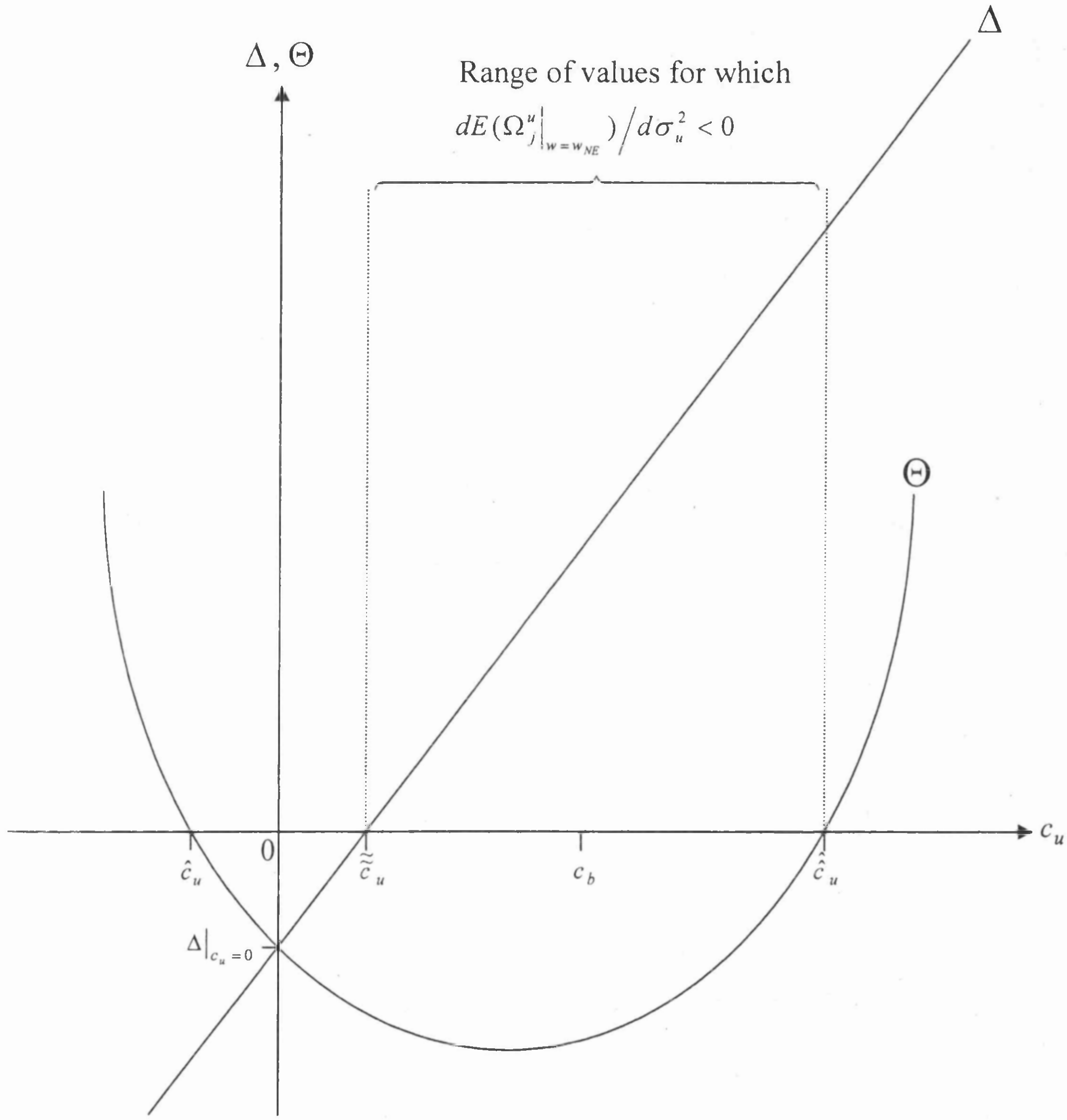
$$\left.\frac{dE(\Omega_j^u\big|_{w=w_{NE}})}{d\sigma_u^2}\right|_{c_u=\tilde{\tilde{c}}_u} = 0 \qquad (121)$$

where $\tilde{\tilde{c}}_u \equiv \dfrac{c_b(1-\alpha)[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]}{[\alpha+\varepsilon(1-\alpha)]\Phi_b}$.

The intuitive explanation for (121) is that when $c_u = \tilde{\tilde{c}}_u$, the externality-mitigating effect and the forecast error effect are precisely equal in absolute value and therefore cancel each other out.

The reader will recall that under the simple rule, when c_u takes a particular value it is efficient for unions to ignore the signal and always to set a nominal wage of zero. This simple-rule special case is that in which $c_u = \tilde{c}_{u,\,Simple\ Rule} \equiv (\gamma-1)/(1-\alpha)$, and although when this expression holds any non-zero response is inefficient, under equilibrium wage-setting the externality leads each union to adjust its wage in response to the signal. Hence in this special case under the simple rule the externality-mitigating effect of a deterioration in signal quality must outweigh the forecast-error

Figure III.5
Identification of the sign of the impact of a marginal deterioration in signal quality on the expected loss of non-atomistic unions under discretion



Notes:
1. It is assumed that $1 < n < \infty$.
2. Note that $\tilde{\tilde{c}}_u \equiv c_b(1-\alpha)[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]/[\alpha+\varepsilon(1-\alpha)]\Phi_b$, and
$\Delta|_{c_u=0} = -c_b(1-\alpha)[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]$.

effect, since the latter effect is necessarily zero. The analogous special case under discretion is that in which $c_u = c_b$. If each union places the same weight on real wage variability, relative to employment variability, as the central bank places on price level variability, relative to employment variability, it follows that when unions are completely uninformed about the shock, the conduct of monetary policy by the central bank must bring about unions' preferred apportionment of the shock's impact between employment and the real wage. Hence when $c_u = c_b$ wage-setting efficiency requires that unions completely ignore signals, and a deterioration in signal quality must be beneficial to unions.

This reasoning leads us to surmise that c_b must be a member of the set of c_u values for which $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ is the case. Sure enough, evaluating Δ and Θ for $c_u = c_b$ we find that their product is negative if there are multiple unions:

$$(\Delta|_{c_u=c_b})(\Theta|_{c_u=c_b}) = -(n-1)^2 c_b^2 \alpha^2 [1 + c_b(1-\alpha)^2]^3 < 0 \, \forall n > 1 \tag{122}$$

The range of c_u values for which $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ is the case, therefore contains c_b, has a positive lower bound of $\approx{c}_u$, and has for its upper bound the positive c_u solution to the equation $\Theta = 0$.

Note that if there is a single union, $\approx{c}_u$, $\hat{c}_u$ and c_b all coincide, implying that there is no positive c_u value for which Δ and Θ differ in sign. This is unsurprising: a worsening of signal quality cannot make the single union better off, since there is no externality to be mitigated. Formally evaluating Δ and Θ for $n = 1$ confirms this:

$$(\Delta|_{n=1})(\Theta|_{n=1}) = c_b^2 (c_u - c_b)^2 (1-\alpha)^4 [\alpha + \varepsilon(1-\alpha)]^2 [1 + c_u(1-\alpha)^2] \tag{123}$$

Expression (123) is positive for all c_u values such that $c_u \neq c_b$, and is zero when $c_u = c_b$ since in this special case the central bank stabilises the unanticipated component of the shock in the most desirable way from the point of view of the union.

Although we have found many points in common between the two scenarios as regards the relationship between signal quality and union welfare, they differ as regards one aspect of the results summarised in (54) and (120). Whereas under the simple rule the special-case values of $\tilde{c}_u$ and $\tilde{\tilde{c}}_u$ are positive only if $\gamma > 1$, their counterparts under discretion are necessarily positive because of the assumption that c_b is positive. This means that whereas under the simple rule it is possible for $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ to be the case for all positive c_u values below a certain critical value (and $\gamma = 1$ was found to be a necessary, but not a sufficient, condition for this to be so), under discretion the $c_b > 0$ restriction means that the range of c_u values for which $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ cannot have zero as its lower bound. A related point is that whereas under discretion there must, provided $n > 1$, be a range of c_u values for which $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ is the case, under the simple rule unions' expected loss can be a monotonic increasing function of σ_u^2, for all admissible c_u values.

For the sake of completeness, we end this subsection by stating the equivalents of (119) and (120) for the case of atomistic unions. Taking the limit as n tends to infinity of both (91a) and (91b), combining the resulting expressions with (10), and differentiating the result with respect to σ_u^2, yields:

$$\frac{d[\lim_{n\to\infty} E(\Omega_j^u|_{w=w_{NE}})]}{d\sigma_u^2} = \left\{ \frac{-c_u\{\varepsilon^2 + c_u[\alpha + \varepsilon(1-\alpha)]^2\}}{\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}^2} + \frac{[c_u + c_b^2(1-\alpha)^2]}{[1 + c_b(1-\alpha)^2]^2} \right\} \beta^2 \qquad (124)$$

It follows from (124) that the atomistic-case equivalent of result (120) is:

$$\frac{d[\lim_{n\to\infty} E(\Omega_j^u|_{w=w_{NE}})]}{d\sigma_u^2} < (>) 0 \quad \text{iff} \quad \Delta'\Theta' < (>) 0 \qquad (125)$$

where: $\Delta' \equiv c_u[\alpha + \varepsilon(1-\alpha)] - c_b\varepsilon(1-\alpha)$

$\Theta' \equiv [1 + c_u(1-\alpha)^2]\Delta' - 2c_u\alpha[1 + c_b(1-\alpha)^2]$.

Figure III.6

Identification of the sign of the impact of a marginal deterioration in signal quality on the expected loss of atomistic unions under discretion

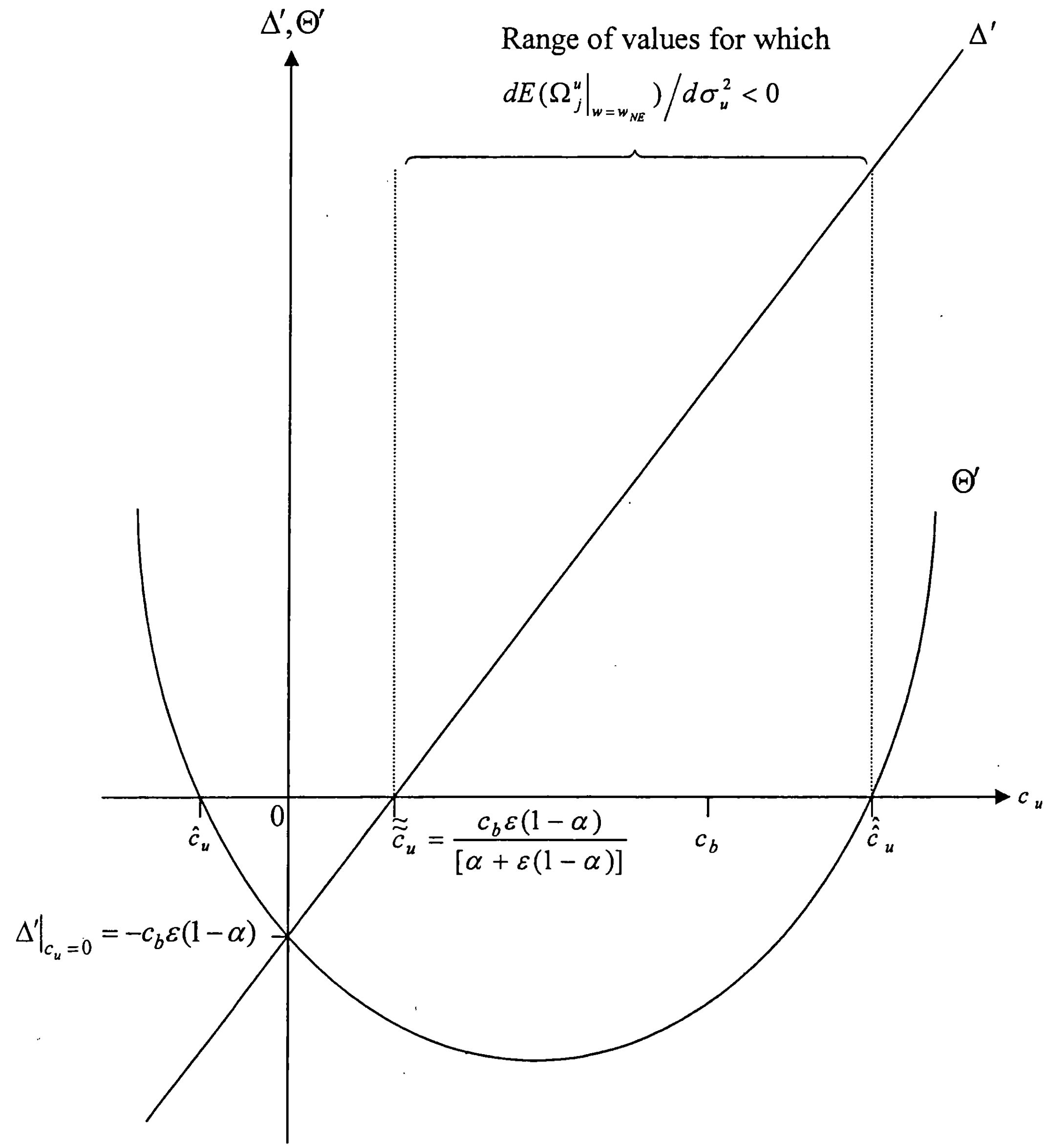


Figure III.6 above gives a graphical representation of this result. Note that as in the case of non-atomistic unions, c_b lies within the range of c_u values for which a deterioration in signal quality is beneficial for the welfare of atomistic unions, and this is for the same reason as given earlier for the non-atomistic case.

III.5.6 Signal Quality and Socially Optimal Delegation

The previous subsection has found that an improvement in the quality of unions' information regarding the productivity shock, as embodied in σ_u^2, the variance of signal noise, can be either beneficial or detrimental to union welfare, depending on the strength of the externality, which depends in turn on the structural parameters n and ε, as well as the weight parameters c_u and c_b. This subsection continues the investigation of the welfare repercussions of unions' information quality by considering how marginal changes in σ_u^2 affect the expected social loss. Rather than address this issue directly, however, it will be more convenient to discuss it in passing while analysing an extended version of the model outlined in previous sections. The extension introduced in this subsection is to endogenise σ_u^2 by making it a choice variable of either the central bank or of the framers of the constitution whose task it is to design the monetary regime. One immediate implication of this is that the socially optimal monetary rule derived in Section III.4 ceases to be a valid yardstick for evaluating the relative performance of the discretionary monetary regime. Indeed, in the remaining sections of this chapter we abstract entirely from rule-based regimes and focus exclusively on discretion.

The relaxation of the implicit assumption of previous sections that signal quality is exogenous thus makes 'economic transparency' a key aspect of the model. It will be recalled from Chapter I that this phrase, coined by Geraats (2002), refers to the disclosure by the authorities of information regarding macroeconomic disturbances, and that the previous contributions to this literature which are most closely related to the analysis of this chapter are those of Cukierman (2001) and Gersbach (2002). The model of this subsection shares in common with these two precursor papers a focus on the implications of economic transparency for the efficacy of stabilisation policy, and like them abstracts from issues relating to the potentially ameliorating effect of economic transparency on a mean inflation bias partly originating in private sector uncertainty regarding the central bank's objectives. Both Cukierman (2001) and Gersbach (2002) assume that the policymaker faces an expectations-augmented Phillips curve (Lucas surprise supply function), so that employment (or output) depends on the realisation of the supply shock and on private-sector expectational errors regarding inflation. If the private sector is uninformed about the shock, expected inflation will be such that the marginal benefit of offsetting the shock's employment impact by means of an inflation surprise will exceed its marginal cost. The drawback to economic transparency identified by these two papers is simply that the adjustment of expected inflation in the light of information about the supply shock causes the marginal benefit and marginal cost of an inflation surprise to be equated, and hence renders attempts to stabilise the economy ineffective.

This subsection differs from Cukierman (2001) and Gersbach (2002) in several important respects. By working with a Phillips curve relationship, both papers abstract from the possibility that private-sector agents might respond to information regarding the supply shock in ways other than by merely adjusting their inflation expectations. The more sophisticated supply-side structure of the model of this chapter overcomes this deficiency. As is evident from equation $(83')$ above, when unions are averse to deviations of both the real wage and employment from their mean values, their equilibrium nominal wage response to the signal involves both an adjustment to incorporate into the wage their revised rational expectation of the price level, and a further adjustment to prevent the full impact of the anticipated component of the productivity shock being borne by employment. It is noteworthy from $(83')$, (88a) and (88b) that in the limiting $c_u \to \infty$ case in which each union is solely concerned to protect its real wage from the impact of the shock, and hence is indifferent to its employment impact, the equilibrium nominal wage is adjusted solely for the price-level expectation, $E[(p|_{w=w_{NE}}) \mid s]$.[47] The models of Cukierman and Gersbach can therefore be thought of as corresponding to a special extreme case of the model of this section, namely the case in which $c_u \to \infty$. As previously noted, the wage-setting externality which arises in our model whenever there are multiple unions (and the goods market is not perfectly competitive) is absent in the limiting $c_u \to \infty$ case.

[47] The second term on the right-hand side of $(83')$ is zero in the limiting $c_u \to \infty$ case.

Hence this chapter also advances the literature on transparency by investigating how this wage-setting externality affects the relationship between economic transparency and the authorities' ability to stabilise the economy. A final respect in which the chapter differs from the Cukierman and Gersbach papers is that it addresses the issue of socially optimal delegation arrangements (specifically, the optimal value for the central bank's weight parameter c_b).

This subsection therefore investigates both the relationship between social welfare and economic transparency, and the implications of that relationship for the optimal choice of central banker. In the interests of clarity it will be useful to begin with a careful discussion of the model's assumptions about the timing of moves. There are in fact several timing scenarios which are captured by the information structure of the model. As in previous subsections, in the very first stage of the multi-stage game society assigns a loss function to the central bank, and endows it with the discretionary power to set the money supply after the determination of w. The departure from our earlier version of the model is that the central bank also has the power to supply unions with information regarding the productivity shock prior to wages being set. Hence, unlike in previous subsections, the central bank now has the ability to influence wage-setting by disclosing information about the shock to unions. In the most straightforward timing scenario consistent with the assumed information structure, the central bank, immediately after it is assigned its loss function, chooses the quality of the productivity-shock signal that it will receive prior to the signing of wage contracts by unions and firms. In other words, the variance of the signal noise term, σ_u^2, is a choice variable of the central bank. (With σ_θ^2 exogenous and fixed, a particular choice of σ_u^2 implies a particular β.) Upon receiving a noisy signal s of the productivity shock (where the noise term u is randomly drawn from the zero-mean normal distribution which has the central bank's choice of σ_u^2 as its variance), the central bank then forms its own best forecast, βs, of the shock and communicates this to unions. (Equivalently, it is the signal itself that is communicated by the central bank to unions, and it is assumed that the unions have learnt the value of σ_u^2, so that the informativeness of the signals is known to them, which consequently enables each union to form the rational expectation $E(\theta \mid s) = \beta s$.) The stages of this scenario subsequent to unions receiving the forecast or signal are the same as for previous versions of the model in which σ_u^2 was exogenous. These stages are the determination of the aggregate nominal wage in the simultaneous-move wage-setting game, followed by the central bank setting the money supply in the light of full information regarding the productivity shock, with firms then simultaneously setting prices, which in turn leads to the determination of the price level, the real wage, employment and output. As in previous sections of this chapter, the information structure is consistent with the money supply being set either prior to, or at the same time as, prices: the important point is that m is set after w has been determined, and that the central bank's information regarding the shock at the time it sets m is superior to the information possessed by unions when setting wages. Note that, just as elsewhere in this chapter, the assumption that the central bank is perfectly informed about θ when setting m is innocuous, since any forecast errors regarding the shock made by the central bank at this juncture would not affect its monetary response to w, and would be irrelevant to union wage decisions taken earlier in the game. This timing scenario is summarised by Time Line III.3 below.

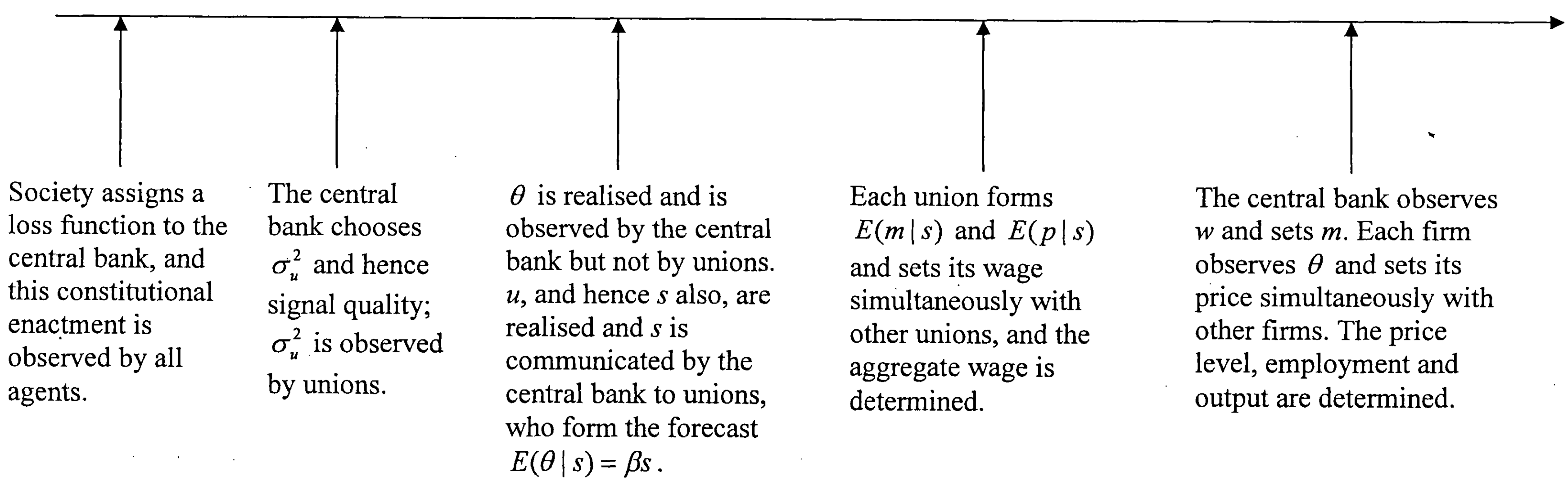
Before moving on, we mention a variant timing scenario consistent with the information structure discussed above, and which consequently leads to the same equilibrium outcomes as the one represented in Time Line III.3. This alternative differs in that the central bank observes the actual realisation of the shock at the time it occurs, but nevertheless chooses to pass on to unions a noisy signal of this shock. In effect, the central bank is envisaged as adding to θ a random noise term drawn from a zero-mean normal distribution with variance σ_u^2 which it has previously chosen. It is assumed for simplicity that unions have come to know the value of this variance (by studying, for instance, the correlation between previous pairs of signal and shock), so that the quality of the signal supplied by the central bank is known to them. The time line for this scenario appears below as Time Line III.4.

With this information structure in mind, we now derive society's optimal choice of central banker using backward induction. The relevant expressions for equilibrium employment variability and price level variability for given values of σ_u^2 and c_b, are (91b) and (103), and substituting these into the loss function assigned to the central bank, namely (9), yields:

Time Line III.3

1. Society assigns a loss function to the central bank, and this constitutional enactment is observed by all agents.
2. The central bank chooses σ_u^2 and hence signal quality; σ_u^2 is observed by unions.
3. θ is realised but is not observed by any party. s is realised and is communicated by the central bank to unions, who form the forecast $E(\theta \mid s) = \beta s$. (Equivalently, the central bank communicates this forecast to unions.)
4. Each union forms $E(m \mid s)$ and $E(p \mid s)$ and sets its wage simultaneously with other unions, and the aggregate wage is determined.
5. The central bank observes w and θ and sets m. Each firm observes θ and sets its price simultaneously with other firms. The price level, employment and output are determined.

Time Line III.4



$$E(\Omega^{cb}\big|_{w=w_{NE}}) = c_b[1+c_b(1-\alpha)^2]\left\{\frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2}\right.$$

$$\left. + \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Phi_b^2\beta\sigma_\theta^2}{c_b^2(1-\alpha)^2[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2}\right\} \quad (126)$$

The central bank which has been assigned a loss function with given weight c_b will choose σ_u^2 to minimise (126). This optimisation task will involve scrutiny of the following derivative:

$$\frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} = \left.\frac{\partial E(\Omega^{cb}\big|_{w=w_{NE}})}{\partial\sigma_u^2}\right|_{\beta \text{ fixed}} + \frac{\partial E(\Omega^{cb}\big|_{w=w_{NE}})}{\partial\beta}\left(\frac{\partial\beta}{\partial\sigma_u^2}\right) \quad (127a)$$

$$\frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} = c_b[1+c_b(1-\alpha)^2]\Gamma\beta^2 \quad (127b)$$

where $$\Gamma \equiv \frac{1}{[1+c_b(1-\alpha)^2]^2} - \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Phi_b^2}{c_b^2(1-\alpha)^2[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2}$$

(127c)

It follows directly from (127b) that $\Gamma > (<) 0$ is a necessary and sufficient condition for the central bank's (unconditional) expected loss to be strictly increasing (decreasing) in σ_u^2. Clearly, a further corollary is that $\Gamma = 0$ is a necessary and sufficient condition for this loss to be independent of σ_u^2.

Before embarking on an economic interpretation of these findings, it is worthwhile enlarging upon a point hinted at in the first paragraph of this subsection. When σ_u^2 is exogenous, and the central bank's sole instrument is consequently the money supply, the best possible outcome from the central bank's point of view is for the impact of each θ shock to be divided between employment and the price level according to (respectively) the weights $c_b(1-\alpha)/[1+c_b(1-\alpha)^2]$ and $-1/[1+c_b(1-\alpha)^2]$, so that the resulting (unconditional) expected loss of the central bank is minimised at $c_b\sigma_\theta^2/[1+c_b(1-\alpha)^2]^2$. (This best possible outcome when σ_u^2 is exogenous will only be attainable if unions ignore signals, either because they are infinitely noisy or because $c_u = \tilde{\tilde{c}}_u$ happens to be the case.) When σ_u^2 is made a choice variable of the central bank, however, the set of possible outcomes changes, as does the set of outcomes attainable by the central bank. The aforementioned best possible outcome when σ_u^2 is exogenous is obviously attainable when σ_u^2 is endogenous, since it can be brought about by making the signal infinitely noisy and by setting the money supply optimally in response to θ. It turns out, however, that it may be possible for the central bank to achieve a superior outcome to this by selecting a finite value for σ_u^2 and hence allowing the equilibrium wage response to the signal to determine the apportionment of the anticipated component of the shock, βs, between employment and the price level. By doing this, the central bank in effect deprives itself of the power to influence, via its setting of the money supply, the component of the equilibrium outcome which relates to βs. (More precisely, although the central bank will still be able to influence, via a monetary surprise, the outcome component relating to βs, it will choose not to do so since the rational expectation incorporated into the wage ensures that the marginal cost of a monetary surprise will exceed its marginal benefit to the central bank.) Note that by choosing a finite value for σ_u^2, the central bank not only relinquishes the ability to use the money supply to determine the employment outcome relating to βs, but also incurs a stochastic inflation bias. Despite this drawback, it nevertheless remains possible that the central bank will find it advantageous to make the signal informative rather than infinitely noisy.

(An alternative way of making the point is the following. The productivity shock variance, σ_θ^2, must be distributed in some way between employment variability and price level variability. If the central bank can choose σ_u^2, it can in effect choose the fraction of σ_θ^2 (specifically, the fraction β), for which this distribution will be determined by equilibrium wage-setting, with the distribution of the remaining fraction, $(1-\beta)$, determined by its own optimising behaviour in setting the money supply in response to the unanticipated component of each shock. Clearly, relative to the scenario considered previously in which σ_u^2 was exogenous, having the ability to select σ_u^2, and hence β, cannot make the central bank worse off, since it can always set σ_u^2 at whatever value it had in the scenario in which it was exogenous.)

Mindful of these facts, we return to the economic interpretation of (127b) and (127c). It is evident from these expressions that the sign of Γ depends on the relative strength of two effects of opposite sign. A marginal increase in σ_u^2, via the resulting marginal decrease in β, reduces the value of the term in $\beta\sigma_\theta^2$ on the right-hand side of (126), and consequently has a beneficial effect on the central bank's expected loss. Counteracting this is the effect of a marginal increase in σ_u^2 on the term in the variance of union forecast errors. Since $\beta\sigma_u^2 = (1-\beta)\sigma_\theta^2$, it is evident that this variance is an increasing function of σ_u^2, and hence that the effect involving the term in $\beta\sigma_u^2$ is detrimental. Which of these two effects is the stronger determines the sign of Γ and hence the sign of the impact of a deterioration in union signal quality on the central bank's expected loss. Note that there are two aspects of the model which matter for the relative strength of the two effects. The first of these aspects is the magnitude of c_u relative to c_b, while the second is the strength of the wage-setting externality.

The first step towards gaining further insight is to obtain an alternative expression for Γ by writing the right-hand side of (127c) as a single term:

$$\Gamma = \frac{\Gamma'}{c_b^2(1-\alpha)^2[1+c_b(1-\alpha)^2]^2[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2} \tag{127d}$$

where:

$$\Gamma' = -c_u^2[\alpha+\varepsilon(1-\alpha)]^2[1+2c_b(1-\alpha)^2]\Phi_b^2 +$$
$$+2c_u c_b^2(1-\alpha)^3[\alpha+\varepsilon(1-\alpha)][c_b\alpha(1-\alpha)+\varepsilon\Phi_b]\Phi_b + c_b^2(1-\alpha)^2[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]^2 \tag{127e}$$

Γ' is quadratic in c_u, and since $\partial^2\Gamma'/\partial c_u^2 < 0$ and $\Gamma'|_{c_u=0} > 0$, it immediately follows that the equation $\Gamma' = 0$ has two real c_u solutions of opposite sign. Denoting these solutions by Γ_1' and Γ_2', they are found to be:

$$\Gamma_1' = \frac{-c_b(1-\alpha)[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]}{[\alpha+\varepsilon(1-\alpha)][1+2c_b(1-\alpha)^2]\Phi_b} \tag{127f}$$

$$\Gamma_2' = \frac{c_b(1-\alpha)[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]}{[\alpha+\varepsilon(1-\alpha)]\Phi_b} \tag{127g}$$

From comparison of (127g) with (121) it is apparent that $\Gamma_2' = \tilde{\tilde{c}}_u$, i.e. that Γ_2' is the value of c_u which leads unions to disregard the signal. From the facts that have been established about Γ_2', it then follows directly that the following condition is both necessary and sufficient to ensure the central bank's expected loss is increasing in σ_u^2:

$$\frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} > 0 \;\text{ iff }\; 0 \le c_u < \tilde{\tilde{c}}_u \tag{128a}$$

It also follows that the necessary and sufficient condition for the central bank's expected loss to be decreasing in σ_u^2 is:

$$\frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} < 0 \;\text{ iff }\; c_u > \tilde{\tilde{c}}_u \tag{128b}$$

Finally, if $c_u = \tilde{\tilde{c}}_u$ the central bank's expected loss is unaffected by a deterioration in the quality of the unions' signal:

$$\frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} = 0 \;\text{ iff }\; c_u = \tilde{\tilde{c}}_u \tag{128c}$$

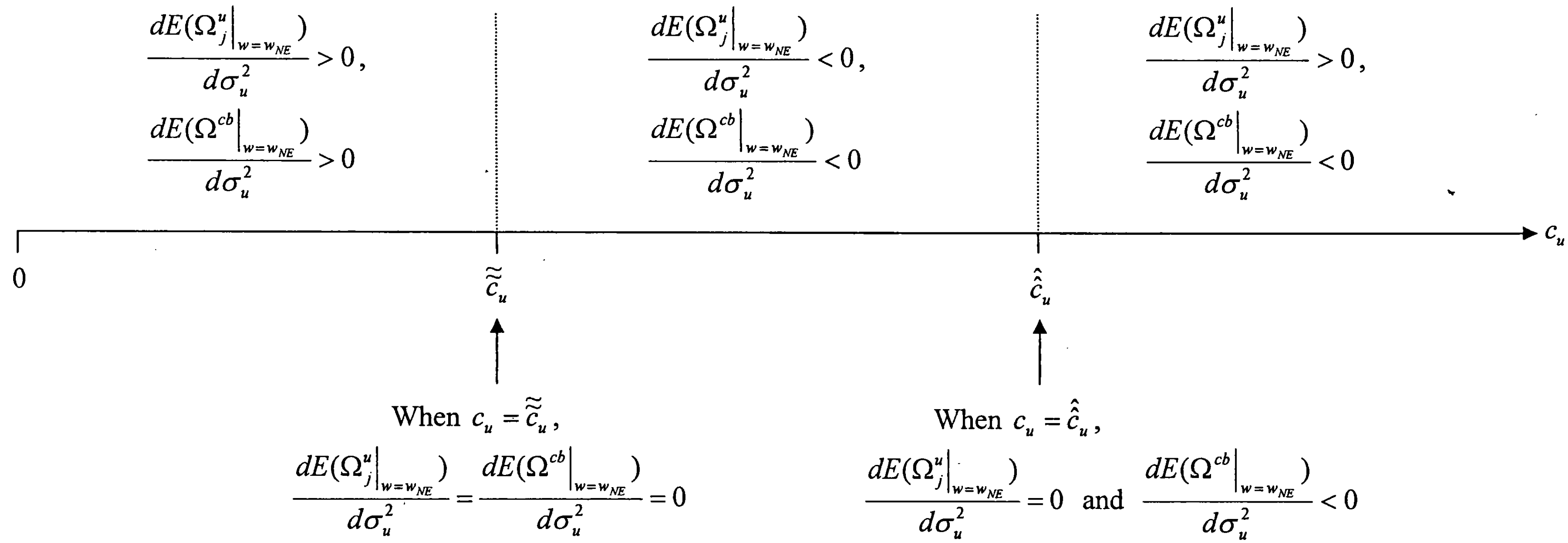
Result (128c) is the most straightforward of these results to explain: since unions ignore the signal when $c_u = \tilde{\tilde{c}}_u$, the macroeconomic outcomes which are of concern to the central bank, and hence its expected loss as well, are independent of σ_u^2. It is noteworthy that $\tilde{\tilde{c}}_u$ is the only admissible value of c_u in respect of which both the expected loss of unions and the expected loss of the central bank are unaffected by changes in σ_u^2. Combining results (128a) to (128c) with the results contained in (120) and (121), the range of possible values for c_u (i.e. the non-negative portion of the real line) is found to be divisible into five sections according to the impact of marginal increases in σ_u^2 on the welfare of unions and of the central bank. This finding is depicted below as Figure III.7, where two particular values of c_u, namely $\tilde{\tilde{c}}_u$ and $\hat{c}_u$ (the positive c_u solution of the equation $\Theta = 0$) constitute 'sections' in themselves.

A striking aspect of the results summarised in Figure III.7 is that there exists a range of c_u values (specifically, the range $\tilde{\tilde{c}}_u < c_u < \hat{c}_u$), for which a deterioration in the quality of the unions' signal of the supply shock can be beneficial to the welfare of both the unions and the central bank. As we shall see, this at first sight counter-intuitive finding can be explained in terms of the mitigating effect which a decline in signal quality has on the wage-setting externality. Results (128a) to (128c) also directly imply the quality of the signal that will be chosen by the central bank when c_u satisfies one or the other of the stated conditions. Since its expected loss is strictly increasing in σ_u^2 when c_u is in the interval $0 \le c_u < \tilde{\tilde{c}}_u$, it follows that the central bank will choose a setting of zero for σ_u^2, so that $s = \theta$ at all times, and the unions are consequently fully informed about the shock when setting wages. Conversely, if $c_u > \tilde{\tilde{c}}_u$ the central bank's optimal strategy is to make signals infinitely noisy (i.e. choose the limiting case in which $\sigma_u^2 \to \infty$), so that each union in setting its wage disregards the particular realisation of s which it observes. In the special $c_u = \tilde{\tilde{c}}_u$ case the central bank's choice of σ_u^2 is indeterminate.

As mentioned earlier, there are two aspects of the model which underlie these results, namely the relative magnitudes of c_u and c_b, and the wage-setting externality. To obtain intuition regarding results (128a) and (128b) it is helpful to begin by isolating the aspect involving the weight parameters. This can be straightforwardly done by considering the version of the model in which the goods market is perfectly competitive, and from which the externality is therefore absent. It will be recalled that $\lim_{\varepsilon \to \infty} \tilde{\tilde{c}}_u = c_b$, so that in this special case results (128a) to (128c) simplify to the following:

$$\lim_{\varepsilon \to \infty} \frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} > 0 \text{ iff } 0 \le c_u < c_b \tag{129a}$$

Figure III.7



$\frac{dE(\Omega^u_j|_{w=w_{NE}})}{d\sigma^2_u} > 0,$
$\frac{dE(\Omega^{cb}|_{w=w_{NE}})}{d\sigma^2_u} > 0$
$\frac{dE(\Omega^u_j|_{w=w_{NE}})}{d\sigma^2_u} < 0,$
$\frac{dE(\Omega^{cb}|_{w=w_{NE}})}{d\sigma^2_u} < 0$
$\frac{dE(\Omega^u_j|_{w=w_{NE}})}{d\sigma^2_u} > 0,$
$\frac{dE(\Omega^{cb}|_{w=w_{NE}})}{d\sigma^2_u} < 0$
c_u
0
$\tilde{\tilde{c}}_u$
$\hat{\hat{c}}_u$
When $c_u = \tilde{\tilde{c}}_u$,
$\frac{dE(\Omega^u_j|_{w=w_{NE}})}{d\sigma^2_u} = \frac{dE(\Omega^{cb}|_{w=w_{NE}})}{d\sigma^2_u} = 0$
When $c_u = \hat{\hat{c}}_u$,
$\frac{dE(\Omega^u_j|_{w=w_{NE}})}{d\sigma^2_u} = 0$ and $\frac{dE(\Omega^{cb}|_{w=w_{NE}})}{d\sigma^2_u} < 0$

$$\lim_{\varepsilon\to\infty} \frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} < 0 \text{ iff } c_u > c_b \tag{129b}$$

$$\lim_{\varepsilon\to\infty} \frac{dE(\Omega^{cb}\big|_{w=w_{NE}})}{d\sigma_u^2} = 0 \text{ iff } c_u = c_b \tag{129c}$$

Hence in the absence of the externality the central bank's optimal strategy when $c_u < c_b$ is to choose $\sigma_u^2 = 0$, so that the signal it provides to unions is perfectly informative, while its optimal strategy when $c_u > c_b$ is to deny unions any useful information about the productivity shock.

The economic intuition for this has already been hinted at in our earlier discussion of the relationship between signal quality and union welfare under discretion, but requires elaboration at this point. When wage-setting is efficient, each union ignores the signal when $c_u = c_b$, since the very fact that the central bank places the same relative weight on price level variability as the representative union places on real wage variability, implies that in the absence of wage adjustments in response to signals, monetary policy brings about the unions' efficient combination of real wage variability and employment variability. However, if $c_u > c_b$ is the case, and unions are uninformed about shocks, the central bank, via its setting of the money supply, brings about too little employment variability, together with too much real wage variability, from the unions' viewpoint. Hence if unions receive a signal which is at all informative (i.e. if σ_u^2 is finite, so that $\beta > 0$), they will form the forecast βs of the shock, as well as the associated rational expectation of the central bank's optimal monetary response to the aggregate wage and βs, and will incorporate this rational expectation into the wage in order to ensure the impact of βs on employment and the real wage conforms to the efficient pattern. This means that if $c_u > c_b$, union wage responses to informative signals will cause employment variability to be higher than were the signals completely uninformative. Worse still, the unions' recognition of the incentives facing the central bank implies that the excessive employment variability will be accompanied by a stochastic inflation bias. The better is signal quality, the more severe will be both these problems, and hence when $c_u > c_b$ the central bank's optimal strategy is to make signals infinitely noisy, so that the apportionment of the impact of each shock between employment and the price level does not depend at all on wage-setting, and instead is determined solely by the central bank's setting of the money supply. It is noteworthy that the set of cases for which $c_u > c_b$ includes the special limiting $c_u \to \infty$ case mentioned earlier, in which unions wish to divert the full impact of the shock on to employment, and consequently react to the signal only by adjusting the wage for their revised expectation of the price level. Thus a special case of our model replicates the result of Cukierman (2001) and Gersbach (2002).

In the converse case in which $c_u < c_b$, if unions are completely uninformed about the shock, the conduct of monetary policy will cause too much employment variability, and too little real wage variability, from unions' point of view. Consequently, making signals more informative will reduce employment variability because of the efficient adjustment of the wage in response to each (non-zero) signal, and the resulting distribution of the anticipated component of each shock between employment and the real wage in accordance with union preferences. While greater economic transparency has a disadvantageous aspect in that it exacerbates the stochastic inflation bias, this drawback does not outweigh its beneficial effect of reduced employment variability. Therefore, when $c_u < c_b$, the central bank will choose full economic transparency (i.e. choose to make signals perfectly informative).

Turning now to investigate how these conclusions change when the goods market is monopolistically competitive, it is useful to begin by once again considering the $c_u = c_b$ case. This we know to be the special case in which wage-setting efficiency requires every union to ignore the signal. However, with a monopolistically competitive goods market this does not happen under equilibrium wage-setting. Improvements in signal quality must therefore have an unambiguously detrimental impact on the central bank's expected loss when $c_u = c_b$, since the resulting worsening of the externality both directly increases employment variability and exacerbates the stochastic inflation bias. Next, we make use of a fact established in our earlier discussion of the relationship between signal quality and union welfare in subsection III.5.5, namely that when the goods market is monopolistically competitive, the externality induces unions to ignore signals in equilibrium if c_u has a particular value. This value is $\tilde{\tilde{c}}_u$, and provided $n > 1$ and ε is finite, $\tilde{\tilde{c}}_u < c_b$ is the case. When $c_u = \tilde{\tilde{c}}_u$ the central bank's expected loss is independent of signal quality.

It is known from previous analysis that when c_u is in the interval $\tilde{\tilde{c}}_u < c_u < c_b$, monetary policy would, in the absence of wage adjustments in reaction to signals, bring about higher employment variability than unions desire, and therefore were wage-setting efficient, improved signal quality would lead to lower employment variability. When the goods market is monopolistically competitive, however, improvements in signal quality do not have this effect: even though when $\tilde{\tilde{c}}_u < c_u < c_b$ union preferences are such that each union would prefer lower employment variability and higher real wage variability than the central bank brings about in the absence of wage movements in reaction to signals, the externality in wage-setting leads to employment variability being greater than were the signal completely uninformative. The introduction of the wage-setting externality into the model, or its strengthening as a result of a reduction in ε or an increase in n, has the effect of shifting $\tilde{\tilde{c}}_u$ closer to the origin and thus enlarging the set of c_u values for which improvements in signal quality worsen the central bank's expected loss, and for which it is therefore optimal for the central bank to pursue a strategy of making signals completely uninformative. For values of c_u such that $c_u < \tilde{\tilde{c}}_u$, the presence of the externality is insufficient to prevent improvements in signal quality having a beneficial effect on employment variability, so that the optimal strategy for the central bank to follow is one of full economic transparency (i.e. perfectly informative signals).

It remains to work out society's optimal choice of the delegation parameter c_b in the light of these findings. To this end, we first of all obtain society's expected loss for a given value of c_b by substituting (91b) and (103) into (8):

$$E(\Omega^s\big|_{w=w_{NE}}) = [c_s + c_b^2(1-\alpha)^2]\left\{\frac{\beta\sigma_u^2}{[1+c_b(1-\alpha)^2]^2}\right. \\ \left. + \frac{c_u^2[\alpha+\varepsilon(1-\alpha)]^2\Phi_b^2\beta\sigma_\theta^2}{c_b^2(1-\alpha)^2[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^2}\right\} \tag{130}$$

Differentiating (130) with respect to σ_u^2 yields:

$$\frac{dE(\Omega^s|_{w=w_{NE}})}{d\sigma_u^2} = [c_s + c_b^2(1-\alpha)^2]\Gamma\beta^2 \tag{131}$$

where Γ is as in expressions (127c) or (127d). Equation (131) immediately reveals that the necessary and sufficient condition for the expected social loss to be increasing in σ_u^2 is the same as for the central bank's expected loss. Hence the social-loss counterparts to (128a), (128b) and (128c) are as follows:

$$\frac{dE(\Omega^s|_{w=w_{NE}})}{d\sigma_u^2} > 0 \text{ iff } 0 \le c_u < \tilde{\tilde{c}}_u \tag{132a}$$

$$\frac{dE(\Omega^s|_{w=w_{NE}})}{d\sigma_u^2} < 0 \text{ iff } c_u > \tilde{\tilde{c}}_u \tag{132b}$$

$$\frac{dE(\Omega^s|_{w=w_{NE}})}{d\sigma_u^2} = 0 \text{ iff } c_u = \tilde{\tilde{c}}_u \tag{132c}$$

These expressions imply that, provided c_b is finite, a marginal increase in σ_u^2 has exactly the same repercussions for the expected social loss as it does for the expected loss of the central bank.[48] Figure III.7 is therefore doubly useful in that it also compares how a deterioration in signal quality affects society's welfare with how it affects the welfare of unions. (All that is required for Figure III.7 to fulfill this purpose is that each of the '*cb*' superscripts be replaced with an '*s*' denoting 'society'.) Hence the at first sight counter-intuitive finding emerges that when c_u is in the interval $\tilde{\tilde{c}}_u < c_u < \hat{\hat{c}}_u$, greater economic transparency (a decrease in the noisiness of the signal) is detrimental to the welfare of both society and of unions, a result which we know from previous discussion is attributable to the role of signal quality (i.e. the unions' forecasting slope, β) in determining the welfare impact of the adverse wage-setting externality. (Recall that β can be thought of as the weight which apportions the productivity shock variance, σ_θ^2, between the variance of the anticipated component of shocks, $\beta\sigma_\theta^2$, and the variance of the unanticipated component of

[48] The proviso that c_b is finite turns out to have some significance in what follows.

shocks, $(1-\beta)\sigma_\theta^2$, with the adverse wage-setting externality only appertaining to the former.)

Having established the relationship between social welfare and signal quality for a given value of the delegation parameter c_b, we can now proceed to derive the socially optimal choice of c_b when signal quality is chosen by the central bank, and therefore is itself a function of c_b. The derivation focuses on a particular value of c_b, namely $\tilde{\tilde{c}}_u \equiv c_b(1-\alpha)[c_b\alpha(1-\alpha)+\varepsilon\Phi_b]/[\alpha+\varepsilon(1-\alpha)]\Phi_b$, since if $c_u < \tilde{\tilde{c}}_u$ an optimising central bank assigned a loss function with weight c_b supplies perfectly accurate signals, while if $c_u > \tilde{\tilde{c}}_u$ it supplies completely uninformative signals. The most straightforward way to solve society's delegation problem is to ascertain first of all what would happen were society to appoint a representative central banker (i.e. were society to set $c_b = c_s$). Were such an appointment to be made, $\tilde{\tilde{c}}_u$ would take a particular value, specifically it would be:

$$\tilde{\tilde{c}}_u\Big|_{c_b=c_s} = \frac{c_s(1-\alpha)[c_s\alpha(1-\alpha)+\varepsilon\Phi_s]}{[\alpha+\varepsilon(1-\alpha)]\Phi_s} \tag{133a}$$

where: $\Phi_s \equiv \Phi_b\big|_{c_b=c_s} = n[1+c_s(1-\alpha)^2]-1$ (133b)

There are three possibilities which we must now consider. Firstly, if it so happens that $c_u > \tilde{\tilde{c}}_u\big|_{c_b=c_s}$, the representative central banker will choose to make signals completely uninformative, and will then use the money supply to apportion the entire impact of the shock between employment and the price level in the way that minimises the expected social loss. Since we established earlier that the signal quality which is optimal for the central bank with finite c_b is also optimal for society, it follows that society's optimal delegation choice when $c_u > \tilde{\tilde{c}}_u\big|_{c_b=c_s}$ is to appoint a representative central banker, and that the resulting monetary regime will be completely opaque.

The second possibility is that $c_u = \tilde{\tilde{c}}_u\big|_{c_b = c_s}$ happens to be the case, so that unions ignore signals, regardless of their quality. In this case the central bank's choice of signal quality is irrelevant to the outcome (and is consequently indeterminate), and society's optimal delegation choice is again a representative central banker so that the socially optimal apportionment of the productivity shock variance between employment variability and price level variability is achieved. Given $c_u \geq \tilde{\tilde{c}}_u\big|_{c_b = c_s}$, therefore, optimal delegation will result in a minimised expected social loss with the following value:

$$E(\Omega^s\big|_{w = w_{NE}, c_b = c_s, c_u \geq \tilde{\tilde{c}}_u|_{c_b = c_s}}) = \frac{c_s \sigma_\theta^2}{[1 + c_s(1-\alpha)^2]} \tag{134}$$

The third possibility is that the value of c_u is sufficiently low that the appointment of a representative central banker results in $c_u < \tilde{\tilde{c}}_u\big|_{c_b = c_s}$ being the case. It follows from (128a) that the representative central banker will then be completely transparent about productivity shocks (i.e. chooses $\sigma_u^2 = 0$), so that each union is fully informed about θ when setting its wage. (The argument invoked earlier that, given c_b is finite, the central bank's optimal choice of σ_u^2 is also socially optimal remains valid.) As described previously, the unions' rational expectation formed in the light of full information regarding θ, and the incorporation of that rational expectation into the equilibrium nominal wage deprives the central bank of an incentive to influence employment via its setting of *m*. We know from a previous subsection that the resulting expression for employment variability is a decreasing function of c_b for $n > 1$, while in the extreme cases of $n = 1$ and atomistic unions it is independent of c_b. Complete transparency also gives rise to a stochastic inflation bias (indeed, this will be the sole component of the expression for price level variability), and this too is known to be a decreasing function of c_b for all $n \geq 1$. It follows, therefore, that if unions are perfectly informed about the shock when setting wages, the resulting expected social loss must be strictly falling in c_b. Furthermore, appointing a conservative central banker (i.e. one for which c_b is finite and exceeds c_s) will not impair the central bank's incentive to be completely transparent when $c_u < \tilde{\tilde{c}}_u\Big|_{c_b = c_s}$, since $\tilde{\tilde{c}}_u$ is an increasing function of c_b for all $n \geq 1$, as is evident from the following derivative:

$$\frac{\partial \tilde{\tilde{c}}_u}{\partial c_b} = \frac{(1-\alpha)[c_b \alpha(1-\alpha)(n-1+\Phi_b)+\varepsilon\Phi_b^2]}{[\alpha+\varepsilon(1-\alpha)]\Phi_b^2} \tag{135}$$

(Note that a necessary proviso for this argument is that c_b is finite, since $\tilde{\tilde{c}}_u$ has been derived on the assumption that this is the case.) The fact that when $c_u < \tilde{\tilde{c}}_u\Big|_{c_b \geq c_s}$ the expected social loss is strictly falling in c_b suggests that the optimal choice of central banker in these circumstances may be an ultraconservative, i.e. one whose sole concern is to stabilise the price level and who is indifferent to employment variability. In our model, the limiting $c_b \to \infty$ case therefore corresponds to this type of central banker. The fact that c_b is infinite for an ultraconservative creates a small difficulty, however, since the expected loss of an ultraconservative central bank is necessarily zero and independent of σ_u^2, and consequently such a central bank will lack an incentive to provide any particular quality of signal to the unions. (If the central bank's sole concern is price-level stability, the stochastic inflation bias associated with the anticipated component of shocks is zero, while if such a central bank chooses a regime of less-than-perfect signal quality, it perfectly neutralises the potential price-level impact of the component of each shock which is unanticipated by unions via an appropriate setting of the money supply.) However, the socially optimal outcome will not be attained unless the ultraconservative central bank does practise full economic transparency. Hence when $c_u < \tilde{\tilde{c}}_u\Big|_{c_b = c_s}$, optimal delegation will involve both the appointment of an ultraconservative central banker and a constitutional requirement which compels the central bank to provide perfectly informative signals to unions. Note that the resulting equilibrium outcome would feature complete stability of the price level together with positive employment variability (and positive variability of the real wage), and that the expected social loss would be given by:

$$\lim_{c_b \to \infty} E(\Omega^s \Big|_{w=w_{NE},\, c_u < \tilde{\tilde{c}}_u \big|_{c_b = c_s}}) = \frac{n^2 c_u^2 (1-\alpha)^2 [\alpha + \varepsilon(1-\alpha)]^2 \sigma_\theta^2}{[\alpha + n(1-a)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}]^2} \qquad (136)$$

Two final points relate to the comparison of (134) with (136). The arguments provided regarding the optimal delegation decision imply that when $c_u < \tilde{\tilde{c}}_u \big|_{c_b = c_s}$ the right-hand side of (136) must be smaller in magnitude than the right-hand side of (134). These two expressions also allow us to identify the circumstances under which society will find it preferable to appoint an ultraconservative (who then practises full transparency) rather than a central banker with representative preferences (who chooses to make signals completely uninformative). Appointing an ultraconservative will be socially optimal if the right-hand side of (134) exceeds the right-hand side of (136). Manipulating the resulting inequality, the necessary and sufficient condition for this to be the case is found to be:

$$c_s > \frac{n^2 c_u^2 (1-\alpha)^2 [\alpha + \varepsilon(1-\alpha)]^2}{[\alpha + n\varepsilon(1-a)]\{\alpha + n\varepsilon(1-a) + 2nc_u(1-\alpha)^2[\alpha + \varepsilon(1-\alpha)]\}} \qquad (137)$$

The right-hand side of (137) is increasing in n for all $n \geq 1$, and is decreasing in ε for all $n > 1$, indicating that for given c_u and α, the set of c_s values which will lead society to appoint an ultraconservative is smaller, the more severe is the externality. This makes perfect sense, since under an ultraconservative both employment variability and (since the stochastic inflation bias is zero) the expected social loss must be larger, the greater the externality's strength. When society finds it optimal to appoint a representative central banker, on the other hand, the expected social loss is necessarily independent of n and ε, since the resulting regime of complete opacity prevents the externality from arising at all. Hence if the two sides of (137) happened to be equal, so that appointing an ultraconservative gives rise to the same expected loss as appointing a representative central banker, a worsening of the externality must tip the balance in favour of the latter. This immediately implies that the stronger is the externality, the smaller must be the set of c_s values which imply that society's optimal delegation choice is an ultraconservative.

III.6 Conclusion

This chapter has built on Chapter II by extending to a stochastic context the analysis of the macroeconomic implications of union wage-setting decisions when firms operate in a monopolistically competitive goods market. The key feature of the chapter has been the addition of stochastic elements to our basic model, and in particular the introduction of a stochastic productivity shock in respect of which unions possess noisy information at the time wages are set. The objective function of the representative union in this chapter assumes an aversion to variability in both employment and the real wage about their respective mean values, and in terms of both individual and collective union welfare gives rise to a trade-off between employment and the real wage as regards stabilisation of the anticipated component of each shock.

A major finding of the chapter is that an adverse macroeconomic externality arises from individual wage decisions made in response to the common signal of the productivity shock. The externality is the consequence of each union not fully internalising the price-level repercussions of its individual wage, and results in real wage variability being inefficiently low (i.e. less than that which would result from fully coordinated wage-setting), and employment variability being inefficiently high. As in the case of the wage-setting externality relating to the mean real wage which was seen to arise in the non-stochastic model, this chapter's externality is weaker, the smaller is the number of unions, and is absent if there is a single economy-wide union. A second parallel with the results of Chapter II is that the presence of imperfect goods-market competition is essential to the existence of the externality, which is weaker the more competitive is the goods market, and indeed is absent when firms are perfect competitors.

As in Chapter II, the externality has been shown to arise under a variety of monetary regimes. The current chapter's principal concern has been to investigate how the externality affects the relationship between welfare and the quality of unions' information about the shock, and the implications of that relationship in turn for the optimal design of a discretionary monetary regime. With regard to these issues, a major finding of the chapter is that under the simple rule, deterioration in signal quality has, a few special cases apart, both a beneficial externality-mitigating effect on union welfare and a detrimental effect involving the resulting higher variance of union forecast errors. For a certain set of parameter values, the externality-mitigating effect on union welfare outweighs the forecast-error effect, so that unions are better off, the worse is the quality of their information about productivity shocks. A similar result has been shown to appertain to the discretionary regime when the representative union's weight parameter c_u takes one of a set of possible values which are functions of the central bank's own weight parameter c_b. Economic intuition for this result has been provided, with one of the key points being the fact that if the two weight parameters are equal, so that the relative preference of unions for employment stability coincides with the central bank's relative preference for it, efficiency in wage-setting requires that unions maintain the nominal wage at its mean value regardless of the signal, since monetary policy in this situation stabilises shocks in the way that unions 'collectively' find most desirable. Since the wage-setting externality leads to wages inefficiently responding to signals when $c_u = c_b$, it follows that there exists a range of c_u values in respect of which a worsening of signal quality is beneficial to union welfare, and that this range includes, and extends on either side of, c_b.

Closely related to this key result is another which relates to the relationship between employment variability and the central bank's weight parameter when the quality of unions' information is held constant. For the extremes of wage-bargaining structure, the impact of the anticipated component of each shock on employment is found to be independent of this weight, either because unions are atomistic, so that changes in c_b do not affect the extent (zero) to which each union internalises the price level impact of its wage decision, or because there is a single union, so that full internalisation occurs regardless of the monetary regime. However, for the case of multiple non-atomistic unions, each of which partially internalises the macroeconomic repercussions of its wage, the weight c_b does matter for employment outcomes relating to the anticipated component of each shock. A higher c_b induces greater internalisation of wage consequences and hence mitigates the externality. Although the component of employment variability relating to the anticipated component of shocks is thus lowered by a marginal increase in c_b, whether greater central bank conservatism reduces employment variability as a whole depends however on the magnitude of the resulting increase in the component of employment variability relating to the unanticipated component of shocks. These results therefore provide a candidate explanation for why empirical studies typically do not find greater central bank conservatism to be associated with higher employment variability.

The chapter identifies another beneficial effect of an increase in the weight placed on inflation by the central bank, namely a reduction in price level variability, whether arising from the anticipated or unanticipated component of shocks. While the reduction in the contribution of union forecast errors to price level variability is a straightforward consequence of greater conservatism leading to stronger emphasis on moderating (via use of the money supply) the impact of such errors on the price level, the effect of greater conservatism on the stochastic inflation bias (the component of price level variability relating to the anticipated component of shocks) is more subtle, since it works purely via its beneficial influence on the expectation of the price level formed by each union conditional on the signal. Taken in conjunction with the aforementioned findings concerning employment variability, the optimal degree of central bank conservatism when signal quality is exogenous is found to be that which exploits the trade-off between, on the one hand, the components of the expected social loss which are falling in c_b (i.e. price level variability and, if unions are non-atomistic, the component of employment variability relating to union forecasts of shocks), and on the other hand the component of employment variability relating to union forecast errors, since this alone is an increasing function of c_b.

The penultimate section of the chapter investigates how these conclusions change when signal quality is endogenised, and is assumed to be a choice variable of the central bank. Society's optimal delegation decision was found to depend upon whether c_u fell below or exceeded a particular critical value, namely the critical value which, were a central banker with representative preferences to be appointed, would result in each union in equilibrium choosing not to adjust its wage in response to the signal, regardless of the signal's value. If c_u happens to take this value, the central bank will have the task of stabilising the entire component of the shock, regardless of its choice of signal quality, and hence a representative central banker is society's optimal choice. If c_u exceeds the critical value, society will again appoint someone with representative preferences, since such an appointee will practise the regime of complete opacity which is socially optimal in these circumstances. Finally, if c_u falls below the critical value, it is in society's interests, because of the resulting amount of employment variability, that the central bank provide unions with a perfectly informative signal of each shock. Appointing a conservative central banker will ensure this. However, because the stochastic inflation bias is strictly decreasing in c_b, and since when unions are fully informed equilibrium employment is beyond this parameter's influence, it turns out that the optimal delegation decision involves both appointing an ultraconservative, and supplementing this with a constitutional provision to ensure such an appointee does provide unions with perfectly informative signals.

Chapter IV

The Macroeconomics of Wage Indexation:

Introduction and Literature Review

IV.1 Introduction

The formal investigation of the macroeconomic repercussions of wage indexation is generally agreed to have commenced with the seminal paper by Gray (1976), while an important extension to this approach was made by Ball (1988), who incorporated monopolistic competition into Gray's framework. Since the central vein of the macroeconomic literature consists of adaptations of these two key papers, it will assist our discussion to set out below a model which embraces the Gray and Ball frameworks as particular cases. Contributions which build on this approach, or on a simpler approach based on Barro and Gordon's (1983a) model, are subsequently discussed, together with their potential shortcomings. The short penultimate section of the literature review is devoted to the relatively small number of papers concerned with multiparameter wage indexation, an idea which is applied to the basic model of this thesis in Chapter VI. The final section summarises and draws conclusions. Throughout, attention is confined to the theoretical literature on closed economy macroeconomics, since the models subsequently developed in Chapters V and VI are, in common with the entirety of this thesis, concerned with the implications for domestic macroeconomic outcomes of choices relating to the nominal wage made at the level of the individual union. In view of this, the substantial body of literature dealing with the question of the optimal degree of wage indexation for exchange rate stabilisation or monetary union purposes is not discussed.[1]

IV.2 The Gray (1976) and Ball (1988) Models

This section sets out a model which encompasses the key contributions of Gray (1976) and Ball (1988). The structural equations of this encompassing model, together with the

[1] A comprehensive survey is provided by van Gompel (1994). Note that space constraints also lead us to disregard wage-indexation papers featuring macroeconomic topics which are not the concern of this thesis: for example, the government budget constraint (Waller and VanHoose, 1989), public debt (Guidotti, 1993), and central-bank reputation in a repeated game (Diana, 2002).

individual firm's implied labour demand, are set out below as expressions (1) to (8). There is no need to comment in detail on the majority of these equations, since they are in most cases identical to the counterpart equations for the wage-setting model developed in earlier chapters:[2]

$$y_i = \alpha l_i + \theta \qquad 0 < \alpha < 1 \tag{1}$$

$$y^S = \int_0^1 y_i di = \alpha l + \theta \tag{2}$$

$$y^D = \gamma(m - p + \phi) \tag{3}$$

$$p = \int_0^1 p_i di \tag{4}$$

$$y_i^D - y^D = -\varepsilon(p_i - p), \qquad \varepsilon > 1 \tag{5}$$

$$l_i^D = \frac{\gamma(m - p + \phi) - \varepsilon(w_i - p) + (\varepsilon - 1)\theta}{[\alpha + \varepsilon(1 - \alpha)]} \tag{6}$$

Note that the specification of the production function, which is common to all firms, is precisely that assumed by Ball. While Gray does not assign this specific functional form to her model's production function, restricting it instead to exhibit diminishing marginal product of labour, the log-linear specification of (1) is a particular case which conforms to the general concave-in-levels specification adopted by her. So far as the specification of aggregate demand is concerned, namely equation (3), note that Gray sets the elasticity parameter γ equal to unity for simplicity, while Ball instead restricts it to be positive. Both papers assume that the money stock, *m*, is fixed: in other words, that there is no

[2] All variables are specified in logarithms except where stated otherwise, and the notation conforms to that used in the rest of this thesis, rather than to the notation of Gray and Ball.

monetary intervention on the part of the authorities.[3] The scenario is therefore exactly equivalent to that of the simple rule considered in Chapter III above. We also draw the reader's attention to the fact that Gray assumes a perfectly competitive goods market: in terms of equation (5), her model therefore corresponds to the limiting case in which $\varepsilon \to \infty$. In common with much of the literature, both Gray and Ball assume that employment at the individual firm is always equal to the individual firm's labour demand.

The equations which have been set out thus far are identical to their counterparts for the wage-setting model of Chapter III. The Gray-Ball framework differs from our model, however, in two particular respects, namely in the specifications of desired labour supply and of the contract nominal wage. Gray allows desired labour supply to be responsive to the real wage:

$$l_i^S = \omega(w_i - p) \qquad \omega \geq 0 \qquad (7)$$

Ball confines his attention to the special $\omega = 0$ case in which labour supply is completely inelastic with respect to the real wage. The specification of the nominal wage is:

$$w_i = \overline{w}_i + x_i(p - Ep) \qquad (8)$$

where $\overline{w}_i$ is the base nominal wage and x_i the degree of indexation to expectational errors in respect of the price level. $\overline{w}_i$ and x_i are stipulated in a contract signed prior to the realisations of the two shocks being observed by the parties to the contract. The expectational error $p - Ep$ is therefore the unforeseen departure of the actual price level from its rational expectation formed at the contract signing date.

Several features of the nominal wage equation (8) are noteworthy. Firstly, the wage is assumed to be indexed symmetrically to both positive and negative expectational errors regarding the price level. This simplifying assumption greatly enhances the model's tractability, at the cost of not reproducing a feature common to virtually all real-world

[3] The assumption of entirely passive monetary policy made by Gray and Ball warrants a simpler specification for aggregate demand which discards the velocity shock and simply assumes instead that the money stock itself is a stochastic variable. This is precisely the specification found in their papers.

indexed wage contracts, namely that the indexation clause becomes operative only when a positive expectational error regarding the price level occurs. The only paper to have formally investigated the implications of such asymmetric wage indexation is Cover and VanHoose (2002). These authors apply this more realistic specification to the Gray model, and for reasons of tractability, assume uniform distributions for the two shocks. They find that if a contract specifies (positive) indexation of the wage to only positive expectational errors, so that workers are in effect insured against a fall in their real wage below its expected value, the consequence is a lower base nominal wage. However, the lower is the base nominal wage, the lower is the expected price level, which in turn affects the probability of a positive expectational error, which alters the desirable degree of asymmetric indexation. Numerical simulations for plausible values of the structural parameters are resorted to in order to study these interaction effects and their implications for the equilibrium degree of wage indexation. Cover and VanHoose find that a sufficiently large ratio of the aggregate demand shock variance to the productivity shock variance will prevent the existence of an equilibrium in which the degree of asymmetric indexation is positive.

The second noteworthy feature of (8) is its assumption that the wage is indexed to expectational errors regarding current inflation, rather than being (more realistically) indexed to lagged expectational errors (i.e. errors regarding inflation in the period immediately preceding that covered by the contract), or to lagged inflation itself. A more sophisticated model would therefore allow for the fact that under a typical indexed contract, weekly or monthly base-wage payments are accompanied by less frequent indexation-clause payments, with the latter being made in respect of inflation. The principal papers investigating this issue are Fischer (1977b) and Jadresic (2002), discussion of which is deferred to a subsequent section of this survey. A final feature of (8) to which attention should be drawn is that indexation to other macroeconomic variables in addition to the price level is not considered.

Gray, Ball, and many other papers in the central vein of the literature assume that the adverse welfare consequences of stochastic shocks are adequately represented by the unconditional expectation of squared deviations in employment from its full-information level, i.e. the employment level which would prevail in the aftermath of shock realisations, were those shocks perfectly foreseen and the contract base nominal wage set appropriately to ensure labour market clearing. The standard specification of the loss of 'wage-setters' (a term which in this context embraces both parties to the contract) at the individual firm is consequently:

$$E\Omega_i = E(l_i - l_{i,FI})^2 \tag{9}$$

where $l_{i,FI}$ denotes full-information (log) employment. (Gray's specification of the loss in terms of the deviation in output from its full-information level is exactly equivalent to this, as of course is a loss function quadratic in real wage deviations from the full-information real wage, as used by Bar-Ilan and Zanello (1996), for instance.) This specification is implicitly underpinned by the assumption that both contracting parties are risk-neutral. It approximates, by means of a second-order Taylor-series expansion, the welfare loss occasioned by departures of employment from its full-information market-clearing level.[4]

Minimisation of (9) by choice of x_i leads to an expression for the optimal degree of indexation of the individual firm's wage to the price level. Since individual firms and their captive labour pools are assumed to be atomistic in Gray, Ball and the subsequent literature, the aggregate degree of indexation, $x = \int_0^1 x_i di$, is assumed to be taken as given by those at firm i who are responsible for the choice of x_i. The resulting symmetric Nash equilibrium degree of indexation is independent of the relative-price elasticity of demand parameter ε:

[4] In the macroeconomic literature, relatively few attempts have been made to ground the wage indexation decision in the maximisation of a specific worker utility function. Bénassy (1995) is an exception. He assumes both workers and their employers have the same logarithmic utility function, and using an overlapping generations model he proceeds to demonstrate that utility maximisation by rational trade unions will lead them to adopt less-than-full wage indexation, thereby showing that the existence of incomplete nominal wage flexibility can be reconciled with utility maximisation by rational agents.

$$x_{NE} = 1 - \frac{\gamma\sigma_\theta^2}{\{(1+\omega)\sigma_\theta^2 + \gamma^2(1-\alpha)[1+\omega(1-\alpha)]\sigma_\phi^2\}} \tag{10}$$

Gray assumes $\gamma = 1$, and evaluating x_{NE} for $\gamma = 1$ yields (differences in notation apart) Gray's equation (15). Setting $\omega = 0$ in (10) yields Ball's equation (13) for the equilibrium degree of (costless) indexation in his model. In other words, the expressions for the Nash equilibrium degree of indexation in Ball and Gray are particular cases of (10) which result when certain values are assigned to γ and ω. Minimisation of (9) subject to the constraint that indexation must be symmetric (i.e. $x_i = x \forall i$)[5], reveals that the Nash equilibrium degree of indexation is efficient in the sense that it is the degree that would be chosen by a benevolent authority concerned to minimise the expected loss of wage-setters at the representative firm.

Equation (10) exhibits several key results of the literature. In particular, (10) prescribes full indexation to the price level when productivity is not stochastic (the $\sigma_\theta^2 = 0$ case). The intuition for this is that in the absence of productivity shocks, the market-clearing real wage must always be equal to its expected value, while movements in the price level are attributable purely to aggregate demand shocks, so that full indexation is therefore necessary to eliminate undesirable departures of the real wage from its expected value. If the economy is subject to both types of shock, (10) prescribes partial wage indexation, with the degree of wage indexation falling as the ratio $\sigma_\phi^2 / \sigma_\theta^2$ decreases. Intuitively, an economy in which productivity shocks predominate, and which are therefore a greater source of price level variability than aggregate demand shocks, will require a low degree of wage indexation, since in the absence of wage indexation, price level movements generally cause the real wage to adjust in the direction considered desirable, so far as the stabilisation of employment at its full-information market-clearing level is concerned. If aggregate demand disturbances are completely absent, so that $\sigma_\phi^2 = 0$, the optimal degree of wage indexation in the Gray model is independent of the productivity shock variance, and depends on the aggregate demand elasticity γ and the labour supply elasticity ω. In

[5] This amounts to minimising squared deviations of aggregate employment from the full-information level.

the special case considered by Gray in which labour supply is completely inelastic, i.e. $\omega = 0$, and γ is at its commonly assumed value of unity, an absence of nominal disturbances ($\sigma_{\phi}^{2} = 0$) requires zero wage indexation for full efficiency, since in this special case the induced change in the price level occasioned by the productivity shock is precisely sufficient to bring about the desired movement in the real wage which ensures employment attains its full-information value.[6]

IV.3 An Alternative Interpretation of the Gray-Ball Result in Terms of the Argument of Blanchard (1979)

The Gray-Ball optimal indexation result given by (10) can alternatively be interpreted as an optimal formula for the exploitation of information. Although it is seldom remarked upon, the Gray-Ball result therefore has aspects in common with the findings of the macroeconomic literature on signal extraction. Papers investigating this issue, of which some discussion has been provided in an earlier chapter of this thesis, generally involve a strategy being chosen by a player conditional on that player's inference about the state of the world, with such inferences being arrived at by applying the simple econometric methodology of signal extraction to a noisy item of information concerning a stochastic variable. In an earlier chapter, for instance, the individual union's nominal wage is set after a forecast of the future productivity shock has been formed on the basis of information contaminated by noise. In the wage indexation literature, the price level plays the role of the potentially noisy item of information from which inferences about stochastic shocks can be made. The difference with the wage-setting literature involving signal extraction are firstly that in the Gray-Ball framework, the informative variable, namely the price level, is realised after the individual player's strategic variable (the degree of indexation) has been chosen, and secondly the aggregate of such strategy

[6] Note that, depending on the values taken by the structural parameters which appear on the right-hand side of (10), the degree of indexation which minimises employment deviations from the full-information market-clearing level may be negative. In the simple $\sigma_{\phi}^{2} = 0$ case, for example, we have $x_{NE}\big|_{\sigma_{\phi}^{2}=0} = 1-[\gamma/(1+\omega)]$, implying that if the elasticity of aggregate demand with respect to price-level movements is sufficiently large (specifically, if $\gamma > (1+\omega)$), negative indexation is optimal.

choices itself influences, via feedback effects, the realised value of the price level itself. As Blanchard (1979) recognises, the optimal indexation scheme seeks to exploit the information content of the price level.[7] By making the nominal wage contingent on the realised value of this informative variable, an outcome closer to the full information outcome can be achieved. If there is only one stochastic shock affecting the economy, the price level becomes perfectly informative of the realisation of that shock, and the appropriate indexation scheme can then bring about the full-information outcome. Thus if there are no productivity shocks, all price movements are attributable to aggregate demand disturbances, and full indexation of the wage to this fully informative variable will ensure ex-post labour market clearing. Conversely, a complete absence of stochastic aggregate demand disturbances renders the price level a perfect indicator of productivity shocks, and then the optimal indexation scheme adjusts the nominal wage by precisely the amount required to ensure that the response in (desired) labour supply to the resulting change in the real wage is equal to the induced change in the demand for labour. With both types of shock present, the price level can be regarded as noisily informative in respect of each of these shocks. For a given variance of the productivity shock, a larger variance of the aggregate demand disturbance renders the price level a noisier indicator of the productivity shock, making higher indexation optimal.[8] Equivalently, for a given aggregate demand shock variance, a larger productivity shock variance increases the noisiness of the price level as regards its informativeness about the aggregate demand shock, and this decline in the reliability of the price level as an indicator of purely nominal disturbances makes it optimal to reduce the linkage between the price level and the nominal wage. The Gray-Ball result thus corresponds to an optimal scheme for exploiting the information content of the price level in order to minimise the deviation of employment from its full-information outcome.

[7] This point is central also to the paper by Adolph and Wolfstetter (1991). That Gray's optimal indexation scheme parameter has affinities to a signal-extraction result is explicitly remarked upon only by Devereux (1987).
[8] As explicitly recognised by Devereux (1987), p.430.

IV.4 Modifications of Gray's Model which do not Engender Externalities

Before proceeding to discuss the existence of externalities in variants of the Gray-Ball model, we briefly review sundry contributions to the literature which, by modifying one or the other of the structural equations of Gray's model, produce results which are argued to throw doubt upon the robustness of her conclusions regarding the optimal degree of wage indexation. Cukierman (1980), for example, shows that Gray's results are crucially dependent on the assumption that employment under contracts is demand-determined. However, as already mentioned elsewhere in this work, the empirical evidence in general supports this assumption that under labour contracts firms retain the 'right to manage'. Bar-Ilan and Zanello (1991), meanwhile, point out that when the structural parameters of the Gray model are assigned plausible values, the resulting numerical value for the optimal degree of indexation is inconsistent with the empirical facts concerning wage indexation in the US. These authors show that the introduction of a term in price-level variability into the objective function assumed by Gray lowers the optimal degree of wage indexation, and thus reduces the extent of the disagreement between theory and the observed low incidence of wage indexation in the US, as well as the low estimated elasticity of contract wages with respect to price-level expectational errors in that country. The flaw in this argument, of course, is that were the degree of indexation endogenised in a model of atomistic agents, every such agent would perceive its individual indexation choice to have a negligible effect on the price level, implying that the expression derived by Bar-Ilan and Zanello would not in fact prevail in equilibrium.

A more interesting paper which falls into this category is Nishimura (1989), which introduces aspects of Blanchard and Kiyotaki (1987) into Gray's framework. In Nishimura's model, the goods market is perfectly competitive, as in Gray, but the labour market is characterised by monopolistically competitive unions, each supplying a special type of labour, with each firm employing members of every union in the economy. In addition to economy-wide shocks, there are labour-type-specific shocks to the individual union's labour demand and supply, with these shocks being imperfectly observed. (Unions observe the composite shocks to their labour demand and supply.) In a passive monetary policy context, Nishimura proceeds to show that in this labour-market scenario, full indexation of the wage to the aggregate price level fails to insulate the economy from purely nominal disturbances, and partial indexation is required to achieve this. Nishimura's conclusion is therefore sharply at variance with that of Gray and the mainstream literature, which finds that full indexation guarantees insulation from purely nominal disturbances.

The validity of Gray's findings has also been questioned by papers which address an issue briefly alluded to above in our discussion of the Gray-Ball model's specification of the nominal wage. Gray, Ball and the bulk of the literature, assume that the wage is indexed to expectational errors regarding the current price level, rather than to lagged expectational errors, or to lagged inflation itself. The robustness of Gray's key result that full indexation stabilises aggregate output when aggregate demand shocks are predominant, and destabilises it when productivity shocks are predominant, to such alternative specifications was investigated by Fischer (1977b). Fischer modified the basic Gray model so that instead of wages being indexed to the expectational error regarding the current period price level (or inflation), the wage is indexed to the adjustment in expectations regarding the price level which has occurred in the immediately preceding period. Temporarily introducing time subscripts in the interests of clarity at this point, Fischer assumed that the wage is indexed fully to $\underset{t-1}{E} p_t - \underset{t-2}{E} p_{t-1}$, the revision in expectations concerning the price level. (Recall that in Gray, the wage is indexed to $p_t - \underset{t-1}{E} p_t$, the expectational error regarding the current period price level.) Fischer found that when the economy featured overlapping two-period contracts, Gray's conclusions were indeed robust to the modification he considered.

Jadresic (2002) departs from Gray and Fischer by considering full indexation of the wage to lagged inflation rather than (less realistically) to expectational errors regarding current inflation or to the lagged adjustment in expected inflation. Jadresic's model, like Fischer's, features uniformly staggered two-period wage contracts, while the shocks are assumed to have a permanent impact on the level of output. For certain values of the structural parameters, including in particular a low responsiveness of inflation to the rate of change of aggregate output adjusted for the productivity shock, Jadresic finds that full indexation of wages to lagged inflation (a typical feature of real-world indexed wage contracts) has a destabilising effect on output both with regard to productivity shocks and to aggregate demand shocks. The conclusions of Gray and Fischer are found to be robust to Jadresic's modification only in cases in which the parameter measuring the responsiveness of inflation to the rate of change of aggregate output, adjusted for the productivity shock, is close to unity, a value which Jadresic argues to be plausible only for very open economies. In addition, Jadresic proceeds to show that when fully accommodating monetary policy is introduced into his model, so that the current money growth rate is no longer fixed as in Gray and Fischer, but responds one-to-one to the previous period inflation rate, output is then found to be immune to real disturbances when there is full indexation of the wage to lagged inflation. This result contrasts sharply with the general finding in the literature building on Gray, that when the wage is indexed to expectational errors regarding current-period inflation, real disturbances are output-destabilising regardless of whether monetary policy accommodates such shocks or not. Jadresic argues furthermore that partial indexation of the wage to lagged inflation also makes both types of shock destabilise output when the structural parameters have particular values, although the destabilising effect is less severe, the lower the degree of partial indexation. Jadresic's model therefore appears to imply that when nominal wages are indexed to lagged inflation, the conclusions arrived at by Gray and Fischer may not be valid. While Jadresic's findings are clearly of potential importance, there needs to develop a wider consensus on the likely values of the model's key structural parameters, however, before the debate on this matter can be viewed as definitively settled.

IV.5 Indexation Externalities in Versions of the Gray-Ball Model with Passive Monetary Policy

The basic Gray-Ball framework outlined above does not give rise to indexation externalities, despite the fact that the choice of indexation degree at the individual firm affects the mean degree of indexation across the economy and hence has repercussions for the variability of the price level. Regardless of the degree of goods market competition, the equilibrium degree of indexation is efficient when adjudged in terms of the assumed loss function of wage-setters. Ball provides an intuitive explanation for this. Although the aggregate degree of indexation affects the individual firm by altering the responsiveness of the price level to both kinds of shock, it turns out that appropriate choice of the individual indexation degree ensures that the impact of unexpected price level movements, due to shocks, on labour demand working through the real wage exactly offsets their impact on labour demand working via real money balances and hence individual product demand. (The latter channel does not operate under perfect competition.) Individually optimal wage indexation therefore ensures that the individual firm's employment is fully insulated from the effects of the indexation decisions taken at other firms, and hence also from the aggregate degree of indexation. For an externality to arise when the loss function of wage setters has the specification it has in Ball, a cost to adopting indexation must be introduced, with this cost differing across firms. Ball shows that when a certain proportion of firms choose not to index at all rather than incur a cost, an increase in that proportion, or in the equilibrium degree of indexation, can be either beneficial or detrimental to the welfare of wage-setters at non-indexed firms. Whether or not the externality works to the disadvantage of the non-indexed wage-setters depends on the relative values of the parameters γ and ε, since the greater price-level variability which is a consequence of higher indexation and/or more firms adopting indexation, affects the individual firm's employment variability via several potentially counteracting channels: the greater real wage variability at non-indexed firms which is caused by higher indexation at the aggregate level, may, in its employment variability impact, be either counteracted or exacerbated by the influence greater wage indexation has, via the induced change in aggregate demand variability, on the variability of individual firm product demand.

Ball's 1988 paper thus initiated one of the principal themes of the subsequent wage indexation literature, namely the investigation of macroeconomic externalities arising from individual indexation decisions. The Gray and Ball papers assumed purely passive monetary policy, however, and hence abstracted from potential links between indexation decisions and macroeconomic outcomes via induced changes in the conduct of policy. A second major concern of the literature, additional to the theme of indexation externalities, has therefore been to extend the Gray-Ball framework to investigate the potentially beneficial effect wage indexation may have on the inflationary bias associated with discretionary monetary policy when a low-mean-inflation policy is time inconsistent. A subsidiary third theme which investigation of the two aforementioned main themes often leads to is the devising of theoretical explanations for why the empirically observed incidence of wage indexation is often low, and as to why, when it is stipulated in wage contracts, the degree of wage indexation is often less than unity. (The Gray and Ball models provide one candidate explanation for the latter, of course.)

It is best to commence our survey of the post-Ball literature with a discussion of Kovanen (1992), Kempf (1998) and Duca and VanHoose (1998a), since unlike the bulk of the later literature, these authors are not concerned to study the impact of wage indexation under discretionary monetary policy, but rather focus on endogenous indexation in a passive monetary policy context. The departure from the Gray model made by Kovanen in his little-noticed paper is to assume that perfectly competitive firms employ two different types of labour, which differ in their marginal products. The Cobb-Douglas production function generally used by the literature is abandoned in favour of a log-linear approximation of the following CES production function: in levels, output is given by $Y = (L^b + N^{cb})^{1/b} e^{\theta}$, where θ is the (log) productivity shock, $(1-b)^{-1}$ is the elasticity of substitution between the two types of labour and $0 < c < 1$ renders L and N the skilled and unskilled types respectively. Kovanen proceeds to demonstrate that if all workers in the economy simultaneously choose their individual degree of indexation, the equilibrium degree is partial, similar in many respects to Gray's result, and common to both types of workers. In other words, productivity differences do not matter for individual indexation choice (although they do matter for a labour type's expected loss in equilibrium), and there are no externalities. However, if one of the two labour types is constrained to have zero indexation, with workers of the other type free to choose their individual degree, indexation externalities do arise. It transpires in Kovanen's model that if skilled workers are the type which can choose their indexation degree, their equilibrium choice then has a negative externality as regards the welfare of the unskilled workers who are constrained to be non-indexed. Conversely, if it is unskilled workers who are free to choose, while skilled workers are constrained, there is a positive indexation externality as regards the welfare of the latter. Of course, the arbitrariness of the assumption that one particular type of labour is constrained and the other not can be criticised, but nevertheless Kovanen's analysis is of considerable value in demonstrating the potential for externalities to arise when the Gray-Ball model is modified.

Turning to Kempf (1998), the element of originality in this paper is the modification of the Ball (1988) model to allow for multiperiod wage contracts, so that each firm's wage-setters must decide upon the degree of indexation of its wage to price-level expectational errors in respect of the price level for several future periods. (Firm *i*'s indexation parameter setting is the same for all periods covered by the contract, with each period having its own particular realisations of both shocks, and the expectational errors all relate to expectations of future price levels formed at the contract-signing date.) The number of periods covered by a contract is assumed to be equal to the number of industry-sectors, which are monopolistically competitive. [9] In order to avoid complications relating to strategic interaction between firms, Kempf assumes that within each sector firms are perfect competitors and hence are atomistic. He proceeds to investigate whether individual indexation choices in the symmetric Nash equilibrium differ from the efficient degree which may be achieved by co-ordinated indexation, and finds that when firms sign wage contracts simultaneously across the entire economy, the equilibrium degree of indexation is efficient, just as it is in the version of Ball (1988) in which indexation is costless. Kempf's most important findings, however, relate to the scenario in which the signing of contracts is staggered across industry sectors, so that while within each sector the firms are identical as regards the set of periods covered by their respective wage contracts, the wage-contract periods of any two sectors are not coincident. For this scenario of overlapping sectoral contract-periods, Kempf finds that, provided both types of shock exist, a particular sector's degree of indexation has spillover effects which increase the frequency of expectational errors in respect of the price level, and which lead necessarily to a larger mean absolute-value deviation of the representative firm's employment from its full-information level. An incentive therefore arises for the individual firm's wage-setters to set a lower degree of indexation than the efficient

[9] A slightly curious feature of Kempf's model is that Ball's relative-product-demand elasticity parameter ε is assigned an inadmissible value of unity. However, although Kempf does not comment on the matter, it would appear that his model is the limiting case as $\varepsilon \to 1$ of a more general model in which ε may take any value in excess of unity.

degree, in order to combat the additional source of employment variability, and for this reason an adverse externality comes to characterise individual indexation decisions.

The third paper which investigates indexation externalities in the Ball (1988) framework is Duca and VanHoose (1998a), which weds the monopolistically competitive firms of Ball's model to the multisector economic structure of Duca and VanHoose's earlier papers, Duca (1987) and Duca and VanHoose (1991). The key feature of these earlier papers is that the economy consists of two sectors which differ in two respects, namely as regards the good produced and labour-market structure. The two sectors produce different goods, but within each sector firms are in perfect competition, while all workers across the economy consume both these goods. The labour market of the classical sector features spot-market hiring of labour, while the non-classical or Keynesian sector features wage contracts. Duca (1987), which does not investigate wage indexation issues at all, uses this basic framework and also assumes that labour supply is elastic with respect to the consumption real wage, and that only aggregate demand disturbances affect the economy (i.e. productivity shocks are ignored by Duca). In the absence of activist monetary policy an aggregate demand disturbance will differ in its impact on the prices of the two goods because of the stickiness of contract nominal wages in the non-classical sector. However, this has implications for employment and output in both sectors because the resulting movement in the consumption real wage will change equilibrium employment in the classical sector. Duca (1987) thus demonstrates that even sectors which have full flexibility of both goods prices and nominal wages are not immune to being affected by aggregate demand disturbances when sticky contract wages elsewhere in the economy create spillover effects on the consumption real wage. While this insight is of value in itself, Duca's paper throws no light on externality issues, since the existence of wage contracts in one sector of the economy is simply assumed, and is not endogenised.[10]

This two-sector, two-good, perfect competition framework is developed in Duca and VanHoose (1991), which allows for multiparameter wage indexation in the non-classical

[10] These wider issues are taken up in Duca and VanHoose (2001), which allows for productivity shocks, aggregate demand shocks, and sector-specific demand shocks, and investigates the repercussions of an increase in the degree of (monopolistic) goods market competition on the decision taken at the individual firm level, to adopt a (non-indexed) wage contract. Since this paper does not consider wage indexation issues at all, it is not appropriate to discuss it further here.

sector's wage contracts, while sector-specific productivity shocks are also introduced. Since Duca and VanHoose (1991) focuses on the optimal multi-parameter indexation scheme for their two-sector scenario; we defer discussion of this paper to a later section of this chapter. For the purposes of our present discussion, however, it is worth remarking that the spillover effects identified by Duca (1987) are found in Duca and VanHoose (1991) to be relevant to optimal indexation.

Duca and VanHoose (1998a), like its 1991 forerunner, modifies Duca (1987), but does so in a different way. While it is assumed that monetary policy is passive, and that there are two sectors which differ according to whether they feature wage contracts or a labour spot market, the assumption made in Duca (1987) that there are just two goods, each of which is produced by perfectly competitive firms, is discarded, with all firms assumed instead to be in monopolistic competition, as in Ball (1988). The proportion of firms hiring labour under contracts, with the contract wage indexed only to the price level, is exogenous and denoted Ω, while the remaining proportion of firms, $1-\Omega$, engage in spot-market hiring. There are no sector-specific shocks in Duca and VanHoose (1998a) and, as in Ball (1988), the productivity shock is common to all firms. Finally it is assumed that labour supply is completely inelastic with regard to the real wage, since the effect which is of interest is not the spillover effect involving equilibrium employment in the classical sector studied in Duca (1987), but rather is a spillover effect relating to the endogenous degree of wage indexation.[11]

The identification of this spillover effect, and its functional relationship with the degree of goods market competition, as represented by ε, is the principal point of Duca and VanHoose's 1998a paper. However, they also contend to have found an externality arising from the choice of indexation degree by wage setters at the individual firm, and consequently appear also to make assertions about the welfare implications of the spillover effect. In what follows, it will be argued that this contention that an externality exists in the Duca and VanHoose (1998a) model is incorrect. The average goods price in

[11] With completely inelastic labour supply, employment and output will be at their market-clearing values in the classical sector, so that no welfare loss can arise in the classical sector as a result of spillover effects on the real wage. (The consumption real wage will be affected by the existence of wage contracts in the non-classical sector, but this is of no consequence for the standard loss function which measures the welfare loss in terms of the squared deviations of employment from its full-information value.)

the sector which features spot labour markets responds in a different manner to the shocks than does the average price of goods in the non-classical sector, and this disparity in response constitutes a spillover effect on the latter sector's consumption real wage. Duca and VanHoose, however, do not confine their discussion to this spillover (a term which encompasses effects which may be neutral in their welfare repercussions) but go further and talk of an associated externality appertaining to the indexation decisions taken at the level of the individual firm. Use of the word 'externality', of course, unambiguously asserts that the decision taken by an individual has a non-neutral welfare effect, whether adverse or beneficial, on others. Such non-neutral welfare effects do not characterise the equilibrium indexation decisions of individuals in Duca and VanHoose (1998a), since it can be shown that the equilibrium indexation degree is in fact efficient from the point of view of the wage-setters at firms in the non-classical sector.[12] Efficiency here is represented by the aggregate indexation degree which minimises the variance of aggregate employment in the non-classical sector. If the trouble is taken to obtain an expression for the variance of aggregate non-classical employment, the degree of indexation which minimises this variance, and is therefore efficient, is found to be:[13]

$$x_{efficient} = \frac{[\alpha(1-\Omega)+\varepsilon(1-\alpha)]}{\{[\alpha(1-\Omega)+\varepsilon(1-\alpha)]\sigma_\phi^2 + [\varepsilon-\alpha(\varepsilon-1)(1-\Omega)]\sigma_\theta^2\}} \tag{11}$$

and this is identical to the equilibrium degree of indexation in Duca and VanHoose's model.[14] The implication is that for given values of the exogenous parameters Ω and ε, no externality derives from individual indexation decisions in the scenario considered by Duca and VanHoose. This is not to say, of course, that changes in the exogenous parameters Ω and ε do not have welfare repercussions: they do, but such parameter changes are not the focus of attention in Duca and VanHoose. Their principal point is that the (partial) degree of equilibrium (and, as we have seen, efficient) indexation of the

[12] Of course, since the labour market in the classical sector always clears, the equilibrium degree of indexation in the non-classical sector is obviously also efficient from the point of view of the economy as a whole.
[13] Using the notation of this chapter.
[14] The denominator of Duca and VanHoose's equation (14) on p.585 of their paper contains a typographical error.

wage to the price level, as given by (11), is dependent on the degree of goods market competition when there exists a sector of the economy which hires labour on a spot market rather than under contracts. This point is most starkly apparent from the fact that if all firms employ labour under contracts (the $\Omega = 1$ case), and/or there is perfect competition, so that all firms produce a homogenous good and the classical sector's average price cannot depart from the average price in the non-classical sector, then the spillover effect is absent and the efficient and equilibrium degree of indexation simplifies to the result obtained by Gray for the case of real-wage-inelastic labour supply.[15] The authors proceed to demonstrate that the degree of wage indexation to the price level decreases as the degree of goods-market competition rises, and contend that this negative relationship would still be found were allowance made for endogenous changes in Ω, the proportion of firms with wage contracts, in response to an increase in ε. In other words, any indirect effect of a change in ε on the equilibrium degree of wage indexation, working via an induced change in Ω, will not outweigh the direct effect of an increase in ε on the degree of indexation. These findings that greater goods market competition should be associated with a lower degree of equilibrium wage indexation, is supported by the empirical evidence, of which some is supplied by Duca and VanHoose's own accompanying econometric study.

[15] While setting $\Omega = 1$ in (11) for the equilibrium and efficient degree of indexation in Duca and VanHoose's model yields Gray's result, taking its limit as $\varepsilon \to \infty$, which amounts to considering the case of perfect competition, does not yield the Gray result. The reason for this is that Duca and VanHoose have assumed Ω to be exogenous, whereas a more sophisticated model would allow for endogeneity of Ω to changes in the relative-product-demand elasticity ε. Under perfect competition, a diversity of labour market institutions which allow marginal cost to differ between sectors would not be possible. In a footnote Duca and VanHoose concede that, in another paper of theirs, later published as Duca and VanHoose (2001), it is seen that the (endogenous) proportion of firms engaged in spot-market hiring of labour increases as a consequence. It is this failure to allow Ω to be endogenous in their 1998a paper which explains the implication of their 1998a model that $\lim_{\varepsilon \to \infty} x_{NE} = (1-\alpha)\sigma_\phi^2 / [(1-\alpha)\sigma_\phi^2 + (1-\alpha+\alpha\Omega)\sigma_\theta^2]$. This expression, and the implication that the perfectly competitive case of their 1998a model does not replicate Gray's result, is not in fact to be found in Duca and VanHoose (1998a). Ironically, the paper's incorrect expression for the equilibrium degree of indexation, namely Duca and VanHoose (1998a) equation (14), does reproduce Gray's result when its limit as $\varepsilon \to \infty$ is taken.

IV.6 Wage Indexation and Macroeconomic Outcomes under Discretionary Monetary Policy

IV.6.1 Introductory Remarks

The previous section has surveyed the relatively few papers which model the macroeconomic impact of wage indexation when the monetary regime is entirely passive, i.e. the authorities refrain entirely from monetary policy, and private sector agents recognise that this is the case. In this section, we discuss the rather more voluminous body of work which investigates wage indexation issues under discretionary monetary policy regimes. This body of literature can be crudely subdivided into two strands. The first of these ignores the potential stabilisation role of monetary policy in response to the observed values of the shocks, or forecasts thereof, and instead focuses on the relationship between wage indexation and any inflation bias. As will be seen below, several papers in this strand do not build on the Gray-Ball framework, but instead are based upon Barro and Gordon (1983a). The second strand explicitly allows for active stabilisation policy by the authorities.

It should also be mentioned that the literature concerned with optimal stabilisation policy when the economy is subject to diverse kinds of macroeconomic shock also often assigns a role to wage indexation. However, detailed discussion of this literature is not warranted here, since its focus is on how the economy can be stabilised when it is subject to shocks rather than on indexation externalities: the degree of wage indexation is typically not endogenised, and the time-inconsistency issue and its associated inflation bias are ignored. Papers in this vein include Fethke and Jackman (1984) and Turnovsky (1987). Fethke and Jackman use an IS-LM AD-AS model to investigate how a fixed-interest-rate rule performs relative to a fixed-money-supply rule when wage indexation affects the responsiveness of aggregate supply to price level movements.[16] Turnovsky

[16] Their principal result is that even when private-sector wage decisions are underpinned by rational expectations, the mean and variance of output differs between the two rules if wages are not made contingent on the realised values of the money supply or interest rate. This result is a counter-example to the policy-neutrality proposition of Sargent and Wallace (1975). In particular, if the degree of wage indexation to the price level is endogenously determined by contracting parties concerned to minimise departures of employment from its full-information level, the component of output variability arising from supply shocks is invariant to the rule's design, but the component of output variability arising from shocks to aggregate demand is sensitive to the choice of policy rule. Despite rational expectations, the optimal choice of rule then depends upon the same considerations as those identified by Poole (1970) for an adaptive-expectations framework.

(1987), meanwhile, only in passing discusses the optimal degree of indexation, and finds that when the degree of wage indexation is an additional policy instrument which can be used in conjunction with activist monetary policy, the optimal degree of indexation is partial, a result similar to Gray's.

IV.6.2 Models in which Monetary Policy is Discretionary but Non-Activist

We begin with those contributions which ignore the existence of stochastic shocks, and consequently also ignore the stabilisation repercussions of wage indexation emphasised by the Gray-Ball approach. Fischer and Summers (1989) is one such classic paper which abstracts from the existence of shocks, and simply assumes the existence of wage indexation as an exogenous feature of the economy. These authors do not follow Gray and Ball, and instead adopt the Barro and Gordon (1983a) set-up in which the authorities can, given naïve private sector expectations, exploit a short-run Phillips curve by delivering surprise inflation and thus occasion a departure of unemployment from its (socially suboptimal) natural rate. Of course, in the rational-expectations equilibrium this incentive to deliver surprise inflation is recognised by the public, and the suboptimally high time-consistent inflation rate is both expected and delivered. By steepening the short-run Phillips curve and hence reducing the marginal benefit, in terms of temporarily reduced unemployment, of a given amount of surprise inflation, wage indexation has a potentially beneficial effect on the inflation bias. Fischer and Summers abstract from the possibility that higher wage indexation might mitigate the social costs of inflation, and (implicitly) argue that indexation measures which combat the undesirable wealth-redistribution effect of inflation (such as indexation of pensions, benefit payments, and debt instruments), lower the marginal social cost of a given inflation surprise, and thus exacerbate the bias.[17]

Ball and Cecchetti (1991) also use the Barro and Gordon model in order to demonstrate the potentially socially beneficial impact of wage indexation in a purely deterministic scenario. They introduce a tax distortion to labour supply in order to generate a trend

[17] Agell and Ysander (1993) is a reply to Fischer and Summers which is largely concerned with the potentially beneficial impact on the inflation bias of indexation of tax brackets to protect real wages from erosion by inflation.

inflation rate, while the key feature of their model is the existence of two-period nominal wage contracts, the signing dates of which are staggered such that half of all the contracts in the economy come up for renegotiation in each period. The wage specified in these contracts is assumed to be 'fixed', in the sense that it is the same in both periods covered by the contract. Wage-setters are assumed to set their individual firm wage so as to minimise deviations of the real wage from its equilibrium value, and given the fixed contract wage assumption, this objective makes it optimal to set the contract nominal wage equal to the mean of the market-clearing wage values for the two periods covered. The presence of a trend inflation rate (i.e. a bias) arising from the time inconsistency of zero-inflation discretionary monetary policy, creates greater dispersion in real wages which directly reduces social welfare.[18] Thus unlike in Fischer and Summers, the role of wage indexation in Ball and Cecchetti's paper is not restricted to its effect of steepening the Phillips curve and thereby mitigating the inflation bias. Instead it has a second, counteracting, effect which outweighs the Phillips-curve-steepening effect, and which operates to increase the bias by making a given inflation surprise less costly to society and hence more tempting to the policymaker. Ball and Cecchetti find that an increase in the proportion of firms which have fully indexed wage contracts is in net terms socially beneficial, in that the direct enhancement of social welfare brought by reduced wage dispersion outweighs the social cost of its accompanying evil, a higher inflation bias.[19] The arguable weaknesses of their paper are twofold. Firstly, their result requires that contract wages in the absence of indexation are 'fixed' rather than 'predetermined': i.e. the contract does not allow for the second-period wage to differ from its first-period counterpart. Ball and Cecchetti's only defence of this crucial assumption is that increased prevalence of wage indexation has been found in empirical research to be associated with greater frequency of wage adjustments in response to inflation. The second arguable shortcoming is the absence of stochastic shocks, which, as the Gray model shows, makes partial wage indexation generally optimal. Ball and Cecchetti's counter-argument to this

[18] Ball and Cecchetti plausibly assume the firm-level individual loss is quadratic in deviations of the real wage from the value the market-clearing real wage would have in the absence of the tax distortion to labour supply.

[19] The obvious existence of other social costs associated with inflation are found by Ball and Cecchetti to render the inflation bias less sensitive to increased prevalence of wage indexation, thereby strengthening their case that wage indexation, while inflationary, is socially beneficial in net terms.

is that the emphasis on the social benefits of increasing the *proportion* of firms with indexed contracts appears to be a robust policy prescription even when the optimal degree of wage indexation to the price level is less than full. This counter-argument may fail, however, if proper account is taken of how, when stochastic shocks are present, more widespread wage indexation may cause greater variability of inflation and thus may exacerbate wage dispersion.

The first papers to introduce discretionary monetary policy into the Gray-Ball framework were those of Devereux (1987, 1989), which feature a tax distortion to labour supply in order to generate an inflation bias. The policy scenario may be described as 'discretionary but non-activist', since the authorities are assumed to set the money stock after contract wages have been set, but before the realisation of stochastic shocks, with the objective of minimising a standard social loss function featuring two terms, a quadratic term in output deviations from socially optimal output (i.e. the output level that would prevail under full information and in the absence of the tax distortion) and a quadratic term in inflation itself. Devereux's principal point is that an increase in the variance of the aggregate demand disturbance can in certain circumstances be beneficial to welfare. The reason for this is that although a higher such variance must necessarily worsen real wage variability, and hence output and employment variability, despite the induced increase in the optimal degree of wage indexation, this adverse effect could be offset, and even outweighed, by the beneficial effect of higher wage indexation on the inflation bias. The most propitious circumstances for this to be the case were found to be low variability of productivity together with a large inflation bias and a low weight on the inflation term in the social loss function.

Devereux's 1987 paper, therefore, was limited in its aims. It confined itself to demonstrating that when the degree of wage indexation is endogenously chosen to minimise employment deviations from its full information value, and discretionary monetary policy is characterised by an inflation bias, higher aggregate demand volatility does not necessarily worsen social welfare. Thus Devereux's paper did not address the question of what is the socially optimal degree of wage indexation in this non-activist scenario, and the related question of whether individual indexation decisions have social externalities. These issues are the principal concern of Waller and VanHoose (1992), a paper which builds on Devereux (1987) in order to demonstrate that the equilibrium indexation choices of atomistic wage-setters are inefficiently low in degree, when evaluated from the perspective of a social loss function comprised of a weighted sum of the variance of output around its socially optimal level, and the square of the trend inflation rate.[20] Waller and VanHoose's result that equilibrium indexation is socially inefficient therefore depends upon their assumption that it is only the trend inflation rate (and not the variability of inflation about that trend) which is detrimental to welfare. Their loss function fails to capture the possibility that it is greater inflation variability, rather than a higher mean, that constitutes the major source of welfare loss to society. A specification of the social loss function which features a quadratic term in actual inflation, rather than a quadratic of the trend, may have led to different conclusions regarding the direction of the externality, as Waller and VanHoose themselves acknowledge in a footnote.[21]

IV.6.3 Models in which Monetary Policy is Discretionary and Activist

Models which allow for active stabilisation policy by the monetary authorities can broadly be subdivided into those which treat the degree of wage indexation as exogenous, and consequently adapt Barro and Gordon (1983a), and those which allow for endogenous indexation, building on the Gray-Ball framework outlined earlier.

In the former category are the papers by Mourmouras (1993, 1997b), which constitute attempts to evaluate the social welfare repercussions of an exogenous change in the degree of wage indexation, by taking into account how wage indexation may possibly affect the marginal social cost of inflation surprises. The key idea in Mourmouras (1993) derives from the theoretical demonstration of Katz and Rosenberg (1983) that greater variability of the real wage caused by increased inflation variability, results in a loss of production efficiency and hence is detrimental to social welfare. Since the implications of

[20] Mourmouras (1997a) is a minor contribution which shows that in the Waller and VanHoose (1992) model, delegation of monetary policy to a central banker whose weight on the squared trend inflation term is greater than society's, reduces the socially optimal degree of indexation, but has no effect on the equilibrium degree of indexation. Thus in the Waller and VanHoose model, the more conservative is the central banker, the smaller is the positive externality appertaining to the individual indexation decision.

[21] Waller and VanHoose (1992) p.1459, footnote 4.

this causal connection between wage indexation and socially detrimental real wage variability are adjudged by Mourmouras not to matter for the degree of wage indexation chosen by wage setters, his papers omit the firm-level optimisation exercises which are the principal concern of Gray, Ball, and Devereux. His focus instead is upon examining the implications of Katz and Rosenberg's argument in the context of the Barro and Gordon (1983a) model: in other words, a Phillips-curve trade-off is assumed, with the authorities able directly to set inflation in response to the realised value of the productivity shock. Mourmouras shows that in his model the variance of the real wage is falling in the degree of wage indexation. Mourmouras therefore contends that since this will reduce the marginal social cost of an inflation surprise, and thus will worsen the policymaker's temptation to spring such a surprise, the effect of higher wage indexation on the inflationary bias is ambiguous, rather than unambiguously beneficial as Fischer and Summers (1989) had argued. Clearly, the validity of Mourmouras' argument depends on there being a perception on the part of the private sector that lower real wage variability is socially beneficial, and hence that exogenous increases in wage indexation, by lowering real wage variability, are (for a given Phillips curve trade-off) inflationary. Thus the beneficial effect of wage indexation on the bias working via its reduction of the marginal benefit of an inflation surprise, is counteracted by a bias-worsening effect working through public perceptions of the temptation facing the policymaker. However, the current consensus is that the resource-misallocation costs of price misperceptions caused by inflation, of which Katz and Rosenberg's production-inefficiency story is an example, are likely to be small (see for example, Cecchetti, 2001). Mourmouras (1997b) extends this author's 1993 paper by allowing the policymaker's weight on inflation stabilisation to vary endogenously with exogenous changes in wage indexation, the dubious rationale for this again being the notion derived from Katz and Rosenberg that the lower real wage variability brought by higher wage indexation reduces the social costs of inflation and hence alters the policymaker's weight parameter. Taking into account this purported effect on the policymaker's preferences, Mourmouras is once again led to the conclusion that the impact of higher wage indexation on mean inflation is of ambiguous sign.

Another limitation of Mourmouras' papers is that changes in the degree of wage indexation are entirely exogenous, rather than determined endogenously by firm-level decisions. The assumption of exogeneity has the disadvantage that it precludes identification of indexation externalities, and furthermore deprives Mourmouras' arguments of applicability to real-world economies in which indexation is chosen by individual agents: in such scenarios, a change in the marginal social cost of inflation surprises, or a change in the policymaker's weight parameter, would induce further adjustments in the degree of wage indexation, interaction effects which are not captured by Mourmouras' model. His analysis is most relevant, therefore, to economies in which the degree of wage indexation is set by an authority distinct from the monetary policymaker. This consideration raises the question of whether there is an optimal degree of wage indexation, considered as a policymaker's instrument, to accompany an inflation-biased discretionary monetary policy regime. This issue is addressed by Milesi-Ferretti (1994) and Crosby (1995).

Milesi-Ferretti's two most important results are as follows. Firstly, when monetary policy can respond to both types of shock, and there is some underlying distortion which creates an inflation bias, an appropriate setting of the wage indexation instrument can reduce the inflation bias and the variance of inflation for a policymaker of given preferences, but that the differences in bias and inflation variance between any two policymakers whose stabilisation preferences vis-à-vis inflation and output differ, are not completely eliminated under optimal indexation by the policymaker. The second result is that with optimal choice of indexation beforehand, the particular weight parameter of the policymaker does not matter for the resulting variability of employment and output. This is because in the absence of wage indexation, employment and output will be more variable, the higher the policymaker's weight on inflation, but it is precisely such inflation-averse policymakers who have less need to set a high degree of wage indexation in order to reduce the bias. A higher weight on inflation does, ceteris paribus, lead to higher employment variability, but it also leads to a lower setting of wage indexation, which for the standard reasons found in the literature, dampens the impact of productivity shocks on employment, aggregate demand shocks being completely neutralised by monetary policy in Milesi-Ferretti's model. It turns out that when policymaker's wage indexation instrument setting is optimal, this is just sufficient to offset the role of the policymaker's weight in determining employment variability.

In an extension, Milesi-Ferretti considers how a governing party's pre-election commitment to a particular setting of the wage indexation instrument will be influenced by the parties' respective weights on inflation: one conclusion which emerges is that a party with a high weight on inflation, and consequently relatively modest bias problem, may have an incentive to set a low (perhaps even zero) degree of wage indexation when an election is imminent, in order to make a rival, less inflation-averse, party a less attractive electoral prospect. (This of course abstracts from time-inconsistency problems relating to the setting of the wage indexation instrument itself, an issue not considered by Milesi-Ferretti.) Crosby (1995) analyses a similar scenario of electoral rivalry, but in his paper the possible settings of the wage indexation instrument are restricted to be either zero or one. Crosby differs from Milesi-Ferretti's extension in that the incumbent party cannot bind the victorious party in an imminent election to a particular wage indexation setting, while discretionary monetary policy is conducted by the elected party in ignorance of the future values of the shocks (i.e. the scenario is 'non-activist'). Crosby assumes that 'left-wing' parties are particularly prone to an inflation bias on account of the public's perception of their preferences. His principal finding is that such left-wingers will often (depending on structural parameters such as the productivity and money-demand shock variances, and the size of the distortion which generates the bias) find it to their electoral advantage to precommit to a regime of full wage indexation. The recent political history of Australia is cited as an example of this.

Having dealt with that portion of the literature which has studied the implications of exogenous changes in the wage indexation parameter when low-mean-inflation activist monetary policy is time-inconsistent, we now turn to the literature on endogenous wage indexation under such a monetary regime. The principal papers here are VanHoose and Waller (1991), Fukuda (1993), Bar-Ilan and Zanello (1996), Hutchison and Walsh (1998), and Lawler (1998).

VanHoose and Waller (1991) consider four scenarios which differ according to the information sets available to atomistic wage-setters and the monetary authorities. The information sets may contain forecasts of the future shocks, with these forecasts being provided by an impartial third party. This assumption therefore evades the issue of whether forecasts published by the authorities will be credible to the private sector when private sector expectations based on such forecasts are a determinant of the subsequent effectiveness of monetary policy. With the forecasts thus assumed to be wholly credible to wage-setters, VanHoose and Waller proceed to examine the equilibrium degree of indexation for two scenarios in which the authorities have no informational advantage over the private sector, with forecasts unavailable in one of those two scenarios, but commonly available in the other, as well as two scenarios in which the authorities do enjoy an informational advantage and observe the shocks with complete accuracy before setting the money stock (the latter two scenarios again differ as to whether forecasts have been made available or not). When the policymaker does not enjoy an informational advantage, Gray's optimal indexation result (or a variant of it featuring the forecast variances of the two shocks, rather than the shock variances themselves) emerges as the equilibrium degree of indexation. Once an informational advantage is conferred on the policymaker, so that the money stock is set conditional on full information regarding both shocks, the equilibrium degree of indexation is found by VanHoose and Waller to be zero, regardless of the availability of the forecasts. This conclusion was based on the assumption that the admissible value of the indexation parameter is confined to the unit interval. Bar-Ilan and Zanello (1996) relax this assumption in a model which, despite numerous superficial differences from VanHoose and Waller, essentially reproduces the latter's third scenario of fully informed authorities together with no forecasts available to wage-setters. Without a lower bound on the degree of wage indexation chosen by individual agents, Bar-Ilan and Zanello find that the equilibrium degree of indexation is unbounded below. In other words, when fully informed monetary policy responds actively to shocks in pursuit of both output stability and price-level stability, wage-setters at the individual firm can always reduce the departure of the firm's employment from its full-information value by indexing the wage negatively to the price level, and the more negative this indexation degree the better.[22] Bar-Ilan and Zanello's result that a lower

[22] Bar-Ilan and Zanello in fact specify the loss function of individual firm wage-setters to be quadratic in deviations of the real wage from the full-information equilibrium real wage. This of course is exactly equivalent to the more common practice standard in the literature of specifying wage-setters' loss in terms of employment deviations from full-information equilibrium employment.

bound must be placed on the admissible degree of individual indexation if an equilibrium is to exist, must also be modified in the light of Lawler (1998), another paper which reconsiders the fully informed activist monetary policy scenario of VanHoose and Waller (1991). Lawler's paper reveals the existence of a Nash equilibrium overlooked by Bar-Ilan and Zanello, in which all firms' wages are fully indexed to the price level. The monetary authorities consequently cannot influence employment by means of inflation surprises and therefore devote policy to stabilising the price level completely, and as a consequence agents at the individual firm are deprived of an incentive to deviate from full indexation. However, the difficulty atomistic agents may have in co-ordinating on full indexation is recognised, especially as individual welfare and social welfare alike are higher in the alternative equilibrium at the lower bound.

This group of papers, therefore, has principally focused upon the existence and characteristics of equilibrium indexation under activist monetary policy, when atomistic agents are concerned to minimise the standard loss function assumed by Gray and Ball. The common finding that the equilibrium degree of indexation is zero when the authorities possess full information on the shocks is interpreted as a possible explanation for the general absence from real economies of wage indexation to the price level.[23] In addition, VanHoose and Waller have a further interesting result, analogous to Devereux's, in their scenario in which forecasts are available to both wage setters and the monetary authorities, but the latter are not fully informed about the shocks when setting the money stock. An increase in the conditional variance of the aggregate demand disturbance can in certain circumstances be beneficial to welfare, because of the induced change in the equilibrium degree of wage indexation. This possibility does not arise when the authorities possess full information, since active monetary policy then neutralises the aggregate demand disturbance completely, making productivity shocks the only source of welfare loss to atomistic agents and hence bringing about zero indexation in equilibrium. The socially optimal degree of indexation, and the related issue of externalities, are not addressed in VanHoose and Waller (1991), the principal concern of which is to demonstrate how the equilibrium degree of indexation in a perfectly competitive

[23] Bar-Ilan and Zanello (1996) is in fact exclusively devoted to providing a theoretical explanation for this empirical stylised fact, and, like Lawler (1998), abstracts from the existence of an inflation bias, so that a welfare evaluation of equilibrium indexation is not pursued.

economy with atomistic wage-setters is sensitive to the information sets of the private sector and the authorities.

This sensitivity of equilibrium outcomes to the economy's information structure is also evident from Fukuda (1993) and Hutchison and Walsh (1998). Fukuda's assumed information structure differs from that of VanHoose and Waller: his paper adapts Devereux (1987) by conferring on the policymaker an informational advantage regarding one of the shocks. As in Devereux, private sector atomistic agents do not possess any information about the future shocks at the time the wage contract is concluded. The policymaker, however, does possess noisy information about the aggregate demand disturbance (but no information at all regarding the productivity shocks) at the time of setting the money stock. A time inconsistency problem which causes an inflation bias is assumed. The equilibrium degree of indexation in this scenario is then found to resemble Gray's result, save that it depends not only on the variances of the productivity and aggregate demand shocks, but also on the variance of the noise disturbance which contaminates the authorities' signal of the aggregate demand disturbance. Fukuda demonstrates that in this setting, an increase in the noisiness of the policymaker's information can in some circumstances be socially beneficial, as a result of the endogenous increase in the degree of wage indexation and consequent mitigation of the inflation bias.

The model set out in the appendix to Hutchison and Walsh (1998) also differs in its information structure from VanHoose and Waller, in that Hutchison and Walsh assume the authorities possess full information on the economy-wide productivity shock, but lack full information on the eventual price level which will follow their monetary response to the productivity shock. (Aggregate demand shocks are not explicitly modelled by Hutchison and Walsh, who assume that the monetary instrument is an intended realisation of the price level, which differs from the actual realisation of the price level by a random control error which cannot be predicted by the authorities.) Hutchison and Walsh make a second departure form the mainstream of the literature by assuming that a single all-encompassing monopoly union sets both the base nominal wage, and the degree of indexation, for all wage contracts in the economy. Their reason for this assumption is that it is argued to provide a better representation of the centralised wage bargaining which characterises New Zealand, the study of the impact of institutional reforms on that country's output-inflation trade-off being the principal focus of Hutchison and Walsh's paper. Since their emphasis is on investigating the relationship between the central bank's weight on inflation and the output-inflation trade-off, rather than how changes in that weight, together with induced changes in the degree of wage indexation, affect any inflation bias, the authors abstract from the existence of such a bias by assuming the central bank's preferred output objective is consistent with the union's employment objective. Thus the welfare issues which are the subject of VanHoose and Waller (1991) and other papers are not addressed by Hutchison and Walsh. It is their result regarding the equilibrium degree of indexation under this alternative information structure which is of interest. They find that the degree of indexation chosen by the single union as Stackelberg leader is a decreasing function of the weight placed on inflation stabilisation in the authorities' loss function, and this is so irrespective of whether or not the union's loss function also features a quadratic term in the deviation of the real wage from its target real wage (the latter is the real wage which ensures the union's employment target is the mean employment level). A theoretical rationale is thus provided for the empirical evidence that New Zealand's output-inflation trade-off increased (i.e. the short-run Phillips curve became flatter) following 1989, the year of reforms which can plausibly be interpreted as having increased the central bank's weight on inflation.

IV.7 Multiparameter Wage Indexation

This penultimate section prepares the ground for Chapter VI by surveying the rather sparse literature on the macroeconomics of multiparameter wage indexation. With this form of indexation, the wage is indexed not only to the price level but also to an additional variable.

The earliest contribution to this vein of literature has already been mentioned, namely Blanchard (1979), a paper which insightfully draws attention to the potential for multiparameter schemes to achieve superior outcomes by exploiting an additional source of information about macroeconomic disturbances. Blanchard's chief concern, however, was to devise a model in which, despite the manifest information-related advantages of multiparameter indexation, structural features of the economy and active stabilisation policy lead conventional single-parameter indexation to be the equilibrium choice of indexation scheme at the level of the individual firm. Just as in Gray and Ball, in Blanchard (1979) firms and workers are both risk neutral, and are parties to a contract which, when expectations regarding prices prove correct, replicates the full-information outcome (i.e. ensures the labour market clears). Productivity shocks to the firms' production function are not explicitly modelled: instead the real disturbance takes the form of an expectational error regarding the price of the composite consumption good relative to the price of a composite input-materials good. Movements in the relative materials price require substitution of labour for materials, or vice versa, for full efficiency, and (given the assumption of demand-determined employment) ex-post labour market clearing. Blanchard shows that a multiparameter indexation scheme which involves indexing the wage fully to the expectational error regarding the consumption good's price (the 'price level') and negatively to the expectational error regarding the relative price of materials, can replicate the full-information outcome. However, if the adoption of such a scheme involves a cost, a restricted wage-indexing rule which involves solely indexing to the price level may be optimal. It is found that an increase in the correlation (whether positive or negative) between the price level and the relative materials price makes it more likely that the restricted rule is optimal, since a higher such correlation makes the price level more informative as regards the real disturbance. A higher variance of the relative price of materials, however, renders it less likely that the restricted rule is superior to the alternative of incurring the cost of also indexing the wage to this relative materials price. Blanchard introduces an aggregate demand equation which features stochastic velocity and monetary shocks, and allows for a stabilisation response by the authorities to such shocks as well as to the real disturbance, the relative price of materials. A distinction is drawn between the actual joint distribution of the two prices (their variance-covariance matrix), and its subjective counterpart upon which the parties to the wage contract base their decisions as to whether to index to both prices or solely to the price level, as well as their decision regarding their degree of indexation to these price(s). The subjective distribution is revised after each contract period closes, and adjustments to the degree of indexation alter the actual distribution by affecting the variance of the price level. In equilibrium the actual and subjective distributions are identical, and no further adjustments to the adopted indexation rule are made. In Blanchard's model, a decision to adopt a restricted indexing rule is self-perpetuating in that it leads to an equilibrium also characterised by such a rule, with the degree of indexing between zero and unity, full indexing being optimal only in the two extreme cases of perfect correlation and no correlation whatsoever between materials and consumer goods prices. Under such a restricted indexing-to-price-level-only rule, it is found that stabilisation policy has the advantageous effect of increasing the correlation of the price level with the relative materials price, thereby making the restricted rule more effective in approximating the full-information outcomes for the real wage and employment.[24]

This important idea of optimal information exploitation is also central to the key paper by Karni (1983) which pointed out, in a model based on Gray (1976), that indexing the contract nominal wage appropriately to both the price level and to aggregate output, would enable the real wage to adjust to its market-clearing value, i.e. the value the real wage would have were full information regarding the productivity shock available at the time wage contracts are concluded. Note that, unlike in Blanchard (1979), the values of the multiparameter indexation parameters are not endogenously determined in Karni, since the latter's implicit concern was to identify their optimal values from the viewpoint of a benevolent authority which possesses the power to impose such a scheme on all firms. Karni differs significantly from Blanchard in assuming that the real shock is a common disturbance to the representative firm's production function, and in abstracting entirely from active stabilisation policy. These features of Karni's model, together with the assumed firm-level objective function, imply that were determination of the indexation parameters to be endogenised, their equilibrium values would be identical to those found to be socially optimal by Karni.

The papers in this strand of literature which are most closely related to Karni's analysis are Duca and VanHoose (1991, 1998b), both of which focus, like Karni, on deriving the multiparameter indexation scheme which replicates the full-information market-clearing

[24] A similar conclusion that stabilisation of aggregate demand disturbances can be welfare-enhancing for the reason that it improves the information content (with respect to real shocks) of the price level is to be found in Adolph and Wolfstetter (1991).

employment level. (Duca and VanHoose in this pair of papers do not endogenise the degrees of indexation.) The major departure from Karni concerns the second variable to which the wage is indexed. In particular, Duca and VanHoose argue that, when accompanied by indexation of the wage to the price level, contractual provisions for productivity-related bonuses and profit-sharing amount to a form of multiparameter indexation, and that this can be crudely but effectively modelled by assuming that the wage is indexed both to the price-level expectational error and to the actual realisation of the productivity shock.

Duca and VanHoose (1991) develops a two-sector model based on Duca (1987), in which each sector produces a single good under perfect competition, with the sectors differing as regards sector-specific productivity shocks, as well as in labour market structure: while one sector features wage contracts, the other has spot-market hiring. Duca and VanHoose demonstrate that in such an economy, optimal multiparameter indexation can reproduce market-clearing outcomes in a manner reminiscent of Karni, and will involve partial indexation to the two goods' prices, together with partial indexation to the productivity shocks, with these indexation parameters dependent on labour supply elasticities as well as the relative sizes of the two sectors. Duca and VanHoose (1998b), like its precursor paper, features two sectors, one with wage contracts, the other with spot-market hiring, while all firms are in monopolistic competition as in Ball (1988). The paper's principal result is that more intense goods market competition reduces the optimal degree of wage indexation to the price level and increases the optimal degree of indexation to a linear combination of factors (including sector-specific productivity and demand shocks) which is related to the marginal product of labour. This finding is argued to be consistent with the empirical evidence for the US labour market in recent decades, where increasing goods market competition has been correlated with reduced incidence of price-level indexing and greater prevalence of profit-sharing in wage contracts.[25]

The only other contribution to the macroeconomics of multiparameter wage indexation which remains to be discussed is Drudi and Giordano (2000). This paper differs

[25] Heo (2003) provides an alternative theoretical explanation of these empirical facts by incorporating efficiency wage considerations into the Gray (1976) model.

substantially from those previously mentioned and from the model developed in Chapter VI below, and therefore a relatively brief description of it is justified. Its chief point of interest from the perspective of this thesis is its approach to modelling the widespread contingency of workers' remuneration on productivity-related outcomes: Drudi and Giordano's modelling approach is simply to assume that the wage is indexed to the productivity shock itself. Their model involves a repeated game in which the private sector (consisting of a single union and a single firm which bargain over the base nominal wage) do not observe the policymaker's type. Union bargaining power increases the mean real wage and hence also mean unemployment, and consequently creates the temptation for a weak policymaker who attaches some weight to employment stabilisation to renege on any precommitment to deliver zero inflation. Drudi and Giordano's principal concern is to investigate, using numerical methods, the welfare repercussions of certain combinations of indexation parameter settings (which for the greater part of their paper are exogenously given), and in particular those parameter values which induce a weak policymaker to deliver zero inflation in the initial stage of the repeated game, in order to acquire a reputation for toughness. Full indexation of wages to productivity shock realisations is found to be potentially beneficial in that the resulting greater employment stability reduces the temptation to spring inflation surprises and hence alleviates the inflation bias. (The model features a trade-off between this effect and the increase in mean unemployment which arises as a result of the higher mean real wage which is necessary to compensate union members for their greater exposure to undesirable real wage variability.)

IV.8 Conclusion

Among the more striking conclusions which emerge from the literature surveyed above are the following. Indexation of wages to the expectational error regarding the current price level can speed up the transmission of shocks to the price level, and hence can exacerbate price-level/inflation variability. On the other hand, wage indexation of this kind can reduce the mean inflation bias associated with discretionary policymaking. As regards stochastic real outcomes, such indexation reduces real wage variability and hence exacerbates employment and output variability in economies in which productivity shocks are dominant relative to aggregate demand shocks. In the light of the criticisms made by Jadresic (2002), however, these conclusions can hardly be considered robust. So far as macroeconomic models featuring endogenous wage indexation are concerned, the literature again does not come to unambiguous conclusions, and is comprised instead of a diverse set of results regarding the character of equilibrium indexation, which has been shown to be sensitive to the information structure of the economy and the degree of goods market competition, as well as to sectoral differences in labour-hiring practices. A modest success of the literature is that several different lines of inquiry have provided theoretical explanations for why zero indexation may be an equilibrium. On the question of macroeconomic externalities appertaining to individual indexation decisions, no very clear conclusions emerge: not particularly convincing assumptions regarding the costs of indexation, or constraints on the ability of particular labour types to index, are necessary to generate such externalities in a passive monetary policy context, while there are grounds to doubt their genuine existence in the multi-sector economies studied by Duca and VanHoose. There are, however, sounder reasons for thinking that a beneficial indexation externality relating to the mean inflation rate may arise under discretionary monetary policy when low-mean-inflation policy is time inconsistent. The model set out in Chapter V below investigates these externality issues by introducing into the standard model an arguably more realistic representation of the objectives of those choosing the degree of wage indexation.

Chapter V: Monopoly Unions, Monopolistically Competitive Firms and Single-Parameter Wage Indexation

V.1 Introduction

As mentioned in the concluding section of Chapter IV, the theme of externalities has figured prominently in the macroeconomic literature on wage indexation. These externalities have been of two essential kinds, namely those relating to the welfare of wage-setters (a term which, it will be recalled, in the relevant strand of literature denotes both workers and their employing firm), and those relating to the welfare of society as a whole. It seems fair to say that the literature has had difficulty in devising theoretical arguments for externalities of the former kind. While Kempf (1998) is moderately successful in this respect, the assumptions made by Ball (1988) and Kovanen (1992) which give rise to such an externality can be criticised as unrealistic and/or arbitrary, while the claim of Duca and VanHoose (1998a) to have pinpointed an externality appears to be incorrect. As regards the second type of wage indexation externality, that relating to social welfare, the work of Waller and VanHoose (1992) suggests that this externality is positive, as a result of the beneficial effect a higher degree of wage indexation may have on the trend inflation rate. However, as pointed out above, Waller and VanHoose assumed a specification for the social loss function which precluded possible social externalities relating to indexation's impact upon the *variability* of inflation about its trend rate.

In view of these criticisms, it therefore appears that further investigation of macroeconomic externalities arising from individual agents' wage indexation decisions is amply warranted. The model set out below pursues this, addressing in particular the efficiency, and social optimality, of individual indexation decisions when agents (specifically, monopoly unions of the kind considered in earlier chapters) attach some weight to the variability of their individual real wage as well as to the variability of their individual employment. By studying the macroeconomic outcomes which result when indexation is the means adopted to minimise our familiar monopoly-union loss function, this chapter addresses an acknowledged weakness of the literature surveyed above, namely its disregard for the real-world role wage indexation plays in enabling workers to lower their exposure to real-wage variability. It will be seen that the arguably more realistic representation of union preferences adopted here does reveal the existence of macroeconomic externalities appertaining to individual wage indexation decisions.

The remainder of this chapter is structured as follows. The next section sets out the model, which shares many features with the wage-setting models considered in earlier chapters. Sections V.3 and V.4 then derive the equilibrium degree of indexation, as well as efficient and socially optimal indexation, for the cases of the simple monetary rule and discretionary monetary policy respectively.[1] Conclusions are drawn in Section V.5.

V.2 The Model

Equations (1) to (9) comprise the essential structure of this chapter's model economy. In the absence of explicit statements to the contrary, the notation used for the following model is the same as that used in previous chapters, with time subscripts again suppressed. The convenient normalisation $p_{-1} \equiv 0$ is once more adopted, so that $\pi = p$.

$$y_i^S = \alpha l_i + \theta, \qquad 0 < \alpha < 1 \qquad (1)$$

where $\theta \sim N(0, \sigma_\theta^2)$.

$$y^S = \int_0^1 y_i^S di \qquad (2)$$

$$y_i^D - y^D = -\varepsilon(p_i - p), \qquad \varepsilon > 1 \qquad (3)$$

where $p = \int_{i=0}^{1} p_i di$.

[1] The simple rule scenario is equivalent to, and might alternatively be described as, a scenario of 'passive monetary policy', in the sense that this term was used in reviewing the indexation literature in Chapter IV. Similarly, the usage of that chapter might similarly lead us to use the term 'activist' as an identifying label for the discretionary regime in which the monetary authorities are able to set the money supply in reaction to shocks. However, in the interests of consistency with the practice of earlier chapters, the terms 'simple rule' and 'discretion' will be used here.

$$y^D = \gamma(m - p + \phi), \qquad \gamma > 0 \tag{4}$$

where $\phi \sim N(0, \sigma_\phi^2)$.

$$l_j^S = 0 \tag{5}$$

$$l_j^D = \int_{(j-1)/n}^{j/n} l_i^D di \Big/ \int_{(j-1)/n}^{j/n} di \tag{6}$$

$$w_i = \overline{w}_i + x_i(p - Ep) \tag{7a}$$

$$w_j = \overline{w}_j + x_j(p - Ep) \tag{7b}$$

$$w = \overline{w} + x(p - Ep) \tag{7c}$$

where $w = \int_0^1 w_i \, di = \frac{1}{n}\sum_{j=1}^{n} w_j$, $\overline{w} = \int_0^1 \overline{w}_i \, di = \frac{1}{n}\sum_{j=1}^{n} \overline{w}_j$, $x = \int_0^1 x_i \, di = \frac{1}{n}\sum_{j=1}^{n} x_j$.

$$\Omega_j^u = l_j^2 + c_u(w_j - p)^2 \tag{8}$$

$$\Omega^s = l^2 + c_s p^2 \tag{9}$$

where $l = \int_0^1 l_i \, di = \frac{1}{n}\sum_{j=1}^{n} l_j$.

The majority of these equations are familiar from previous chapters, and therefore do not require discussion, equations (7a), (7b) and (7c) being the only exceptions in this respect. Equation (7a) states that the nominal wage, w_i, set down in the contract between the individual firm and its associated monopoly union, consists of a base nominal wage, $\overline{w}_i$, and an indexation term, $x_i(p - Ep)$, where x_i is the degree of indexation and $(p - Ep)$ is the expectational error regarding the price level for the period covered by the wage contract. Equation (7b) is simply the individual union's equivalent of (7a), while (7c) is their aggregate-level counterpart. The chosen specification of the contract wage means that the more realistic approaches of Jadresic (in which indexation is to lagged inflation) and Cover and VanHoose (in which indexation is asymmetric) are not pursued in this chapter, partly for reasons of superior tractability, but more importantly because our focus is on establishing the existence of macroeconomic externalities when aversion to real wage variability influences the chosen degree of wage indexation. A second departure from the model of Chapter III is that it is now assumed that no information about the realised values of the shocks is available to the contracting parties at the time their agreement is concluded and the contract parameters $\overline{w}_i$ and x_i are determined. Thus neither unions nor firms receive an informative signal of the future realisation of the productivity shock at the contract-signing stage of the game, and the private sector's information at that stage is confined to the structural features of the economy embodied in equations (1) to (9), which include, of course, the distributions of the two shocks.[2] Were we to follow the practice of earlier chapters and allow unions to observe, at the contract-signing stage, a noisy signal of the productivity shock, this would not alter the basic results obtained below regarding the nature of the indexation externalities. In such a model featuring both productivity shock signals and indexation, equilibrium outcomes would involve adjustment of the base nominal wage in response to the signal just as described in Chapter III. The sole difference to the results presented below would be the non-dependence of union indexation decisions on that component of the productivity shock variance which relates to the anticipated component of shocks. Just as in the model which follows, indexation choices when signals are informative would be a function of the variance of union forecast errors of the productivity shock, but unlike what will follow below, this forecast-error variance will differ from the variance of the shock itself. The inclusion of signals would therefore merely result in the replacement of σ_θ^2 with $(1-\beta)\sigma_\theta^2$ wherever σ_θ^2 occurs in the expressions reported below,[3] and would not affect at all the insights yielded by the model.

[2] This amounts to assuming that the β parameter of Chapter III is now identically zero: $\beta \equiv 0$.

[3] Recall that the variance of union forecast errors is $E(\theta - \beta s)^2 = (1-\beta)\sigma_\theta^2$.

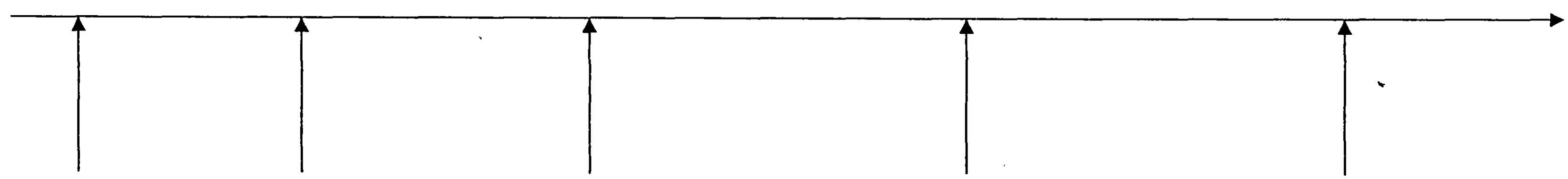
In the interests of clarity, and for consistency with the practice of earlier chapters, two time-line diagrams are presented below which represent alternative scenarios which are consistent with the assumed information structure, and which consequently lead to the same equilibrium outcomes. Note that although these time-lines assume that the money supply is set in the final stage of the game, and in particular after the aggregate nominal wage has been determined, it would make no difference to the results of the next section relating to the simple-rule scenario were the actual implementation of the rule to take place instead prior to, or simultaneously with, the determination of w. Similarly, in the discretionary monetary policy scenario considered in Section 4, the precise time at which m is set could be altered without affecting the reported results, provided the assumption is maintained that the unions do not observe m (and thus do not possess any information on the basis of which conditional expectations of the two shocks may be formed) prior to making their indexation choices.

The union loss function, namely equation (8), is identical to that adopted in Chapter III, and hence abstracts from any desire on the part of the representative union to raise the mean real wage above its market-clearing level. This simplifying assumption is entirely justified, since this chapter's focus is on indexation externalities relating to the stochastic aspects of the economy, i.e. to the variances of the price level, real wage, and employment.[4] As we know from Chapter III, with $c_u > 0$ the specification given by (8) amounts to assuming that the individual union is averse to variability of both employment and the real wage about their expected market-clearing values, while in the $c_u = 0$ case the union is only averse to employment variability. An immediate implication of (8), therefore, is that the individual union will set the contract base nominal wage at the value which is consistent with expected labour-market clearing.

Having set out the model's structural equations, we now derive reduced-form expressions for the price level, real wage and employment. To this end, we must initially

[4] As in Chapters II and III, specifying a mean real-wage objective greater than the expected market-clearing real wage of zero would merely alter the constant term in the expression for the equilibrium price level, and in the case of the discretionary monetary regime would cause the model to be characterised by a mean inflation bias. This would not affect in any way the conclusions to be drawn concerning the relationship between wage indexation and the stochastic aspects of the economy.

Time Line 1



The monetary regime is determined and is observed by all agents.

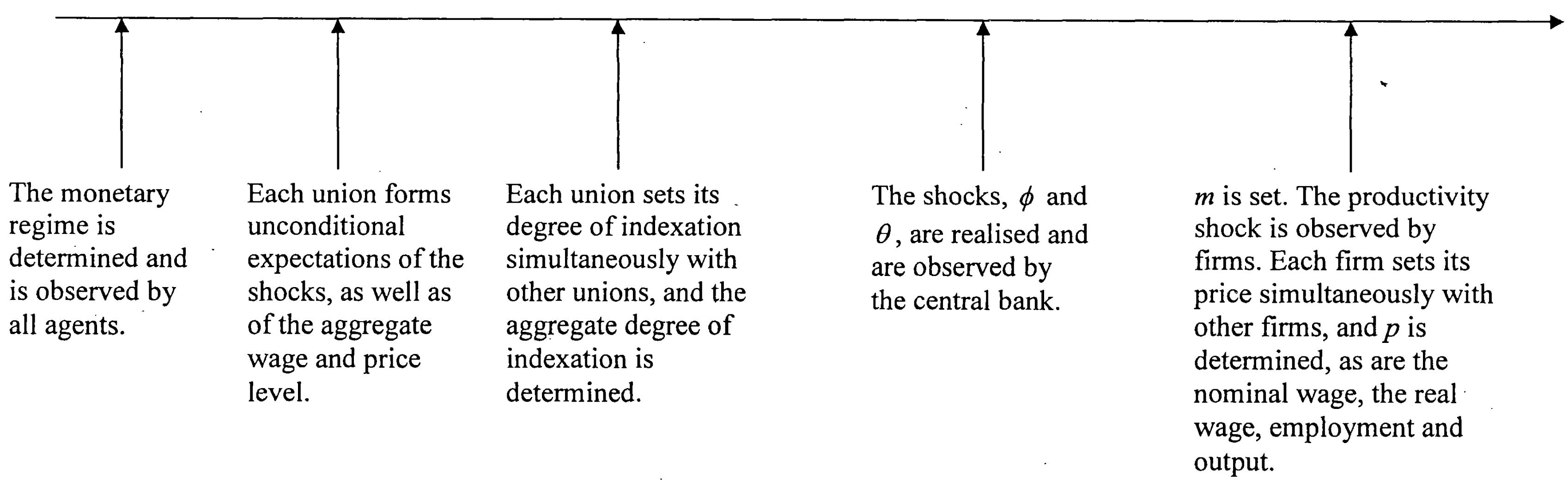
The shocks ϕ and θ are realised and are observed by the central bank, but are not observed by unions.

Each union forms unconditional expectations of the shocks, as well as of the aggregate wage and price level.

Each union sets its degree of indexation simultaneously with other unions, and the aggregate degree of indexation is determined.

m is set. The productivity shock is observed by firms. Each firm sets its price simultaneously with other firms, and p is determined, as are the nominal wage, the real wage, employment and output.

Time Line 2



The monetary regime is determined and is observed by all agents.
Each union forms unconditional expectations of the shocks, as well as of the aggregate wage and price level.
Each union sets its degree of indexation simultaneously with other unions, and the aggregate degree of indexation is determined.
The shocks, ϕ and θ, are realised and are observed by the central bank.
m is set. The productivity shock is observed by firms. Each firm sets its price simultaneously with other firms, and p is determined, as are the nominal wage, the real wage, employment and output.

obtain an expression for aggregate supply in terms of the price level, the money supply, and the stochastic shocks. Our first step is to note that, as in Chapter III, profit-maximisation leads to the following equation for the individual firm's labour demand:[5]

$$l_i^D = \frac{\gamma(m - p + \phi) - \varepsilon(w_i - p) + (\varepsilon - 1)\theta}{[\alpha + \varepsilon(1 - \alpha)]} \tag{10}$$

We have already noted that the base nominal wage, $\overline{w}_i$, will be set equal to the expected market-clearing nominal wage, i.e. that which ensures that $E(l_i^D) = l_i^S = 0$. Hence using (10), $\overline{w}_i$, $\overline{w}_j$ and $\overline{w}$ are found to be:

$$\overline{w}_i = \overline{w}_j = \overline{w} = \left(\frac{1}{\varepsilon}\right)[\gamma Em + (\varepsilon - \gamma)Ep] \tag{11}$$

Appropriate substitutions involving (1), (7a), (10) and (11) then lead to the following expression for firm i's output as a function of x_i, the realised values of the shocks, and the expectational errors of m and p:[6]

$$y_i^S = \frac{\gamma\alpha[m - Em - (p - Ep) + \phi] + \varepsilon\alpha(1 - x_i)(p - Ep) + \varepsilon\theta}{[\alpha + \varepsilon(1 - \alpha)]} \tag{12}$$

Aggregate supply is then obtained by integrating individual firm outputs over the unit interval:

$$y^S = \frac{\gamma\alpha[m - Em - (p - Ep) + \phi] + \varepsilon\alpha(1 - x)(p - Ep) + \varepsilon\theta}{[\alpha + \varepsilon(1 - \alpha)]} \tag{13}$$

[5] Equation (12) is derived in Appendix II.1.

[6] As in previous chapters, employment under the contract is assumed to be demand-determined.

Equating (13) with the aggregate demand equation, (4), obtaining the implied expectation of p in terms of Em and $\overline{w}$,[7] and substituting this for Ep as well as the right-hand side of (11) for $\overline{w}$, leads to the following reduced-form expression for the price level:

$$p = \frac{\gamma(1-\alpha)(m+\phi) + \alpha(1-x)Em - \theta}{[\gamma(1-\alpha) + \alpha(1-x)]} \tag{14}$$

A noteworthy point about (14) is that it implies that the price level is undefined when $x = [\alpha + \gamma(1-\alpha)]/\alpha$, and this is consequently an inadmissible value for x. The subsequent analysis therefore implicitly assumes that $x \neq [\alpha + \gamma(1-\alpha)]/\alpha$, a fact which will play a minor role in the discussion provided below. Another implication of (14) is that $Ep = Em$, and hence the individual union's expectational error in respect of p is:

$$p - Ep = \frac{\gamma(1-\alpha)(m - Em + \phi) - \theta}{[\gamma(1-\alpha) + \alpha(1-x)]} \tag{15}$$

Combining $\overline{w}_i = \overline{w}_j = \overline{w} = Em$ with (6), (7b), (10), and (15) then yields the following reduced-form expressions for the individual union's real wage and employment:

$$w_j - p = -\frac{(1-x_j)[\gamma(1-\alpha)(m - Em + \phi) - \theta]}{[\gamma(1-\alpha) + \alpha(1-x)]} \tag{16a}$$

$$l_j = \frac{1}{[\gamma(1-\alpha) + \alpha(1-x)][\alpha + \varepsilon(1-\alpha)]} \Big\{ \gamma[\alpha(1-x) + \varepsilon(1-\alpha)(1-x_j)](m - Em + \phi)$$

$$+ \{\gamma - \varepsilon(1-x_j) + (\varepsilon - 1)[\gamma(1-\alpha) + \alpha(1-x)]\}\theta \Big\} \tag{16b}$$

The aggregate counterparts to (16a) and (16b) are:

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[7] This expectation is $Ep = [\gamma(1-\alpha)Em + \alpha\overline{w}]/[\alpha + \gamma(1-\alpha)]$. In combination with (11), this implies that $\overline{w}_i = \overline{w}_j = \overline{w} = Em = Ep$.

$$w-p=-\frac{(1-x)[\gamma(1-\alpha)(m-Em+\phi)-\theta]}{[\gamma(1-\alpha)+\alpha(1-x)]} \tag{17a}$$

,

$$l=\frac{\gamma(1-x)(m-Em+\phi)\ +(\gamma-1+x)\theta}{[\gamma(1-\alpha)+\alpha(1-x)]} \tag{17b}$$

V.3 Wage Indexation under the Simple Rule

V.3.1 Outline of the Simple Rule Scenario

We focus to begin with on the scenario considered by Ball (1988), in which the authorities abstain entirely from monetary intervention in response to the stochastic shocks, or, equivalently, possess no information about the shocks when setting their instrument, the money supply. This scenario therefore corresponds to the simple rule of previous chapters, according to which the money supply *m* is kept fixed. An alternative interpretation is that *m* is the mean value of the money supply, with the aggregate demand disturbance, ϕ, representing, when non-zero, a stochastic deviation from its mean value of the money supply itself. It is convenient (and, as before, innocuous) to normalise *m* at zero, and consequently we have $m = Em = Ep = 0$.[8] The base nominal wage is therefore also zero, and the equations for union *j*'s nominal wage and for the price level simplify to the following expressions:

$$w_j = x_j p \tag{7b'}$$

$$p=\frac{\gamma(1-\alpha)\phi-\theta}{[\gamma(1-\alpha)+\alpha(1-x)]} \tag{14'}$$

With *m* fixed, it is apparent from (14′) that, provided $x<[\alpha+\gamma(1-\alpha)]/\alpha$, a higher degree of aggregate wage indexation increases the extent to which the price level deviates

[8] See the first paragraph of Section 3.1.(i) of Chapter III for a discussion of the $\overline{m}=0$ normalisation.

from its mean value in response to shocks.[9] It is also useful to state at this point the variances of the price level, of the individual union's real wage and employment, and of their aggregate counterparts, as functions of x and the structural parameters, under the simple rule. These expressions are obtained by squaring and taking expectations of (14′), (16a), (16b), (17a) and (17b):

$$Ep^2 = \frac{\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{18}$$

$$E(w_j - p)^2 = \frac{(1-x_j)^2[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{19a}$$

$$El_j^2 = \frac{\gamma^2[\alpha(1-x)+\varepsilon(1-\alpha)(1-x_j)]^2\sigma_\phi^2 + \{\gamma - \varepsilon(1-x_j) + (\varepsilon-1)[\gamma(1-\alpha)+\alpha(1-x)]\}^2\sigma_\theta^2}{[\alpha+\varepsilon(1-\alpha)]^2[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{19b}$$

$$E(w-p)^2 = \frac{(1-x)^2[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{20a}$$

$$El^2 = \frac{(1-x)^2\gamma^2\sigma_\phi^2 + (\gamma-1+x)^2\sigma_\theta^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{20b}$$

It is apparent from these equations that when both types of shock are present, there is greater scope to reduce real wage variability via wage indexation, than there is to reduce employment variability by means of it. Indeed, regardless of the values of σ_ϕ^2 and σ_θ^2, real wage variability can always be completely eliminated by indexing the wage fully to the price level, since $x_j = 1$ stabilises union j's real wage perfectly, and $x = 1$ is similarly

[9] From (14′), $\partial p/\partial x = \alpha[\gamma(1-\alpha)\phi - \theta]/[\gamma(1-\alpha)+\alpha(1-x)]^2$. Since $\partial p/\partial\phi > 0$ and $\partial p/\partial\theta < 0$ provided $x < 1 + [\gamma(1-\alpha)/\alpha]$, it follows that when this condition is satisfied (as it always is in our model) a higher degree of aggregate wage indexation increases the responsiveness of the price level to both types of shock. This point is also apparent from (18), since $\partial Ep^2/\partial x = 2\alpha[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2][\gamma(1-\alpha)+\alpha(1-x)]^{-3} > 0$ provided $x < 1 + [\gamma(1-\alpha)/\alpha]$.

effective in the aggregate case. The essential reason for this is that the stochastic shocks can only affect the real wage indirectly via their impact on the price level. This differs sharply from the relationship between the shock variances and employment variability, since employment cannot be perfectly stabilised by means of wage indexation when both σ_ϕ^2 and σ_θ^2 are positive. As is apparent from equations (16b) and (17b), the productivity shock exerts both a direct influence on employment, and an indirect influence on it via its impact on the price level and hence on the real wage and aggregate demand. While it is possible to neutralise totally the employment impact of productivity shocks by setting x_j (or, in the aggregate case, x) appropriately, this setting of the indexation parameter necessarily differs from unity and therefore implies that the real wage, and consequently employment too, cannot at the same time be completely insulated from velocity shocks.

V.3.2 Efficient Wage Indexation

From the collective viewpoint of unions, the efficient degree of indexation, x^*, is that which minimises union *j*'s expected loss, given that every union does set this particular *x*, (i.e. given that indexation is constrained to be symmetric). For the simple rule scenario, this expected loss is obtained by combining (19a) and (19b) with the unconditional expectation of (8), and imposing $x_j = x \forall j$:

$$E\Omega^u\Big|_{x_j = x\forall j} = \frac{\gamma^2(1-x)^2[1+c_u(1-\alpha)^2]\sigma_\phi^2 + [(\gamma-1+x)^2 + c_u(1-x)^2]\sigma_\theta^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{21}$$

Minimising (21) by choice of *x*, the efficient degree of aggregate wage indexation is found to be:

$$x^* = 1 - \frac{\gamma\sigma_\theta^2}{\{\gamma^2(1-\alpha)[1+c_u(1-\alpha)^2]\sigma_\phi^2 + [1+c_u(1-\alpha)]\sigma_\theta^2\}} \tag{22}$$

This expression has been derived on the assumption that the value of at least one of the two shock variances is finite and positive. With this fact in mind, we state the efficient degree of indexation when one or the other of the two shock variances takes an extreme value:

$$\lim_{\sigma_\phi^2 \to \infty} x^* = x^*\Big|_{\sigma_\theta^2 = 0} = 1 \tag{23a}$$

$$\lim_{\sigma_\theta^2 \to \infty} x^* = x^*\Big|_{\sigma_\phi^2 = 0} = \frac{1-\gamma+c_u(1-\alpha)}{1+c_u(1-\alpha)} \tag{23b}$$

The following derivatives also provide insight into the relationship between x^* and the variances of the shocks:

$$\frac{\partial x^*}{\partial \sigma_\phi^2} = \frac{\gamma^3(1-\alpha)[1+c_u(1-\alpha)^2]\sigma_\theta^2}{\{\gamma^2(1-\alpha)[1+c_u(1-\alpha)^2]\sigma_\phi^2+[1+c_u(1-\alpha)]\sigma_\theta^2\}^2} \tag{24a}$$

Note with regard to (24a): $\partial x^*/\partial \sigma_\phi^2 > 0$ provided $\sigma_\theta^2 > 0$, while $(\partial x^*/\partial \sigma_\phi^2)\big|_{\sigma_\theta^2 = 0} = 0$.

$$\frac{\partial x^*}{\partial \sigma_\theta^2} = \frac{-\gamma^3(1-\alpha)[1+c_u(1-\alpha)^2]\sigma_\phi^2}{\{\gamma^2(1-\alpha)[1+c_u(1-\alpha)^2]\sigma_\phi^2+[1+c_u(1-\alpha)]\sigma_\theta^2\}^2} \tag{24b}$$

This derivative can be unambiguously signed when velocity shocks are present: specifically, $\partial x^*/\partial \sigma_\theta^2 < 0$ provided $\sigma_\phi^2 > 0$, while $(\partial x^*/\partial \sigma_\theta^2)\big|_{\sigma_\phi^2 = 0} = 0$.

These results reveal that there are several aspects of the efficient degree of indexation in our model which tally with the findings of previous contributions to the literature on wage indexation. In particular, there is the immediate implication of (22) that the efficient degree of indexation does not exceed unity, while (23a) is an instance of a result common to many papers, including those of Gray (1976) and Ball (1988), namely that full indexation of the wage to the price level is desirable when velocity shocks completely predominate.[10] A point worth making with regard to the latter result, however, is that

[10] To eliminate entirely the possibility of confusing the reader, we emphasise that full indexation to the price level is only ever efficient if $\sigma_\theta^2 = 0$, and that if $\sigma_\theta^2 > 0$, $x^* < 1$ is then the case. In the passage to

while (23a) indicates that full indexation is efficient whenever the ratio of the two shock variances, $\sigma_\theta^2/\sigma_\phi^2$, is zero, the two extreme cases which give rise to $\sigma_\theta^2/\sigma_\phi^2 = 0$ will not have identical outcomes under full indexation. Clearly, in the $\sigma_\theta^2 = 0$ case $x = 1$ completely insulates not only the real wage but also aggregate demand[11] from velocity shocks, so that complete stability of both the real wage and employment is achieved. In the limiting case in which $\sigma_\phi^2 \to \infty$, by contrast, efficiency requires x to be unity simply because velocity shocks are overwhelmingly the major source of variability in outcomes: the resulting high exposure of employment to productivity shocks is a price worth paying to avoid real outcomes being exposed to velocity shocks. Similar arguments arise in respect of the results contained in (23b), i.e. the efficient degree of indexation when the ratio $\sigma_\theta^2/\sigma_\phi^2$ tends in the limit to infinity. The fact that efficiency involves the same degree of indexation when $\sigma_\theta^2 \to \infty$ (with $\sigma_\phi^2 > 0$ of course) as when velocity shocks are absent does not mean that real outcomes under efficient indexation are identical in the two extreme cases. Indeed, it is intuitively obvious that when indexation is efficient the $\sigma_\phi^2 = 0$ case must result in a smaller expected union loss than the limiting case in which $\sigma_\theta^2 \to \infty$, since the variances of both shocks are larger in the latter.

More generally, it is clear from (24a) and (24b) that, provided both types of shock exist, the efficient degree of indexation is closer to unity, the smaller is the ratio $\sigma_\theta^2/\sigma_\phi^2$. The reason for this is that efficiency involves the optimal exploitation of a trade-off between real wage variability and employment variability. As σ_ϕ^2 increases marginally relative to σ_θ^2, a higher degree of indexation is desirable in order to reduce real wage variability, both (when $c_u > 0$) for its own sake, and because of the contribution it makes to employment variability. However, both the direct impact of the productivity shocks on

which this footnote relates, as well as in the remainder of this chapter, the phrase 'velocity shocks completely predominate' is used as a convenient means of referring to both the case in which $\sigma_\theta^2 = 0$ and the limiting case in which $\sigma_\phi^2 \to \infty$.

[11] Under the simple rule, aggregate demand is given by $y^D = \gamma[\alpha(1-x)\phi + \theta]/[\gamma(1-\alpha) + \alpha(1-x)]$, and its variance by $E(y^D)^2 = \gamma^2[\alpha^2(1-x)^2\sigma_\phi^2 + \sigma_\theta^2]/[\gamma(1-\alpha) + \alpha(1-x)]^2$. In this context, note also that equation (10) indicates that firm i's labour demand is a simple monotonic increasing function of aggregate demand.

employment, and their indirect impact working via the price level and aggregate demand[12], are exacerbated by a marginal increase in x towards unity. Efficiency involves adjusting x, as σ_ϕ^2 increases, so as to ensure the beneficial marginal effect on real wage variability is equal to the detrimental marginal effect on employment variability.

This trade-off will be further discussed in a little while as part of the intuitive explanation of the form taken by the x^* expression itself. However, before embarking on this, it is worthwhile examining first of all the relationship between x^* and c_u. For the two extreme admissible values of this weight parameter, it is found that:

$$\lim_{c_u \to \infty} x^* = 1 \tag{25a}$$

$$x^*\Big|_{c_u=0} = \frac{\gamma^2(1-\alpha)\sigma_\phi^2 + (1-\gamma)\sigma_\theta^2}{\gamma^2(1-\alpha)\sigma_\phi^2 + \sigma_\theta^2} \tag{25b}$$

Differentiating x^*, as given by (22), with respect to c_u we have:

$$\frac{\partial x^*}{\partial c_u} = \frac{\gamma(1-\alpha)[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]\sigma_\theta^2}{\{\gamma^2(1-\alpha)[1+c_u(1-\alpha)^2]\sigma_\phi^2 + [1+c_u(1-\alpha)]\sigma_\theta^2\}^2} \tag{26}$$

The explanation for results (25a), (25b) and (26) is straightforward. In the limiting $c_u \to \infty$ case, the representative union's sole concern is to secure a stable real wage, and this can always be achieved via coordinated indexation of unity. In the opposite extreme case of $c_u = 0$, minimisation of employment variability is union *j*'s sole objective, and with both types of shock present, the setting of x which minimises the variance of employment will depend upon the relative magnitudes of the shock variances. This is the familiar lesson of Gray (1976) and Ball (1988), both of which papers assume, of course,

[12] Using the expression for $E(y^D)^2$ stated in the previous footnote, we find that: $\partial^2 E(y^D)^2/\partial\sigma_\theta^2\partial x = 2\alpha/[\gamma(1-\alpha)+\alpha(1-x)]^3$. Hence the contribution made by σ_θ^2 to employment variability is larger, the closer to unity is x. (This assumes $x < [\alpha + \gamma(1-\alpha)]/\alpha$, an inequality which does hold under both efficient and equilibrium indexation.)

that the objective of indexation is the achievement of employment stability. It is not at all surprising, therefore, that (25b) is identical to the efficient degree of (costless) indexation in Ball's model.[13] In her paper, Gray, assumes that the γ parameter of our model is equal to unity, and when (25b) is evaluated for $\gamma = 1$, the resulting expression for $x^*\big|_{c_u=0,\gamma=1}$ turns out to be the optimal degree of indexation in Gray's model when labour supply is completely inelastic. Finally, note that (26) indicates $\partial x^*/\partial c_u > 0$ to be the case provided $\sigma_\theta^2 > 0$. This is attributable to the fact that under symmetric indexation, real wage variability is lower, the closer is x to unity (as is evident from (20a)). Hence the larger is the relative weight placed by unions on real wage variability, the closer to unity is x^*. In the absence of productivity shocks, full indexation is always efficient, regardless of c_u, which explains why (26) implies that $(\partial x^*/\partial c_u)\big|_{\sigma_\theta^2=0} = 0$.

To acquire intuition regarding our x^* expression, (i.e. equation (22)), it is helpful to consider initially the special case in which the economy is subject only to productivity shocks. The efficient degree of indexation when velocity shocks do not exist has already been presented above as part of expression (23b), to which we refer the reader once again. Relevant also to the analysis is an entity which will be familiar from Chapter III, and which was originally derived in Appendix II.2, namely the symmetric-wage labour demand curve. It is convenient to repeat its equation here[14]:

$$l_j\big|_{w_j=w\forall j} = \frac{-[w_j - (p\big|_{w_j=w\forall j})] + \theta}{(1-\alpha)} \tag{27}$$

Since in this chapter the nominal wage of union j is (given the normalisations) $w_j = x_j p$, (27) may be alternatively written as follows:

[13] Equation (25b) is identical, apart from notational differences, to equation (11) of Ball (1988), p.303.
[14] At this point we add subscripts to some of the variables in this equation in order to emphasise the underlying assumption of symmetry in wages.

$$l_j\big|_{x_j=x\forall j} = \frac{(1-x_j)(p\big|_{x_j=x\forall j})+\theta}{(1-\alpha)} \qquad (27')$$

Equation (27′) implicitly assumes that $x_j = x\forall j$, and hence will be referred to as the symmetric-indexation labour demand curve.[15] For concreteness, the case of a positive productivity shock, $\theta > 0$, will be studied: needless to say, an explanation which parallels that given here could be provided for the case in which $\theta < 0$.[16] The symmetric-indexation labour demand curve when $\theta > 0$ is depicted below in Figure V.1. Since this diagram is closely related to Figure III.1 of Chapter III, it will require very few explanatory remarks. It differs from Figure III.1 in only one important respect, namely that $\beta \equiv 0$ is now assumed to be the case, so that the expectation of the symmetric-wage labour demand curve which appears in Figure III.1 implicitly passes through the origin in Figure V.1. (Since it does little to advance the discussion, the expected symmetric-indexation labour demand curve is omitted entirely in order to reduce diagrammatic clutter.)

Under symmetric indexation, the outcome in the labour market(s) in which union j is the monopoly union, and indeed the outcome at the aggregate level, must be located on the labour demand curve depicted in Figure V.1. Each point on the curve corresponds to a particular degree of symmetric indexation. If $x_j = 1\forall j$, so that $x = 1$, the outcome will be at the vertical intercept of the labour demand curve, the real wage will take its expected market-clearing value of zero, and will be unaffected by the shock, the impact of which will be borne entirely by employment. As x decreases from unity, the outcome migrates gradually down the curve. The x value which ensures the outcome is at the horizontal-axis intercept of the curve is $1-\gamma$, and this x value is positive if, and only if, $\gamma < 1$. The

[15] Like (27), (27′) is a special case of the aggregate-level labour demand curve, and therefore may be alternatively written with l and x respectively in place of l_j and x_j. The important point regarding (27) and (27′) is that $w_j = w\forall j$, which in this chapter amounts to $\overline{w}_j = \overline{w}\forall j$ and $x_j = x\forall j$.

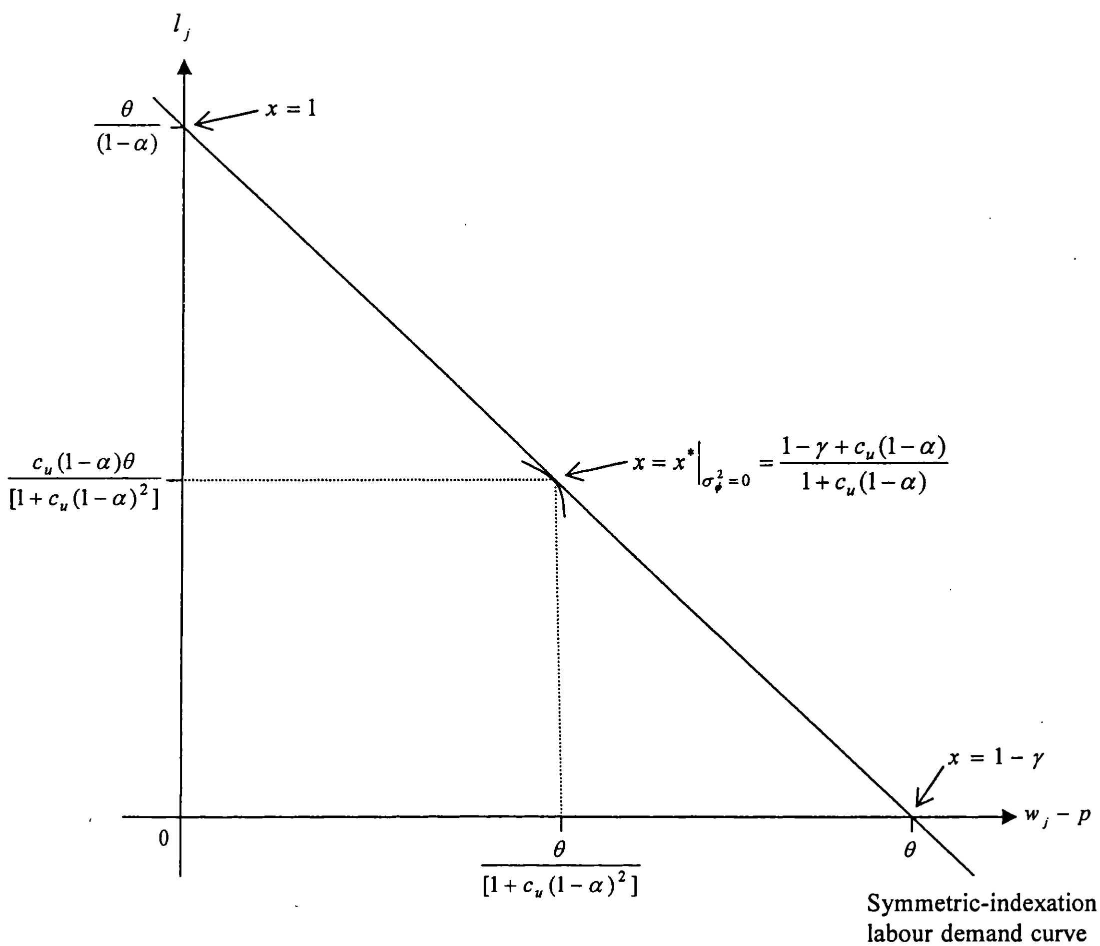
[16] As for the $\theta = 0$ case, this is simply that in which the symmetric-indexation labour demand curve passes through the origin. Since there is no expectational error regarding the price level in this case (i.e. $p\big|_{\theta=0} = Ep = 0$), the indexation clause in the wage contract does not come into play, and the wage is at its mean level of zero. Labour-market clearing occurs, and the outcome is located at the origin.

efficient x, of course, is that which ensures the outcome is at the tangency point between the isoloss map and the symmetric-indexation labour demand curve, and from (23b) this x is known to be $x^*\big|_{\sigma_\phi^2=0} = [1-\gamma+c_u(1-\alpha)]/[1+c_u(1-\alpha)]$. It is clear that when $\gamma<1$ (and $c_u>0$), the outcome at the tangency point is brought about by an x value which is positive. If $\gamma>1$, part of the curve lying above the horizontal axis is then associated with negative values of x, and the possibility then arises that the tangency itself is located at a point associated with a non-positive x value. The closer to zero is union j's weight parameter c_u, the more horizontally elongated are the ellipses which make up the union's isoloss map. In other words, the stronger is union j's relative aversion to employment variability (i.e. the lower is c_u) the lower down the curve will the tangency point be. Hence for a given γ value in excess of unity, the tangency point will only be located on the part of the curve associated with negative x values if c_u is sufficiently low.

These facts are of assistance in interpreting a key aspect of our result for the efficient degree of indexation in the absence of velocity shocks, namely that it is zero if, and only if, c_u happens to satisfy the equation $c_u=\widetilde{c}_u$, where $\widetilde{c}_u \equiv (\gamma-1)/(1-\alpha)$. This finding is obviously closely related to one of the key results of Chapter III, namely that when $c_u=\widetilde{c}_u$, the efficient nominal wage is always zero, regardless of the realised value of the signal, and regardless also of signal quality. It will be recalled that the model of this chapter only differs from that of Chapter III in that it assumes signals to be completely uninformative ($\beta\equiv 0$), and in that it also allows for wage indexation and velocity shocks (and for the moment the latter are assumed absent in any case). Given the nature of these differences, it is hardly surprising that if efficiency in wage-setting requires that $w=0$ at all times, the efficient degree of indexation should turn out to be zero, since with a base nominal wage of zero, a setting of zero for x clearly ensures that the efficiency requirement identified in Chapter III for the $c_u=\widetilde{c}_u$ special case is met. Underpinning this result is the fact that when $c_u=\widetilde{c}_u$, the movement in the price level occasioned by a non-zero realisation of the productivity shock is, in the absence of an adjustment of the aggregate nominal wage, precisely what is required to ensure the outcome is located at

Figure V.1

Employment and real wage outcomes from efficient indexation



Notes:

1. It is assumed for illustrative purposes that $\theta > 0$.
2. If $\sigma_\phi^2 = 0$ is the case, each of the indicated points on the symmetric-indexation labour demand curve will be the outcome brought about by the stated value of x. If $\sigma_\phi^2 > 0$, each indicated point will, if the stated value of x is set, be the mean outcome (i.e. it will be the outcome if $\phi = 0$ transpires).

the tangency point between the isoloss map and the symmetric-wage labour demand curve.

Effective use can also be made at this juncture of arguments devised in Chapter III to explain the efficient wage response to a non-zero signal when $c_u \neq \widetilde{c}_u$. As we saw in that chapter, if $c_u > \widetilde{c}_u$ is the case, a non-zero productivity shock has, in the absence of any nominal wage adjustment (i.e. if $w_j = 0 \forall j$), too weak an impact on employment and too strong an impact on the real wage. When a positive shock is expected, i.e. when $E(\theta \mid s) = \beta s > 0$, efficiency requires that the fall in the price level caused by the shock's anticipated component be accompanied by a decrease in the nominal wage, so that this component's impact on the real wage is dampened, and its impact on employment is strengthened, by just enough to ensure that the ideal distribution of the impact across these outcome variables is achieved. Plainly, this reasoning is applicable also to our finding that the efficient degree of indexation is positive when $c_u > \widetilde{c}_u$, since positive indexation ensures that the nominal wage adjusts negatively in response to the fall in the price level resulting from a positive productivity shock. The converse case in which $c_u < \widetilde{c}_u$ is that in which a positive adjustment of the nominal wage in response to a positive shock is desirable, since in the absence of any wage adjustment the shock's impact on the real wage will be weaker, and its impact on employment stronger, than the impact pattern which the unions prefer. It therefore follows that the efficient degree of indexation should be negative when $c_u < \widetilde{c}_u$, since negative indexation ensures that the price level fall caused by a positive shock results, as desired, in an increase in the nominal wage.

One implication of the arguments propounded in the previous paragraphs is that, in the absence of velocity shocks, efficient indexation of the wage to the price level must bring about the same expected union loss as efficient wage-setting when signals are perfectly informative (the $\beta = 1$ case). Combining our $x^{*}\big|_{\sigma_{\phi}^{2}=0}$ result, as given by (23b), with (19a), (19b) and (8), we find that:

$$E\left(\Omega_j^u\Big|_{\sigma_\phi^2=0,\, x_j=x^*\forall j}\right)=\frac{c_u\sigma_\theta^2}{[1+c_u(1-\alpha)^2]} \tag{28}$$

This is identical to the (unconditional) expected loss which results from efficient wage-setting under full information, as given by equation (23) of Chapter III, evaluated for $\beta=1$. This in turn implies that efficient indexation gives rise to a smaller expected loss than that achieved by an efficient nominal wage adjustment in response to an imperfect signal. The reason for this, of course, is that if aggregate demand is not subject to velocity shocks, the realised value of the price level allows the value of the productivity shock to be inferred perfectly. Compared to noisy signals, therefore, the price level is a superior source of information which, although unavailable at the time wage contracts are signed, can nevertheless be exploited, by means of efficient indexation, to bring about the desired movement in the aggregate nominal wage.

It is time to investigate how the existence of velocity shocks affects the efficient degree of indexation. Figure V.1 remains relevant to the analysis, as does the equation of the symmetric-indexation labour demand curve for a given realisation of θ, namely equation (27′). Given θ, the presence of velocity shocks implies that, with a single exception, each value of x does not give rise for certain to a particular outcome point on the symmetric-indexation labour demand curve for that θ realisation, but rather only gives rise to it on average. If $x_j=1-\gamma,\forall j$, for instance, the outcome will only on average be located at the intersection point of the labour demand curve with the horizontal axis; in other words the horizontal intercept will be the location of the outcome when the realised value of ϕ happens to be zero. If ϕ is not zero, the actual outcome when $x_j=1-\gamma,\forall j$ will be positioned elsewhere on the curve. (The set of possible outcomes, given the positive θ shock assumed by the diagram, will be normally distributed around the mean outcome.) Consequently, $x_j=1-\gamma,\forall j$ does not stabilise employment perfectly when velocity shocks exist: with $\sigma_\phi^2>0$ it merely ensures productivity shocks do not affect employment, and that employment is zero on average. This is similarly true of $x=[1-\gamma+c_u(1-\alpha)]/[1+c_u(1-\alpha)]$, which now ensures that the outcome is at the tangency point only on average, i.e. when $\phi=0$. The single exception to this general truth that, when $\sigma_\phi^2 > 0$, each x value only on average results in a particular outcome is $x = 1$, since full indexation perfectly stabilises the real wage irrespective of whether velocity shocks are present or not. Furthermore, as the symmetric degree of indexation approaches unity from below, the variances of both the real wage and employment, for a given realisation of θ, diminish monotonically, as can be seen from the following expressions, obtained using (16a) and (16b) with $x_j = x \forall j$ imposed:

$$\frac{\partial E[(w_j - p)\big|_{x_j = x \forall j} \mid \theta]^2}{\partial x} = \frac{-2(1-x)\gamma^3(1-\alpha)^3\sigma_\phi^2}{[\gamma(1-\alpha)+\alpha(1-x)]^3} \tag{29a}$$

$$\frac{\partial E(l_j\big|_{x_j = x \forall j} \mid \theta)^2}{\partial x} = \frac{-2(1-x)\gamma^3(1-\alpha)\sigma_\phi^2}{[\gamma(1-\alpha)+\alpha(1-x)]^3} \tag{29b}$$

In terms of Figure V.1, therefore, the existence of velocity shocks implies that as x approaches unity from below, the mean outcome travels steadily up the labour demand curve, with the distribution of possible outcomes about the mean outcome gradually narrowing. Note also that result (24a) implies that the introduction of velocity shocks into the model leads the efficient x to exceed the x which causes the mean outcome to be located at the tangency point depicted in Figure V.1. (In other words, we know from (24a) that if $\sigma_\phi^2 > 0$, $x^* > [1-\gamma + c_u(1-\alpha)]/[1 + c_u(1-\alpha)]$ is then the case.) Hence when $\sigma_\phi^2 > 0$, the efficient degree of indexation does not ensure that the mean outcome (i.e. the outcome if ϕ turns out to be zero) is located at the tangency point. The presence of velocity shocks causes the efficient mean outcome to be positioned further up the symmetric labour demand curve than the tangency point, and therefore will involve a given realisation of θ having, on average, a larger impact on employment and a smaller impact on the real wage, than under efficient indexation in the absence of velocity shocks. To make the point in another way, efficient indexation when $\sigma_\phi^2 > 0$ results on average in the productivity shock not being divided between the real wage and employment in the most desirable way from the unions' viewpoint. The implication of this is that, under efficient indexation, the larger is the variance of the velocity shock, the greater is the contribution made by productivity shocks to employment variability, and the smaller is their contribution to real wage variability. The reason why it is efficient for unions to incur this displacement of the mean outcome away from the tangency point is that the higher x which causes this displacement results in a smaller variance of the actual outcome around the mean as a result of velocity shocks. Clearly, this is the indexation literature's familiar trade-off between on the one hand achieving greater stability in the face of velocity shocks, and on the other exacerbating the impact of productivity shocks on employment.

We complete our discussion of efficient indexation under the simple rule with an alternative diagrammatic representation of the aforementioned trade-off. Every admissible value of x uniquely gives rise to a particular combination of aggregate real wage variability, as given by (20a), and aggregate employment variability, as given by (20b). The set of possible such combinations constitutes a locus of possible outcomes in a two-dimensional space with the variance of the aggregate real wage and the variance of aggregate employment on the axes. Note that this locus also constitutes the set of possible outcomes for the individual union, given that indexation is symmetric (i.e. given that $x_j = x \forall j$). The part of the locus which is of interest to us is depicted in Figure V.2.

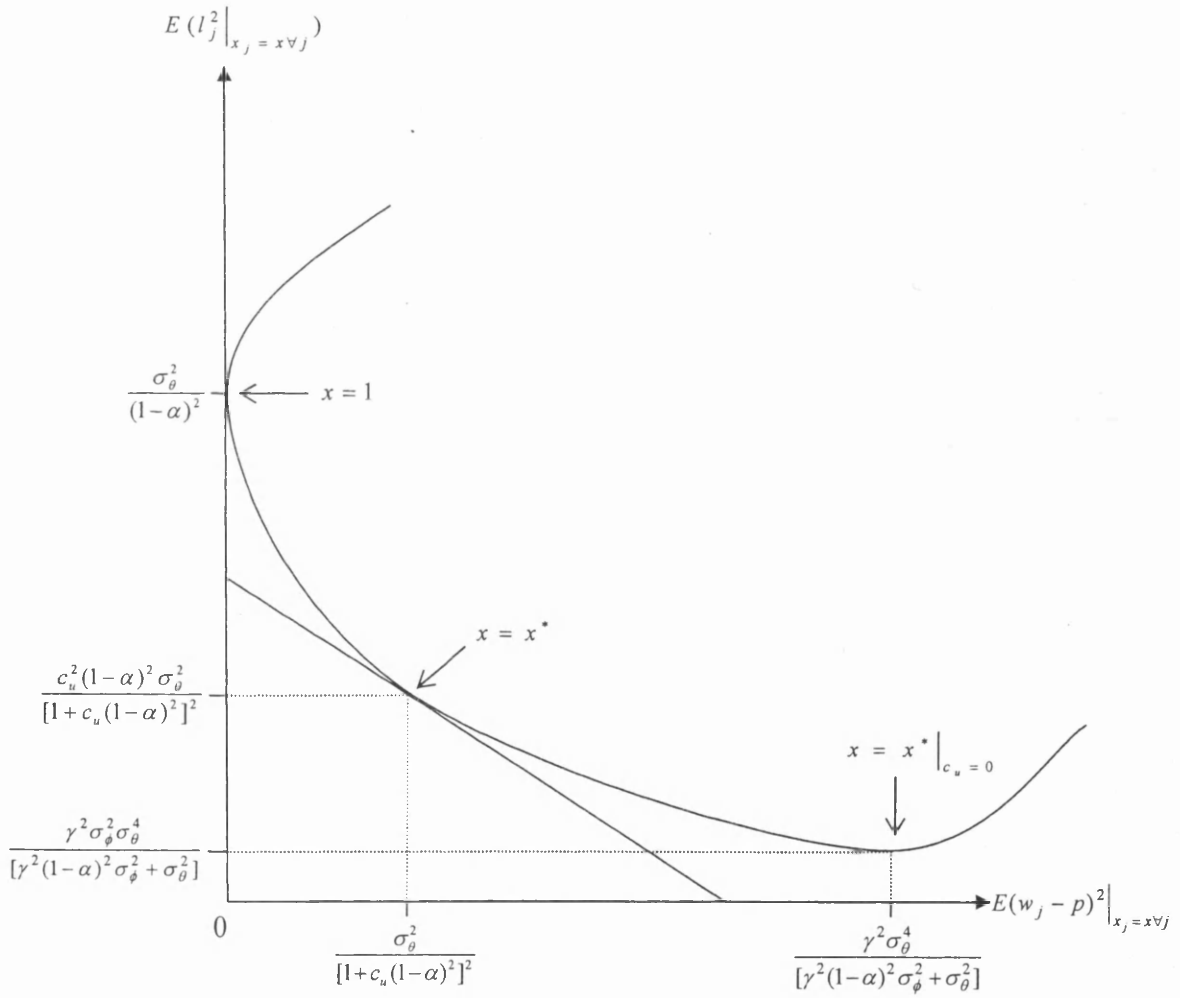
The slope of the locus is given by expression (31) below, and involves the ratio of the following derivatives of (20a) and (20b) with respect to x:

$$\frac{\partial E(w-p)^2}{\partial x} = \frac{-2(1-x)\gamma(1-\alpha)[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]}{[\gamma(1-\alpha)+\alpha(1-x)]^3} \tag{30a}$$

$$\frac{\partial El^2}{\partial x} = \frac{2\gamma[-(1-x)\gamma^2(1-\alpha)\sigma_\phi^2 + (\gamma-1+x)\sigma_\theta^2]}{[\gamma(1-\alpha)+\alpha(1-x)]^3} \tag{30b}$$

$$\frac{dEl^2}{dE(w-p)^2} = \frac{\partial El^2/\partial x}{\partial E(w-p)^2/\partial x} = \frac{[(1-x)\gamma^2(1-\alpha)\sigma_\phi^2 + (1-x-\gamma)\sigma_\theta^2]}{(1-x)(1-\alpha)[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]} \tag{31}$$

Figure V.2
The locus of possible outcomes from symmetric indexation in the simple rule scenario



Notes:
1. The depicted isoloss contour has slope $-c_u$.

The only point in the diagram which is both on the locus and on the vertical axis is the outcome pair which results when full indexation is symmetrically adopted by every union. This is unsurprising, since it is known that unity is the only setting of x which entirely eliminates real wage variability. The locus neither intersects nor has a tangency point with the horizontal axis, a fact that is consistent with the argument put forward earlier that, given both types of shock exist, it is not possible to stabilise employment perfectly by means of wage indexation. The point on the locus which involves minimal employment variability is brought about by the value of x which is efficient in the costless-indexation version of Ball's model (i.e. by $x^*\big|_{c_u=0}$, as given by (25b)). It is apparent from (30a) and (30b) that as x rises in value from Ball's efficient solution value, the outcome migrates gradually up the negatively sloped portion of the locus.[17] The indifference map of the representative union in this space consists of a bliss point at the origin and straight line isoloss curves, with slope $-c_u$. Setting (31) equal to $-c_u$ and solving for x, amounts, of course, to an alternative means of finding the efficient solution x^*. In diagrammatic terms, therefore, x^* is the value of x which causes the outcome to be located at the tangency point between the indifference map and the possible-outcomes locus, and hence ensures that the trade-off embodied in the slope of the locus is optimally exploited from the unions' collective viewpoint.

[17] It is straightforward to show that as x rises above unity, both the variance of employment and the variance of the real wage increase, and indeed tend to infinity as x approaches its only inadmissible value of $[\alpha+\gamma(1-\alpha)]/\alpha$. Hence as x increases from unity, the outcome migrates in a north-easterly direction along the upper of the two positively sloped portions of the locus. If x exceeds $[\alpha+\gamma(1-\alpha)]/\alpha$, both the real wage variance and the employment variance are then falling in x, so that as x increases further the outcome will move down the lower of the two positively sloped portions of the locus, and will gradually converge on a particular point which is the outcome in the limiting case in which $x\to\pm\infty$. (Only a small part of this positively sloped portion has been drawn.) The limiting values of these variances as x becomes infinitely large or infinitely negative are, respectively, $\lim_{x\to\pm\infty} El^2=[\gamma^2\sigma_\phi^2+\sigma_\theta^2]/\alpha^2$ and $\lim_{x\to\pm\infty} E(w-p)^2=[\gamma^2(1-\alpha)^2\sigma_\phi^2+\sigma_\theta^2]/\alpha^2$. In general descriptive terms, therefore, the locus of possible outcomes is smoothly curved and elongated in a north-easterly direction, with a discontinuity which corresponds to the setting of x at $[\alpha+\gamma(1-\alpha)]/\alpha$. (The latter is the only real value of x which does not have associated with it a point on the locus.) However, since our interest in this chapter principally concerns the subset of x values bounded by the Ball model efficient solution and by unity, i.e. the interval $1-\gamma\sigma_\theta^2[\gamma^2(1-\alpha)\sigma_\phi^2+\sigma_\theta^2]^{-1}\le x\le 1$, only that part of the locus corresponding approximately to this range of values is depicted in Figure V.1.

V.3.3 Equilibrium Wage Indexation

It remains to be seen, however, whether the individual union will have an incentive to set its degree of indexation equal to the efficient degree, i.e. whether $x_j = x^*$ can prevail in equilibrium. Our first step towards deriving the individual union's optimal x_j, for given *x*, is to obtain union *j*'s expected loss as a function of x_j, x and the structural parameters. After substituting (19a) and (19b) into the (unconditional) expectation of (8), this is found to be:

$$E\Omega_j^u = \frac{1}{[\alpha+\varepsilon(1-\alpha)]^2[\gamma(1-\alpha)+\alpha(1-x)]^2}\Bigg[\ \gamma^2\{[\alpha(1-x)+\varepsilon(1-\alpha)(1-x_j)]^2$$

$$+c_u[\alpha+\varepsilon(1-\alpha)]^2(1-\alpha)^2(1-x_j)^2\}\sigma_\phi^2+\Big[\{\gamma-\varepsilon(1-x_j)+(\varepsilon-1)[\gamma(1-\alpha)+\alpha(1-x)]\}^2$$

$$+c_u[\alpha+\varepsilon(1-\alpha)]^2(1-x_j)^2\Big]\sigma_\theta^2\ \Bigg] \tag{32}$$

In deriving its individually optimal degree of indexation, union *j* regards the indexation decisions of the other $(n-1)$ unions as beyond its influence. Individual optimality requires that account be taken of both the direct effect of x_j on union *j*'s expected loss, as given by the partial derivative $\partial E\Omega_j^u/\partial x_j$, and the indirect effect x_j has upon it, working through the contribution x_j makes to the realised value of *x*. This latter indirect effect involves the fact that $\partial x/\partial x_j = 1/n$, and is given by the second term in the following total derivative:

$$\frac{dE\Omega_j^u}{dx_j} = \left.\frac{\partial E\Omega_j^u}{\partial x_j}\right|_{x \text{ fixed}} + \left(\frac{1}{n}\right)\left(\frac{\partial E\Omega_j^u}{\partial x}\right) \tag{33}$$

Although the direct effect can be straightforwardly obtained by differentiating (32) with respect to x_j, it is helpful to the discussion to state explicitly the derivatives of (19a) and

(19b), namely $\partial E(w_j - p)^2 / \partial x_j$ and $\partial El_j^2 / \partial x_j$, since the direct effect is simply their weighted sum, the weight in question, of course, being c_u:

$$\frac{\partial E(w_j - p)^2}{\partial x_j} = \frac{-2(1-x_j)[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{34a}$$

$$\frac{\partial El_j^2}{\partial x_j} = \frac{2\varepsilon}{[\alpha+\varepsilon(1-\alpha)]^2[\gamma(1-\alpha)+\alpha(1-x)]^2}\left\{ -\gamma^2(1-\alpha)[\alpha(1-x)+\varepsilon(1-\alpha)(1-x_j)]\sigma_\phi^2 + \{\gamma-\varepsilon(1-x_j)+(\varepsilon-1)[\gamma(1-\alpha)+\alpha(1-x)]\}\sigma_\theta^2 \right\} \tag{34b}$$

The indirect effect, meanwhile, involves the derivative of (32) with respect to x, or, equivalently, the weighted sum of expressions (30a) and (30b), the relevant weight again being c_u.

Expressions (33), (34a) and (34b) have prepared the ground for the derivation of the symmetric Nash equilibrium degree of indexation, x_{NE}, for the general case in which n may take any of its admissible set of values. Before deriving x_{NE} for the large subset of cases for which $n > 1$, however, we must derive separately the equilibrium solution for the case of atomistic unions. The reason for doing so is that the x_{NE} expression for the $n > 1$ subset which will be presented subsequently can only be regarded as legitimate if it reduces in the limit, as $n \to 1$ and as $n \to \infty$, to the equilibrium expressions which have been derived for the $n = 1$ and atomistic cases respectively. The equilibrium solution for the $n = 1$ case has in fact already been derived implicitly. It is clear that x is a choice variable of a single economy-wide union, and since that union will set the same degree of indexation in each of the wage contracts to which it is a party (i.e. $x_i = x \forall i$ and $x_j = x \forall j$ hold), it follows that $E(\Omega_j^u|_{n=1}) = E(\Omega_j^u|_{x_j = x})$ is the case. Hence the individually optimal indexation choice of a single union is the efficient solution, as given by (22), so that $x_{NE}|_{n=1} = x^*$. As for the atomistic case, this is the case in which the individual union believes that its x_j choice has a negligibly small impact on x, and consequently takes x as given in choosing x_j. In other words the second term on the right-hand side of (33) is assumed to be identically zero, so that the first-order condition for the atomistic union's individually optimal x_j choice becomes $\partial E\Omega_j^u / \partial x_j = 0$. Differentiating (32) with respect to x_j, with x treated as fixed, equating the result to zero, and imposing $x_j = x$ in union j's resulting first-order condition, yields an equation which implicitly defines the symmetric Nash equilibrium degree of indexation in the atomistic case. The unique solution in the atomistic case is then found to be:

$$x_{NE,\,atomistic\,case} = 1 - \frac{\gamma \varepsilon \sigma_\theta^2}{[\gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2]} \tag{35}$$

When this expression is compared with its counterpart for the efficient-indexation case, namely equation (22), it is apparent that, in general, the degree of indexation chosen by atomistic unions in equilibrium exceeds the efficient value. This indicates the existence of an adverse externality appertaining to the individual union's indexation decision.

Proceeding now on the assumption that $1 < n < \infty$, so that the second term on the right-hand side of (33) is non-zero, the first-order condition for union j's optimal choice of indexation can be obtained, namely $dE\Omega_j^u / dx_j = 0$. It is not worthwhile explicitly setting out this expression here, since our interest lies rather with the equation which implicitly defines x_{NE}. This latter equation, which is to be found in Appendix V.1, is obtained by imposing $x_j = x$ in union j's first-order condition. The solution is:

$$x_{NE} = \frac{K_1\sigma_\phi^2 + K_2\sigma_\theta^2 - [(K_1\sigma_\phi^2 + K_2\sigma_\theta^2)^2 - 4(n-1)\alpha K_3 K_4]^{1/2}}{2(n-1)\alpha K_3} \tag{36}$$

where:

$$\mathrm{K}_1 \equiv \gamma^2(1-\alpha)[\gamma\alpha + [(n-1)\alpha + \Lambda]\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}]$$

$$\mathrm{K}_2 \equiv \gamma\alpha[1-\varepsilon(n-1)] + [(n-1)\alpha + \Lambda]\{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}$$

$$\mathrm{K}_3 \equiv \gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2$$

$$\mathrm{K}_4 \equiv \gamma^2(1-\alpha)[\gamma\alpha + \Lambda\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}]\sigma_\phi^2 + \{(1-\gamma)(\gamma\alpha + \varepsilon\Lambda) + \Lambda c_u[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2$$

$$\Lambda \equiv (n-1)\alpha + n\gamma(1-\alpha)$$

Note that this is one of two solutions to the equation which implicitly defines x_{NE}. The other solution is disregarded since, as demonstrated in Appendix V.1, its value in the limit as $n \to 1$, and also in the limit as $\varepsilon \to \infty$, is not the efficient degree x^*, nor does this second solution's value in the limit as $n \to \infty$ reduce to the atomistic-case x_{NE}, as given by (35). The solution reported above as (36) is satisfactory in these respects.[18] Specifically, we find that:

$$\lim_{n\to 1} x_{NE} = x^* \qquad (37a)$$

$$\lim_{n\to\infty} x_{NE} = x_{NE,\ atomistic\ case} \qquad (37b)$$

$$\lim_{\varepsilon\to\infty} x_{NE} = x^* \qquad (37c)$$

It is clear from comparison of (36) with (22) that when $n > 1$, the equilibrium degree of indexation differs, in general, from the efficient degree, indicating that non-atomistic unions' indexation decisions are characterised by an adverse externality. Bearing in mind that, special cases apart, $x_{NE,\ atomistic\ case}$ is greater than x^*, and also that the wage-setting externality of Chapter III was found to be stronger, the larger is *n*, and to be weaker, the larger is ε, intuition suggests that $\partial x_{NE}/\partial n > 0$ and $\partial x_{NE}/\partial \varepsilon < 0$ must be the case. Although it has not been possible to devise a general proof of this for all admissible combinations of parameter values, it is easily shown to hold when velocity shocks are

[18] This is demonstrated in Appendix V.1.

absent (i.e. in the $\sigma_\phi^2 = 0$ case), and a demonstration of this is contained in Appendix V.1. The equilibrium indexation result for the $1 < n < \infty$ case has in fact been stated here largely for the sake of completeness and for consistency with the practice of other chapters. Since the key aspects of the externality can be identified by focusing solely on the atomistic-case solution, (35), the discussion which follows concentrates on this less complicated case. To reduce the notational burden, the subscript 'atomistic case' is dropped for the remainder of this section, and it is to be understood that 'x_{NE}' refers to expression (35).

It is appropriate to begin our discussion of the externality by reminding the reader that, as mentioned earlier, the variance of the price level under the simple rule is an increasing function of x.[19] It follows from this fact that a major consequence of the externality is that in equilibrium, price-level variability is higher than it would be under efficient indexation. Furthermore, the derivatives (30a) and (30b) indicate that in equilibrium real wage variability must be lower, and employment variability higher, than under efficient indexation. The expected loss of each union is therefore larger than it would be were unions able to coordinate their individual indexation choices on the efficient degree x^*.[20] The macroeconomic repercussions of the indexation externality are therefore qualitatively very similar to those of the wage-setting externality identified in Chapter III. This qualitative correspondence between the two externalities will be further discussed below. In order to facilitate such a discussion, however, it is necessary first of all to set out various expressions which cast light upon the relationship between the equilibrium indexation solution and the model's structural parameters.

When the ratio of the shock variances, $\sigma_\theta^2/\sigma_\phi^2$, is zero, either because productivity shocks are absent or because velocity shocks completely predominate, x_{NE} is equal to

[19] This is technically true only when $x < 1 + [\gamma(1-\alpha)/\alpha]$. (See footnote 9 above.) However, this condition always holds so far as the efficient and equilibrium values of x are concerned.

[20] It is easily shown to be the case that $E(\Omega_j^u|_{x_j = x_{NE} \forall j}) > E(\Omega_j^u|_{x_j = x^* \forall j})$ if the trouble is taken to obtain these expressions by evaluating (21) for $x = x^*$ and $x = x_{NE}$, as given respectively by (22) and (35).

unity, which from (23a) we know to be the efficient degree of indexation in these two extreme cases:[21]

$$\lim_{\sigma_\phi^2 \to \infty} x_{NE} = x_{NE}\big|_{\sigma_\theta^2 = 0} = 1 \tag{38a}$$

In the converse extreme case in which $\sigma_\theta^2 / \sigma_\phi^2 \to \infty$, either because velocity shocks are absent or because productivity shocks completely predominate, x_{NE} is found to differ from the efficient indexation degree, as given by (23b):

$$\lim_{\sigma_\theta^2 \to \infty} x_{NE} = x_{NE}\big|_{\sigma_\phi^2 = 0} = \frac{\varepsilon(1-\gamma) + c_u[\alpha + \varepsilon(1-\alpha)]}{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]} \tag{38b}$$

The derivatives of x_{NE} with respect to σ_ϕ^2 and σ_θ^2 are also of interest:

$$\frac{\partial x_{NE}}{\partial \sigma_\phi^2} = \frac{\gamma^3 \varepsilon(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2}{[\gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2]^2} \tag{39a}$$

$$\frac{\partial x_{NE}}{\partial \sigma_\theta^2} = \frac{-\gamma^3 \varepsilon(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\sigma_\phi^2}{[\gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2]^2} \tag{39b}$$

Results (39a) and (39b) reveal that as the significance of velocity shocks, as a source of macroeconomic instability, rises relative to that of productivity shocks (i.e. as the ratio $\sigma_\theta^2 / \sigma_\phi^2$ falls), in equilibrium each union increases its degree of indexation in response to the increased importance of velocity shocks. (This pair of results is consistent, of course, with the general finding of the literature.)

As for the relationship between x_{NE} and the elasticity parameter ε, it has already been established in expression (37c) that $\lim_{\varepsilon \to \infty} x_{NE} = x^*$: this result therefore indicates that the

[21] In interpreting (38a) and (38b), note that x_{NE} has been derived on the assumption that at least one of the two shock variances is positive.

externality does not arise if the goods market is perfectly competitive. Furthermore, it is apparent from the following derivative (which is unambiguously negative provided $0 < c_u < \infty$ and provided $\sigma_\theta^2/\sigma_\phi^2 > 0$) that the externality is weaker, the more competitive is the goods market:

$$\frac{\partial x_{NE}}{\partial \varepsilon} = -\frac{c_u \gamma \alpha \sigma_\theta^2[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]}{[\gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha+\varepsilon(1-\alpha)]\}\sigma_\theta^2]^2} \tag{40}$$

As in Duca and VanHoose (1998a), therefore, the equilibrium degree of indexation is falling in the degree of goods-market competition. However, whereas in Duca and VanHoose this arises because greater goods-market competition diminishes the impact of cross-sectoral spillovers on the real wage of workers who are subject to wage contracts, the qualitatively similar result in our model has a different source, namely that greater goods-market competition mitigates the negative externality which characterises the indexation choices of atomistic unions.

So far as the relationship between x_{NE} and c_u is concerned, we find that if c_u takes either of the two extremes of its set of admissible values, equilibrium indexation is efficient, as is evident from comparison of the following expressions with (25a) and (25b):

$$\lim_{c_u \to \infty} x_{NE} = 1 \tag{41a}$$

$$x_{NE}\big|_{c_u=0} = \frac{\gamma^2(1-\alpha)\sigma_\phi^2 + (1-\gamma)\sigma_\theta^2}{\gamma^2(1-\alpha)\sigma_\phi^2 + \sigma_\theta^2} \tag{41b}$$

The derivative of x_{NE} with respect to c_u is unambiguously positive, provided $\sigma_\theta^2/\sigma_\phi^2 > 0$:

$$\frac{\partial x_{NE}}{\partial c_u} = \frac{\gamma\varepsilon[\alpha+\varepsilon(1-\alpha)]\sigma_\theta^2[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]}{[\gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha+\varepsilon(1-\alpha)]\}\sigma_\theta^2]^2} \tag{42}$$

Expressions (38a) to (42) collectively reveal that four features of this chapter's model are essential to the existence of the indexation externality. The first and second of these features are a monopolistically competitive goods market (i.e. $\varepsilon < \infty$) and an aversion on the individual union's part to both employment variability and real wage variability (i.e. $0 < c_u < \infty$). The importance of the third and fourth features is concisely captured in the requirement that $\sigma_\theta^2 / \sigma_\phi^2 > 0$ hold. In other words, for the externality to arise not only must the economy be subject to productivity shocks (i.e. $\sigma_\theta^2 > 0$ must be the case), but in addition the velocity shock variance must not be so large as to render productivity shocks a comparatively negligible source of price level and aggregate demand instability (i.e. $\sigma_\phi^2 < \infty$ is the fourth essential feature). Clearly, the first three of these requirements have elsewhere been identified as necessary conditions for the existence of a wage-setting externality in the version of our basic model analysed in Chapter III. This indicates that the underlying source of the inefficiency in atomistic unions' decisions is common to both scenarios. Note, however, that the fourth of the above necessary conditions for the existence of the indexation externality also has a partial parallel in the model of Chapter III, namely in the fact that the wage-setting externality only exists if the variance of signal noise is finite ($\sigma_u^2 < \infty$). This correspondence is not surprising, since in both scenarios an externality only arises if there exists a variable which carries information about the productivity shock and which can induce an adjustment in nominal wages. In the present chapter's model this role is performed by the price level, which, of course, becomes completely uninformative about the productivity shock in the limit as $\sigma_\phi^2 \to \infty$.

Our immediate concern is to explain how productivity shocks, monopolistic competition, and the assumed specification of the union loss function, together serve to engender the externality. In pursuit of this, it is instructive to abstract initially from the existence of velocity shocks, so that the equilibrium degree of indexation is given by (38b). As we know, productivity shocks must occasion a departure of either employment or the real wage, or both, from their mean values of zero, while indexation is a mechanism for allocating the impact of a non-zero productivity shock between the real wage and employment. The individual union which is averse to variability in both these outcome variables chooses its degree of indexation so as to allocate the shock between them in the most favourable way, given the constraints the union faces. In doing so, each atomistic union perceives (correctly) that the price level repercussions of its individual indexation decision are negligibly small, and recognises also that with monopolistic competition in the goods market, the product demand of its employer firms will not vanish if their prices exceed the prevailing average price. Consequently, the individual union perceives that it may potentially benefit from setting an individual degree of wage indexation which differs from the aggregate (i.e. average) degree. It so happens that if all other unions cooperatively abide by the efficient degree of indexation, union j will be able to improve its individual expected loss, relative of course to the expected loss outcome associated with efficient indexation by all unions, by choosing an individual degree of indexation which is greater than the efficient degree. By increasing x_j above x^*, the individual union can trade somewhat lower real wage variability for somewhat higher employment variability, and thereby improve its expected welfare outcome. Of course, this incentive to deviate from x^* is faced by every union, and the resulting adverse repercussion on price-level variability means that in equilibrium all end up worse off than had coordination on x^* occurred.

The familiar concept of the perceived labour demand curve can once again be deployed to provide a diagrammatic representation of the externality's source. As in previous chapters, there exists a perceived labour demand curve for each combination of productivity shock realisation, θ, and nominal wage set symmetrically by all other unions other than union j. Since atomistic unions take the aggregate nominal wage itself as given, there will, given θ, be an infinite set of possible perceived curves, one for every possible realisation of the aggregate nominal wage. Two members of this infinite set are of particular interest, however: the perceived labour demand curve when the aggregate degree of indexation (and hence also, given θ, the aggregate nominal wage) is efficient, and its equilibrium counterpart for the case in which all other unions set x_{NE}. It is convenient at this point to set out the equations of these two curves:

$$l_j\big|_{x_k = x^*\forall k\neq j} = \frac{1}{[\alpha+\varepsilon(1-\alpha)]}\left[\frac{\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\theta}{[1+c_u(1-\alpha)^2]} - \varepsilon(w_j - p)\big|_{x_k = x^*\forall k\neq j}\right] \tag{43a}$$

$$l_j\big|_{x_k = x_{NE}\forall k\neq j} = \frac{1}{[\alpha+\varepsilon(1-\alpha)]}\left[\frac{\{\varepsilon^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\}\theta}{\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} - \varepsilon(w_j - p)\big|_{x_k = x_{NE}\forall k\neq j}\right] \tag{43b}$$

(43a) and (43b) are simply the versions of equations (49a) and (49b) of Chapter III which arise in the limit as $n \to \infty$, with the notation slightly modified to reflect the fact that these entities now relate to symmetric indexation[22] by all unions other than union *j*, and to a full-information environment. (Although union *j* has no information whatsoever at the time it sets x_j, it nevertheless does know that its real wage and employment, given θ, and given every other union sets x^* or x_{NE}, must respectively be related according to (43a) and (43b). It is therefore entirely appropriate to say that union *j* faces a perceived labour demand curve at the time it chooses x_j, despite its ignorance of θ at that juncture.)

The two perceived labour demand curves given by (43a) and (43b), together with the symmetric-indexation labour demand curve given by (27′) appear in Figure V.3 below, which to avoid an element of repetition depicts the case of a negative realisation of θ. An aggregate degree of indexation of unity would place the outcome on the vertical axis, and as x decreases gradually from unity the location of the outcome migrates up the aggregate-level labour demand curve, which coincides, of course, with the symmetric-indexation labour demand curve.[23] The efficient outcome is at *a*, and given efficient aggregate indexation, union *j* can, by appropriately adjusting x_j, ensure its individual outcome is located at any particular point on its perceived labour demand curve passing through *a*. By increasing x_j above x^* sufficiently, union *j* can place its outcome at the tangency point *b*, which is closer than the efficient outcome to the bliss point. The Nash

[22] With full information regarding the productivity shock, symmetric indexation is merely a special form of symmetric wage-setting.
[23] When $x = 1-\gamma$, the aggregate outcome is located at the aggregate labour demand curve's intercept with the horizontal axis.

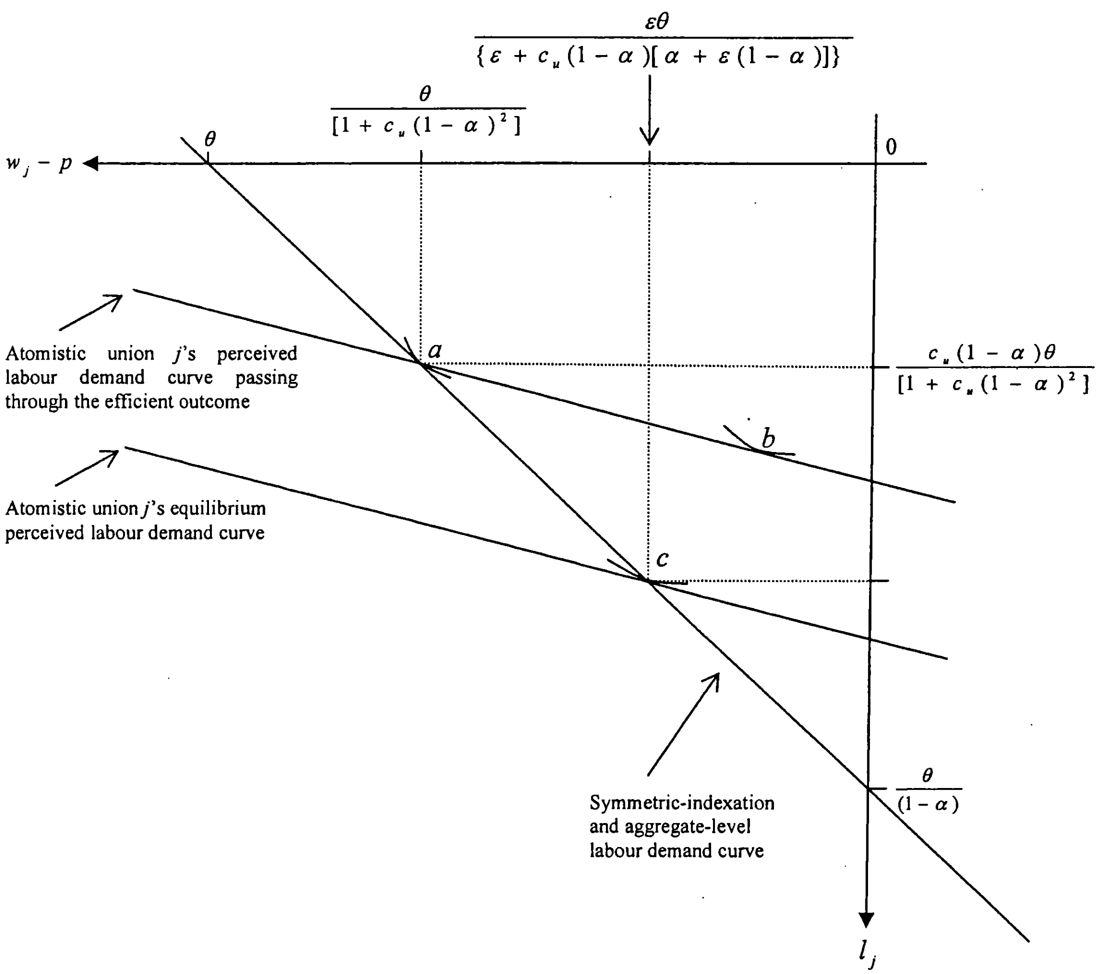
equilibrium outcome is located at c, since this is the only point on the symmetric-indexation labour demand curve which is also a point of tangency between the atomistic union's perceived labour demand curve through that point and the isoloss map.

Figure V.3 is also useful in explaining why it is essential to the externality that c_u be positive and finite. The reason why the externality does not arise in the $c_u = 0$ case and in the limit case of $c_u \to \infty$, is because in these extreme cases the nature of the representative union's isoloss map is such as to render irrelevant to the union's welfare the trade-off faced by union *j* between reducing the impact of a given θ shock on the real wage, and exacerbating that shock's impact on employment. In these extreme cases the isoloss curves are parallel straight lines, and hence no tangency point exists between either the aggregate level labour demand curve or any perceived labour demand curve and the isoloss map. Furthermore, bliss outcomes are not given by the origin but rather by the horizontal axis (when $c_u = 0$) or the vertical axis (when $c_u \to \infty$). In the latter $c_u \to \infty$ case, union j can achieve its sole objective of perfect real wage stability by setting $x_j = 1$, regardless of the value of σ_θ^2 (or, for that matter, σ_ϕ^2). Hence there is no possibility of an externality since every union can insulate itself fully from the repercussions of other unions' indexation decisions. In the $c_u = 0$ case union *j* can, in the absence of velocity shocks, similarly achieve its bliss outcome of perfect employment stability by choosing x_j so as to place its individual outcome at the intersection point of the aggregate-level labour demand curve with the horizontal axis. (Given efficient aggregate indexation in this $c_u = 0$ case, so that $x = x^*\big|_{\sigma_\phi^2 = 0,\, c_u = 0} = 1 - \gamma$, union *j*'s perceived labour demand curve will pass through the aggregate-level curve's horizontal intercept, which is, of course, the efficient outcome when $c_u = 0$.) Although the presence of velocity shocks ($\sigma_\phi^2 > 0$) will mean that this ideal outcome of perfectly stable employment can only be achieved on average, it will not alter union *j*'s incentive to set $x_j = x^*\big|_{c_u = 0}$ and thus ensure its mean outcome is on the horizontal axis.

One further aspect of the special case in which $\sigma_\phi^2 = 0$ requires comment before we proceed to explain how the existence of velocity shocks affects the externality. If we take

Figure V.3

Labour demand curves for the case of a negative productivity shock



Notes:

1. In the absence of velocity shocks ($\sigma_\phi^2 = 0$), the efficient outcome is at a and the symmetric Nash equilibrium is at c.
2. The vertical-axis coordinate of point c is $c_u[\alpha + \varepsilon(1-\alpha)]\theta / \{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}$.

the trouble to combine our $x_{NE}|_{\sigma_\phi^2=0}$ result, namely equation (38b), with the semi-reduced from for p (which will be given by (14) with $\sigma_\phi^2 = 0$ and the appropriate normalisations imposed), and the general real wage equation, $w_j - p = (x_j - 1)p$, the equilibrium real wage in the $\sigma_\phi^2 = 0$ special case is found to be as follows:

$$(w-p)|_{\sigma_\phi^2=0, x=x_{NE}} = \frac{\varepsilon\theta}{\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}} \tag{44}$$

If we compare this with equation (40a) of Chapter III, it is apparent that in the absence of velocity shocks equilibrium indexation by atomistic unions brings about exactly the same real wage that results when the wage is set in response to a perfectly informative signal of θ. In other words, when velocity shocks do not exist, and the equilibrium price level therefore conveys perfect information regarding θ, the indexation externality has precisely the same strength as the wage-setting externality in the perfect-signal $\beta = 1$ case. The essential reason for this equivalence is that in both scenarios each atomistic union takes the aggregate nominal wage as given. Since when $\sigma_\phi^2 = 0$ and $\beta = 1$ the two scenarios only differ in the way that the aggregate nominal wage affects the price level, wage-setting and indexation decisions which are uninfluenced by the relationship between these aggregate variables must lead to exactly the same equilibrium outcomes.

In general, of course, velocity shocks do exist, and signals of productivity shocks are noisy ($\beta < 1$), so that the equivalence identified in the previous paragraph only holds between extreme cases of each scenario. With $\sigma_\phi^2 > 0$, union j's choice of x_j will determine the average position of its real wage and employment outcome pair, for a given realisation of θ, on the relevant perceived labour demand curve. The actual outcome will depend upon the particular realisation of ϕ, and hence the distribution of possible outcomes around the average outcome will depend on the velocity shock variance σ_ϕ^2. By increasing its individual degree of indexation marginally from the value given by the right-hand side of (38b)[24], union j displaces the average outcome in Figure V.3 away from the tangency point (which is at c, if we assume that other unions symmetrically set their indexation parameters equal to (38b)), but at the same time also causes the distribution of actual outcomes around the average outcome to narrow. Optimal indexation by union j involves adjusting x_j to ensure that, for a given realisation of θ, the marginal benefit of reducing the variance of actual outcomes around the mean outcome, is equal to the marginal cost of incurring a larger departure of the mean outcome from the tangency point at c.

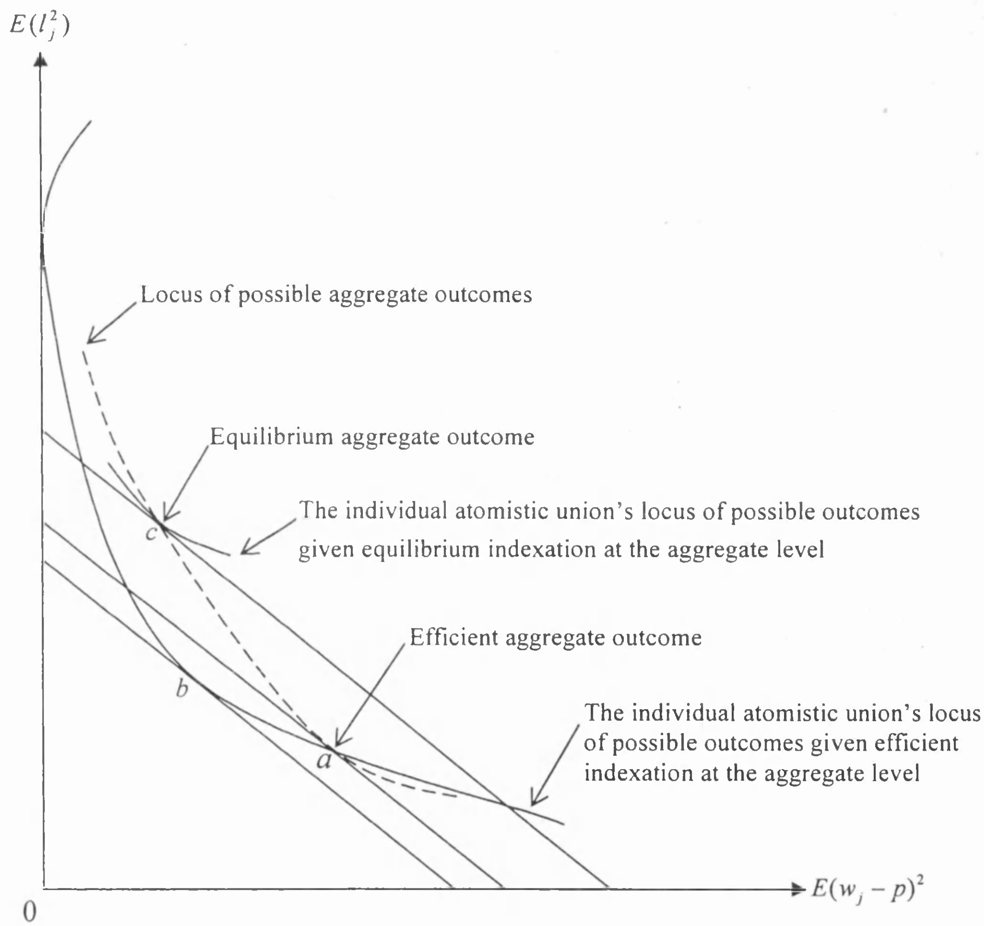
The individual union's optimal indexation choice when velocity shocks are present can alternatively be analysed using the concept of a locus of possible outcomes in variance of real wage, variance of employment space. The aggregate version of this locus has already been depicted in Figure V.2. Given a particular degree of aggregate indexation, x, each atomistic union faces its own locus of possible individual outcomes which intersects the aggregate locus at the outcome point which is brought about by x.[25] Two members of the infinite set of possible individual loci are of particular interest, namely that which obtains when $x = x^*$, and its counterpart for the case of $x = x_{NE}$. The relevant portions of these two loci are illustrated below in Figure V.4, together with part of the aggregate locus which was more fully depicted in Figure V.2. Given symmetric indexation, $x_j = x \forall j$, the individual union's outcome combination of employment variability and real wage variability must be located on the aggregate locus. Given efficient aggregate indexation, $x = x^*$, union j's individual locus is not tangent to its isoloss map at the efficient outcome at a, but rather at b. By increasing x_j above x^* sufficiently, union j can trade increased individual employment variability for lower real wage variability, and thus attain an

[24] The right-hand side of (38b), of course, is the symmetric Nash equilibrium degree of indexation only when $\sigma_\phi^2 = 0$. Note that for the $\theta < 0$ realisation to which Figure V.3 relates, the equilibrium perceived labour demand curve for the $\sigma_\phi^2 > 0$ case is not depicted, but would lie below the equilibrium perceived labour demand curve for the $\sigma_\phi^2 = 0$ case shown passing through c. The tangency point between the representative union's isoloss map and the equilibrium perceived labour demand curve for the $\sigma_\phi^2 > 0$ case would not be on the aggregate-level labour demand curve, but below it and to its left.

[25] In Appendix V.2, an expression for the slope of the atomistic union's locus of individual outcomes is presented and discussed.

Figure V.4

Efficient and equilibrium outcomes and the loci of possible outcomes



Notes:

1. Each of the depicted linear isoloss curves has slope equal to $-c_u$.
2. Only part of each locus of possible outcomes is depicted.
3. The aggregate outcome is at point a when $x = x^*$, and is at point c when $x = x_{NE}$.

individual outcome at b which is superior to that which obtains at the aggregate level when $x = x^*$. Since, given $x = x^*$, every union would face the locus of possible outcomes passing through a and b, it follows that $x_{NE} = x^*$ cannot be the case. The symmetric Nash equilibrium outcome is that point on the aggregate locus which also happens to be a tangency point between the isoloss map and the individual locus that prevails in equilibrium, and this point is labelled c in Figure V.4.

V.3.4 Socially Optimal Indexation

This final subsection relating to the simple-rule scenario investigates whether union indexation decisions give rise to externalities which affect the welfare of society as a whole. To this end, it is necessary to derive the socially optimal degree of wage indexation. Combining (18) and (20b), the expressions for the variances of the price level and aggregate employment, with the social loss function, (9), yields the (unconditional) expected social loss as a function of x:

$$E\Omega^s = \frac{\gamma^2[(1-x)^2 + c_s(1-\alpha)^2]\sigma_\phi^2 + [(\gamma - 1 + x)^2 + c_s]\sigma_\theta^2}{[\gamma(1-\alpha) + \alpha(1-x)]^2} \tag{45}$$

Minimising (45) by choice of x, the socially optimal degree of wage indexation, denoted $\hat{x}$, is found to be:

$$\hat{x} = 1 - \frac{[\gamma^2 c_s \alpha(1-\alpha)^2 \sigma_\phi^2 + (\gamma^2 + c_s \alpha)\sigma_\theta^2]}{\gamma[\gamma^2(1-\alpha)\sigma_\phi^2 + \sigma_\theta^2]} \tag{46}$$

Several points are noteworthy about (46). The higher is the weight which society attaches to the variance of the price level, the lower is the socially optimal degree of wage indexation. In the case in which society's weight, c_s, is zero, (46) reduces to the efficient solution of Ball's model when indexation is costless. This is not surprising, since with $c_s = 0$ the social loss function consists solely of the variance of employment. More generally, for the symmetric Nash equilibrium degree of indexation under the simple rule to be socially optimal requires both that unions be indifferent to real wage variability, and that the social loss be independent of price-level variability, so that $c_u = c_s = 0$ holds. In this special case, equations (22), (35) and (46) are such that: $\hat{x} = x^* = x_{NE} = 1 - \{\gamma\sigma_\theta^2 / [\gamma^2(1-\alpha)\sigma_\phi^2 + \sigma_\theta^2]\}$. If $c_u > 0$, the equilibrium degree of wage indexation will not be socially optimal. The proof is straightforward: (22) and (46) imply respectively that $\partial x^*/\partial c_u > 0$ and $\partial\hat{x}/\partial c_s < 0$, and since it has already been established that $x^* > x_{NE}$, it follows that $x_{NE}(c_u > 0) > x^*(c_u > 0) > \hat{x}(c_s \geq 0)$, where $x(\;)$ denotes a functional relationship. Furthermore, since it is easily shown that $\partial E\Omega^s/\partial x > 0$ if $x > \hat{x}$, it follows that social welfare must be falling in the degree of wage indexation when $x > \hat{x}$. Consequently, in the simple-rule scenario of this section, which abstracts from any inflation bias associated with discretionary policy-making, the externality arising from the indexation decisions of atomistic unions is adverse both from the point of view of unions themselves and from that of society as a whole.

This finding confirms the suspicion voiced by Waller and VanHoose (1992) that if social welfare is falling in the variance of inflation, rather than simply in the trend inflation rate as assumed in their model, the direction of the wage-indexation externality need not be positive. The result reported above suggests that the beneficial externality relating to the mean inflation bias could be significantly counteracted by the adverse externality relating to the variability of inflation, rendering ambiguous in sign the direction of the net external effect of individual indexation decisions on social welfare.

V.4 Wage Indexation under Discretionary Monetary Policy

V.4.1 Outline of the Discretionary Monetary Policy Scenario

In this section it is assumed that the authorities possess full information on both shocks when setting the money supply to minimise the social loss, as given by (9). Substituting the reduced-form expressions for the price level, (14), and aggregate employment, (17b), into (9), allows the social loss to be expressed as a function of *m*, *x* and the two shocks:

$$\Omega^s = \frac{[\gamma(1-x)(m-Em+\phi)+(\gamma-1+x)\theta]^2 + c_s[\gamma(1-\alpha)(m+\phi)+\alpha(1-x)Em-\theta]^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \quad (47)$$

Minimising (47) by choice of m, the authorities' (and society's) optimal monetary response to the shocks, given x, is found to be:

$$m^* = -\phi - \frac{[(1-x)(\gamma-1+x)-c_s(1-\alpha)]\theta}{\gamma[(1-x)^2+c_s(1-\alpha)^2]} \quad (48)$$

This expression implies that the representative union's rational expectation of the authorities' setting of the money supply is $Em = 0$. Equation (48) also exhibits the standard result that when the authorities possess full information on the aggregate demand shock, their optimal response is to offset it completely, since such shocks cause employment and the price level to move in the same direction, and consequently do not create conflicting objectives for monetary policy.[26]

Substituting (48) into (14) yields the price level under discretion for a given x:

$$p = -\frac{(1-x)\theta}{[(1-x)^2+c_s(1-\alpha)^2]} \quad (49)$$

The variance of the price level is:

$$Ep^2 = \frac{(1-x)^2\sigma_\theta^2}{[(1-x)^2+c_s(1-\alpha)^2]^2} \quad (50)$$

An immediate implication of (49) is that $Ep = 0$. Furthermore, whereas under the simple rule a marginal increase in x, when x is currently less than unity, results in an increased

[26] In other words, neutralising the potential impact of the aggregate demand shock on the price level necessarily neutralises its potential impact on employment as well.

responsiveness of the price level to productivity shocks,[27] it is apparent from (49) that when the authorities respond to shocks and attach some weight to price-level stabilisation, the price level's sensitivity to productivity shocks is not necessarily increasing in x, when $x \leq 1$:

$$\frac{\partial p}{\partial x} = \frac{[c_s(1-\alpha)^2 - (1-x)^2]\theta}{[c_s(1-\alpha)^2 + (1-x)^2]^2} \tag{51}$$

$$\frac{\partial Ep^2}{\partial x} = \frac{2(1-x)[(1-x)^2 - c_s(1-\alpha)^2]\sigma_\theta^2}{[(1-x)^2 + c_s(1-\alpha)^2]^3} \tag{52}$$

Equations (51) and (52) reveal that under discretion, if society's weight on the variance of the price level, c_s, is sufficiently low to ensure $c_s < (1-x)^2/(1-\alpha)^2$, a marginal increase in x increases the responsiveness of the price level to productivity shocks and hence worsens price-level variability. Conversely, if society places a sufficiently high weight on price-level variability, such that $c_s > (1-x)^2/(1-\alpha)^2$, a marginal increase in x reduces the absolute movement in the price level occasioned by a given productivity shock, leading to lower price-level variability. The reason for this has to do with the effect a marginal increase in x has on the monetary reaction function of the optimising central bank. Given x, socially optimal monetary policy distributes the productivity shock variance, σ_θ^2, between price-level variability and employment variability in the most desirable way for society. The value of x, together with the structural parameters α and σ_θ^2, determines the opportunity set of employment variability, price-level variability outcome pairs facing the central bank. Whether the changes in this set occasioned by a marginal change in x induces the central bank to select an outcome pair involving higher or lower variability of the price level than previously, depends on society's preference parameter c_s, as well as on the value of x prior to the change.

[27] As stated in footnote 9, this is generally the case when $x < [\alpha + \gamma(1-\alpha)]/\alpha$. However, our attention can justifiably be confined to less-than-full indexation since in the case of the simple rule x^* and x_{NE} are less than unity.

Preparatory to the derivation of the efficient and equilibrium degrees of indexation in subsequent subsections, we end these initial remarks regarding the discretion scenario by stating expressions for the individual union's real wage and employment, their aggregate counterparts, and their respective variances. Substituting (48) into (16a) and (16b) yields:

$$w_j - p = \frac{(1-x_j)(1-x)\theta}{[(1-x)^2 + c_s(1-\alpha)^2]} \tag{53a}$$

$$l_j = \frac{\{\varepsilon(x_j - x)(1-x) + c_s(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\theta}{[\alpha + \varepsilon(1-\alpha)][(1-x)^2 + c_s(1-\alpha)^2]} \tag{53b}$$

The variances of these variables are respectively:

$$E(w_j - p)^2 = \frac{(1-x_j)^2(1-x)^2\sigma_\theta^2}{[(1-x)^2 + c_s(1-\alpha)^2]^2} \tag{54a}$$

$$El_j^2 = \frac{\{\varepsilon(x_j - x)(1-x) + c_s(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}^2\sigma_\theta^2}{[\alpha + \varepsilon(1-\alpha)]^2[(1-x)^2 + c_s(1-\alpha)^2]^2} \tag{54b}$$

The aggregate counterparts to (53a), (53b), (54a) and (54b) are as follows:

$$w - p = \frac{(1-x)^2\theta}{[(1-x)^2 + c_s(1-\alpha)^2]} \tag{55a}$$

$$l = \frac{c_s(1-\alpha)\theta}{[(1-x)^2 + c_s(1-\alpha)^2]} \tag{55b}$$

$$E(w - p)^2 = \frac{(1-x)^4\sigma_\theta^2}{[(1-x)^2 + c_s(1-\alpha)^2]^2} \tag{56a}$$

$$El^2 = \frac{c_s^2(1-\alpha)^2\sigma_\theta^2}{[(1-x)^2 + c_s(1-\alpha)^2]^2} \tag{56b}$$

V.4.2 Efficient Indexation

We are now well placed to derive and discuss the efficient degree of indexation from the collective viewpoint of unions. As in the case of the simple rule analysed earlier, the efficient degree of indexation is the value of x which minimises the individual union's expected loss, given that this x is adopted by every union in the economy. When indexation is thus constrained to be symmetric across unions (i.e. $x_j = x \forall j$), the individual union's real wage and employment will be respectively equal to their aggregate counterparts, as given by the right-hand sides of (55a) and (55b). The expected loss of union j, under symmetric indexation is therefore obtained by combining (56a) and (56b) with (8):

$$E\Omega_j^u\Big|_{x_j = x\forall j} = \frac{[c_s^2(1-\alpha)^2 + c_u(1-x)^4]\sigma_\theta^2}{[(1-x)^2 + c_s(1-\alpha)^2]^2} \tag{57}$$

The efficient degree of wage indexation, denoted x^*, is the x which minimises (57). The two solutions to the first-order condition $d(E\Omega_j^u\big|_{x_j = x\forall j})/dx = 0$ are:

$$x_1^* = 1 - \left(\frac{c_s}{c_u}\right)^{1/2} \tag{58a}$$

$$x_2^* = 1 + \left(\frac{c_s}{c_u}\right)^{1/2} \tag{58b}$$

Several aspects of (58) require comment. Firstly, unlike under the simple rule, the precise magnitude of the productivity shock variance is irrelevant to the efficient degree of indexation. This is because the complete neutralisation of all aggregate demand disturbances by monetary policy reduces the unions' collective decision problem to one of how best to apportion the sole remaining source of stochastic variability between the real wage and employment, and hence the magnitude of that variability becomes irrelevant. Secondly, whereas under the simple rule the efficient degree of indexation is unique, under discretion there are two efficient solutions, which differ only as regards whether a negative or positive square root term is added to unity. (Note that the former solution, (58a), is negative if $c_s > c_u$.) The discussion which follows below generally concentrates on the solution with the negative square root, since casual empiricism suggests that in the real world wage indexation in excess of unity is very seldom encountered. (In the interests of simplicity of notation, the distinguishing subscript is henceforth dropped, so that x^* is to be understood as denoting expression (58a).)

Focusing on (58a), therefore, it is apparent that apart from the special cases in which society attaches no weight to price-level variability (the $c_s = 0$ case), or unions attach no weight to employment variability (the limiting $c_u \to \infty$ case), less-than-full indexation is efficient. The essential reason for this is that with the authorities neutralising the velocity shocks completely, stabilisation of the price level in response to productivity shocks also stabilises the real wage. In the $c_s = c_u$ case, the representative union cares just as much as does society about the relative variability of employment, relative of course to their respective other objective. Consequently, when $c_s = c_u$, monetary policy brings about the most desirable amount of employment variability from unions' viewpoint, and necessarily also brings about the most desirable amount of accompanying real wage variability, given the constraint that indexation be symmetric across all unions. Hence zero aggregate indexation is efficient in such circumstances, since it does not interfere with monetary policy's ability to bring about the best attainable combination of employment and real wage variability given symmetric indexation. In the $c_s < c_u$ case, each union places a smaller relative weight on employment variability than does society, and consequently, were unions to adopt zero wage indexation, the resulting amount of employment variability brought about by policy would be too low, and the associated variability of the real wage too high, than in the best outcome attainable by means of coordinated indexation. By increasing x from zero, when $c_s < c_u$, a desirable reduction in real wage variability can be secured at the cost of somewhat higher employment variability, and this trade-off is optimally exploited when $x = x^* > 0$. A similar argument applies in the converse $c_s > c_u$ case. In this case the representative union cares relatively more about employment variability than does society, and so with zero wage indexation, monetary policy would bring about an inefficiently high degree of employment variability, together with inefficiently low variability of the real wage, from the collective viewpoint of unions. When $c_s > c_u$ unions therefore find it desirable that the wage be indexed negatively to the price level, so that the responsiveness of the real wage to the productivity shock is enhanced, and the stability of employment increased.

It is clear that there is a close parallel between this efficient indexation result and our earlier findings with regard to efficient wage-setting under discretion, obtained in Section 5 of Chapter III. In both cases, when the preferences of the representative union are perfectly aligned with those of the central bank, in the sense that both place the same relative weight on employment stability, efficiency requires that the nominal wage not adjust at all, regardless of the realised value of the productivity shock, since the central bank's monetary response to θ, and the resulting movement in the price level, brings about the most desirable division of the shock between the real wage and employment. If preferences are not perfectly aligned, efficiency requires that the aggregate wage be adjusted in response to the informative variable, whether that variable be a signal of θ, or the value of the price level in the period covered by the wage contract. The efficient adjustment in w incorporates the rational expectation of the authorities' monetary response to both θ and w. This ensures that the authorities have no incentive to set m at any value other than that which, in conjunction with the efficient wage, w^*, ensures the productivity shock (or, if $\beta < 1$ in the wage-setting scenario, the shock's anticipated component) is divided between the real wage and employment in the ideal way from the unions' collective point of view. Indeed, substituting x^*, as given by (58), into the real wage equation, (55a), and comparing the result with equation (79a) of Chapter III, we find that under discretion the real wage brought about by efficient indexation differs from the value it has under efficient wage-setting only insofar as signal quality is imperfect (i.e. $\beta < 1$ is the case):

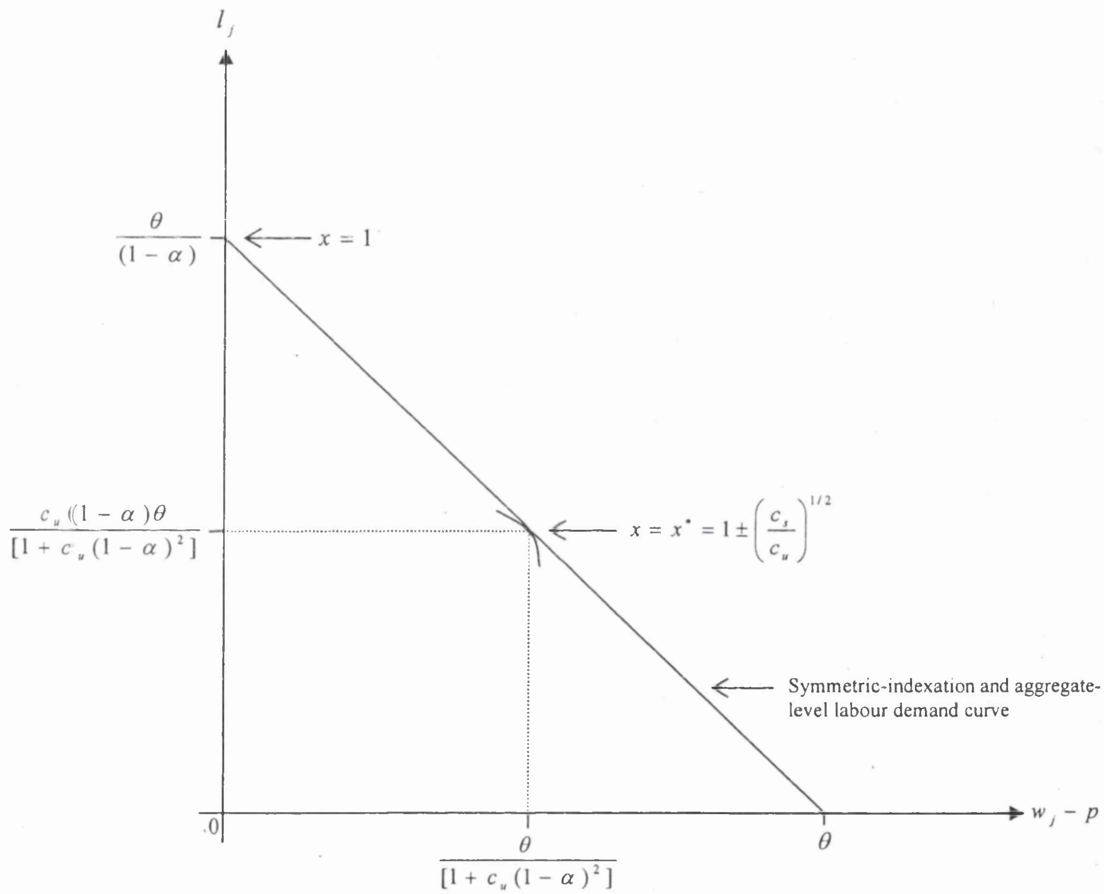
$$(w-p)\big|_{x=x^*} = \frac{\theta}{[1+c_u(1-\alpha)^2]} \tag{59}$$

It follows from this that efficient indexation under discretion yields the same expected union loss as efficient indexation under the simple rule when velocity shocks are absent. (In other words, under discretion $E\Omega_j^u\big|_{x_j=x^*\forall j}$ is equal to the right-hand side of (28).) A further corollary is that when monetary policy entirely neutralises velocity shocks, and any signal received by unions at the time of signing wage contracts is less than perfectly informative (i.e. $\beta < 1$), efficient indexation must yield superior outcomes (from unions' point of view) to efficient wage-setting in response to the signal.

A diagrammatic representation of the efficient outcome for a given positive realisation of θ is provided below as Figure V.5. The diagram depicts the aggregate-level labour demand curve, which coincides, of course, with the symmetric-indexation labour demand curve. Figure V.5 is very similar to the corresponding diagram for the simple-rule scenario, namely Figure V.1. In particular, for a given θ, the aggregate-level curve has the same vertical and horizontal intercept in the two scenarios, since these values are, respectively, simply the employment and real wage outcomes which result when either of these variables bears the full burden of the θ shock. Despite the close similarity, however, Figure V.5 does differ from Figure V.1 in several respects. For instance, under discretion when $\theta > 0$, the aggregate curve does not extend into parts of the diagram which involve a negative outcome for either the real wage or employment. (Note that it is also evident from equations (55a) and (55b) that a positive shock cannot have a negative impact on these variables under discretion.) The essential reason for this is that for a positive shock to be associated with a fall in employment, the real wage would have to increase by more than the full value of the shock, and this circumstance could only be brought about by the central bank deliberately exacerbating the shock's negative impact on the price level by reducing the money supply by a sufficiently large amount. However, the very fact that this outcome involves a fall in both the price level and in employment implies that the monetary contraction which would propagate it cannot be an optimal strategy for a

Figure V.5

The aggregate-level labour demand curve under discretion



Notes:

1. $\theta > 0$ is assumed for illustrative purposes.
2. The outcome located at the aggregate-level labour demand curve's horizontal intercept is attained in the limit as $x \to \pm\infty$.

central bank concerned to minimise the social loss function, (9), since such an outcome can always be improved upon by increasing the money supply marginally in order to reduce the extent by which both these outcome variables depart from their socially optimal values of zero. The key point here is that under optimal monetary policy the impact of a productivity shock on employment must differ in sign from its impact on the price level. It follows from this that a positive shock cannot have a negative impact on the real wage either, since a fall in the real wage would necessarily be accompanied by increases in both employment and the price level. Hence apart from its two intercepts with the axes, the aggregate level labour demand curve depicted in Figure V.5 must lie entirely above the horizontal axis and to the right of the vertical. By means of coordinated indexation, unions can ensure the outcome is located at any particular point on the curve. The unique value of x which places the outcome on the vertical axis, and hence achieves complete real-wage stability, is unity. Every other point on this opportunity set can be attained by either of two values, namely $x = 1 \pm r$, where r is a non-zero real number. Successive increases or decreases in x relative to unity cause the location of the outcome to migrate down the labour demand curve, with the horizontal-axis intercept, and hence complete employment stability, attained in the limit as $x \to \pm\infty$. The efficient x, of course, is that which ensures the outcome is at the curve's tangency point with the representative union's isoloss map.[28]

V.4.3 Equilibrium Indexation

In deriving the equilibrium degree of wage indexation under discretion, use must be made of the total derivative of union j's expected loss with respect to x_j, which was stated earlier in Section V.3 as equation (33). As in the case of the simple rule, it is necessary first of all to derive the equilibrium solution for the extreme case of atomistic unions, in order to have a criterion by which to judge the validity of solutions obtained

[28] The line of reasoning contained in this paragraph also implies that under discretion a negative productivity shock cannot have a positive impact on either the real wage or employment. Hence the aggregate-level labour demand curve in the $\theta < 0$ case has negative intercepts with both axes, and between these intercepts lies entirely below the horizontal axis and to the left of the vertical. $x = 1$ places the outcome at the vertical intercept, and successive increases from unity, or decreases below it, cause the aggregate outcome to migrate up the curve, with the horizontal axis attained in the limit as $x \to \pm\infty$.

for the general $n>1$ case. The requirement that any solution for the general case reduce in the limit as $n \to \infty$ to the x_{NE} solution obtained separately for the atomistic case is one of two criteria which it must satisfy for it to be regarded as legitimate, the other criterion, of course, being the requirement that in the limit as $n \to 1$, the solution for the $n>1$ case reduce to the known equilibrium expression for the case of a single union. (As under the simple rule, the x_{NE} solution for $n=1$ is known, since for a single union x_j is replaced by x in its expected loss expression, and hence $x_{NE}\big|_{n=1} = x^*$ must be the case.)

Our first step must be to obtain the (unconditional) expected loss of union j (whether atomistic or not) for given settings of x_j and x, by combining (54a) and (54b) with (8):

$$E\Omega_j^u = \frac{[\{\varepsilon(x_j - x)(1-x) + c_s(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2 + (1-x_j)^2(1-x)^2 c_u[\alpha+\varepsilon(1-\alpha)]^2]\sigma_\theta^2}{[\alpha+\varepsilon(1-\alpha)]^2[(1-x)^2 + c_s(1-\alpha)^2]^2} \tag{60}$$

In deriving the solution for the atomistic case, the derivative of the aggregate degree of indexation, x, with respect to the individual union's indexation choice is assumed to be identically zero: $\partial x/\partial x_j \equiv 0$. Bearing this in mind, a point worth noting about (60) is that full indexation at the aggregate level (i.e. $x=1$) renders the individual union's expected loss independent of its individual indexation parameter x_j. Consequently, under discretion $x_j = 1 \forall j$ must be a symmetric Nash equilibrium in the atomistic case. This result is also to be found in Lawler (1998), which examines a special case of the model of this section, namely that in which $c_u = 0$. The result arises here for the same reason as in Lawler: if every other union has adopted full indexation, union j has nothing to gain from deviating and setting $x_j \neq 1$. (Equally, however, union j has nothing to lose from deviating and setting $x_j \neq 1$, and thus the $x_{NE} = 1$ equilibrium may require some coordinating mechanism if it is to be brought about. Indeed a persuasive case can be made for regarding this equilibrium as implausible, since it is easily shown to result in a value for the expected union loss which is greater than the value of the loss in the other equilibria.[29])

Proceeding on the assumptions that $\partial x/\partial x_j \equiv 0$ and $x \neq 1$, minimisation of (60) by choice of x_j yields the individual atomistic union's optimal choice of indexation, for given x:

$$x_j^{**} = \frac{(1-x)\{\varepsilon^2 x + c_u[\alpha+\varepsilon(1-\alpha)]^2\} - c_s\varepsilon(1-\alpha)[\alpha+\varepsilon(1-\alpha)]}{(1-x)\{\varepsilon^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\}} \tag{61}$$

Imposing $x_j = x \forall j$ in (61) and solving for x, the symmetric Nash equilibria are found to be:

$$x_{NE,1,\,atomistic\ case} = 1 - \left\{\frac{c_s\varepsilon(1-\alpha)}{c_u[\alpha+\varepsilon(1-\alpha)]}\right\}^{1/2} \tag{62a}$$

$$x_{NE,2,\,atomistic\ case} = 1 + \left\{\frac{c_s\varepsilon(1-\alpha)}{c_u[\alpha+\varepsilon(1-\alpha)]}\right\}^{1/2} \tag{62b}$$

As in the case of efficient indexation, there are two solution values, corresponding to positive and negative square roots of the term $c_s\varepsilon(1-\alpha)/c_u[\alpha+\varepsilon(1-\alpha)]$.

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[29] The proof of this statement follows straightforwardly from the properties of the expected union loss under symmetric indexation, namely expression (57). Evaluated for $x=1$, the first and second derivatives with respect to x of this function are respectively zero and negative: $\left(\partial E\Omega_j^u\big|_{x_j=x\forall j}\big/\partial x\right)\Big|_{x=1} = 0$, $\left(\partial^2 E\Omega_j^u\big|_{x_j=x\forall j}\big/\partial x^2\right)\Big|_{x=1} = -4\sigma_\theta^2/c_s(1-\alpha)^4$. Symmetric indexation of unity therefore maximises the expected union loss. Each of the two atomistic-case Nash equilibria, (62a) and (62b), lies between unity and the corresponding efficient degree of indexation, namely x_1^* (equation (58a)) and x_2^* (equation (58b)) respectively. $E\Omega_j^u\big|_{x_j=x\forall j}$ is a continuous function of x, since it is the ratio of two polynomials in x and its denominator is positive for all real values of x. It therefore follows that $x=1$ must result in a higher value of the expected union loss than the other two atomistic-case equilibria. Whether $x=1$ merely maximises the expected loss locally, or does so globally, depends on the values of c_u and α. Since $E\Omega_j^u\big|_{x_j=1\forall j} = \sigma_\theta^2/(1-\alpha)^2$ and $\lim_{x\to\pm\infty} E\Omega_j^u\big|_{x_j=1\forall j} = c_u\sigma_\theta^2$, it is clear that $x=1$ is a global maximum if (and only if) $c_u(1-\alpha)^2 < 1$.

Turning now to the broad subset of cases for which n is such that $1 < n < \infty$, the second term on the right-hand side of (33) which involves $\partial x / \partial x_j$ will now be non-zero. The equation which implicitly defines the symmetric Nash equilibrium is obtained by replacing x_j with x (i.e. by imposing $x_j = x \forall j$) in the first-order condition $dE\Omega_j^u / dx_j = 0$ for union j's optimal choice of x_j. This equation is not of great interest in itself, and consequently is not reported here: it is to be found instead in Appendix V.3. Our interest rather lies with the x values which solve this equation, of which there are five. Three of these do not pass muster as candidates for symmetric Nash equilibria. Two members of this unsatisfactory trio can be disregarded because they are non-real (a proof of this is provided in Appendix V.3), while the third solution involves full indexation to the price level ($x = 1$), and can be ignored on the grounds that it is inconsistent with optimising behaviour by the individual non-atomistic union. (In Appendix V.3 it is shown that were every other union to adopt an indexation degree of unity, union j would maximise its expected loss, at least locally, were it also to set $x_j = 1$. Hence $x_j = x \forall j$ cannot be a Nash equilibrium if unions are non-atomistic. This result indicates that our earlier finding that $x_j = 1 \forall j$ is a Nash equilibrium in the atomistic case, is crucially dependent on the assumption that each individual union perceives the contribution made by its individual indexation choice to the aggregate degree of indexation to be negligibly small, and therefore in choosing x_j disregards the external effects of its choice.) It turns out that of the five solutions to the equation formed by imposing $x_j = x \forall j$ on union j's first-order condition, only the following satisfy the criteria set out above for a solution to be considered legitimate.

$$x_{NE,1} = 1 - \left\{ \frac{c_s(1-\alpha)\{(n-1)\varepsilon - (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} + \chi_1^{1/2}}{2(n-1)c_u[\alpha + \varepsilon(1-\alpha)]} \right\}^{1/2} \tag{63a}$$

$$x_{NE,2} = 1 + \left\{ \frac{c_s(1-\alpha)\{(n-1)\varepsilon - (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} + \chi_1^{1/2}}{2(n-1)c_u[\alpha + \varepsilon(1-\alpha)]} \right\}^{1/2} \tag{63b}$$

where:

$$\chi_1 \equiv c_s^2(1-\alpha)^2\left\{ \{(n-1)\varepsilon + (n+1)c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2 + 8(n-1)c_u\alpha[\alpha+\varepsilon(1-\alpha)] \right\}$$

In Appendix V.4 it is shown using L'Hôpital's rule that the limits of (63a) and (63b) as $n \to 1$ are respectively equal to (58a) and (58b). Obtaining the limits of (63a) and (63b) as $n \to \infty$ and as $\varepsilon \to \infty$ is straightforward:

$$\lim_{n\to\infty} x_{NE,1} = x_{NE,1,\,atomistic\ case} \tag{64a}$$

$$\lim_{n\to\infty} x_{NE,2} = x_{NE,2,\,atomistic\ case} \tag{64b}$$

$$\lim_{\varepsilon\to\infty} x_{NE,1} = x_1^* \tag{65a}$$

$$\lim_{\varepsilon\to\infty} x_{NE,2} = x_2^* \tag{65b}$$

Appendix V.4 also contains proofs of the following results regarding the derivatives of expressions (63a) and (63b) with respect to n and ε:

$$\frac{\partial x_{NE,1}}{\partial n} > 0 \,\forall n > 1 \tag{66a}$$

$$\frac{\partial x_{NE,2}}{\partial n} < 0 \,\forall n > 1 \tag{66b}$$

$$\frac{\partial x_{NE,1}}{\partial \varepsilon} < 0 \,\forall n > 1 \tag{67a}$$

$$\frac{\partial x_{NE,2}}{\partial \varepsilon} > 0 \,\forall n > 1 \tag{67b}$$

For the same reasons as stated earlier, we largely confine our attention to the solution with the negative square root, and therefore x_{NE} is henceforth to be understood as referring to (63a). There are several aspects of this solution which accord with results reported in the earlier literature on wage indexation under activist monetary policy. Firstly, as in Hutchison and Walsh (1998), the equilibrium degree of indexation is, in general, a decreasing function of the monetary authorities' weight parameter c_s.[30] Secondly, (63a) is also consistent with the finding of Lawler (1998) that if unions attach no weight to the variability of the real wage, and care solely about stabilising employment, the degree of equilibrium indexation is unbounded below. However, if the law or social convention places a lower bound on the set of values that x may take (by, for instance, restricting x to the unit interval), (63a) then suggests that in the $c_u = 0$ case the equilibrium degree will be zero.

More significant than these points in common between x_{NE} and earlier contributions to the literature, however, is that the equilibrium degree of indexation in general departs from the efficient degree. It is apparent that the externality appertaining to individual unions' indexation decisions which arises in the simple rule scenario also arises when the money supply is adjusted in a discretionary fashion in response to the realised values of the shocks and of the aggregate nominal wage. In both scenarios the indexation externality has the same effect in qualitative terms, namely it results in an increase in the

[30] The principal qualitative difference between Hutchison and Walsh's result for the equilibrium degree of indexation and the corresponding result in this chapter, is that whereas in Hutchison and Walsh equilibrium indexation is a decreasing function of the central bank's relative weight on the quadratic inflation term in its objective function (the equivalent of this chapter's c_s), regardless of whether the union's relative weight parameter (the equivalent of our c_u) is positive or zero, in this chapter x_{NE} is a decreasing function of c_s only if c_u is positive. In other words the difference in qualitative results relates solely to the extreme case in which $c_u = 0$. (Hutchison and Walsh do not discuss the converse extreme case in which $c_u \to \infty$ in the limit, nor do they provide an expression for equilibrium indexation when $c_u > 0$ from which this limit case can be ascertained, providing instead a verbal description of how the introduction of a term in the real wage into the union objective function alters their indexation result.) Note that the difference in results is not attributable to the fact that Hutchison and Walsh assume a perfectly competitive goods market and a single economy-wide union, since the imposition of these features on our model implies x_{NE} will equal the efficient solution, x^*, as given by (58a), which like (63a) is also not a decreasing function of c_s if c_u is zero. The source of the difference, rather, is that whereas this chapter (and indeed this thesis in its entirety) assumes the central bank's setting of its monetary instrument is not subject to control errors, Hutchison and Walsh assume this choice variable (which in their model is the intended price level) is subject to a random additive disturbance term.

variability of employment, and a decrease in the variability of the real wage, relative to their efficient levels. (Note that this occurs under discretion regardless of whether price-level variability is increasing or decreasing in x when $x = x^*$.) That the externality has the same ultimate source in the two scenarios is evident from the fact that if the goods market is perfectly competitive (the limit case in which $\varepsilon \to \infty$), and/or if unions are indifferent to either real wage variability or employment variability (i.e. if c_u is not a finite positive number), equilibrium indexation is efficient and $x_{NE} = x^*$ is the case.[31] Fundamental to the indexation externality's existence in both scenarios, therefore, are two factors: firstly, the potential under monopolistic competition for the individual firm's price to differ from the average price; secondly, the incentives which face union *j*, given efficient indexation by every other union, to adjust x_j away from x^* and towards unity in order to obtain lower individual real wage variability at the acceptable cost of higher individual employment variability. The externality mechanism differs superficially between the two regimes in that under discretion the individual non-atomistic union partially (or, if $n = 1$, fully) internalises the monetary-response implications of its indexation choice, whereas this is obviously not an issue under the simple rule. In both scenarios, union *j*'s concern is with the price-level repercussions of its setting of x_j, and under discretion this implies that the responsiveness of the money supply to both the shocks, and (via *w*) to *x*, must be taken into account in making indexation choices. One

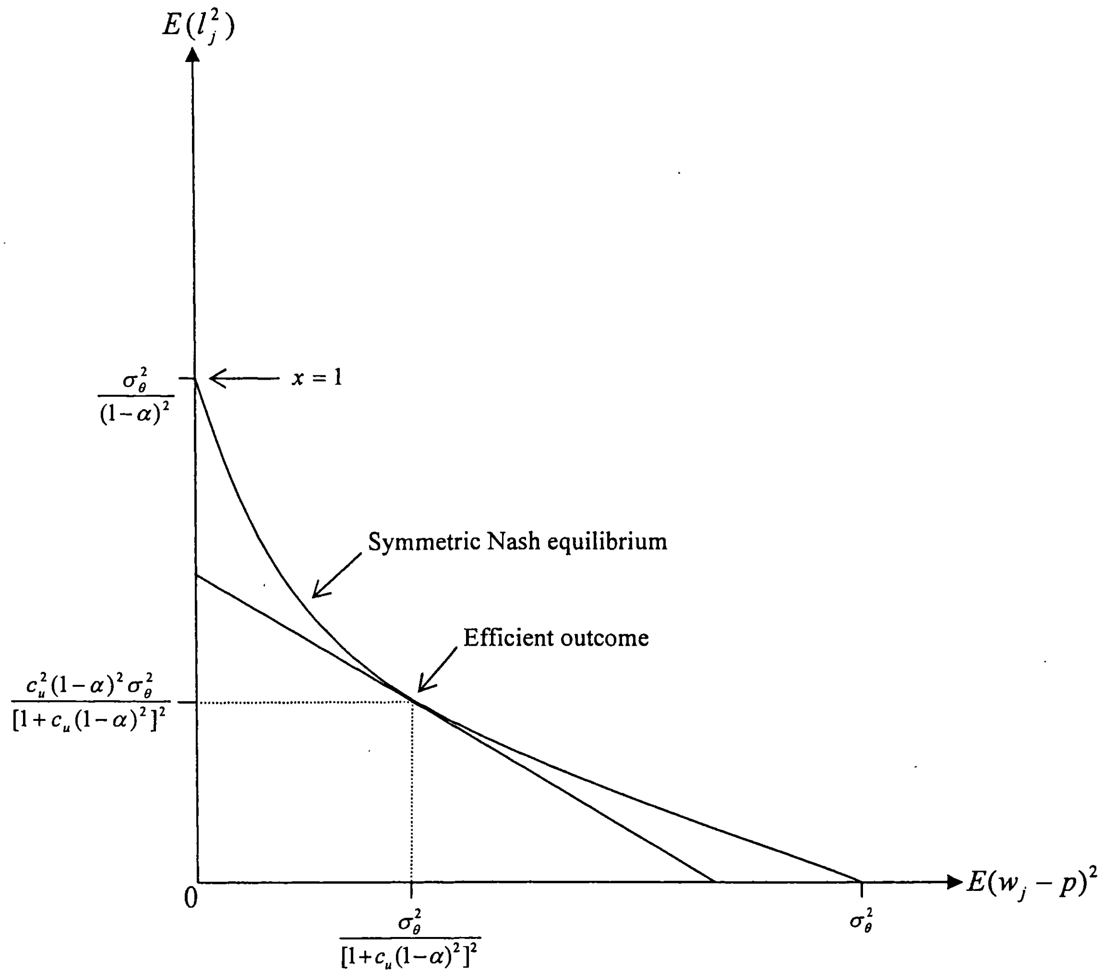
[31] Note that although casual scrutiny of (63a) suggests that $x_{NE}\big|_{c_s=0} = x^*\big|_{c_s=0} = 1$, and hence that the externality does not arise in the special case in which the social loss function lacks a term in inflation, this conclusion is not in fact warranted, since the central bank's optimal setting of the money supply, as given by (48), has been derived on the implicit assumption that the restrictions $c_s = 0$ and $x = 1$ do not simultaneously hold. (In other words, *m*, as given by (48), can be validly evaluated for cases in which $c_s = 0$, provided $x \neq 1$, and for cases in which $x = 1$, provided $c_s > 0$, but has no value (indeed, is undefined) in the special case in which both $c_s = 0$ and $x = 1$.) When $x = 1$, the central bank is powerless to influence the aggregate real wage and hence employment, and consequently if $c_s = 0$ lacks an incentive to set any particular *m*. This reasoning implies that the two efficient-indexation solutions, (58a) and (58b), have also been derived under the implicit assumption that $c_s > 0$, and indeed it is apparent from (57) that the representative union's expected loss under symmetric indexation is independent of *x* in the special $c_s = 0$ case. Note also that the individual union's expected loss, as given by (60), is similarly independent of its individual indexation choice x_j in the special case in which both $c_s = 0$ and $x = 1$. (Indeed, $E\Omega_j^u\big|_{c_s=0,x=1}$ is not defined.) This implies that the x_{NE} results, (62a), (62b), (63a) and (63b), have also been derived while tacitly assuming that $c_s > 0$.

consequence of this is that when unions are non-atomistic, the strength of the externality, and the resulting variability of both employment and the real wage in equilibrium, depends upon society's relative-weight parameter c_s. This point will be readily apparent to the reader who peruses expressions (55a) and (55b) while mentally substituting x_{NE} (whether given by (63a) or by (63b)) for x. This dependence of real equilibrium outcomes when $1 < n < \infty$ on the relative weight placed by society on price-level stabilisation ought not to surprise us in the least, since it is entirely consistent with our findings in earlier chapters that the nature of the monetary regime can influence the real wage and employment when strategic considerations underpin union wage-setting decisions. While this obviously raises the issue of optimal delegation arrangements (i.e. the extent to which the central bank's weight parameter should differ from that of society), this theme will not be pursued here, not least because it will involve traversing a good deal of ground already covered in earlier chapters.

We bring this discussion of the indexation externality under the discretionary regime to a close by commenting on its possible diagrammatic representations. Were we to depict its effects on the labour market which is monopolised on the supply side by union j, the resulting diagram would, in the case of a negative productivity shock, very closely resemble Figure V.3, which, it will be recalled, illustrates the effect of the externality under the simple rule in a particular period when the productivity shock happens to be negative. (Clearly diagrams for the two scenarios would also closely resemble each other in the case of a positive productivity shock.) The only difference would relate to the aggregate-level (or symmetric-indexation) labour demand curve, which, as mentioned earlier, under discretion does not extend above the horizontal axis or beyond the vertical when the realisation of θ is negative. Note that since each atomistic union completely disregards the consequences of its x_j choice for the behaviour of the money supply, the externality's strength under discretion must in the atomistic case be the same as under the simple rule when velocity shocks are absent. (This is easily verified by substituting x_{NE}, whether given by (63a) or (63b), into (55a): the resulting real wage is found to be identical to expression (44).) Consequently, in the equivalent of Figure V.3 for discretion,

Figure V.6

The aggregate-level and symmetric-indexation locus of possible outcomes under discretion



Notes:

1. Each of the depicted linear isoloss curves has slope equal to $-c_u$.
2. The outcome located at the aggregate-level locus' horizontal intercept is attained in the limit as $x \to \pm\infty$.

the perceived labour demand curves for atomistic union j would be identical to (i.e. have the same slope and intercepts with the axes as) those depicted in Figure V.3. (Note that under discretion, whereas the aggregate-level labour demand curve is confined to a single quadrant of the diagram, the atomistic union's perceived labour demand curve crosses both axes.)

Finally, we provide as Figure V.6 a simplified version of the discretionary regime counterpart to Figure V.4. The complete neutralisation of velocity shocks by monetary policy, and the consequent possibility of achieving perfect employment stability, implies that, unlike its simple-rule counterpart, the locus of aggregate outcomes under discretion meets the horizontal axis. Under both regimes, the vertical intercept of the locus involves an employment variability outcome of $\sigma_\theta^2/(1-\alpha)^2$, which is brought about by symmetric indexation of unity, i.e. $x_j = 1 \forall j$. Under discretion, however, the locus lacks positively sloped portions, and apart from its vertical intercept, each point on the locus is associated with two values of x, specifically with $x = 1 \pm r$, where r is a real number. The horizontal axis is attained in the limit as $x \to \pm\infty$. The efficient outcome is located at the tangency point with the isoloss map: note that this point in Figure V.6 must be closer to the origin than the corresponding tangency point in Figure V.4 on account of the entire elimination of velocity shocks. In both diagrams, the equilibrium outcome is situated further up the locus than the efficient outcome. (We refrain from superimposing on Figure V.6 the loci of possible individual outcomes faced by an atomistic union given efficient or equilibrium aggregate indexation, since these would resemble their Figure V.4 counterparts in all material particulars.)

V.4.4 Socially Optimal Indexation

The socially optimal degree of wage indexation under discretion remains to be investigated. Since society is now assumed to possess two instruments with which to minimise its expected loss, the socially optimal monetary reaction to shocks, and the accompanying socially optimal degree of indexation are implicitly the solutions to the two simultaneous first-order conditions $\partial E\Omega^s/\partial m = \partial E\Omega^s/\partial x = 0$, where the expectations operator E is unconditional. An equivalent approach which yields exactly the same answer is to assume that society delegates the conduct of monetary policy to a representative central banker, and then derives its optimal degree of indexation in the knowledge that m, Ep^2 and El^2 will respectively be given by equations (48), (50) and (56b). The latter approach is obviously convenient here, and combining (50) and (56b) with the unconditional expectation of (9), the expected social loss, given optimal monetary policy, is found to be the following function of x:

$$E\Omega^s = \frac{c_s \sigma_\theta^2}{[(1-x)^2 + c_s(1-\alpha)^2]} \tag{68}$$

Setting the derivative of $E\Omega^s$ with respect to x equal to zero, and solving for x, reveals that $x=1$ maximises the social loss. With full indexation, monetary policy is powerless to influence real variables, and therefore is optimally devoted to stabilising the price level completely. Thus the variance of inflation is minimised at zero when $x=1$. Productivity shocks have their strongest possible impact on employment under full indexation, however, and in this situation the variance of employment is therefore maximised. It turns out that this also ensures the social loss is maximised, despite the zero inflation variance.

It is easily shown that $\partial E\Omega^s/\partial x > 0$ if $x<1$, and $\partial E\Omega^s/\partial x < 0$ if $x>1$. Hence under discretion the closer to unity is the aggregate degree of indexation, the greater is the social loss. Since both the symmetric Nash equilibria are closer to $x=1$ than their efficient-solution counterparts, it follows that the indexation choice of the individual union has an adverse externality with regard to social welfare, in addition to its adverse externality as regards the welfare of unions themselves. Although equation (68) indicates that $\lim_{x\to\pm\infty} E\Omega^s = 0$, and therefore suggests that the socially optimal degree of indexation under discretion is infinitely large or infinitely negative, it would be unwise to use this finding as a basis for policy recommendations for real-world economies. The reason for sounding this cautionary note is that there is, as yet, no known instance of a real economy in which the aggregate degree of indexation has taken a value outside the unit interval.

Consequently, there is no means of knowing how well models in the Gray-Ball lineage faithfully reproduce the behaviour of the price level, real wage and employment which would arise in the real world were x to take a value far below zero or greatly in excess of unity. In view of this substantial empirical uncertainty, it seems best to confine our attention here to the socially optimal degree of indexation implied by the model, given the fact that the feasible set of values for x is generally restricted by social convention to be the unit interval. In the presence of such a restriction, and in the light of the result reported above that $\partial E\Omega^s / \partial x > 0$ provided $x < 1$, it is clear that the socially optimal value for x is zero.

V.5 Conclusion

This chapter has adapted our basic framework of monopolistically competitive firms and unions which are averse to both real wage variability and employment variability by allowing for endogenous wage indexation. An externality in the individual union's indexation choice has been identified, and has been shown to have close affinities to the externality in the individual union's choice of nominal wage (conditional on a signal received regarding a productivity shock) which formed the principal focus of investigation earlier in the thesis. In particular, as in the case of the wage-setting externality of Chapter III, the indexation externality has been found to be weaker, the stronger is competition in the goods market and the smaller is the number of unions. As in Chapter III, the externality is absent if the goods market is perfectly competitive, or if there is a single economy-wide union, or if each individual union does not regard departures of its real wage or employment from their (unconditional) mean values as inherently undesirable. The indexation externality has an adverse effect on union welfare, and is manifested in an inefficiently low degree of real wage variability in equilibrium, together with an inefficiently high degree of associated employment variability. The externality has similarly been shown to be adverse in its impact on social welfare, as represented by a standard social loss function featuring a term in the variance of inflation. However, in assessing the social welfare repercussions of endogenous indexation, the analysis of this chapter has abstracted from the existence of a mean inflation bias, and therefore has not taken into account the finding of Waller and VanHoose (1992) that atomistic agents' individual indexation decisions are also characterised by a positive externality which operates to reduce the mean inflation bias. The direction of the net externality regarding social welfare appears to be ambiguous: the negative indexation externality which exacerbates price level/inflation variability will at least partially offset the beneficial externality which reduces the bias, and indeed may well outweigh it, so that the net externality is negative, with the equilibrium degree of indexation being suboptimally high, rather than suboptimally low as suggested by Waller and VanHoose.

The next chapter further investigates indexation externalities under conditions of monopolistic goods-market competition by extending the analysis of this chapter to multiple-parameter indexation schemes, according to which the nominal wage is indexed to another aggregate variable in addition to the price level.

Chapter VI: Monopoly Unions, Monopolistically Competitive Firms and Multiparameter Wage Indexation

VI.1 Introduction

This final major chapter continues the investigation of indexation externalities begun in Chapter V by extending the analysis to multiparameter indexation schemes. In such a scheme, the nominal wage is indexed to a second macroeconomic variable in addition to the price level. As pointed out in Chapter IV, several previous contributions to the stabilisation-policy literature have argued that multiparameter indexation, by exploiting potential sources of information which are neglected by conventional schemes which index the wage only to the price level, can lead to superior macroeconomic outcomes. In this context, the optimal outcomes are generally assumed to be those associated with labour-market clearing. This assumption is to be found in Gray (1976), Ball (1988) and the related literature, and the principal papers on multiparameter indexation have similarly assumed that the primary purpose of wage indexation is to bring the real wage and employment more closely into alignment with their market-clearing values.[1] As previously pointed out in this thesis, this assumption amounts to a view that the parties who are responsible for determining the nominal wage (whether directly via wage-setting, or indirectly via indexation arrangements), are indifferent to real wage variability per se. The implications of relaxing this assumption in a model of endogenous multiparameter indexation are the key theme of this chapter. Consistent with the other parts of this work which feature stochastic shocks, we investigate below the consequences of alternatively assuming that real wage variability and employment variability are both regarded as inherently undesirable by the monopoly unions which set the wage-contract parameters governing the determination of the nominal wage.

The remainder of the chapter is organised as follows. Section 2 sets out the structural equations of the chapter's basic model. Section 3, the longest section of the chapter, is largely concerned with the simple-rule scenario. It derives and explains in detail the efficient multiparameter scheme in which the wage is directly indexed to the productivity shock as well as to the price level, and follows this with an analysis of the externalities which arise when the indexation parameters are endogenously

[1] Drudi and Giordano (2000) is an exception in this respect.

determined. The analysis is then briefly repeated for an alternative regime-scenario in which the central bank sets the money supply after observing the realised values of both shocks. (For consistency with previous chapters, this regime is referred to as 'discretion'.) Sections 4 and 5 present counterpart results for two alternative multiparameter indexation schemes, namely indexation of wages to both aggregate output and the price level, as advocated by Karni (1983), and another variant in which the real variable to which wages are indexed is employment.

VI.2 The Model

The structural equations are virtually identical to those of the previous chapter, and for convenience are set out below once again.

$$y_i^S = \alpha l_i + \theta, \qquad 0 < \alpha < 1 \qquad (1)$$

where $\theta \sim N(0, \sigma_\theta^2)$.

$$y^S = \int_0^1 y_i^S di \qquad (2)$$

$$y_i^D - y^D = -\varepsilon(p_i - p), \qquad \varepsilon > 1 \qquad (3)$$

where $p = \int_{i=0}^{1} p_i di$.

$$y^D = \gamma(m - p + \phi), \qquad \gamma > 0 \qquad (4)$$

where $\phi \sim N(0, \sigma_\phi^2)$.

$$l_j^S = 0 \qquad (5)$$

$$l_j^D = \int_{(j-1)/n}^{j/n} l_i^D di \Bigg/ \int_{(j-1)/n}^{j/n} di \qquad (6)$$

$$w_i = \overline{w}_i + x_i(p - Ep) + b_i\theta \quad (7a)$$

$$w_j = \overline{w}_j + x_j(p - Ep) + b_j\theta \quad (7b)$$

$$w = \overline{w} + x(p - Ep) + b\theta \quad (7c)$$

where $w = \int_0^1 w_i \, di = \frac{1}{n}\sum_{j=1}^{n} w_j$, $\overline{w} = \int_0^1 \overline{w}_i \, di = \frac{1}{n}\sum_{j=1}^{n} \overline{w}_j$, $x = \int_0^1 x_i \, di = \frac{1}{n}\sum_{j=1}^{n} x_j$, and $b = \int_0^1 b_i \, di = \frac{1}{n}\sum_{j=1}^{n} b_j$.

$$w_i = \overline{w}_i + x_i(p - Ep) + b_i'(y - Ey) \quad (8a)$$

$$w_j = \overline{w}_j + x_j(p - Ep) + b_j'(y - Ey) \quad (8b)$$

$$w = \overline{w} + x(p - Ep) + b'(y - Ey) \quad (8c)$$

where w, $\overline{w}$ and x are as previously defined, and $b' = \int_0^1 b_i' \, di = \frac{1}{n}\sum_{j=1}^{n} b_j'$.

$$w_i = \overline{w}_i + x_i(p - Ep) + b_i''(l - El) \quad (9a)$$

$$w_j = \overline{w}_j + x_j(p - Ep) + b_j''(l - El) \quad (9b)$$

$$w = \overline{w} + x(p - Ep) + b''(l - El) \quad (9c)$$

where $b'' = \int_0^1 b_i'' di = \frac{1}{n}\sum_{j=1}^{n} b_j''$.

$$\Omega_j^u = l_j^2 + c_u(w_j - p)^2 \quad (10)$$

$$\Omega^s = l^2 + c_s p^2 \quad (11)$$

where $l = \int_0^1 l_i \, di = \frac{1}{n}\sum_{j=1}^{n} l_j$.

Apart from the nominal wage equations, the above are identical to the structural equations of Chapter V, and therefore need not be commented upon. For each set of wage equations, (7a, 7b, 7c), (8a, 8b, 8c) and (9a, 9b, 9c), three interrelated equations specify the nominal wage at firm i and union j, as well as their aggregate counterpart, for the particular multiparameter indexation scheme in question. The equation trio (7a, 7b, 7c) assumes economy-wide direct indexing of the wage to the productivity shock as well as to the price level, and is intended to capture in a simple way the widespread practice of making wages and other forms of contractual remuneration partly contingent on productivity-related outcomes (by means, for instance, of profit-related bonuses). Equation trio (8a, 8b, 8c) is the equivalent set of equations for the Karni indexation scenario in which wages are indexed to aggregate output as well as to the price level.[2] (Note that Karni confined his attention to a situation in which the parameters b_i', b_j' and b' are decided by a benevolent authority concerned to maximise social welfare, and consequently did not endogenise these parameters.) The final trio, (9a, 9b, 9c), analogously considers multiparameter indexation to both aggregate employment and the price level.

The assumed information structure and timing of moves are the same as in the single-parameter indexation model of Chapter V. The reader who wishes to refresh his or her memory of these aspects of the model is therefore advised to consult at this point Time Lines 1 and 2 of Chapter V.

VI.3 Multiparameter Wage Indexation with Direct Indexation to the Productivity Shock

VI.3.1 The Simple-Rule Monetary Regime

VI.3.1(i) Introductory Remarks and Reduced Forms

This section begins the analysis of the macroeconomics of multiparameter indexation by assuming that every union's wage (and hence every firm's wage also) is indexed both to the price level and directly to the productivity shock. The relevant

[2] The reader may be prompted to ask why the analysis does not postulate an indexation scheme in which firm i's wage is indexed to its individual output or employment, and union j's wage is similarly indexed to its employer firms' output or employment, rather than to their aggregate counterparts as assumed here. Unfortunately, for the version of the model in which unions are not constrained to index their wages symmetrically to aggregate variables, formal analysis cannot progress very far with these plausible alternatives since for the specifications in question it is not possible to derive a closed-form expression for the price level.

nominal wage equations are therefore (7a), (7b) and (7c). Our first task is to derive a semi-reduced form for the price-level, expressing that entity as a function of m, $\overline{w}$, x, b, and the realisations of the shocks. This will be a multi-purpose expression in the sense that it will also be relevant to the analysis of discretionary monetary policy in Section VI.3.2. Since the individual firm's labour demand for given m, p and nominal wage w_i, must be the same as in Chapters III and V, we once again have:

$$l_i^D = \frac{\gamma(m-p+\phi)-\varepsilon(w_i-p)+(\varepsilon-1)\theta}{[\alpha+\varepsilon(1-\alpha)]} \tag{12}$$

Since desired labour supply, (5), and the individual union's loss function, (10), are here the same as in Chapters III and V, the base nominal wage of both the individual firm and individual union is again given by:

$$\overline{w}_i = \overline{w}_j = \left(\frac{1}{\varepsilon}\right)[\gamma Em+(\varepsilon-\gamma)Ep] \tag{13}$$

Appropriate substitutions involving (1), (7a), (12) and (13) then yield firm i's output as a function of the indexation parameters specified in its wage contract, as well as of the shocks and of m, p and their expected values at the contract-signing stage:

$$y_i^S = \frac{\gamma\alpha[m-Em-(p-Ep)+\phi]+\varepsilon\alpha(1-x_i)(p-Ep)+\varepsilon(1-\alpha b_i)\theta}{[\alpha+\varepsilon(1-\alpha)]} \tag{14}$$

The aggregate counterpart to (14) is obtained by integrating individual firm outputs over the unit interval:

$$y^S = \frac{\gamma\alpha[m-Em-(p-Ep)+\phi]+\varepsilon\alpha(1-x)(p-Ep)+\varepsilon(1-\alpha b)\theta}{[\alpha+\varepsilon(1-\alpha)]} \tag{15}$$

Note that (15) implies that mean aggregate output is zero: $Ey = Ey^S = 0$. Equating

(15) with (4) and solving for p then yields the semi-reduced form for the price level:[3]

$$p = \frac{\gamma(1-\alpha)(m+\phi)+\alpha(1-x)Ep-(1-\alpha b)\theta}{[\gamma(1-\alpha)+\alpha(1-x)]} \tag{16}$$

In the next section we allow m to be determined at the discretion of the central bank after it has observed the values of the shocks. The current section's concern, however, is with the simple-rule regime in which m is kept fixed regardless of the values taken by other variables. As in previous parts of the thesis concerned with this monetary scenario, m can be innocuously assigned any suitable value. As in previous chapters, the most convenient normalisation is $m=0$, which is therefore adopted for the remainder of this section. With $m=Em=0$, equation (16) immediately implies that $Ep=0$, and consequently equations (13) and (16) simplify to the following:

$$\overline{w}_i = \overline{w}_j = 0 \tag{13'}$$

$$p = \frac{\gamma(1-\alpha)\phi-(1-\alpha b)\theta}{[\gamma(1-\alpha)+\alpha(1-x)]} \tag{16'}$$

The variance of the price level as a function of the two aggregate indexation parameters is:

$$Ep^2 = \frac{\gamma^2(1-\alpha)^2\sigma_\phi^2+(1-\alpha b)^2\sigma_\theta^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \tag{17}$$

A further consequence of our $m=0$ normalisation is that equations (7a), (7b) and (7c) simplify as follows:

$$w_i = x_i p + b_i\theta \tag{7'a}$$

$$w_j = x_j p + b_j\theta \tag{7'b}$$

$$w = xp + b\theta \tag{7'c}$$

[3] Note that in solving the equation $y^S = y^D$ to derive p as a function of m, $Em = Ep$ is found to be the case.

Appropriately combining (12), (13′) and (16) with (7′b) and (7′c) then yields the following reduced-form expressions for the variances of union j's real wage and employment and their aggregate counterparts:

$$E(w_j - p)^2 = \frac{\gamma^2(1-\alpha)^2(1-x_j)^2\sigma_\phi^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} + \left\{\frac{(1-x_j)(1-\alpha b)}{[\gamma(1-\alpha)+\alpha(1-x)]} + b_j\right\}^2 \sigma_\theta^2 \qquad (18a)$$

$$El_j^2 = \frac{1}{[\alpha+\varepsilon(1-\alpha)]^2[\gamma(1-\alpha)+\alpha(1-x)]^2}\Big[\ \gamma^2[\varepsilon(1-\alpha)(1-x_j)+\alpha(1-x)]^2\sigma_\phi^2$$

$$+\{[\gamma-\varepsilon(1-x_j)](1-\alpha b)+[\gamma(1-\alpha)+\alpha(1-x)][\varepsilon(1-b_j)-1]\}^2\sigma_\theta^2\ \Big] \qquad (18b)$$

$$E(w-p)^2 = \frac{\gamma^2(1-\alpha)^2(1-x)^2\sigma_\phi^2 + [1-x+\gamma(1-\alpha)b]^2\sigma_\theta^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \qquad (19a)$$

$$El^2 = \frac{\gamma^2(1-x)^2\sigma_\phi^2 + [\gamma(1-b)-(1-x)]^2\sigma_\theta^2}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \qquad (19b)$$

VI.3.1(ii) Efficient Multiparameter Indexation

The efficient indexation scheme is the (x_j, b_j) pair which minimises the representative union's expected loss when indexation is constrained to be symmetric, i.e. when $x_j = x\forall j$ and $b_j = b\forall j$ must be the case. Hence the efficient pair of indexation parameters, denoted (x^*, b^*), is the (x, b) pair which solves the following pair of simultaneous equations:

$$\frac{dE(\Omega_j^u\big|_{x_j=x, b_j=b})}{dx_j} = 0 \qquad (20a)$$

$$\frac{dE(\Omega_j^u\big|_{x_j=x,b_j=b})}{db_j}=0 \qquad (20b)$$

The relevant expression for $E(\Omega_j^u\big|_{x_j=x,b_j=b})$ is obtained by combining (19a) and (19b) with the unconditional expectation of (10). This expression is not stated here, since it is not of great interest in itself, and for the same reason we also omit the derivatives corresponding to the left-hand sides of the first-order conditions (20a) and (20b). Our major interest lies rather with the following unique efficient solution pair:

$$x^*=1 \qquad (21a)$$

$$b^*=\frac{1}{[1+c_u(1-\alpha)^2]} \qquad (21b)$$

Under this efficient multiparameter indexation scheme, the nominal wage for a particular period is therefore found to be:[4]

$$w\big|_{x=x^*,b=b^*}=\frac{\theta}{[1+c_u(1-\alpha)^2]} \qquad (22)$$

It is instructive to compare (22) with equation (19) of Chapter III, noting in doing so that, since in this section we are abstracting entirely from the existence of informative signals of shocks, the unions' expectation of the price level at the time wage contracts are concluded is zero. It is apparent from the comparison that efficient multiparameter indexation replicates almost exactly the efficient nominal wage when unions possess perfect information regarding both shocks. The efficient nominal wage under fully informed wage-setting only differs from the efficient nominal wage under multiparameter indexation to the extent that union expectations of the price level, at the time that wage contracts are signed, differ across the two scenarios. (In other words, efficiency leads to the same mark-up of the nominal wage over the expected

[4] The left-hand side of (22) could alternatively be written as the individual union's wage given efficient indexation, i.e. as $w_j\big|_{x_j=x^*\forall j,b_j=b^*\forall j}$

price-level in both scenarios.) An immediate implication is that the efficient multiparameter indexation scheme, as given by (21a) and (21b), ensures that the representative union's labour-market outcome is located at the tangency point between the aggregate-level (or symmetric-wage) labour demand curve and the union isoloss map. Hence the efficient multiparameter scheme has the same diagrammatic representation in ($w_j - p$, l_j) space as the efficient single-parameter scheme when velocity shocks are absent. Consequently, Figure V.1 of the previous chapter also depicts, in all essential respects, the outcome under efficient multiparameter indexation, the only differences being in respect of the parameter settings which that diagram indicates to be associated with particular points on the symmetric-indexation labour demand curve. With $x = 1$, a setting of zero for b would place the outcome at the aggregate labour demand curve's vertical intercept. As b increases gradually from zero, the aggregate outcome migrates gradually down the curve, attaining the horizontal intercept when $b = 1$.[5]

There are a few more aspects of (21a) and (21b) which call for comment. Firstly, in common with previous contributions to the macroeconomics of multiparameter wage indexation, the variances of the two shocks do not play a role in determining the equilibrium degree of indexation to either the price level or the productivity shock. This result contrasts with the general finding of the literature on single-parameter indexation that the efficient degree of indexation to the price level is a function of the relative magnitudes of the two shocks' variances. The reason for this difference between the two veins of literature is that with two shocks, two (or more) indexation instruments allow the desirable degree of neutralisation of aggregate demand disturbances[6] to be achieved by appropriate setting of one parameter, allowing the other parameter to achieve the optimal wage response to the productivity shock, regardless of the precise magnitude of that shock's variance. Because our model assumes for simplicity, like Karni (1983), that every firm employs labour under

[5] If b is negative, the outcome for $\theta > 0$ will be located beyond the aggregate-level labour demand curve's vertical intercept, since with $x = 1$, $b < 0$ implies that the real wage falls when a positive productivity shock occurs, thereby exacerbating the shock's employment impact. If $b > 1$ when $x = 1$, the indexation scheme causes employment to fall in response to $\theta > 0$, with the real wage increasing by more than the full amount of the productivity shock. Hence in terms of Figure V.1 for $\theta > 0$, with $x = 1$, settings of b in excess of unity place the outcome on the portion of the aggregate-level curve which lies below the horizontal axis.

[6] Complete neutralisation if every firm has labour contracts, as in Karni (1983), partial neutralisation if some firms employ labour in a spot market, so that cross-sectoral spillovers arise, as in Duca and VanHoose (1998b).

contracts, and omits a second sector with spot-market labour hiring which is central to Duca and VanHoose (1991, 1998b), the efficient degree of indexation to the price level is unity.

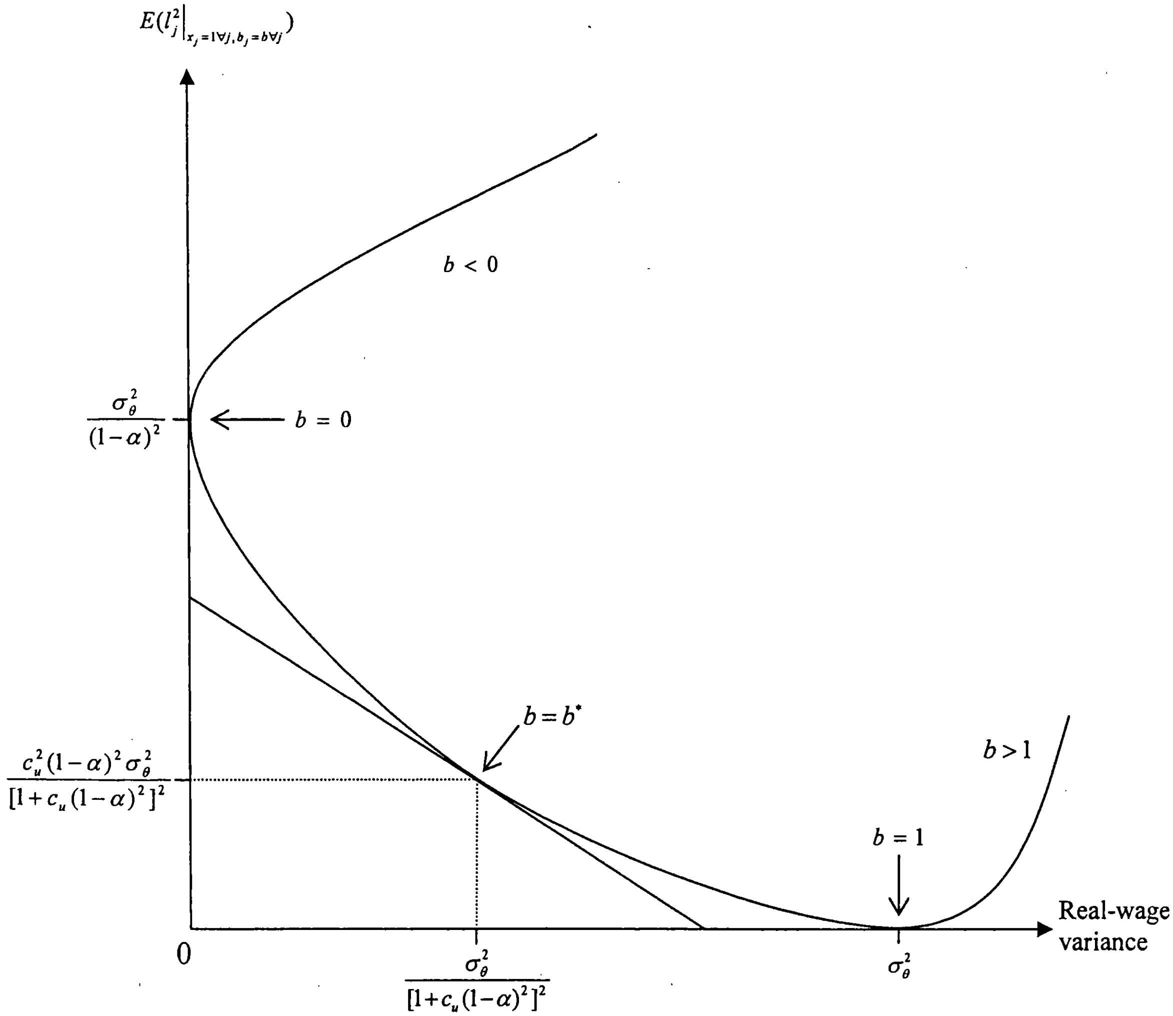
Secondly, provided $c_u > 0$, the efficient degree of indexation to the productivity shock is less than unity. This reflects the fact that under symmetric indexation with $x = 1$, setting b closer to unity makes the real wage more responsive to productivity shocks and hence brings lower employment variability at the expense of greater real wage variability. A higher c_u therefore causes the value of b^* to move closer to zero, while in the extreme limiting case in which $c_u \to \infty$, real wage variability is the sole source of welfare loss to each union, and (21a) and (21b) then prescribe $(x^* = 1, b^* = 0)$, hence ensuring complete rigidity of the real wage. Conversely, if union j's sole concern is employment variability ($c_u = 0$), equation (21b) indicates that efficiency will involve full indexation to the productivity shock, so as to ensure that the real wage and employment are at their market-clearing values (of θ and zero respectively), with complete employment stability being secured as a consequence. This result that in the $c_u = 0$ case (21a) and (21b) become $(x^* = 1, b^*|_{c_u=0} = 1)$ is in fact very closely related to Karni's result that, when the elasticity of labour supply with respect to the real wage is zero, complete stability of employment can be achieved by indexing the wage fully to both the price level and to aggregate output.[7]

It is clear, therefore, that there are numerous close parallels between efficient multiparameter indexation and efficient indexation only to the price level in the absence of velocity shocks. Just as Figure V.1 requires little modification to depict labour-market outcomes under multiparameter indexation, so too is Figure V.2 concerning outcomes in $(E(w_j - p)^2,\ El_j^2)$ space readily adaptable in this respect. Since $x = 1$ prevents velocity shocks from being a source of variability in either the real wage or employment, the locus of possible outcomes for $x = 1$ depicted in Figure VI.1 below touches both axes. As indicated, appropriate settings of b allow either of these graphical intercepts to be attained, while as b increases from zero towards unity

[7] Karni's paper contains a series of algebraic errors, commencing with his equation (10), which results in the equation which is of interest to us, his equation (15), also being slightly incorrect. After making the necessary corrections, Karni's equation (15) prescribes full indexation of the nominal wage to expectational errors regarding both aggregate output and the price level, when the objective is to minimise output variability (or, what amounts to the same thing, minimise employment variability) around its mean value.

Figure VI.1

The locus of possible aggregate outcomes under multiparameter indexation



Notes:

1. Variable on horizontal axis is the individual union's real-wage variance, given symmetric indexation, $E[(w_j - p)^2|_{x_j=1\forall j, b_j=b\forall j}]$.
2. The depicted isoloss curve has slope equal to $-c_u$.

the actual outcome in this space migrates down the negatively sloped portion of the locus. Values of b outside the unit interval place the outcome on one or other of the two positively sloped branches of the opportunity set, with variability of both the real wage and employment tending to the infinitely large as $b \to \pm\infty$. These facts will alternatively be apparent to the reader from the following expressions, obtained by setting x at unity in equations (19a) and (19b):

$$E(w-p)^2\Big|_{x=1} = b^2\sigma_\theta^2 \tag{23a}$$

$$El^2\Big|_{x=1} = \frac{(1-b)^2}{(1-\alpha)^2}\sigma_\theta^2 \tag{23b}$$

The derivatives of these expressions with respect to b will prove useful to the subsequent exposition:

$$\frac{\partial E(w-p)^2\Big|_{x=1}}{\partial b} = 2b\sigma_\theta^2 \tag{24a}$$

$$\frac{\partial El^2\Big|_{x=1}}{\partial b} = \frac{2(b-1)}{(1-\alpha)^2}\sigma_\theta^2 \tag{24b}$$

Equations (24a) and (24b) imply that the derivative with respect to b of the representative union's expected loss, given symmetric indexation on $x=1$, is:

$$\frac{\partial E\Omega_j^u\Big|_{x=1}}{\partial b} = \frac{2\{b[1+c_u(1-\alpha)^2]-1\}\sigma_\theta^2}{(1-\alpha)^2} \tag{24c}$$

It immediately follows from (24c) that:

$$\frac{\partial E\Omega_j^u\Big|_{x=1}}{\partial b} < (>) 0 \quad \text{iff} \quad b < (>) b^* \tag{25}$$

Unsurprisingly, the expected union loss under the efficient multiparameter indexation scheme is the same as under efficient indexation to the price level in the absence of velocity shocks.[8] Hence we have:

$$E\left(\Omega_j^u\Big|_{x_j=1\forall j, b_j=b^*\forall j}\right)=\frac{c_u\sigma_\theta^2}{[1+c_u(1-\alpha)^2]} \tag{26}$$

VI.3.1(iii) Equilibrium Multiparameter Indexation

In accordance with the practice adopted in Chapter V, our investigation of equilibrium indexation begins by deriving solutions for the two extremes of wage-bargaining structure. As we know, the equilibrium solution for the case of a single economy-wide union always coincides with the efficient solution. Using (x_{NE}, b_{NE}) to denote the symmetric Nash equilibrium parameter pair, we consequently conclude that $x_{NE}\big|_{n=1}=1$ and that $b_{NE}\big|_{n=1}=b^*$, as given by (21b). As for the other extreme case, its key aspect is that each atomistic union assumes that the influence of its individual indexation parameter settings on their aggregate counterparts is negligibly small, and hence takes x and b as given in working out its individually optimal indexation strategy. This optimal (x_j, b_j) pair is the solution to the following pair of simultaneous first-order conditions:

$$\left.\frac{\partial E\Omega_j^u}{\partial x_j}\right|_{x\text{ fixed}}=0 \tag{27a}$$

$$\left.\frac{\partial E\Omega_j^u}{\partial b_j}\right|_{b\text{ fixed}}=0 \tag{27b}$$

Combining (18a) and (18b) with (10) and differentiating the resulting expected union loss with respect to x_j and b_j, with x and b treated as given, yields expressions for (27a) and (27b) which can be solved for union *j*'s individually optimal (x_j, b_j) pair.

[8] The relevant equation in Chapter V is numbered (28).

These solutions are not in themselves of much interest, and are consequently omitted for the sake of brevity, as are the related pair of equations which implicitly define the atomistic-case (x_{NE}, b_{NE}) solution pair, and which are obtained by imposing symmetry in indexation scheme choice (i.e. $x_j = x \forall j$, $b_j = b \forall j$) on union *j*'s first-order conditions. The unique symmetric Nash equilibrium solution pair for the atomistic case is found to be:

$$x_{NE,\,atomistic\,case} = 1 \tag{28a}$$

$$b_{NE,\,atomistic\,case} = \frac{\varepsilon}{\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}} \tag{28b}$$

This pair of equations differs from the efficient pair, as given by (21a) and (21b). It follows immediately from this fact that there is an adverse externality to each individual atomistic union's multiparameter indexation decision. In particular, while every such union chooses, as efficiency requires, to index fully to the price level in order to insulate both its real wage and its employment from the impact of velocity shocks, straightforward manipulation reveals that $b_{NE,\,atomistic\,case} < b^*$, indicating that each union in equilibrium chooses to make its nominal wage less responsive to productivity shock realisations than is efficient. Since both b^* and $b_{NE,\,atomistic\,case}$ lie within the unit interval, and since (24a) and (24b) respectively indicate that when $0 < b < 1$, real wage variability is increasing in *b*, and employment variability is decreasing in *b*, it follows that in the atomistic case the externality results in an inefficiently high degree of employment variability, together with inefficiently low variability of the real wage. This finding obviously has strong affinities with some of the key results of previous chapters, and it is not in the least surprising to note that (28b) implies that $\partial b_{NE,\,atomistic\,case} / \partial \varepsilon > 0$, indicating that stronger goods-market competition reduces the departure of $b_{NE,\,atomistic\,case}$ from b^*, and hence operates in a familiar fashion to mitigate the externality. Another point in common with previous results is that it is apparent from (21b) and (28b) that b^* and $b_{NE,\,atomistic\,case}$ coincide if $c_u = 0$ or if $c_u \to \infty$ in the limit.

The equilibrium expected loss of atomistic unions is given by:

$$E(\Omega_j^u\big|_{x=x_{NE,\ atomistic\ case},\ b=b_{NE,\ atomistic\ case}}) = \frac{c_u\{\varepsilon^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\}\sigma_\theta^2}{\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2} \tag{29}$$

Comparison of (29) with equation (46) of Chapter III reveals that for the case of atomistic unions, the equilibrium expected loss from multiparameter indexation is the same as the expected loss which results when each union has perfect information regarding both shocks when setting its wage.[9] An explanation for this finding will be provided shortly, together with a somewhat more detailed discussion of the externality, focusing in particular on its source in the trade-off faced by the individual union in choosing b_j.

Before doing so, however, we derive the symmetric Nash equilibrium for the general case in which n can take any of its admissible values, namely $n \geq 1$. The individual union's optimal choice of indexation parameters, denoted (x_j^{**}, b_j^{**}), is the (x_j, b_j) solution pair to the following pair of simultaneous first-order conditions:

$$\frac{\partial E\Omega_j^u}{\partial x_j} + \left(\frac{1}{n}\right)\left(\frac{\partial E\Omega_j^u}{\partial x}\right) = 0 \tag{30a}$$

$$\frac{\partial E\Omega_j^u}{\partial b_j} + \left(\frac{1}{n}\right)\left(\frac{\partial E\Omega_j^u}{\partial b}\right) = 0 \tag{30b}$$

The second term on the left-hand side of each of these first-order conditions is the indirect effect on union j's loss of its setting of its individual indexation parameters. This indirect effect is the contribution the pair (x_j, b_j) makes to the induced change in the aggregate indexation parameter pair, (x, b), and is smaller the larger the number of unions, since $\partial x/\partial x_j = \partial b/\partial b_j = 1/n$.

[9] This expected loss is obtained by setting $\sigma_u^2 = 0$ and $\beta = 1$ in equation (46) of Chapter III. Note that although velocity shocks were ignored in deriving our results for equilibrium wage-setting in Chapter III, the very same expressions would have been obtained had we assumed instead that unions possess perfect information about the velocity shock when setting wages.

Obtaining an expression for $E\Omega_j^u$ by substituting (18a) and (18b) into the unconditional expectation of (10), and differentiating the result with respect to x_j, b_j, x and b, yields the first-order conditions given by (30a) and (30b), expressed in terms of the model's structural parameters, as well as x_j, b_j, x, and b. In the interests of brevity these equations are omitted, since what is of interest is the resulting (x_j^{**}, b_j^{**}) solution pair when $x = 1$, namely:[10]

$$x_j^{**}\Big|_{x=1} = 1 \tag{31a}$$

$$b_j^{**}\Big|_{x=1} = \frac{[\alpha + n\varepsilon(1-\alpha)][\alpha(1-b)+\varepsilon(1-\alpha)]}{(1-\alpha)\{\varepsilon[\alpha+n\varepsilon(1-\alpha)] + nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]^2\}} \tag{31b}$$

Imposing $b_j^{**} = b \,\forall\, j$ in (31b), the symmetric Nash equilibrium is solved for as:

$$x_{NE} = 1 \tag{32a}$$

$$b_{NE} = \frac{[\alpha + n\varepsilon(1-\alpha)]}{\{\alpha + n\varepsilon(1-\alpha) + nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]\}} \tag{32b}$$

The resulting value of the equilibrium expected union loss is obtained by setting $x = x_{NE} = 1$ and $b = b_{NE}$, as given by (32b), in (19a) and (19b), and substituting the resulting expressions into the unconditional expectation of (10):

$$E(\Omega_j^u\Big|_{x=x_{NE},b=b_{NE}}) = \frac{c_u\{[\alpha+n\varepsilon(1-\alpha)]^2 + n^2c_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]^2\}\sigma_\theta^2}{\{\alpha+n\varepsilon(1-\alpha)+nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]\}^2} \tag{33}$$

Evaluating b_{NE}, as given by (32b) for $n = 1$, and finding its limit as $n \to \infty$, we find that $b_{NE}\big|_{n=1} = b^*$ and $\lim_{n\to\infty} b_{NE} = b_{NE,\,atomistic\,case}$, as given respectively by (21b) and (28b).

[10] It is explained below why attention is confined to solutions which arise when $x = 1$. Note that taking the limit as $n \to \infty$ of $b_j^{**}\big|_{x=1}$, as given by (31b), would provide us with the expression for the atomistic union's individually optimal b_j, given b, which we chose to omit earlier.

Thus (32a) and (32b) are consistent with our earlier results relating to the extreme cases of wage-bargaining structure. Note, however, that whereas in both the single-union and atomistic cases the symmetric Nash equilibrium is known to be unique, the surmised uniqueness of the only known solution pair, (32a) and (32b), for cases such that $1 < n < \infty$, remains unproven.[11]

It is straightforward to show that, in general, b_{NE} is less than b^* for all admissible values of n other than $n = 1$, and when viewed in conjunction with (25) it is apparent from this finding that the adverse externality found earlier to characterise the atomistic case generally arises whenever there is more than one union in the economy. It is noteworthy that the departure of b_{NE} from b^* is greater, and hence that the externality is more severe, the larger is *n*, as is apparent from the following derivative, which is strictly negative provided $\varepsilon < \infty$ and $0 < c_u < \infty$:

$$\frac{\partial b_{NE}}{\partial n} = \frac{-c_u \alpha (1-\alpha)^2 [\alpha + \varepsilon(1-\alpha)]}{\{\alpha + n\varepsilon(1-\alpha) + nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]\}^2} < 0 \tag{34}$$

As in the atomistic case, the externality's strength is generally diminishing in the degree of goods-market competition, as represented by ε:

$$\frac{\partial b_{NE}}{\partial \varepsilon} = \frac{n(n-1)c_u \alpha (1-\alpha)^3}{\{\alpha + n\varepsilon(1-\alpha) + nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]\}^2} \tag{35}$$

It is clear that $\partial b_{NE}/\partial \varepsilon > 0$ for all $n > 1$, provided $0 < c_u < \infty$. The correspondence with our findings for the atomistic case also extends further to the externality's absence in the extreme cases in which $\varepsilon \to \infty$ in the limit, and c_u is either zero or approaches infinity in the limit:

[11] Advanced mathematical software (*Mathematica*) has been used in an attempt to solve the pair of equations which implicitly define all the possible (x_{NE}, b_{NE}) solution pairs. This equation pair is simply union *j*'s first-order conditions, given by (30a) and (30b), with $x_j = x$ and $b_j = b$ imposed. The symmetric Nash equilibrium given by (32a) and (32b) is the only obvious solution pair to these simultaneous equations, and while *Mathematica* does not arrive at a definitive set of solutions to them, despite being allowed numerous days of uninterrupted evaluation time to do so, it is perhaps of greater significance that the program does not find any other solution pairs additional to the pair (32a) and (32b). Our surmise that this pair is a unique symmetric Nash equilibrium for the general $n \geq 1$ case therefore appears to be highly plausible.

$$\lim_{\varepsilon \to \infty} b_{NE} = b^* \tag{36}$$

$$b_{NE}\Big|_{c_u=0} = b^*\Big|_{c_u=0} = 1 \tag{37a}$$

$$\lim_{c_u \to \infty} b_{NE} = \lim_{c_u \to \infty} b^* = 0 \tag{37b}$$

VI.3.1(iv) Discussion of the Source of the Indexation Externality

As in previous chapters, the externality arises because each individual union, in deciding how its nominal wage adjusts in response to the realised value of θ, does not fully internalise the price-level repercussions of its strategy choice. Differentiating (17) with respect to *b*, the variance of the price level is found to be a decreasing function of *b* provided $b < 1/\alpha$, a condition which holds both in equilibrium and under efficient indexation:

$$\frac{\partial E(p^2\big|_{x_j=1\forall j})}{\partial b} = \frac{-2\alpha(1-\alpha b)\sigma_\theta^2}{\gamma^2(1-\alpha)^2} \tag{38}$$

Since we have already established that $b_{NE} < b^*$ provided $n > 1$, and that $\partial b_{NE}/\partial n < 0$, it is clear that the externality leads to increased price-level variability, and that this increase is greater, the larger the number of unions. This exacerbation of price-level variability does not affect the equilibrium real wage or its variance, of course, since the full indexation of each union's wage to the price level insulates the real wage completely from this effect. However, under the simple rule the higher variability of the price level necessarily causes aggregate demand to be more variable in equilibrium than under efficient indexation, and it is through this aggregate-demand channel that the externality's adverse impact on the price-level variance leads to inefficiently high employment variability.

The reason why the externality is weaker, the fewer the number of unions, is straightforward: namely that the smaller is *n*, the greater is the internalisation by union *j* of the effect its choice of b_j has on the price level. This is a familiar theme from previous chapters, and it ought not to surprise us that the externality can be explained in terms of a departure of the individual union's perceived trade-off between its real wage and employment, from the trade-off prevailing at the aggregate level. Although union j is ignorant of the realised values of the shocks at the time it sets its indexation parameters, it knows that setting $x_j = 1$ entirely eliminates the uncertainty regarding real outcomes which arises from the existence of velocity shocks. Hence union j perceives that, given efficient indexation by every other union (so that $b_k = b^* \forall k \neq j$, and, with $x_j = 1$, $x = 1$ also), there exists a set of possible outcomes for its individual labour market, each of which is associated with a particular setting of b_j. The concept which comes to mind in this context, of course, is the perceived labour demand curve which passes through the efficient outcome. Indeed, only minor modifications need be made to Figure V.3 of Chapter V in order for it to illustrate the source of the externality under multiparameter indexation. Since yet another diagram differing form Figure V.3 in only minor respects would clearly be superfluous, we confine ourselves here to verbal remarks regarding the perceived labour demand curves under multiparameter indexation.[12] The key point is the following. Of the infinite set of possible perceived labour demand curves when $x_j = 1 \forall j$, there is only one such curve whose tangency point with the union isoloss map is coincident with its intersection point with the aggregate-level labour demand curve. The curve which is unique in this respect is the perceived labour demand curve which prevails when $b_k = b_{NE} \forall k \neq j$, and this is therefore the only member of the set which can induce union j to follow the other unions in its setting of b_j.[13]

The slope of the perceived labour demand curve is the trade-off faced by union j between on the one hand reducing the impact of a non-zero productivity shock on employment by means of a higher setting of b_j, and on the other directly increasing that shock's real-wage impact, and is given by:

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[12] Note that Figure V.3 of Chapter V has been drawn on the assumption that unions are atomistic. The version of this diagram which would relate to the present discussion concerned with multiparameter indexation when $1 < n < \infty$ would therefore indicate the equilibrium at c as having different real wage and employment coordinates than those stated in Figure V.3. For the atomistic case, however, the equilibrium values for the real wage and employment would be precisely those indicated in Figure V.3.

[13] In the terminology of earlier chapters, this perceived labour demand curve which prevails given $b_k = b_{NE} \forall k \neq j$ (and $x = 1$) is the 'equilibrium perceived labour demand curve'.

$$\left.\frac{dl_j}{d(w_j - p)}\right|_{x_j=1\forall j} = \frac{-[\alpha + n\varepsilon(1-\alpha)]}{n(1-\alpha)[\alpha + \varepsilon(1-\alpha)]} \tag{39}$$

It is clear that this slope is flatter (less negative), and hence that the trade-off is more favourable to union *j*, the larger is *n*. The fewer the number of unions, therefore, the closer does the trade-off faced by the individual union approximate that which exists at the aggregate level.[14] Differentiating the right-hand side of (39) with respect to ε immediately reveals that, provided $n > 1$, this slope is falling in ε. Hence the slope of union *j*'s perceived labour demand curve under multiparameter indexation exhibits the same characteristics as its counterparts in earlier parts of this work.[15] (The reason for the externality's sensitivity to the degree of goods-market competition is the same as in previous chapters, and therefore need not be repeated here.)

VI.3.1(v) Equilibrium Multiparameter Indexation and Full-Information Outcomes

As noted in the Introduction to this chapter, a major theme of the literature on multiparameter indexation is the idea that it may, by exploiting sources of information which are unavailable at the time contract wages are set, enable the outcomes that would result from fully informed wage-setting to be replicated. It is obviously of interest, therefore, to enquire whether the equilibrium multiparameter indexation scheme in the present chapter's model of endogenous indexation brings about the same outcomes as would result were the model's representative agent fully informed about the macroeconomic shocks when choosing its strategy. As a step towards answering this question, we set out at this point the expression for the individual union's expected loss under full information:

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[14] Note that evaluating (39) for $n = 1$ yields, as we would expect, the slope of the aggregate-level labour demand curve: $[dl_j/d(w_j - p)]\big|_{x_j=1\forall j, n=1} = -1/(1-\alpha)$

[15] $\partial[dl_j/d(w_j - p)]\big|_{x_j=1\forall j}\big/\partial\varepsilon = -(n-1)\alpha/n[\alpha + \varepsilon(1-\alpha)]^2$. This is negative for all $n > 1$: hence, provided $n > 1$, a higher degree of goods-market competition steepens union *j*'s perceived labour demand curve and reduces the extent of its departure from the aggregate-level curve. Note in addition that $\lim_{\varepsilon\to\infty}\left([dl_j/d(w_j - p)]\big|_{x_j=1\forall j}\right) = -1/(1-\alpha)$, so that under perfect competition the curve perceived by the individual union coincides with the aggregate curve, a familiar result from previous chapters.

$$E(\Omega_j^u\big|_{w=w_{NE},FI}) = \frac{c_u\{(\gamma\alpha+\varepsilon\Lambda)^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\Lambda^2\}\sigma_\theta^2}{\{\gamma\alpha+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^2} \tag{40}$$

This expression is simply equation (44) of Chapter III, evaluated for the extreme case of perfect information (i.e. for $\sigma_u^2 = 0$, $\beta = 1$), and with subscript '*FI*' adopted as a convenient notation for the full-information scenario.

A comparison of (40) with (33), the equilibrium expected union loss when indexation is to p and θ, elicits several very interesting points. Firstly, whereas we established earlier in this chapter that, in the two extreme cases of a single union and atomistic unions, equilibrium multiparameter indexation results in the same expected union loss as fully informed wage-setting, this is not the case for $1 < n < \infty$. It is straightforward to show that the equilibrium expected loss is unambiguously greater under fully informed wage-setting than under multiparameter indexation, provided $1 < n < \infty$.[16]

The second point of interest provides a clue towards an explanation for this finding. Whereas it is apparent from (33) that the equilibrium expected loss under multiparameter indexation is completely independent of the aggregate demand elasticity parameter, γ, for all $n \geq 1$, perusal of (26), (29) and (40) reveals that the full-information equilibrium expected loss is independent of γ only in the two extreme cases of atomistic unions and a single economy-wide union. In the multiparameter indexation scenario, full indexation to the price level completely neutralises aggregate output from aggregate demand disturbances and ensures that the price level always adjusts by just enough to equate aggregate demand and aggregate supply. In other words, with $x = 1$, output is determined purely by the supply-side parameters α and b, and by the productivity shock realisation, θ. Under fully

[16] The following is a proof. Equating (33) and (40) and solving for *n*, the only admissible finite solution value (i.e. the only $n \geq 1$) to the equation $E(\Omega_j^u\,|_{x=x_{NE},\,b=b_{NE}}) = E(\Omega_j^u\,|_{w=w_{NE},FI})$, is found to be $n = 1$. (There is another solution to this equation which is easily shown to be necessarily less than unity, and which therefore is ignored as inadmissible.) Since it has already been established that the equilibrium expected loss, as a function of *n*, is minimised in both scenarios when $n = 1$, we examine the second derivatives with respect to *n* of these loss expressions evaluated at $n = 1$. Since

$\partial^2 E(\Omega_j^u\big|_{w=w_{NE},FI})\big/\partial n^2\Big|_{n=1} = 2c_u^2\alpha^2[\alpha+\gamma(1-\alpha)]^2\sigma_\theta^2/\gamma^2[\alpha+\varepsilon(1-\alpha)]^2[1+c_u(1-\alpha)^2]^3 >$

$\partial^2 E(\Omega_j^u\big|_{x=x_{NE},b=b_{NE}})\big/\partial n^2\Big|_{n=1} = 2c_u^2\alpha^2(1-\alpha)^2\sigma_\theta^2/[\alpha+\varepsilon(1-\alpha)]^2[1+c_u(1-\alpha)^2]^3$, it follows that

$E(\Omega^u_{w=w_{NE},FI}) > E(\Omega^u_{x=x_{NE},b=b_{NE}})$ for all finite $n > 1$.

informed wage-setting, by contrast, there are only two cases in which equilibrium output is entirely determined by the supply-side of the economy. The first such case is that of atomistic unions. Because the impact of an individual union's wage on the price level, and hence on real money balances, is negligibly small, γ is completely disregarded by the individual union when choosing its wage, and thus plays no role in determining the response of output to the known value of the productivity shock. The price level then adjusts endogenously to bring aggregate demand into equality with aggregate supply. The second case in which supply-side factors solely matter for equilibrium output is that of a single monopoly union. With full information on both shocks, the union will take full account of the response of the price level to its setting of the aggregate nominal wage, and hence will be able to bring about its preferred combination of real wage and employment, given the known value of the productivity shock. Output in the $n=1$ case is consequently determined purely by supply-side factors, and, as in the atomistic case, an adjustment in the price level occurs to ensure that aggregate demand and aggregate supply are equal. For intermediate values of n between these two extreme cases, i.e. for $1<n<\infty$, each union recognises that its nominal wage decision will have a non-negligible impact on the price level and hence on aggregate demand via real money balances. This leads the union's nominal wage to be functionally dependent on γ. However, if $n>1$ the union does not fully internalise the price-level implications of its wage decision, and finds it optimal to deviate from the collectively efficient nominal wage, with the result that the real wage, employment, and hence output are not independent of the aggregate demand parameter γ.

It is straightforward to show that a higher value for γ, i.e. greater sensitivity of aggregate demand to the price level, induces the individual union to take greater account of the price level in setting its wage. The externality is thus less severe, and the equilibrium expected loss under full information smaller, the larger is γ, as can be seen from the following derivative, which, provided $0<c_u<\infty$, is unambiguously negative for all $n>1$:

$$\frac{\partial E(\Omega_j^u\big|_{w=w_{NE},FI})}{\partial\gamma}=\frac{-2c_u^2(n-1)^2\alpha^3[\alpha+\varepsilon(1-\alpha)][\alpha+\gamma(1-\alpha)]\sigma_\theta^2}{\{\alpha\gamma+\varepsilon\Lambda+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}^3} \tag{41}$$

It is also of interest that in the limiting case in which $\gamma\to 0$, i.e. the case in which aggregate demand tends towards a complete lack of responsiveness to movements in the price level, unions' wage-setting behaviour, regardless of *n*, tends towards the behaviour of atomistic unions, with the externality consequently as severe as it possibly can be: $\lim_{\gamma\to 0}E(\Omega_j^u\big|_{w=w_{NE},FI})=\lim_{n\to\infty}E(\Omega_j^u\big|_{w=w_{NE},FI})$, as given by the right-hand side of (29). In the converse extreme case in which $\gamma\to\infty$, so that aggregate demand becomes extremely responsive to price level movements, the sensitivity of the price level to the individual non-atomistic union's setting of its wage approximates very closely its sensitivity to the setting of b_j (when $x_j=1\forall j$) in the indexation scenario, and indeed we find that $\lim_{\gamma\to\infty}E(\Omega_j^u\big|_{w=w_{NE},FI})=E(\Omega_j^u\big|_{x=x_{NE},b=b_{NE}})$, as given by (33).[17]

In both scenarios, union *j* in setting its instrument takes into account two channels through which its setting of that instrument can affect its individual welfare. The first of these is the real wage channel. For a given value of the productivity shock, a particular setting of the wage or of b_j contributes to determine union *j*'s real wage directly in both scenarios. Under full information, however, the chosen wage also does so via the induced change in the price level, whereas under multiparameter indexation $x_j=1$ ensures this indirect effect is absent. Under fully informed wage-setting, therefore, the real wage channel matters for union welfare in two ways: directly, because of the term in the real wage in the representative union's loss function, (10), and indirectly via the real wage's role in determining employment. Under multiparameter indexation only the former of these two is of substance. The

[17] Since $E(\Omega_j^u\big|_{x=x_{NE},b=b_{NE}})$ is independent of γ, and $E(\Omega_j^u\big|_{w=w_{NE},FI})$ is a continuous function of γ, our findings that $\partial E(\Omega_j^u\big|_{w=w_{NE},FI})\big/\partial\gamma<0\;\forall\, n>1$,
and that $\lim_{\gamma\to 0}E(\Omega_j^u\big|_{w=w_{NE},FI})=\lim_{n\to\infty}E(\Omega_j^u\big|_{w=w_{NE},FI})=\lim_{n\to\infty}E(\Omega_j^u\big|_{x=x_{NE},b=b_{NE}})$,
and that $\lim_{\gamma\to\infty}E(\Omega_j^u\big|_{w=w_{NE},FI})=E(\Omega_j^u\big|_{x=x_{NE},b=b_{NE}})$, together with $\partial E(\Omega_j^u\big|_{x=x_{NE},b=b_{NE}})\big/\partial n>0\;\forall\, n>1$,
constitute an alternative proof that the equilibrium expected union loss under fully informed wage-setting is unambiguously higher than the equilibrium expected union loss under indexation to *p* and θ, provided $0<c_u<\infty$ and $1<n<\infty$.

second channel involves aggregate demand, which affects union j's welfare by being one of the factors which determine employment. This aggregate demand channel exists in both scenarios and operates via the contribution of union j's instrument, whether the wage itself or b_j, to price level determination, and hence to the determination also of real money balances and aggregate demand.

The conclusion which emerges from the above analysis is that when the underlying economic structure gives rise to a negative externality in the individual union's setting of its instrument, the impact of this externality on unions' equilibrium expected loss via the aggregate demand channel is less severe in the multiparameter indexation scenario than under fully informed wage-setting. This is because full indexation of wages to the price level maximises the responsiveness of aggregate demand to price-level movements, which in turn induces each non-atomistic union to exercise greater caution in setting its instrument, b_j, than in the full-information scenario. While the two scenarios also differ in how the externality works via the real wage channel, it turns out that the difference in the aggregate demand channel suffices to ensure that the externality in wage-setting under full information is always worse than the externality in indexation-parameter choice, provided of course that $0 < c_u < \infty$ and that unions are non-atomistic.

VI.3.1(vi) Socially Optimal Multiparameter Indexation

We end this section by briefly considering the socially optimal multiparameter indexation scheme. Substituting (17) and (19b) into the unconditional expectation of (11) yields the expected social loss in terms of x and b. Differentiating this expression with respect to x and b in order to obtain a pair of simultaneous first-order conditions, and solving the latter, the unique socially optimal setting of the aggregate indexation parameter pair is found to be:

$$\hat{x} = 1 - \frac{c_s \alpha (1-\alpha)}{\gamma} \tag{42a}$$

$$\hat{b} = 1 \tag{42b}$$

In order to gain some intuition for this result, note from (19b) that complete stabilisation of employment can by achieved by setting both indexation parameters at unity, i.e. $x = b = 1$. Equations (42a) and (42b) indicate that when price-level variability is immaterial to the social loss (i.e. when $c_s = 0$), $x = b = 1$ would indeed be the socially optimal indexation scheme. With $c_s > 0$, however, full indexation to both the price level and the productivity shock leads to suboptimally high price-level variability. By decreasing x marginally from unity whilst maintaining $b = 1$, a reduction in price-level variability can be obtained which, in its impact on social welfare, outweighs the undesirable increase in employment variability thereby incurred.[18] This trade-off is optimally exploited when $x = \hat{x}$, as given by (42a).

Since the solution pair (42a), (42b) differs from the equilibrium pair given by (32a) and (32b), it is apparent that unions' equilibrium indexation decisions also have a negative externality with regard to social welfare. Although it is ambiguous whether or not equilibrium multiparameter indexation leads to higher employment variability than the socially optimal scheme, it is straightforward to show that if society attaches a positive weight to the variability of the price level, so that $c_s > 0$, the equilibrium is necessarily characterised by suboptimally high price-level variability. This point can be quickly demonstrated as follows. The variance of the price level under the socially optimal scheme is:

$$Ep^2\Big|_{x=\hat{x},b=\hat{b}} = \frac{\gamma^2(\gamma^2\sigma_\phi^2 + \sigma_\theta^2)}{(\gamma^2 + c_s\alpha^2)^2} \tag{43a}$$

Considered as a function of c_s, this expression will be at a maximum in the limiting case in which $c_s \to 0$:

$$\lim_{c_s \to 0} Ep^2\Big|_{x=\hat{x},b=\hat{b}} = \sigma_\phi^2 + \frac{\sigma_\theta^2}{\gamma^2} \tag{43b}$$

[18] It is apparent from (17) that $\partial Ep^2/\partial x > 0$ if (and only if) $x < 1 + [\gamma(1-\alpha)/\alpha]$. This result is essentially the same as that mentioned in Footnote 9 of Chapter V, a fact which is not surprising since it is clear from (17) that the value of b cannot affect the sign of the derivative $\partial Ep^2/\partial x$.

The equilibrium scheme features $x=1$, and our point can be made by evaluating the price-level variance, (17), for $x=1$:

$$Ep^2\Big|_{x=1} = \sigma_\phi^2 + \frac{(1-\alpha b)^2}{\gamma^2(1-\alpha)^2}\sigma_\theta^2 \tag{43c}$$

It is clear that (43b) and (43c) together imply that $b<1$ is a sufficient (but not a necessary) condition for price-level variability to be suboptimally high from society's viewpoint, and as we have seen, this condition is always satisfied in equilibrium provided $c_u > 0$.[19]

VI.3.2 The Discretionary Monetary Regime

VI.3.2(i) Introductory Remarks and Derivation of the Monetary Reaction Function

The purpose of this section is to show that the key results reported in the previous section for the simple-rule scenario also arise when the central bank has the discretionary power to adjust the money supply in response to the observed values of the shocks and after the realisation of the aggregate nominal wage. Consequently, in this scenario the individual union in deciding upon its optimal strategy must take into account the influence of its indexation parameter choices on the central bank's monetary reaction function, since this entity will clearly matter for the realised values of both the price level and aggregate demand, and hence for the real wage and employment.

Our first step, therefore, must be to derive the central bank's optimal setting of the money supply for given values of *x*, *b* and the two shocks. Relevant to this end is equation (16), which expresses the price level as a function of *m*, and which implies that $Ep = Em$. Substituting (16) for *p* and (7a) for w_i in firm *i*'s labour-demand

[19] Note that if $c_u = 0$, so that $x_{NE} = b_{NE} = 1$, equilibrium price-level variability will be equal to the socially optimal value only in the limit as $c_s \to 0$, and will otherwise exceed it. (If the reader is nonplussed as to why we have considered the limit as $c_s \to 0$ in this passage rather than simply evaluate price-level variability for $c_s = 0$, note that this is owing to the fact that when $c_s = 0$ society is indifferent to price-level variability, and the concept of a socially optimal price-level variance then obviously has no validity.)

equation, (12), and aggregating across firms yields aggregate employment in terms of m, Em, x, b and the shocks:

$$l = \frac{1}{[\gamma(1-\alpha)+\alpha(1-x)]}\left\{\gamma(1-x)(m+\phi)+[\gamma(1-b)-(1-x)]\theta+\frac{\alpha(1-x)[\varepsilon(1-x)-\gamma]}{[\alpha+\varepsilon(1-\alpha)]}Em\right\} \quad (44)$$

Since the central bank observes all variables (as well as the aggregate indexation parameters x and b) before setting its instrument, the derivation of its optimal setting of m involves the following steps. Squaring our expressions for p and l, as given respectively by (16) and (44), substituting these squares into the social loss function, (11), and differentiating the result with respect to m, allows the central bank's first-order condition $\partial\Omega^s/\partial m = 0$ to be formed. Applying the unconditional expectations operator to this first-order condition establishes that $Em = 0$, and after taking this into account, the solution is found to be:[20]

$$m^* = -\phi + \frac{\{(1-x)[1-x-\gamma(1-b)]+c_s(1-\alpha)(1-\alpha b)]\}}{\gamma[(1-x)^2+c_s(1-\alpha)^2]}\theta \quad (45)$$

Appropriate substitutions involving (6), (7'b), (7'c),[21] (12), (16) and (44) then yield the following expressions for the price level and its variance, as well as the variances of union j's real wage and employment and their aggregate counterparts:

$$p = \frac{-(1-x)(1-b)\theta}{[(1-x)^2+c_s(1-\alpha)^2]} \quad (46)$$

$$Ep^2 = \frac{(1-x)^2(1-b)^2\sigma_\theta^2}{[(1-x)^2+c_s(1-\alpha)^2]^2} \quad (47)$$

[20] Note that both p (equation (16)) and l (equation (44)) are undefined when $x = 1+[\gamma(1-\alpha)/\alpha]$, and consequently m^* is derived here on the assumption that $x \neq 1+[\gamma(1-\alpha)/\alpha]$.

[21] Note that since $Ep = Em = 0$ has been found to be the case under discretion, equation (13) implies that the wage equations (7a), (7b) and (7c) simplify to (7'a), (7'b) and (7'c), just as under the simple rule.

$$E(w_j - p)^2 = \left\{ \frac{(1-x_j)(1-x)(1-b)}{[(1-x)^2 + c_s(1-\alpha)^2]} + b_j \right\}^2 \sigma_\theta^2 \tag{48a}$$

$$El_j^2 = \frac{1}{[\alpha + \varepsilon(1-\alpha)]^2[(1-x)^2 + c_s(1-\alpha)^2]^2} \Big\{ \varepsilon(1-x)[(1-x)(1-b_j) - (1-x_j)(1-b)]$$
$$+ c_s(1-\alpha)[\alpha(1-b) + \varepsilon(1-\alpha)(1-b_j)] \Big\}^2 \sigma_\theta^2 \tag{48b}$$

$$E(w-p)^2 = \left[\frac{(1-x)^2 + c_s(1-\alpha)^2 b}{(1-x)^2 + c_s(1-\alpha)^2} \right]^2 \sigma_\theta^2 \tag{49a}$$

$$El^2 = \frac{c_s^2(1-\alpha)^2(1-b)^2\sigma_\theta^2}{[(1-x)^2 + c_s(1-\alpha)^2]^2} \tag{49b}$$

VI.3.2(ii) Efficient Multiparameter Indexation under Discretion

The efficient indexation scheme satisfies the pair of simultaneous first-order conditions given by equations (20a) and (20b), with the expected union loss under symmetric indexation now given by (10) appropriately combined with (49a) and (49b). It turns out that there is an infinite set of (x^*, b^*) solution pairs to the first-order conditions, and this infinite set can be succinctly represented as every (x,b) pair which satisfies the following equation:

$$x = 1 \pm \left[\frac{c_s\{1 - b[1 + c_u(1-\alpha)^2]\}}{c_u} \right]^{1/2} \tag{50}$$

where $b \leq \frac{1}{[1 + c_u(1-\alpha)^2]}$ and $b \neq \frac{1}{[1 + c_u(1-\alpha)^2]}\left[1 - \frac{c_u\gamma^2(1-\alpha)^2}{c_s\alpha^2}\right]$.

Were unions symmetrically to set any particular admissible value of b which satisfies the restrictions stated in the previous line, the efficient outcome would be brought about by every union symmetrically indexing its wage to the price level in accordance with equation (50).[22] By substituting (50) into (49a) and (49b), and the resulting expressions into the unconditional expectation of (10), it is easily shown that every possible (x^{*},b^{*}) efficient solution pair results in the same value of the expected union loss, and that this value is given by the right-hand side of equation (26) above. This is not in the least surprising, since as we know, when velocity shocks are of no concern, efficient indexation which takes into account the behaviour of the price level, and the authorities' monetary reaction function (if any), can always ensure the productivity shock's impact is distributed between the real wage and employment in the most desirable way from unions' collective viewpoint. It ought not to surprise us either, therefore, that two of the infinite set of efficient solution pairs have been encountered previously in this work. Setting b equal to zero in (50), we find that $(x^{*}=1\pm(c_s/c_u)^{1/2},\, b^{*}=0)$ is an efficient solution pair, and this is precisely the efficient degree of single-parameter indexation which was identified in Chapter V for the discretion scenario. Secondly, setting x equal to unity in (50) and solving for b reveals that the unique efficient multiparameter indexation scheme for the simple-rule scenario is one of the infinite set of efficient schemes under discretion. The reason why there is an infinite set of efficient solutions, of course, is that the complete neutralisation of velocity shocks by the central bank creates a degree of freedom as regards the choice of x and/or b which will bring about the efficient outcome for any particular realisation of the productivity shock. With these shocks the only source of macroeconomic variability, one of the two indexation instruments available to unions is superfluous, so far as achieving the efficient combination of employment and real wage variability is concerned.

VI.3.2(iii) Equilibrium Multiparameter Indexation under Discretion

We begin this section by deriving the symmetric Nash equilibrium when unions are atomistic. The relevant first-order conditions defining union j's optimal (x_j,b_j) choice are therefore given by equations (27a) and (27b) above. Combining

[22] The restriction that the b which unions symmetrically adopt be such as to satisfy the inequality $b\leq[1+c_u(1-\alpha)^2]^{-1}$ arises because non-real values are assumed to be inadmissible for x. The restriction that b not be equal to $[1+c_u(1-\alpha)^2]^{-1}\{1-[c_u\gamma^2(1-\alpha)^2/c_s\alpha^2]\}$ arises because $[\alpha+\gamma(1-\alpha)]/\alpha$ is not an admissible value for x.

expressions (48a) and (48b) with the unconditional expectation of (10), differentiating this expected loss with respect to x_j and b_j, with x and b taken as given, and imposing symmetry (i.e. $x_j = x \forall j$, $b_j = b \forall j$) on the resulting first-order conditions, yields a pair of simultaneous equations which implicitly defines every (x_{NE}, b_{NE}) solution pair for the atomistic case. As under efficient indexation, there is an infinite set of solutions, and each of these symmetric Nash equilibria is a (x,b) pair which satisfies the following equation:

$$x = 1 \pm \left[\frac{c_s(1-\alpha)[\varepsilon - b\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}]}{c_u[\alpha + \varepsilon(1-\alpha)]} \right]^{1/2} \qquad (51)$$

where $b \leq \dfrac{\varepsilon}{\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}}$,

and $b \neq \dfrac{1}{\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}} \left\{ \varepsilon - \dfrac{c_u \gamma^2 (1-\alpha)[\alpha + \varepsilon(1-\alpha)]}{c_s \alpha^2} \right\}$.

Provided the aggregate (i.e. average) degree of indexation of wages to the productivity shock is less than $\varepsilon / \{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}$, therefore, there exists a particular degree of indexation to the price level (specifically, that given by (51)), which, if it is expected to prevail by the individual atomistic union, leads the latter to adopt this expected (x,b) pair as its own indexation scheme.[23] The reason why there is an infinite set of equilibria is ultimately because both the individual union's perceived trade-off between the real wage and employment in respect of marginal adjustments in x_j, and the similar trade-off it faces in respect of marginal adjustments in b_j, are independent of x and b in the atomistic case. This means that regardless of the particular value union j's expectations of x and b happen to have, it has an incentive to set x_j and b_j respectively equal to these expected values, provided x and b are related according to (51). Every such (x,b) pair constitutes a symmetric Nash

[23] The reason why the restriction $b \leq \varepsilon / \{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}$ must hold is that the equilibrium x implied by (51) is otherwise not a real number. Note, however, that one particular b value which satisfies this condition cannot feature as part of the equilibrium multiparameter indexation scheme, since it would require the symmetric degree of indexation to the price level to take the inadmissible value of $[\alpha + \gamma(1-\alpha)]/\alpha$. This particular b value is that stated in the second restriction immediately following equation (51).

equilibrium, and may be denoted $(x_{NE}, b_{NE})_{atomistic\ case}$.[24] It is clear that, provided $0 < c_u < \infty$, equation (51) differs from (50), and hence that, in general, our familiar indexation externality also arises under discretion. (As usual, the externality is absent when $c_u = 0$, and when $c_u \to \infty$ or $\varepsilon \to \infty$ in the limit.)

A further noteworthy aspect of these findings for the atomistic case under discretion is that every possible (x_{NE}, b_{NE}) pair results in the same expected union loss, and that this is given by (29), the expected loss for the equilibrium under the simple rule, which we know to be identical to the expected loss under discretion when atomistic unions simultaneously index their wages only to the price level. Because of the simultaneity of individual unions' (x_j, b_j) choices, an obvious implication of the fact that the expected loss is the same for all possible equilibria, is that there are no strong *a priori* grounds for arguing that any particular one of the infinite set of (x_{NE}, b_{NE}) solution pairs is more likely than others to arise. Notwithstanding this, however, there are two members of the infinite set which might be argued to have focal-point characteristics. The first of these is the solution which involves $b_{NE} = 0$, with x_{NE} consequently identical to expression (62a) or (62b) of Chapter V. This single-parameter solution appears slightly more plausible than others on account of its relative simplicity as an indexation scheme. The second solution pair which may be a focal point is that which features $x_{NE} = 1$, so that b_{NE} takes the value it has under the simple rule, and hence is given by equation (28b) of this chapter. The principal reason for arguing that atomistic unions may focus on this solution is simply that it can be thought of as prompted by a rule-of-thumb that the wage ought always to be indexed fully to the price level to protect real outcomes from aggregate demand disturbances (even when these are known to be neutralised by monetary policy), while the other indexation parameter serves as an allocator of the productivity shock's impact between the real wage and employment. There is a second argument in its favour as a potential focal point, however, namely that when unions are non-atomistic there is a unique equilibrium multiparameter indexation scheme, and this unique solution pair

[24] Since it is generally clear that the atomistic-case solution is the subject of discussion in the ensuing passage, we omit the cumbersome 'atomistic case' subscript below, apart from when it serves a useful purpose.

involves full indexation to the price level (i.e. $x_{NE} = 1$ when $1 < n < \infty$). We now turn to discuss this solution for the $1 < n < \infty$ case.

Appropriately combining (48a) and (48b) once again to obtain $E\Omega_j^u$, and proceeding in an analogous fashion to that described earlier for the atomistic case, save that allowance is now made for the fact that $\partial x/\partial x_j$ and $\partial b/\partial b_j$ are no longer assumed to be zero, but are instead both equal to $1/n$, enables the first-order conditions (30a) and (30b) to be derived. Imposing symmetry ($x_j = x\forall j, b_j = b\forall j$) on these equations, and solving for *x* and *b*, the unique real symmetric Nash equilibrium solution pair under discretion when $1 < n < \infty$ is found to be identical to the only equilibrium identified earlier for the simple-rule scenario, namely that given by equations (32a) and (32b).[25] Taking the limit of (32b) as $n \to 1$ reveals that in this extreme case the unique equilibrium solution pair is a member of the infinite set of efficient solutions given by (50). For $n > 1$, however, the equilibrium solution pair differs, in general, from the efficient solution involving $x = 1$. (The only exceptions are the usual extreme cases in which the externality does not arise, i.e. $c_u = 0$, and the limiting $c_u \to \infty$ and $\varepsilon \to \infty$ cases.) Hence the multiparameter indexation externality found to characterise the atomistic case also arises when $1 < n < \infty$.[26] Furthermore, since for $1 < n < \infty$ the unique equilibrium under discretion is identical to the only known equilibrium under the simple rule, it follows that the equilibrium expected union loss under discretion must be given by (33). It is straightforward to show that this loss is necessarily smaller than the equilibrium expected union loss which results from fully informed wage-setting under discretion. (The latter loss is given by equation (94) of Chapter III, evaluated for $\sigma_u^2 = 0$ and $\beta = 1$.) Clearly, our earlier finding in respect of the simple-rule scenario that the externality is weaker under multiparameter indexation than when wages are set in response to full information about the shocks is also true of discretion. This is ultimately attributable to the trade-off faced by union *j* being more favourable under fully informed wage-setting than

[25] For the sake of completeness, we mention that there are also two other solution pairs to the pair of simultaneous equations which implicitly define (x_{NE}, b_{NE}), but that these are disregarded since they both involve a non-real value for *x*. Specifically, these two solution pairs are given by $(x = 1 \pm (1-\alpha)[-nc_s/(n-1)]^{1/2},\ b = 1)$.

[26] Note that in the limit as $n \to \infty$, (32a) and (32b) reduce to a particular case subsumed by (51), namely that for which $x = 1$.

under multiparameter indexation, regardless of the nature of the monetary regime, so that in general the externality is always stronger under the former when $1 < n < \infty$.

We end this section with some comments on the uniqueness of the symmetric Nash equilibrium when $1 < n < \infty$. The complexity of union j's first-order conditions for its optimal (x_j, b_j) pair makes it very difficult to devise satisfactory intuitive explanations for this uniqueness, and for why the unique equilibrium involves each union indexing fully to the price level. Nevertheless, it is clear that the equilibrium's uniqueness has to do with the fact that each non-atomistic union's strategy choice has a non-negligible influence on the monetary reaction function, and hence also on price-level behaviour. It can be shown that union j's perceived trade-offs between real wage and employment with respect to marginal adjustments in both x_j and b_j are functionally dependent on x and hence on its own choice of x_j.[27] It so happens that because other unions' strategy choices are beyond its influence, union j's best strategy, given its rational expectation of other unions' decisions, is unique, and involves setting x_j at unity. This ensures that its real wage is fully insulated from both monetary surprises and the indexation choices of other unions, and allows b_j to be devoted to the task of determining the impact of productivity shocks on its individual labour-market outcomes. Of course, every other union faces exactly the same incentives, and hence the unique equilibrium under discretion when $1 < n < \infty$ features $x_{NE} = 1$.

VI.4 Multiparameter Indexation of Wages to the Price Level and Output

VI.4(i) Introductory Remarks and Reduced Forms

This relatively brief section demonstrates that qualitatively very similar results to those described in Section VI.3 arise when the specification of the multiparameter indexation scheme is given by equations (8a), (8b) and (8c). For convenience, this kind of multiparameter indexation will be referred to as Karni indexation, since, as in Karni (1983), it involves each firm's wage being indexed to aggregate output as well as to the price level. In the interests of brevity, we confine attention to the simple-rule

[27] If one takes the trouble to derive expressions for these perceived trade-offs, they are found to be independent of b and b_j.

scenario, and omit all details of the derivations on account of their very close similarity to the steps described in previous sections.

Adopting our usual simple-rule normalisation of the fixed money supply at zero, the following semi-reduced forms for the price level, its variance, and the variances of union *j*'s real wage and employment, and of their aggregate counterparts, are found to arise. (Note that $m = 0$ once again leads to $Em = Ep = \overline{w}_i = \overline{w}_j = \overline{w} = Ey = 0$ being the case.)

$$p = \frac{\gamma(1-\alpha+\alpha b')\phi - \theta}{[\gamma(1-\alpha+\alpha b') + \alpha(1-x)]} \tag{52}$$

$$Ep^2 = \frac{\gamma^2(1-\alpha+\alpha b')^2\sigma_\phi^2 + \sigma_\theta^2}{[\gamma(1-\alpha+\alpha b') + \alpha(1-x)]^2} \tag{53}$$

$$E(w_j - p)^2 = \frac{\gamma^2[\alpha b_j'(1-x) - (1-\alpha+\alpha b')(1-x_j)]^2\sigma_\phi^2 + (\gamma b_j' + 1 - x_j)^2\sigma_\theta^2}{[\gamma(1-\alpha+\alpha b') + \alpha(1-x)]^2} \tag{54a}$$

$$El_j^2 = \frac{\gamma^2\varphi_1^2\sigma_\phi^2 + \varphi_2^2\sigma_\theta^2}{[\alpha+\varepsilon(1-\alpha)]^2[\gamma(1-\alpha+\alpha b') + \alpha(1-x)]^2} \tag{54b}$$

where: $\varphi_1 \equiv \alpha(1-\varepsilon b_j')(1-x) + \varepsilon(1-x_j)(1-\alpha+\alpha b')$

$\varphi_2 \equiv \gamma[\varepsilon(1-b_j') - \alpha(\varepsilon-1)(1-b')] - \varepsilon(1-x_j) + \alpha(\varepsilon-1)(1-x)$

$$E(w-p)^2 = \frac{\gamma^2(1-\alpha)^2(1-x)^2\sigma_\phi^2 + (\gamma b' + 1 - x)^2\sigma_\theta^2}{[\gamma(1-\alpha+\alpha b') + \alpha(1-x)]^2} \tag{55a}$$

$$El^2 = \frac{\gamma^2(1-x)^2\sigma_\phi^2 + [\gamma(1-b') - (1-x)]^2\sigma_\theta^2}{[\gamma(1-\alpha+\alpha b') + \alpha(1-x)]^2} \tag{55b}$$

Chapter VII

Conclusion

This thesis has investigated the nature of wage-setting and indexation externalities in an economy featuring potentially non-atomistic monopoly trade unions and monopolistically competitive firms, as well as the implications of such externalities for the optimal design of the monetary regime, including information-disclosure practices in respect of supply shocks. It has been assumed throughout that labour is homogenous, and that each union is the sole representative of the immobile potential workforce of its associated employer firms. The thesis consequently belongs to a particular strand of literature within the broader corpus of research on wage-related macroeconomic externalities, namely that which includes Bratsiotis and Martin (1999), Soskice and Iversen (2000) and Coricelli et al. (2006), and which is distinct from related strands which assume that each union is either inflation-averse and/or represents a particular type of skilled labour (so that the unions are themselves monopolistically competitive). A principal aim of this work has been to provide a thorough analysis of the underlying source of these externalities. However, in addition to doing so, it has also sought to extend their investigation to a stochastic framework in which unions either possess noisy information regarding productivity shocks at the time wages are set, or practise wage indexation in an attempt to allocate the impact of shocks between the real wage and employment in accordance with the pattern which they prefer. The four chapters of the thesis which report research results are therefore divisible into two groups according to whether the chapters in question largely focus on synthesising, and providing new insights into, closely related models which are already established in the literature (Chapter II), or alternatively contribute more substantially to macroeconomic theory by introducing into precursor models, such as those of Karni (1983), Ball (1988) and Herrendorf and Lockwood (1997), various features to be found in other lines of research (Chapters III, V and VI).

The second chapter of the thesis investigated the optimal design of the monetary regime when the basic model common to all the research chapters lacks stochastic shocks. The analysis was conducted in respect of two alternative specifications of the union loss function, which differ according to whether the loss function's real wage term is linear or quadratic. Commencing with the simple-rule scenario in which the money supply is always kept fixed, an investigation was undertaken of the externality which arises under simultaneous wage-setting, when each union would individually benefit were it to receive a real wage and employment combination that differs from the combination that would be brought about by means of efficient coordinated wage-setting by all unions. This externality was explained in terms of a difference between the slope of the labour demand curve which each union perceives itself to face, and the slope of the aggregate-level labour demand curve, with these slopes being identical for a single economy-wide union, or if the goods market is perfectly competitive. The externality was generally found to be stronger, the larger the number of unions, and the less competitive the goods market. The analysis was also undertaken in respect of various monetary rules which prescribe a particular response to the realised value of an aggregate variable which is functionally dependent on the aggregate nominal wage: specifically, rules specified in terms of monetary responses to the price level, nominal income or employment were analysed. Importantly, and unlike previous papers to have considered such rules, no arbitrary restrictions were imposed on the set of values potentially taken by the monetary-response parameter in each of these rules. It was found that a rule which implies a high degree of monetary accommodation of wages can induce non-atomistic unions to set the market-clearing nominal wage. This key result was explained in terms of the effect which the rule's specified monetary response to the aggregate variable has on the slope of the individual non-atomistic union's perceived labour demand curve. A parallel result was found to obtain for the delegation scenario in which a particular objective function is assigned to a discretionary central bank. If this objective function features a particular *negative* relative weight on inflation, the implied functional relationship between the money supply and wages, and the resulting perceived trade-off between real wage and employment faced by each non-atomistic union, are then such as to induce it to set the market-clearing nominal wage. This result, implicit in the model of Coricelli et al. (2004a, 2006), was argued to have been overlooked by them on account of their having restricted the central bank's weight parameter to be non-negative. It was also argued that the result ought perhaps to be regarded as a theoretical curiosity, since a non-negativity restriction is entirely plausible when central bankers are empowered to conduct monetary policy in accordance with their own personal preferences. (Although this restriction is somewhat less justifiable when the extent of their independence is limited to their choice of instrument settings, and does not extend to the formulation of policy goals.) The chapter concluded by emphasising the potential relevance of the Cubitt (1997) critique to this strand of literature, namely that the underlying wage-setting externality which enables design features of the monetary regime to influence real outcomes may not arise if unions, in formulating their real wage and employment targets, take into account the constraints implied by the economy's structure.

The third, fifth and sixth chapters extended this analysis of wage-related externalities to the case of an economy subject to stochastic productivity and velocity shocks. In addition to the introduction of shocks, two other key features were also added to the framework of the first chapter. The first of these is the specification of the representative union's loss function, which captures the intuitively plausible idea that each union is averse to variability in both its employment and its real wage. The second additional key feature is the assumption that unions commonly receive a potentially noisy signal of the productivity shock prior to the determination of contract nominal wages. (This assumption was relevant only to the wage-setting scenario of Chapter III, and not to the indexation scenarios considered in later chapters.) In each chapter, the analysis was conducted in respect of both a simple-rule scenario in which the money supply is kept fixed, regardless of shock realisations, and an activist scenario in which the central bank adjusts its instrument in a discretionary fashion after wage contracts have been concluded, and in the light of full information regarding the realised values of aggregate shocks. For both regime scenarios, adverse externalities were found to appertain to union wage-setting and indexation decisions, with the symmetric Nash equilibrium nominal wage differing, in general, from the efficient wage that would arise were unions to abide by an agreement to coordinate their decisions: as a consequence, real wage variability is inefficiently low, and employment variability inefficiently high. These externalities were shown to have numerous qualitative affinities with the externality which arises in the non-stochastic model of Chapter II: in particular their severity is reduced by stronger goods-market competition and a smaller number of unions, while each externality was again found to be explicable in terms of a departure of the individual union's perceived labour demand curve (appropriately defined) from its aggregate counterpart.

The wage-setting externality of Chapter III was found to relate only to the component of each productivity shock that is anticipated by unions, and hence to be dependent on the quality of the signals that unions receive. This finding leads directly to one of the third chapter's principal contributions to the literature, namely that in the presence of the identified wage-setting externality, a change in information quality has potentially important implications for the variability of employment and output. A marginal deterioration in signal quality has, in general, two offsetting effects on employment variability: a beneficial effect involving mitigation of the externality, and a detrimental effect involving the increased variance of union forecast errors regarding productivity shocks. It was shown that under both the simple rule and the activist regime, there may exist a subset of admissible values for the representative union's relative-weight parameter, for which the beneficial externality-mitigating effect of greater signal noise on employment variability may outweigh the detrimental forecast-error effect. Another of the key results of this part of the thesis is that when unions are non-atomistic, greater central-bank conservatism can reduce both employment variability and price-level variability, a theoretical result that fits better with the empirical stylised facts than do the theoretical predictions of the Rogoff model. The result is attributable to the effect of greater conservatism, and the implied change in the monetary reaction function, on the individual union's wage-setting behaviour. In particular, greater conservatism mitigates the wage-setting externality and hence reduces both the component of employment variability relating to the anticipated component of shocks, and the associated stochastic inflation bias. These two beneficial effects, together with a reduction in the component of price-level variability arising from union forecast errors of shocks, can outweigh greater conservatism's only drawback in this situation, namely that the component of employment variability relating to union forecast errors becomes suboptimally large. The final part of Chapter III addressed the issue of optimal delegation, and envisaged a scenario involving economic transparency: in other words, whereas previously the quality of signals received by unions had been assumed to be exogenous, the variance of signal noise was instead assumed to be a choice-variable of the central bank. In this situation, a particular value of the central bank's weight parameter was identified as crucially significant for the optimal choice of central banker. Depending on how the actual union weight parameter compares with this critical value, the optimal appointee is found to be either a representative central banker (who would choose entirely to deny unions useful information regarding shocks), or an ultraconservative instructed to practise full transparency.

The remaining chapters of the thesis analysed the externalities which arise in respect of union indexation decisions. In this part of the thesis, the assumption that unions receive a common signal of the productivity shock was discarded, with allowance instead being made for indexation of the nominal wage either solely to the price level (Chapter V), or to both the price level and a second aggregate variable (Chapter VI). These chapters advance the literature by demonstrating that externalities may well arise when unions are averse to variability in both employment and the real wage. (In previous papers the existence of externalities has depended on arguably rather arbitrary assumptions in respect of supply-side features, while in the case of multiparameter indexation no previous contribution to the meagre literature on this topic has considered a model in which the degrees of indexation are endogenously determined.) For the single-parameter scenario, it was found that the externality leads to an inefficiently high degree of indexation in equilibrium, and is found to be much the same in its qualitative effects under both of the monetary regimes considered. Although there are some minor differences in results between the two regime scenarios, these differences are attributable to the neutralisation of velocity shocks achieved under activist policy, so that indexation then resembles fully informed wage-setting, in that it becomes purely a matter of ensuring the nominal wage adjusts in such a way as to bring about the individual union's preferred productivity-shock impact-pattern across employment and the real wage. This is also true of the various multiparameter indexation schemes that were considered, since full indexation of the wage to the price level ensures complete insulation of real outcomes from velocity shocks, so that the efficient impact pattern may be achieved by an appropriate degree of indexation to either the shock, or to an aggregate variable which depends upon it, such as output or employment. It was found that endogenous multiparameter indexation exhibits an adverse externality with characteristics familiar from other parts of the thesis. The finding of greatest interest with regard to this issue, however, was that, provided unions are non-atomistic, the multiparameter-indexation externality is less severe than the externality which arises under fully informed wage-setting. This was explained in terms of the fact that the price level is more sensitive to each union's strategy choice under such a scheme, than it is when wages are adjusted directly in response to signals.

The overall message of this work, therefore, is that the combination of a unionised labour market and monopolistic competition gives rise to macroeconomic externalities which potentially have considerable significance for the optimal design of monetary institutions. It seems appropriate to end by suggesting several possible further extensions to the analysis. One rather obvious gap in the literature is that it has yet to be demonstrated that the quadratic union loss function used in much of the thesis (and which, it will be recalled, implies an aversion to variability in both the real wage and employment about their mean values), is consistent with any standard utility function which might be postulated for a representative union member. (Intuition suggests that risk-aversion as regards both real-income and hours-of-work outcomes may plausibly be assumed to characterise such individuals.)

Numerous (but by no means all) sections of the thesis have assumed that the central bank can influence monetary conditions, and hence aggregate demand, at more frequent intervals than wages can be adjusted. As mentioned in Chapter I, the extent to which this assumption is appropriate remains controversial, and hence modified versions of our wage-setting models in which adjustments to the monetary instrument have a lagged, rather than immediate, effect on aggregate demand seem worthy of investigation, especially if combined with either the potential incidence of control errors in the setting of the monetary instrument, or the introduction of 'intrinsic' uncertainty (i.e. regarding a discretionary policymaker's type, and hence the implied monetary response to shocks and the aggregate wage). Another simplifying assumption which might be relaxed concerns the joint distribution of the (log) productivity shock and noise term in the unions' observed signal, $s=\theta+u$. An alternative formulation in which θ and u are multiplicatively, rather than additively, related (so that $s=\theta u$, with $E(u)=1$) might be attempted, although the success of this venture will obviously depend on whether the resulting model's tractability is seriously impaired by any complications relating to the representative union's implied optimal forecasting equation. Further modifications might also be made in respect of this thesis' abstraction, in common with much of the literature, from firm-union bargaining over the real wage and employment. A version of the model of Chapter III which allows for an initial bargaining stage, with the representative union's bargaining power functionally dependent on the strength of the expected productivity shock, βs, might well yield further insights.

Several possibilities for future work also arise in respect of the indexation models presented in the thesis. Firstly, the findings of Hutchison and Walsh (1998), as well as preliminary work by the present author, suggest that when unions practise single-parameter wage indexation, a scenario in which an activist central bank observes aggregate shocks but is subject, in its instrument setting, to random control errors, is not equivalent to a passive monetary policy scenario in which aggregate shocks do not induce a policy response. Despite unions being exposed to outcome-uncertainty as a result of unpredictable movements in aggregate demand in both scenarios, the fact that the (intended) instrument setting is adjusted in response to the foreseen component of the productivity shock in one scenario, but not in the other, appears to lead to significant differences in the equilibrium degree of indexation. Further investigation of this result is therefore obviously warranted. Secondly, the fact that figures for aggregate output are typically subject to major revisions following their initial publication, suggests the need for a version of the endogenous Karni indexation model of Chapter VI in which y is not (as therein assumed) observable with complete accuracy at the date on which the contract wage is paid. Thirdly, one other obvious proposal for future work involving the indexation models of this thesis is to address the critical observations of Jadresic (2002), by conducting the analysis with wages indexed to lagged inflation, instead of the expectational error regarding inflation for the period covered by the wage contract.

It is fitting that our very last paragraph be devoted to the potentially most fruitful class of modifications of the models presented in previous chapters. These involve relaxing the assumptions made throughout this work, and largely in accordance with the prevailing practice in the literature, that the productivity shock is identical across firms, and that agents only observe a common signal of each aggregate shock, and do not additionally observe a firm- or union-specific private signal of that aggregate shock. However, as shown recently in James and Lawler (2007), allowing for firm-specific shocks or signals which have a non-zero common component introduces further subtle issues which may be significant for optimal monetary-regime design. This particular paper shows that when perfectly competitive firms are subject to idiosyncratic productivity shocks, and the wage at each firm is set at the value that is expected to clear the labour market, greater dedication of monetary policy to price-level stabilisation also achieves greater employment stability. This is attributable to the fact that at each firm in such an economy, deviations of employment from the market-clearing level can only occur as a result of expectational errors regarding the price level. It would be a relatively straightforward step to introduce into this setting monopolistic competition and union concerns to stabilise the real wage as well as employment. Space constraints, however, compel us to leave this task to the future.

Appendices to Chapter II

Appendix II.1: Derivation of the Individual Firm's Profit-Maximising Price and Labour Demand

Firm *i* observes the productivity shock, θ, as well as the aggregate price level p and the realised value of the money supply, *m*, at the time it choose its individual price, p_i, to maximise its profits, with the contract nominal wage, w_i, predetermined. The firm's price decision then implies it has a particular demand for labour services. The key equations for the firm's optimisation exercise are the versions in levels of its production function and individual product-demand function:

$$Y_i^S = C_0 L_i^\alpha e^\theta \qquad \text{(A.II.1.1)}$$

$$Y_i^D = C_1 Y^D \left(\frac{P_i}{P}\right)^{-\varepsilon} \qquad \text{(A.II.1.2)}$$

Where capital letters denote quantities in levels and *e* is the base of the natural logarithm. C_0 and C_1 are constants which may be normalised appropriately in order to eliminate constants from the expressions for the individual firm's optimal price and labour demand. Since these innocuous normalisations are of no interest, all constants are henceforth suppressed from the expressions in this appendix.

Firm *i*'s profit in levels is:

$$\Pi_i = Y_i P_i - L_i W_i \qquad \text{(A.II.1.3)}$$

By making use of the fact that $Y_i^S = Y_i^D = Y_i$, i.e. continuous market-clearing characterises the goods market, and also of the fact that (A.II.1.1.) implies that $L_i = Y_i^{1/\alpha} e^{-\theta/\alpha}$, firm *i*'s profit may, after appropriate substitutions, be written as follows:

$$\Pi_i = Y^D P^\varepsilon P_i^{(1-\varepsilon)} - \left(Y^D P^\varepsilon P_i^{-\varepsilon} e^{-\theta}\right)^{1/\alpha} W_i \qquad \text{(A.II.1.4)}$$

Firm *i*'s first-order condition for its profit-maximising price is therefore:

$$\frac{\partial \Pi_i}{\partial P_i} = (1-\varepsilon)Y^D P^\varepsilon P_i^{-\varepsilon} + \left(\frac{\varepsilon}{\alpha}\right)\left(Y^D P^\varepsilon P_i^{-(\varepsilon+\alpha)} e^{-\theta}\right)^{1/\alpha} W_i = 0 \qquad \text{(A.II.1.5)}$$

Solving for the optimal P_i, it is found to be:

$$P_i^* = \left[W_i^\alpha (Y^D P^\varepsilon)^{(1-\alpha)} e^{-\theta}\right]^{1/[\alpha+\varepsilon(1-\alpha)]} \qquad \text{(A.II.1.6)}$$

Substituting $Y^D = (Me^\phi/P)^\gamma$ into (A.II.1.6) and taking logs, the optimal log price of firm i is:

$$p_i^* = \frac{\{(1-\alpha)[\gamma(m+\phi)+(\varepsilon-\gamma)p]+\alpha w_i - \theta\}}{[\alpha+\varepsilon(1-\alpha)]} \qquad \text{(A.II.1.7)}$$

Combining this expression with the logarithmic versions of (A.II.1.1) and (A.II.1.2) allows firm i's optimal log demand for labour to be derived:

$$l_i^D = \frac{[\gamma(m+\phi-p)-\varepsilon(w_i-p)+(\varepsilon-1)\theta]}{[\alpha+\varepsilon(1-\alpha)]} \qquad \text{(A.II.1.8)}$$

For the sake of completeness we demonstrate that P_i^*, as given by (A.II.1.6), does satisfy the second-order condition for a maximum. The second derivative of Π_i with respect to P_i is:

$$\frac{\partial^2 \Pi_i}{\partial P_i^2} = \varepsilon(\varepsilon-1)Y^D P^\varepsilon P_i^{-(\varepsilon+1)} - \frac{\varepsilon(\varepsilon+\alpha)}{\alpha^2}\left(Y^D P^\varepsilon P_i^{-(\varepsilon+2\alpha)} e^{-\theta}\right)^{1/\alpha} W_i \qquad \text{(A.II.1.9)}$$

Π_i is maximised when $P_i = P_i^*$ if, and only if, $\left(\partial^2 \Pi_i / \partial P_i^2\right)_{P_i=P_i^*} < 0$. Making use of (A.II.1.9) and of the facts that W_i, Y^D, P and P_i are all strictly positive, it follows that the second-order condition for a maximum is satisfied if (and only if):

$$\left(P_i^*\right)^{[\alpha+\varepsilon(1-\alpha)]/\alpha} < \frac{(\varepsilon+\alpha)[(Y^D P^\varepsilon)^{(1-\alpha)} e^{-\theta}]^{1/\alpha} W_i}{\alpha^2(\varepsilon-1)}$$

Substituting for P_i^*, as given by (A.II.1.6), and simplifying, this condition boils down to: $\alpha[\alpha(\varepsilon-1)-1]<\varepsilon$. This condition does hold, since $\varepsilon>1$ and $0<\alpha<1$. Hence the second-order condition for P_i^* to be the P_i which maximises Π_i is indeed satisfied.

Appendix II.2: Derivation of the Symmetric-Wage Labour Demand Curve

As explained in the text, the term 'symmetric-wage labour demand curve' is used in this thesis to refer to the labour demand curve in real wage, employment space when every union sets the same wage ($w_j = w \forall j$). The symmetric-wage labour demand curve is a special case of the aggregate-level labour demand curve. To derive the equations of these two curves, we make use of the semi-reduced form for the price level, (12)[1], and the aggregate counterpart to (7) (the individual union's labour demand curve), after modifying these expressions for the velocity shock, ϕ, and the productivity shock, θ.

$$p = \frac{\gamma(1-\alpha)(m+\phi)+\alpha w-\theta}{[\alpha+\gamma(1-\alpha)]} \qquad \text{(A.II.2.1)}$$

$$l = \frac{\gamma(m+\phi-p)-\varepsilon(w-p)+(\varepsilon-1)\theta}{[\alpha+\varepsilon(1-\alpha)]} \qquad \text{(A.II.2.2)}$$

(A.II.2.1) implies that the aggregate real wage is:

$$w-p = \frac{\gamma(1-\alpha)(w-m-\phi)+\theta}{[\alpha+\gamma(1-\alpha)]} \qquad \text{(A.II.2.3)}$$

Substituting (A.II.2.1) into (A.II.2.2), we have:

$$l = \frac{-\gamma(w-m-\phi)+(\gamma-1)\theta}{[\alpha+\gamma(1-\alpha)]} \qquad \text{(A.II.2.4)}$$

The slope of the aggregate-level labour demand curve is given by:

$$\frac{dl}{d(w-p)} = \frac{dl/dw}{d(w-p)/dw} = -\frac{1}{(1-\alpha)} \qquad \text{(A.II.2.5)}$$

[1] All equation numbers in appendices relating to Chapter II which lack an 'A' prefix refer to equations in Chapter II.

Since $l_j^S = 0$ implies that market-clearing employment is zero, the market-clearing nominal wage under symmetric wage-setting implied by (A.II.2.4) is:

$$w_{market-clearing} = m + \phi + \frac{(\gamma - 1)}{\gamma}\theta \qquad \text{(A.II.2.6)}$$

Substituting (A.II.2.6) into (A.II.2.3), the market-clearing real wage is found to be:

$$(w - p)_{market-clearing} = \theta \qquad \text{(A.II.2.7)}$$

This finding implies that the intercept of the aggregate-level labour demand curve is $\theta/(1-\alpha)$. Hence the aggregate-level labour demand curve's equation is:

$$l = \frac{-(w-p)+\theta}{(1-\alpha)} \qquad \text{(A.II.2.8)}$$

Although in deriving this expression use has been made of semi-reduced forms for the simple rule, it is in fact independent of the monetary regime and could alternatively have been derived by making use of the semi-reduced forms for one of the other monetary regimes. The aggregate-level labour demand curve simply relates average employment to the average real wage and the realised value of the productivity shock, and the only structural parameter which matters for this relationship is the productivity-related parameter α. The symmetric-wage labour demand curve is a special case of the aggregate-level curve, since it assumes that nominal wage-setting is symmetric. It is therefore convenient to write the equation of the symmetric-wage labour demand curve as follows:

$$l_j\Big|_{w_j = w \forall j} = \frac{-(w_j - p)+\theta}{(1-\alpha)} \qquad \text{(A.II.2.9)}$$

This equation has been expressed in terms of union *j*'s labour demand and real wage, rather than their aggregate counterparts, in order to stress the fact that the real outcomes for the individual union must lie on the symmetric-wage labour demand curve whenever there is symmetric wage-setting, $w_j = w \forall j$. Thus the symmetric Nash equilibrium in wages, as well as any outcome arising from coordinated wage-setting, must satisfy equation (A.II.2.9).[2]

[2] If equation (A.II.2.9) is satisfied, equation (A.II.2.8) must necessarily also be satisfied.

Appendix II.3: Derivation of the Individual Union's Perceived Labour Demand Curve under the Price Level Rule for the Non-Stochastic Model

As explained in Chapter II, union *j*'s perceived labour demand curve is the set of points in employment, real wage space attainable by union *j* given every other union sets the same nominal wage. There is an infinite set of possible perceived labour demand curves, one for every nominal wage on which the other $n-1$ unions could coordinate their wage-setting. However the two perceived labour demand curves which are of particular interest are those passing through the efficient and equilibrium outcomes. This appendix derives the equations of these curves for the case of the price level rule, as given by (14), and when the union loss function is given by (29). Our task is to derive the vertical intercept and slope coefficient in the equation:

$$l_j^D = C_0 + \frac{dl_j^D}{d(w_j - p)}(w_j - p) \tag{A.II.3.1}$$

Equations (7), (14) and (15) imply that demand for union *j*'s labour as a function of its individual wage, w_j, and the aggregate wage, *w*, is:

$$l_j^D = \frac{\alpha[\varepsilon + \gamma(\tau_1 - 1)]w - \varepsilon[\alpha + \gamma(1-\alpha)(1-\tau_1)]w_j}{[\alpha + \varepsilon(1-\alpha)][\alpha + \gamma(1-\alpha)(1-\tau_1)]} \tag{A.II.3.2}$$

Equation (15) implies that union *j*'s real wage is:

$$w_j - p = w_j - \frac{\alpha w}{[\alpha + \gamma(1-\alpha)(1-\tau_1)]} \tag{A.II.3.3}$$

The slope coefficient in (A.II.3.1) is the ratio of the derivatives with respect to w_j of (A.II.3.2) and (A.II.3.3):

$$\frac{dl_j^D}{d(w_j - p)} = \frac{dl_j^D / dw_j}{d(w_j - p)/dw_j} = \frac{-(\gamma\alpha + \varepsilon\Lambda) + \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1}{[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]} \tag{A.II.3.4}$$

This slope coefficient is the same for every possible perceived labour demand curve.

With the union's loss function given by (29), the efficient outcome is $l_j^* = l_u - c_u'(1-\alpha)$, $(w_j - p)^* = (1-\alpha)[c_u'(1-\alpha) - l_u]$. Substituting these values together with (A.II.3.4) into (A.II.3.1) allows us to solve for the vertical intercept C_0. For the case of the price level rule, the equation of union j's perceived labour demand curve which passes through the efficient outcome is therefore found to be:

$$l_j^D = \frac{(n-1)\alpha[\alpha+\gamma(1-\alpha)(1-\tau_1)][l_u - c_u'(1-\alpha)] + \{-(\gamma\alpha+\varepsilon\Lambda)+\gamma[\alpha+n\varepsilon(1-\alpha)]\tau_1\}(w_j-p)}{[\alpha+\varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1)]}$$

(A.II.3.5)

For the case of the simple rule, $m_r = \overline{m}$ (i.e. $\tau_1 = 0$), this becomes:

$$l_j^D\Big|_{\tau_1=0} = \frac{(n-1)\alpha[\alpha+\gamma(1-\alpha)][l_u - c_u'(1-\alpha)] - (\gamma\alpha+\varepsilon\Lambda)(w_j-p)}{[\alpha+\varepsilon(1-\alpha)]\Lambda} \qquad \text{(A.II.3.6)}$$

Employment and the real wage in the symmetric Nash equilibrium outcome are given by (67a) and (67b) respectively, and combining these with (A.II.3.1) and (A.II.3.4) yields the equation of union j's perceived labour demand curve which passes through the equilibrium outcome:

$$l_j^D = \frac{-1}{[\alpha+\varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]}\Bigg\{ \{\gamma\alpha+\varepsilon\Lambda-\gamma[\alpha+n\varepsilon(1-\alpha)]\tau_1\}(w_j-p)$$
$$+ (n-1)\alpha[\alpha+\gamma(1-\alpha)(1-\tau_1)]\left[\frac{c_u'[\alpha+\varepsilon(1-\alpha)][\Lambda-n\gamma(1-\alpha)\tau_1]}{\{\gamma\alpha+\varepsilon\Lambda-\gamma[\alpha+n\varepsilon(1-\alpha)]\tau_1\}} - l_u\right]\Bigg\}$$

(A.II.3.7)

In the case of the simple rule ($\tau_1 = 0$) this becomes:

$$l_j^D\Big|_{\tau_1=0} = \frac{-1}{[\alpha+\varepsilon(1-\alpha)]\Lambda}\left\{(\gamma\alpha+\varepsilon\Lambda)(w_j-p) + (n-1)\alpha[\alpha+\gamma(1-\alpha)]\left[\frac{c_u'[\alpha+\varepsilon(1-\alpha)]\Lambda}{(\gamma\alpha+\varepsilon\Lambda)} - l_u\right]\right\}$$

(A.II.3.8)

Appendix II.4: The Socially Optimal Wage-Contingent Monetary Rule

This appendix derives the socially optimal monetary rule, specified as a response to the aggregate nominal wage. It is assumed that $1 < n < \infty$, that the rule, m_r, is fully credible (so that $Em_r = m_r$), and that the individual union's loss function is given by (29). Using (7) and (12), the demand for union *j*'s labour and its real wage, expressed as functions of m_r and $w = (1/n)\sum_{j=1}^{n} w_j$, are:

$$l_j^D = \frac{\gamma[\alpha + \varepsilon(1-\alpha)]m_r + \alpha(\varepsilon - \gamma)w - \varepsilon[\alpha + \gamma(1-\alpha)]w_j}{[\alpha + \varepsilon(1-\alpha)][\alpha + \gamma(1-\alpha)]} \qquad \text{(A.II.4.1)}$$

$$w_j - p = w_j - \frac{[\gamma(1-\alpha)m_r + \alpha w]}{[\alpha + \gamma(1-\alpha)]} \qquad \text{(A.II.4.2)}$$

The total derivatives of these expressions with respect to w_j are:

$$\frac{dl_j^D}{dw_j} = \frac{1}{[\alpha + \gamma(1-\alpha)]}\left\{\gamma\left(\frac{dm_r}{dw_j}\right) - \frac{(\gamma\alpha + \varepsilon\Lambda)}{n[\alpha + \varepsilon(1-\alpha)]}\right\} \qquad \text{(A.II.4.3)}$$

$$\frac{d(w_j - p)}{dw_j} = \frac{1}{n[\alpha + \gamma(1-\alpha)]}\left[\Lambda - n\gamma(1-\alpha)\left(\frac{dm_r}{dw_j}\right)\right] \qquad \text{(A.II.4.4)}$$

Using (29), union *j*'s first-order condition for its optimal wage, w_j^{**}, is:

$$\frac{dE\Omega_j^u}{dw_j} = 2E\left[(l_j - l_u)\left(\frac{dl_j}{dw_j}\right) - c_u' \frac{d(w_j - p)}{dw_j}\right] = 0 \qquad \text{(A.II.4.5)}$$

where E is the rational expectations operator. Substituting (A.II.4.3) and (A.II.4.4) into (A.II.4.5) with $l_j = l_j^D$, and noting that the rule is assumed to be credible, and that union *j*'s rational expectation of the wages set by other unions is its own optimal wage, we may impose $w_j = w \forall j$ to obtain an equation which implicitly defines the symmetric Nash equilibrium nominal wage, w_{NE}:

$$w_{NE} = m_r\Big|_{w=w_{NE}} + \frac{[\alpha + \gamma(1-\alpha)]}{\gamma\{n\gamma[\alpha + \varepsilon(1-\alpha)][d(m_r|_{w=w_{NE}})/dw_j] - \gamma\alpha - \varepsilon\Lambda\}}\Bigg[\{n\gamma[\alpha + \varepsilon(1-\alpha)] \times$$
$$[c_u'(1-\alpha) - l_u][d(m_r|_{w=w_{NE}})/dw_j]\} + (\gamma\alpha + \varepsilon\Lambda)l_u - c_u'[\alpha + \varepsilon(1-\alpha)]\Lambda \Bigg] \quad \text{(A.II.4.6)}$$

Equation (A.II.4.1) implies that employment in the symmetric Nash equilibrium (which involves $w_j = w_{NE} \forall j$) is:

$$l\Big|_{w=w_{NE}} = \frac{\gamma(m_r|_{w=w_{NE}} - w_{NE})}{[\alpha + \gamma(1-\alpha)]} \quad \text{(A.II.4.7)}$$

The associated price level outcome is:

$$p\Big|_{w=w_{NE}} = \frac{\gamma(1-\alpha)(m_r|_{w=w_{NE}}) + \alpha w_{NE}}{[\alpha + \gamma(1-\alpha)]} \quad \text{(A.II.4.8)}$$

Substituting (A.II.4.7) and (A.II.4.8) into the social loss function, (9′), yields:

$$\Omega^{soc}\Big|_{w=w_{NE}} = \frac{\gamma^2(m_r|_{w=w_{NE}} - w_{NE})^2 + c_s[\gamma(1-\alpha)(m_r|_{w=w_{NE}}) + \alpha w_{NE}]^2}{[\alpha + \gamma(1-\alpha)]^2} \quad \text{(A.II.4.9)}$$

If we conjecture that the optimal rule is of the form[3] $m_r = (dm_r/dw)w$, substitute this and (A.II.4.6) into (A.II.4.9), and minimise the equilibrium social loss by choice of the rule parameter, dm_r/dw, the optimal rule emerges as:

[3] It turns out that if we include a constant term in the conjectured optimal rule, so that $m_r = \overline{m} + (dm_r/dw)w$, the optimal setting of the constant $\overline{m}$ is zero if dm_r/dw is chosen optimally. To shorten this appendix we therefore omit $\overline{m}$ from the conjectured optimal rule.

$$m_r^* = \frac{\{c_u'[\alpha + \varepsilon(1-\alpha)]\Lambda - (\gamma\alpha + \varepsilon\Lambda)l_u\}}{\gamma[\alpha + \varepsilon(1-\alpha)][c_u'(1-\alpha) - l_u]} w \qquad \text{(A.II.4.10)}$$

It is straightforward to convert this into the optimal price level rule, $m_r^* = \tau_1^* p$, where τ_1^* is given by (71). To do so, we make use of the fact that the semi-reduced from for p, namely $p = [\gamma(1-\alpha)m_r + \alpha w]/[\alpha + \gamma(1-\alpha)]$, implies that $w = \{[\alpha + \gamma(1-\alpha)]p - \gamma(1-\alpha)m_r\}/\alpha$. Substituting this into (A.II.4.10) and manipulating to isolate m_r on the left-hand side, yields $m_r^* = \tau_1^* p$. Similar procedures involving the semi-reduced form for output, $y = \gamma\alpha(m_r - w)/[\alpha + \gamma(1-\alpha)]$, allow the derivation of the optimal nominal income rule, $m_r^* = \tau_2^*(y+p)$, and the optimal employment rule, $m_r^* = \tau_3^* l$, where τ_2^* and τ_3^* are respectively given by equations (100) and (108).

Appendix II.5: Equilibrium Wage-Setting in the Non-Stochastic Model with Endogenised Union Real Wage Objective

The wage-setting model with union loss function given by (30) described in Chapter II is modified here to allow for an initial stage in which unions simultaneously choose their individual real wage objectives. The real wage objective for union j will be denoted $w_{u,j}^{real}$, so that its individual loss now becomes:

$$\Omega_j^u = (l_j - l_u)^2 + c_u(w_j - p - w_{u,j}^{real})^2 \qquad \text{(A.II.5.1)}$$

The preference parameters l_u and c_u are here regarded as primitive and immutable. As explained in Chapter II, once l_u has been assigned a particular value, symmetric wage-setting and goods-market clearing implies that every union's employment and real wage outcome will be related according to the aggregate-level labour demand curve, $l = -(w-p)/(1-\alpha)$. Hence if l_u happens to be the employment outcome of the symmetric Nash equilibrium of the wage-setting game, the associated real wage outcome will be $(w-p)|_{w=w_{NE}} = -(1-\alpha)l_u$. Given this structural constraint, and the assumption that l_u is a fundamental component of union j's preferences, it follows that insofar as $w_{u,j}^{real} \neq -(1-\alpha)l_u$, this individual real wage objective has an element of arbitrariness to it. This appendix investigates union j's equilibrium choice of $w_{u,j}^{real}$ when each and every union recognises the existence of this structural constraint and this fact is common knowledge. Any of the monetary regimes considered in Chapter II will serve our purpose here, and for convenience we make use of the price level rule, $m_r = \tau_1 p$. The individual union will foresee that once an aggregate real wage objective, w_u^{real} (where $w_u^{real} = n^{-1}\sum_{j=1}^{n} w_{u,j}^{real}$) has been determined, the equilibrium employment outcome of the subsequent simultaneous-move wage-setting game will be given by equation (123b) of Chapter II, which we reproduce here, appropriately renumbered, for convenience:

$$l_j\big|_{w=w_{NE}} = \frac{\{\gamma\alpha + \varepsilon\Lambda - \gamma[\alpha + n\varepsilon(1-\alpha)]\tau_1\}l_u - c_u[\alpha + \varepsilon(1-\alpha)][\Lambda - n\gamma(1-\alpha)\tau_1]w_u^{real}}{\gamma\alpha(1-\tau_1) + \{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}[\Lambda - n\gamma(1-\alpha)\tau_1]}$$

(A.II.5.2)

Union *j* will recognise that in equilibrium l_j and $(w_j - p)$ must be related according to the equation of the symmetric-wage labour demand curve, and taking this into account its rational expectation of its loss function, (A.II.5.1), at the time unions are simultaneously choosing their individual real wage objectives, can be rewritten as follows:

$$E(\Omega_j^u\big|_{w=w_{NE}}) = (l_j\big|_{w=w_{NE}} - l_u)^2 + c_u[-(1-\alpha)l_j\big|_{w=w_{NE}} - w_{u,j}^{real}]^2 \quad \text{(A.II.5.3)}$$

where E is the rational expectations operator. Union *j* foresees that its $w_{u,j}^{real}$ choice can influence its loss not only directly, but also indirectly via equilibrium employment, to the determination of which $w_{u,j}^{real}$ contributes via the aggregate real wage objective w_u^{real}.

Substituting (A.II.5.2) into (A.II.5.3), and minimising by choice of $w_{u,j}^{real}$, taking other unions' choices of real wage objective as given, yields a first-order condition for union *j*'s optimal choice of $w_{u,j}^{real}$. By imposing $w_{u,j}^{real} = w_u^{real}\ \forall j$ in this first-order condition (which we omit because of its length), an equation implicitly defining the symmetric Nash equilibrium w_u^{real} is obtained. Solving this equation, the individual union's equilibrium choice of real wage objective is found to be:

$$w_{u,NE}^{real} = -(1-\alpha)l_u \quad \text{(A.II.5.4)}$$

Hence, irrespective of *n*, each union in equilibrium chooses a real wage objective which is consistent with the employment objective of l_u. Note that this conclusion also applies, with some uninteresting minor complications, when the loss function is given by (28), i.e. features both a quadratic and a linear term in the real wage, the reason being that given l_u, c_u and c_u' are common to all unions, union *j*'s implied real wage objective (i.e. the real wage coordinate of its bliss point) , $(c_u'/c_u)+v_{u,j}$, has an element of arbitrariness to it, and it is the notional real wage objective, $v_{u,j}$, which is the source of this arbitrariness. As mentioned in Chapter II, things are rather different in the case of union loss function (29), which features no quadratic term in the real wage but rather only a linear real wage term, since with this specification the preference parameter which is the ultimate source of the wage-setting externality, namely the weight c_u', is primitive, and therefore cannot be endogenised.

Appendices to Chapter III

Appendix III.1: Derivation of the Individual Union's Perceived Labour Demand Curve under the Simple Rule for the Stochastic Model

In the stochastic version of the model, union *j*'s perceived labour demand curve is the set of points in employment, real wage space attainable by union *j* given every other union sets the same nominal wage, and given also that union *j*'s rational expectation of the shock proves correct (i.e. given $\beta s = \theta$). As in the non-stochastic model, given the conditional expectation of the shock, $E(\theta \mid s) = \beta s$, there is an infinite set of possible perceived labour demand curves, one for every nominal wage on which the other $n-1$ unions could coordinate their wage-setting. We confine our attention, however, to the two perceived labour demand curves which are of particular interest, namely those passing through the efficient and equilibrium outcomes. We assume the prevailing monetary regime is the simple rule, according to which the money supply is kept fixed, and as in Section 3 of Chapter III this appendix normalises *m* at zero, and abstracts from velocity shocks.

For each of the two perceived labour demand curves, our task is to derive the vertical intercept and slope coefficient in the equation:

$$E(l_j^D \mid s) = C_0 + \frac{dl_j^D}{d(w_j - p)} E[(w_j - p) \mid s] \qquad \text{(A.III.1.1)}$$

Equations (6), (13′) and (15) of Chapter III imply that union *j*'s conditional expectation of the demand for its labour, expressed as a function of its individual wage, w_j, and the aggregate wage, *w*, is:

$$E(l_j^D \mid s) = \frac{(\gamma - 1)\beta s}{[\alpha + \gamma(1-\alpha)]} + \frac{\{\alpha(\varepsilon - \gamma)w - \varepsilon[\alpha + \gamma(1-\alpha)]w_j\}}{[\alpha + \varepsilon(1-\alpha)][\alpha + \gamma(1-\alpha)]} \qquad \text{(A.III.1.2)}$$

Equation (15) implies that union *j*'s conditional expectation of its real wage is:

$$E[(w_j - p) \mid s] = \frac{\beta s}{[\alpha + \gamma(1-\alpha)]} + \frac{\{[\alpha + \gamma(1-\alpha)]w_j - \alpha w\}}{[\alpha + \varepsilon(1-\alpha)][\alpha + \gamma(1-\alpha)]} \qquad \text{(A.III.1.3)}$$

The slope coefficient in (A.III.1.1) is the ratio of the derivatives with respect to w_j of (A.III.1.2) and (A.III.1.3):

$$\frac{dE(l_j^D \mid s)}{dE[(w_j - p) \mid s]} = \frac{dE(l_j^D \mid s)/dw_j}{dE[(w_j - p) \mid s]/dw_j} = \frac{-(\gamma\alpha + \varepsilon\Lambda)}{[\alpha + \varepsilon(1-\alpha)]\Lambda} \qquad \text{(A.III.1.4)}$$

This slope coefficient is the same for every possible perceived labour demand curve. The (expected) efficient outcome in the stochastic model of Chapter III is given by the expectation, conditional on s, of equations (20a) and (20b): this expected outcome involves employment of $E(l_j|_{w_j = w^* \forall j} \mid s) = c_u(1-\alpha)\beta s/[1 + c_u(1-\alpha)^2]$, together with a real wage of $E[(w_j - p)|_{w_j = w^* \forall j} \mid s] = \beta s/[1 + c_u(1-\alpha)^2]$. Substituting these values together with (A.III.1.4) into (A.III.1.1) allows us to solve for the vertical intercept C_0. The equation of union j's perceived labour demand curve which passes through the efficient outcome is therefore found to be:

$$E(l_j|_{w_k = w^* \forall k \neq j} \mid s) = \frac{1}{[\alpha + \varepsilon(1-\alpha)]\Lambda}\left[\frac{\{\gamma\alpha + \varepsilon\Lambda + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\Lambda\}\beta s}{[1 + c_u(1-\alpha)^2]} - (\gamma\alpha + \varepsilon\Lambda)E[(w_j - p)|_{w_k = w^* \forall k \neq j} \mid s]\right] \qquad \text{(A.III.1.5)}$$

The equation of union j's equilibrium perceived labour demand curve (i.e. that which passes through the equilibrium outcome, which involves employment and real wage given by the expectations, conditional on s, of, respectively, (38a) and (38b)) is derived in a similar fashion and is found to be:

$$E(l_j\big|_{w_k = w_{NE}\forall k\neq j} \mid s) = \frac{1}{[\alpha+\varepsilon(1-\alpha)]\Lambda}\left[\frac{\{(\gamma\alpha+\varepsilon\Lambda)^2 + c_u[\alpha+\varepsilon(1-\alpha)]^2\Lambda^2\}\beta s}{\{\gamma\alpha+\varepsilon\Lambda + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\Lambda\}} + \right.$$

$$\left. -(\gamma\alpha+\varepsilon\Lambda)E[(w_j - p)\big|_{w_k = w_{NE}\forall k\neq j} \mid s] \right] \quad \text{(A.III.1.6)}$$

Appendix III.2: The Impact of a Marginal Deterioration in Signal Quality on the Expected Union Loss under the Simple Rule in the $\gamma < 1$ and $\gamma > 1$ Cases

For convenience we begin by setting out, and appropriately renumbering, expression (54) of Chapter III, namely the necessary and sufficient condition for a marginal deterioration in signal quality to be beneficial (detrimental) to the welfare of non-atomistic unions:

$$\frac{dE(\Omega_j^u|_{w=w_{NE}})}{d\sigma_u^2} < (>) 0 \text{ iff } \Delta\Theta < (>) 0 \qquad \text{(A.III.2.1)}$$

where $\Delta \equiv (1-\gamma)(\gamma\alpha + \varepsilon\Lambda) + c_u[\alpha + \varepsilon(1-\alpha)]\Lambda$

$\Theta \equiv -(n-1)c_u\alpha[\alpha + \gamma(1-\alpha)]^2 + [1-\gamma + c_u(1-\alpha)]\{\gamma\alpha + \varepsilon\Lambda + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\Lambda\}$

An intuitive interpretation of this expression for the $\gamma = 1$ case has been presented in Chapter III, and the reader who is familiar with the relevant passage will be aware that the two special cases of the representative union's relative-weight parameter denoted $\tilde{c}_u$ and $\tilde{\tilde{c}}_u$ figure prominently in that discussion. The special case in which $c_u = \tilde{c}_u$, where $\tilde{c}_u \equiv (\gamma-1)/(1-\alpha)$, is that in which in the absence of any adjustment in the aggregate wage, the movement in the price level which results from a non-zero productivity shock, serves to allocate that shock between the real wage and employment in the most desirable way from the representative union's viewpoint. Efficient wage-setting therefore requires that each union disregard signals if their preferences are such that $c_u = \tilde{c}_u$. The other special case involves $c_u = \tilde{\tilde{c}}_u$, where $\tilde{\tilde{c}}_u \equiv (\gamma-1)(\gamma\alpha + \varepsilon\Lambda)/[\alpha + \varepsilon(1-\alpha)]\Lambda$, and is that in which each union in equilibrium disregards the signal and always maintains a constant wage, even though efficiency requires that its wage be adjusted in response to this item of information. Since c_u cannot take negative values, the possibility that $c_u = \tilde{c}_u$ or $c_u = \tilde{\tilde{c}}_u$ does arise if γ happens to exceed unity, but is precluded if $\gamma < 1$ is the case. For these reasons, the greater part of this appendix is devoted to the case in which $\gamma > 1$, to which we now turn.

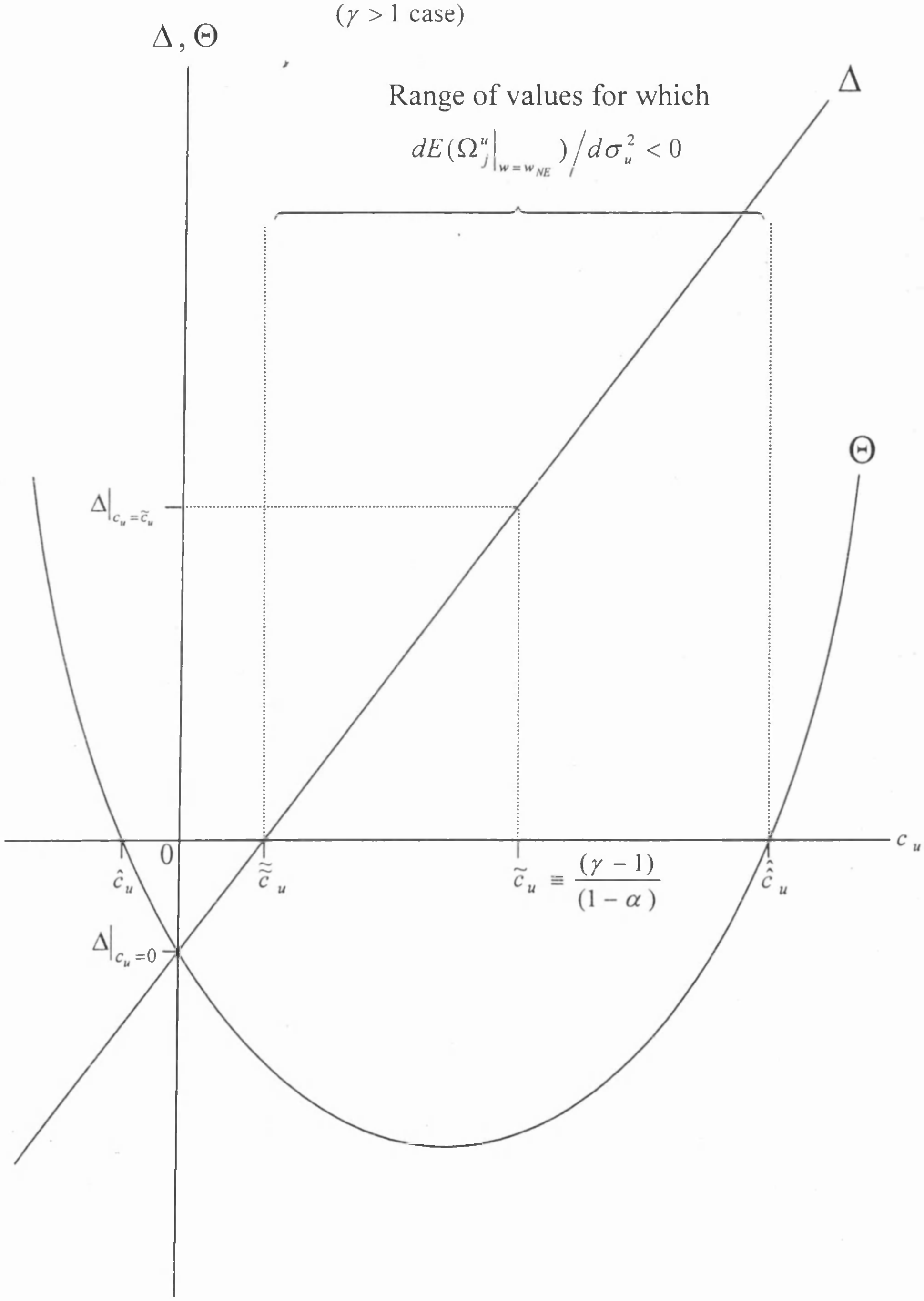
As mentioned in Chapter III, Δ is equal to minus one times the numerator of the coefficient on βs in the equilibrium nominal wage equation. In the light of the fact that unions in equilibrium are led to disregard the signal when $c_u = \tilde{\tilde{c}}_u$, it is therefore unsurprising to find that $\Delta|_{c_u=\tilde{\tilde{c}}_u} = 0$. It then follows from (A.III.2.1) that $[dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2]|_{c_u=\tilde{\tilde{c}}_u} = 0$, indicating that changes in signal quality do not affect union welfare under equilibrium wage-setting when $c_u = \tilde{\tilde{c}}_u$. The interpretation of Θ is less straightforward. It is noteworthy, however, that if $c_u = \tilde{c}_u$ (so that $[1-\gamma + c_u(1-\alpha)]|_{c_u=\tilde{c}_u} = 0$), we find that provided $n > 1$, $\Delta|_{c_u=\tilde{c}_u} > 0$ and $\Theta|_{c_u=\tilde{c}_u} < 0$. This implies that when $\gamma > 1$, $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2|_{c_u=\tilde{c}_u} < 0$ for all $n > 1$. This last result is in full accordance with the intuitive arguments advanced in Chapter III that, provided $n > 1$, deterioration in signal quality must be beneficial to the welfare of unions when $c_u = \tilde{c}_u$ is the case.

To provide further insight into result (A.III.2.1) for the $\gamma > 1$ case, we now refer to its graphical representation in Figure A.III.1 below. The fact that $\gamma > 1$ is assumed ensures not only that $\tilde{c}_u > 0$ and $\tilde{\tilde{c}}_u > 0$, but also that the horizontal-axis intercept of Δ has a positive value, and that the solutions (denoted $\hat{c}_u$ and $\hat{\hat{c}}_u$) of the equation $\Theta = 0$ are real, with the numerically smaller solution negative, $\hat{c}_u < 0$, and the other solution positive, $\hat{\hat{c}}_u > 0$.[1] (There is nothing to be gained expositionally from stating the algebraic expressions for $\hat{c}_u$ and $\hat{\hat{c}}_u$, and consequently we refrain from doing so.) It follows immediately from Figure A.III.1 that, when $\gamma > 1$, the necessary and sufficient condition for $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ stated in (A.III.2.1), namely that $\Delta\Theta < 0$, is satisfied if c_u is such that $\tilde{\tilde{c}}_u < c_u < \hat{\hat{c}}_u$.[2] In other words, when $\gamma > 1$ it is necessarily the case, provided $n > 1$, that there is a range of c_u values for which

[1] The proof that $\hat{c}_u < 0$ and that $\hat{\hat{c}}_u > 0$ when $\gamma > 1$ is as follows. Since $\partial^2\Theta/\partial c_u^2 > 0$, the parabola of Θ considered as a function of c_u is upright (and is in fact so for all admissible values of γ, and not just for $\gamma > 1$). Since it is easily shown using (A.III.2.1) with $\gamma > 1$ that $\Theta|_{c_u=0} < 0$ and $(\partial\Theta/\partial c_u)|_{c_u=0} < 0$, and that $\Theta|_{c_u=\tilde{c}_u} < 0$ and that $(\partial\Theta/\partial c_u)|_{c_u=\tilde{c}_u} > 0$, it then follows that when $\gamma > 1$, $\hat{c}_u < 0$ and $\hat{\hat{c}}_u > \tilde{c}_u > 0$, as depicted in Figure A.III.1.

[2] It is apparent from Figure A.III.1 that, when $\gamma > 1$, $\Delta\Theta < 0$ if $c_u < \hat{c}_u$, where $\hat{c}_u < 0$. However, this is disregarded since it requires that c_u take an inadmissible negative value.

Figure A.III.1
Identification of the sign of the impact of a marginal deterioration in signal quality on the expected union loss under the simple rule
($\gamma > 1$ case)



Notes:
1. It is assumed that $1 < n < \infty$ and $\gamma > 1$. The diagram does not apply to the atomistic case, since Δ and Θ have been derived on the assumption that n is finite.
2. Note that $\tilde{\tilde{c}}_u \equiv (\gamma-1)(\gamma\alpha+\varepsilon\Lambda)/[\alpha+\varepsilon(1-\alpha)]\Lambda$, $\Delta|_{c_u=\tilde{c}_u} = (n-1)(\gamma-1)\alpha[\alpha+\gamma(1-\alpha)]/(1-\alpha)$ and $\Delta|_{c_u=0} = -(\gamma-1)(\gamma\alpha+\varepsilon\Lambda)$.

improvements in signal quality are detrimental to unions' welfare. If c_u is outside this range then better-quality information either benefits unions (this is the case when $0 < c_u < \tilde{\tilde{c}}_u$ or when $c_u > \hat{\hat{c}}_u$) or has no impact on their welfare (the case when $c_u = \tilde{\tilde{c}}_u$ or when $c_u = \hat{\hat{c}}_u$).

Intuitive arguments can readily be provided for the existence of a range of admissible c_u values for which $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ is the case when $\gamma > 1$. For instance, the fact that this range of values, namely $\tilde{\tilde{c}}_u < c_u < \hat{\hat{c}}_u$, does not include c_u values close to zero, or above a certain value ($\hat{\hat{c}}_u$) is understandable in the light of the point established in Chapter III that the externality is weaker when the representative union is relatively indifferent to one of the two terms in its loss function: hence in the $\gamma > 1$ case at least, if c_u is sufficiently low or sufficiently high, the beneficial externality-mitigating effect of a deterioration in signal quality must be weak because the externality itself is relatively weak, and thus the detrimental forecast-error effect must be the stronger of the two.

A second point concerning the $\tilde{\tilde{c}}_u < c_u < \hat{\hat{c}}_u$ set of values in this $\gamma > 1$ case is that $\tilde{c}_u$ lies within this interval. This is entirely to be expected, since from our earlier discussion of the $c_u = \tilde{c}_u$ special case it will be recalled that this is the case in which efficiency requires that unions always set a wage of zero regardless of the signal: hence informative signals must be detrimental to unions' welfare under equilibrium wage-setting when $c_u = \tilde{c}_u$, and an increase in the noisiness of the signal must make unions better off. A clear implication of this argument is that there must be a range of c_u values extending to either side of $\tilde{c}_u$, for which the beneficial externality-mitigating effect outweighs the detrimental forecast-error effect, with the lower ($\tilde{\tilde{c}}_u$) and upper ($\hat{\hat{c}}_u$) bounds of this range being those values at which these two effects of a marginal increase in the signal-noise variance are exactly equal in absolute value.

There is a third aspect of this result for the $\gamma > 1$ case for which intuition can be provided, namely that $\tilde{\tilde{c}}_u$ should be one of the two values at which the externality-mitigating effect of an increase in the signal-noise variance should be exactly neutralised by the forecast-error effect. As mentioned in Chapter III, when $c_u = \tilde{\tilde{c}}_u$ the externality works in such a way as to cause unions to disregard the signal regardless of its quality. Hence changes in signal quality must have a zero effect on the union loss in this special case. However, it is worth making the additional and more subtle point that, while an increase in σ_u^2 does not induce any change in union wage-setting behaviour when $c_u = \tilde{\tilde{c}}_u$, it nevertheless does mitigate the externality, since the degree of inefficiency in wage-setting clearly does depend on signal quality. To see why this is so, note that if signal quality is very poor (i.e. σ_u^2 is large relative to σ_θ^2, so that β is close to zero), equilibrium wage-setting behaviour which ignores signals cannot be enormously harmful to union welfare, since the low β means that the efficient wage response to a non-zero signal is of small absolute magnitude in any case. Furthermore, the better is signal quality, the larger (in absolute magnitude) the efficient wage response to a given non-zero signal, and hence the more costly is it for unions to disregard that signal. It follows that when $c_u = \tilde{\tilde{c}}_u$, the inefficiency associated with keeping the wage always fixed at zero, regardless of the signal's value, must be smaller, the noisier signals happen to be: hence an increase in the signal noise variance must mitigate the externality when $c_u = \tilde{\tilde{c}}_u$, even though it does not alter unions' equilibrium wage-setting behaviour. The reason why the externality-mitigating effect exactly counterbalances the forecast-error effect when $c_u = \tilde{\tilde{c}}_u$, is that the coefficients on the variances of the anticipated and unanticipated components of the shock, i.e. on $\beta\sigma_\theta^2$ and $\beta\sigma_u^2$, in the expected union loss, as given by (44), are identical when $c_u = \tilde{\tilde{c}}_u$. Consequently, the reapportionment of the shock variance σ_θ^2 between $\beta\sigma_\theta^2$ and $\beta\sigma_u^2$, occasioned by a change in σ_u^2 does not have any repercussions for union welfare.[3]

We now turn to the $\gamma < 1$ case. Thankfully, the detailed discussions of the cases in which $\gamma \geq 1$ that have been provided in Chapter III and earlier in this Appendix, will enable us to deal relatively briefly with the $\gamma < 1$ case. Just as in the $\gamma \geq 1$ cases, when $\gamma < 1$ the sign of the impact of an increase in the signal noise variance on union welfare is ambiguous, and depends on the values taken by the other structural parameters, c_u, n, ε and α. Among the consequences of γ being less than unity are

[3] Note that $\beta\sigma_\theta^2 + \beta\sigma_u^2 = \sigma_\theta^2$.

the following. Firstly, considered as functions of c_u, $\Delta|_{\gamma<1}$ and $\Theta|_{\gamma<1}$, as given by (A.III.2.1), have a common positive vertical-axis intercept.[4] Secondly, the equation $\Theta|_{\gamma<1} = 0$ may lack real solutions: if this happens to be the case, it would then be impossible for union welfare to be reduced as a result of an improvement in signal quality, since $(\Delta|_{\gamma<1})(\Theta|_{\gamma<1}) > 0$ would be the case for all $c_u > 0$.

These first two points, together with the fact that the graph of Θ is an upright parabola, indicate a third consequence of γ being less than unity, namely that if the equation $\Theta|_{\gamma<1} = 0$ does have real solutions, it will only be when these are positive that there will arise the interesting result that $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ for a certain range of c_u values (this range will be bounded by the values of the two positive solutions). It is this situation in which $\Theta|_{\gamma<1} = 0$ has positive real solutions that is depicted below as Figure A.III.2. It is of interest to note that, unlike in the $\gamma = 1$ case, if there exists a range of c_u values for which $dE(\Omega_j^u|_{w=w_{NE}})/d\sigma_u^2 < 0$ in the $\gamma < 1$ case, the lower bound of this range of values must then be strictly positive. This is evident from the fact that the parabola of $\Theta|_{\gamma<1}$ has a vertical intercept, but note that it is also implied by the fact that $\tilde{c}_u|_{\gamma<1} < 0$, since this in turn implies that in the limit as $c_u \to 0$, it cannot be that both the externality-mitigating effect and the forecast-error effect of an increase in the signal-noise variance will be zero. The economic reason for this is that when $\gamma < 1$, employment is not completely insulated from the impact of the unanticipated component of shocks, and consequently the variance of forecast errors must be contributing to employment variability as well as to real wage variability. Thus whereas the externality-mitigating effect does tend to zero as $c_u \to 0$ in the limit, regardless of the value of γ, when $\gamma < 1$ the forecast-error effect does not tend to zero as $c_u \to 0$, and consequently when $\gamma < 1$ the forecast-error effect must be the stronger of the two below some positive c_u value.

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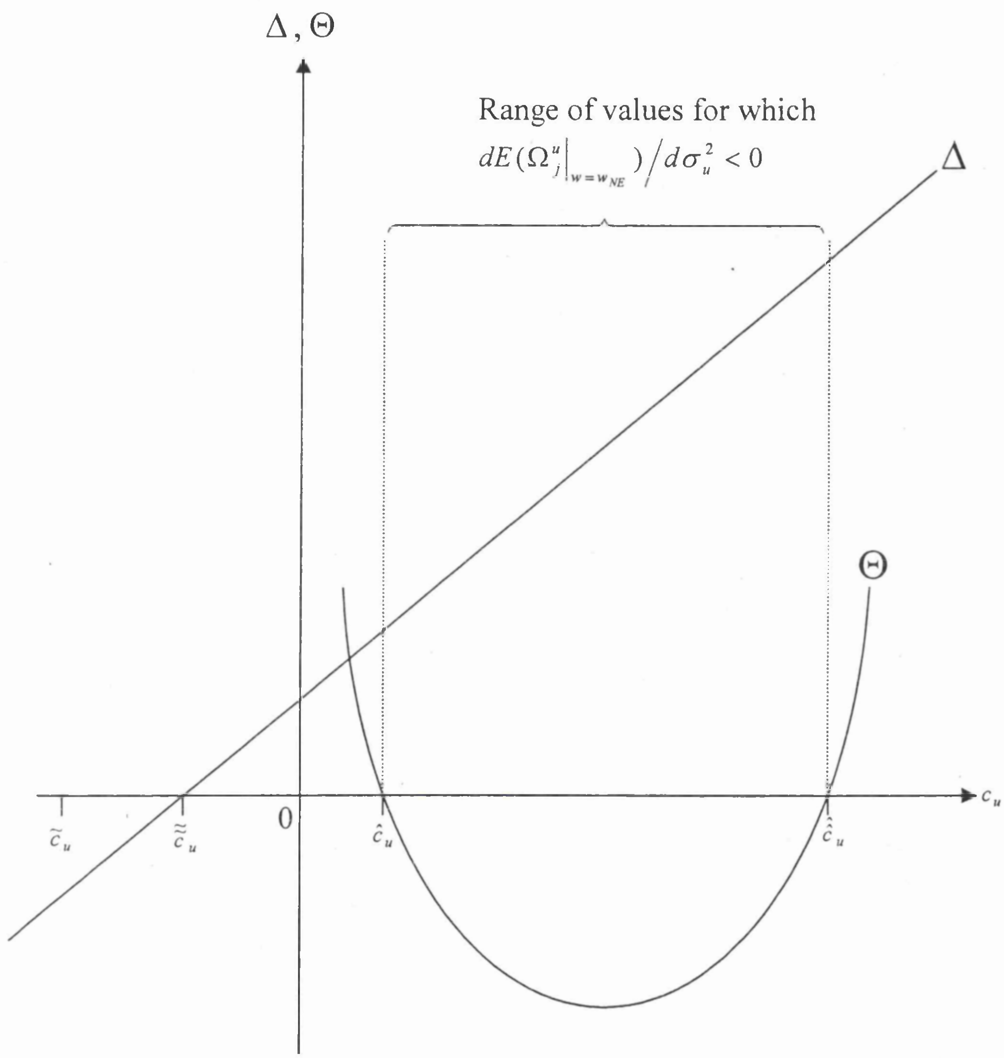
[4] Δ and Θ have the same vertical-axis intercept regardless of the value taken by γ, as is readily apparent from (A.III.2.1): $\Delta|_{c_u=0} = \Theta|_{c_u=0} = (1-\gamma)(\gamma\alpha + \varepsilon\Lambda)$.

Figure A.III.2
Identification of the sign of the impact of a marginal deterioration in signal quality on the expected union loss under the simple rule

($\gamma < 1$ case)



Notes:
1. It is assumed that $1 < n < \infty$ and $\gamma < 1$.
2. It is assumed for illustrative purposes that the equation $\Theta = 0$ has two positive real roots (i.e. that $\hat{c}_u > 0$ and $\hat{\hat{c}}_u > 0$).

Appendix III.3: The Relationship between Composite Parameter Ψ and the Central Bank's Weight Parameter c_b

As stated in equation (100) of Chapter III, Ψ denotes the following expression:

$$\Psi \equiv \frac{c_b(1-\alpha)[c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b]^3}{(n-1)c_u^2\alpha[\alpha+\varepsilon(1-\alpha)]^2[1+c_b(1-\alpha)^2]^3\Phi_b} \qquad \text{(A.III.3.1)}$$

where $\Phi_b \equiv n[1+c_b(1-\alpha)^2]-1$.

In analysing the functional relationship between Ψ and c_b we must bear in mind that Ψ has been derived on the assumption that $n>1$. Since in the limit as $c_b \to 0$ the denominator of Ψ remains strictly positive and its numerator tends to zero, it follows that $\lim_{c_b\to 0}\Psi = 0$. The limiting value of Ψ as $c_b \to \infty$ is strictly positive, as is evident from the following expression:

$$\lim_{c_b\to\infty}\Psi = \frac{\{\alpha+n\varepsilon(1-\alpha)+nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]\}^3}{n(n-1)c_u^2\alpha(1-\alpha)^4[\alpha+\varepsilon(1-\alpha)]^2} \qquad \text{(A.III.3.2)}$$

In analysing the derivative of Ψ with respect to c_b, it is useful to rewrite Ψ as follows:

$$\Psi = \frac{(1-\alpha)}{(n-1)c_u^2\alpha[\alpha+\varepsilon(1-\alpha)]^2}\left(\frac{c_b}{\Phi_b}\right)\Psi'^3 \qquad \text{(A.III.3.3)}$$

where $\Psi' \equiv \dfrac{c_b\alpha(1-\alpha)+\{\varepsilon+c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\Phi_b}{[1+c_b(1-\alpha)^2]}$.

Hence we have:

$$\frac{d\Psi}{dc_b} = \frac{(1-\alpha)}{(n-1)c_u^2\alpha[\alpha+\varepsilon(1-\alpha)]^2}\left[\left(\frac{d(c_b/\Phi_b)}{dc_b}\right)\Psi'^3+3\left(\frac{c_b}{\Phi_b}\right)\Psi'^2\left(\frac{d\Psi'}{dc_b}\right)\right] \qquad \text{(A.III.3.4)}$$

where $\dfrac{d(c_b/\Phi_b)}{dc_b} = \dfrac{(n-1)}{\Phi_b^2}$ and $\dfrac{d\Psi'}{dc_b} = \dfrac{(1-\alpha)[\alpha+\varepsilon(1-\alpha)][1+c_u(1-\alpha)^2]}{[1+c_b(1-\alpha)^2]^2}$.

Since Ψ', $d(c_b/\Phi_b)/dc_b$ and $d\Psi'/dc_b$ are all strictly positive, it follows that $d\Psi/dc_b > 0$ must be the case. This completes our demonstration of all the statements made about Ψ in Chapter III, namely that Ψ has a limiting value of zero as $c_b \to 0$, is a monotonic increasing function of c_b for all admissible values of c_b (i.e. for all $c_b > 0$), and has a positive finite asymptotic value as $c_b \to \infty$.

Appendices to Chapter V

Appendix V.1: Solutions to the Equation Implicitly Defining the Symmetric Nash Equilibrium under the Simple Rule

The total derivative of union *j*'s real wage variance with respect to x_j, evaluated for the case of symmetric indexation, $x_j = x\forall j$, is given by the following expression:

$$\left.\frac{dE(w_j - p)^2}{dx_j}\right|_{x_j = x\forall j} = \frac{-2(1-x)[n\gamma(1-\alpha)+(n-1)\alpha(1-x)][\gamma^2(1-\alpha)^2\sigma_\phi^2+\sigma_\theta^2]}{n[\gamma(1-\alpha)+\alpha(1-x)]^3}$$

(A.V.1.1a)

The counterpart expression for union *j*'s employment variance is:

$$\left.\frac{dEl_j^2}{dx_j}\right|_{x_j = x\forall j} = \frac{-2\{\gamma[\alpha+n\varepsilon(1-\alpha)]+(n-1)\varepsilon\alpha(1-x)\}[\gamma^2(1-\alpha)(1-x)\sigma_\phi^2+(1-\gamma-x)\sigma_\theta^2]}{n[\alpha+\varepsilon(1-\alpha)][\gamma(1-\alpha)+\alpha(1-x)]^3}$$

(A.V.1.1b)

Imposing the restriction that indexation is symmetric on union *j*'s first-order condition for its individually optimal x_j choice yields an equation which implicitly defines the symmetric Nash equilibrium degree of indexation x_{NE}. This first-order condition, of course, equates with zero the total derivative with respect to x_j of union *j*'s expected loss, and therefore will involve the weighted sum of (A.V.1.1a) and (A.V.1.1b), the relevant weight being c_u. Following simplification,[1] the equation which implicitly defines x_{NE} can be written as follows:

$$f(x,n,\varepsilon) \equiv Z_1\sigma_\phi^2 + Z_2\sigma_\theta^2 = 0 \qquad \text{(A.V.1.2)}$$

where:

$$Z_1 \equiv -\gamma^2(1-\alpha)(1-x)\{Z_3 + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)][n\gamma(1-\alpha)+(n-1)\alpha(1-x)]\}$$

$$Z_2 \equiv (\gamma-1+x)Z_3 - (1-x)c_u[\alpha+\varepsilon(1-\alpha)][n\gamma(1-\alpha)+(n-1)\alpha(1-x)]$$

[1] The simplifying step is to multiply both sides of union *j*'s first-order condition (with $x_j = x\forall j$ imposed) by $n[\alpha+\varepsilon(1-\alpha)][\gamma(1-\alpha)+\alpha(1-x)]^3/2$.

$Z_3 \equiv \gamma[\alpha + n\varepsilon(1-\alpha)] + (n-1)\varepsilon\alpha(1-x)$

The values of x which solve (A.V.1.2) appear below as $x_{NE,1}$ and $x_{NE,2}$:[2]

$$x_{NE,1} = \frac{K_1\sigma_\phi^2 + K_2\sigma_\theta^2 - K_5^{1/2}}{2(n-1)\alpha K_3} \qquad \text{(A.V.1.3a)}$$

$$x_{NE,2} = \frac{K_1\sigma_\phi^2 + K_2\sigma_\theta^2 + K_5^{1/2}}{2(n-1)\alpha K_3} \qquad \text{(A.V.1.3b)}$$

where:

$K_1 \equiv \gamma^2(1-\alpha)\big[\gamma\alpha + [(n-1)\alpha + \Lambda]\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\big]$

$K_2 \equiv \gamma\alpha[1-\varepsilon(n-1)] + [(n-1)\alpha + \Lambda]\{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}$

$K_3 \equiv \gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2$

$K_4 \equiv \gamma^2(1-\alpha)\big[\gamma\alpha + \Lambda\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\big]\sigma_\phi^2 + \{(1-\gamma)(\gamma\alpha + \varepsilon\Lambda) + \Lambda c_u[\alpha + \varepsilon(1-\alpha)]\}\sigma_\theta^2$

$K_5 \equiv (K_1\sigma_\phi^2 + K_2\sigma_\theta^2)^2 - 4(n-1)\alpha K_3 K_4$

$\Lambda \equiv (n-1)\alpha + n\gamma(1-\alpha)$

Our objective is to establish whether in the limit as $n \to 1$, and also in the limit as $\varepsilon \to \infty$, the right-hand sides of (A.V.1.3a) and (A.V.1.3b) are equal to the efficient solution for the simple rule, as given by equation (22) of Chapter V. We also need to ascertain whether in the limit as $n \to \infty$ expressions (A.V.1.3a) and (A.V.1.3b) are equal to $x_{NE,\,atomistic\,case}$, as given by equation (35) of Chapter V. Commencing therefore with the values of $x_{NE,1}$ and $x_{NE,2}$ in the limit as $n \to 1$, it is noteworthy for the derivation of these values that:

$$\lim_{n\to 1} K_5^{1/2} = \lim_{n\to 1} (K_1\sigma_\phi^2 + K_2\sigma_\theta^2) = \gamma[\alpha + \varepsilon(1-\alpha)]\{\gamma^2(1-\alpha)[1 + c_u(1-\alpha)^2]\sigma_\phi^2 + [1 + c_u(1-\alpha)]\sigma_\theta^2\} \qquad \text{(A.V.1.4)}$$

The limit as $n \to 1$ of the numerator of $x_{NE,1}$ is consequently zero, and hence:

[2] Throughout these appendices to Chapter V, $r^{1/2}$ denotes the positive square root of r, and $-r^{1/2}$ its negative counterpart.

$$\lim_{n\to 1} x_{NE,1} = \frac{0}{0} \qquad \text{(A.V.1.5a)}$$

The limit as $n \to 1$ of the numerator of $x_{NE,2}$ is non-zero however, and hence:

$$\lim_{n\to 1} x_{NE,2} = \frac{2[\lim_{n\to 1}(\mathrm{K}_1\sigma_\phi^2 + \mathrm{K}_2\sigma_\theta^2)]}{0} = \infty \qquad \text{(A.V.1.5b)}$$

It is clear from (A.V.1.5b) that $x_{NE,2}$ does not meet meet one of the criteria set out in Chapter V which any solution to (A.V.1.2) must satisfy if it is to be regarded as a symmetric Nash equilibrium, namely that $\lim_{n\to 1} x_{NE} = x^*$. It remains to be seen whether $x_{NE,1}$ meets this criterion, however, since (A.V.1.5a) is inconclusive in this respect and necessitates the use of L'Hôpital's rule. The derivative of the numerator of $x_{NE,1}$ with respect to n is:

$$\frac{\partial(x_{NE,1}\,\text{Numerator})}{\partial n} = \frac{\partial \mathrm{K}_1}{\partial n}\sigma_\phi^2 + \frac{\partial \mathrm{K}_2}{\partial n}\sigma_\theta^2 - \left\{ (\mathrm{K}_1\sigma_\phi^2 + \mathrm{K}_2\sigma_\theta^2)\left(\frac{\partial \mathrm{K}_1}{\partial n}\sigma_\phi^2 + \frac{\partial \mathrm{K}_2}{\partial n}\sigma_\theta^2\right) - 2\alpha \mathrm{K}_3\left[\mathrm{K}_4 + (n-1)\frac{\partial \mathrm{K}_4}{\partial n}\right] \right\}\mathrm{K}_5^{-1/2} \qquad \text{(A.V.1.6)}$$

Since $\lim_{n\to 1} \mathrm{K}_5^{-1/2} = \lim_{n\to 1} (\mathrm{K}_1\sigma_\phi^2 + \mathrm{K}_2\sigma_\theta^2)^{-1}$, equation (A.V.1.6) implies that:

$$\lim_{n\to 1} \frac{\partial(x_{NE,1}\,\text{Numerator})}{\partial n} = \frac{2\alpha \mathrm{K}_3(\lim_{n\to 1}\mathrm{K}_4)}{\lim_{n\to 1}(\mathrm{K}_1\sigma_\phi^2 + \mathrm{K}_2\sigma_\theta^2)} \qquad \text{(A.V.1.7)}$$

where:

$$\lim_{n\to 1}\mathrm{K}_4 = \gamma[\alpha + \varepsilon(1-\alpha)]\{\gamma^2(1-\alpha)[1 + c_u(1-\alpha)^2]\sigma_\phi^2 + [1 - \gamma + c_u(1-\alpha)]\sigma_\theta^2\}$$

(A.V.1.8)

The derivative of the denominator of $x_{NE,1}$ with respect to n is independent of n:

$$\frac{\partial(x_{NE,1}\,\text{Denominator})}{\partial n} = 2\alpha \mathrm{K}_3 \tag{A.V.1.9}$$

Applying L'Hôpital's rule then yields the following result:

$$\lim_{n\to 1} x_{NE,1} = \frac{\lim_{n\to 1}[\partial(x_{NE,1}\,\text{Numerator})/\partial n]}{\lim_{n\to 1}[\partial(x_{NE,1}\,\text{Denominator})/\partial n]} = \frac{\lim_{n\to 1}\mathrm{K}_4}{\lim_{n\to 1}(\mathrm{K}_1\sigma_\phi^2 + \mathrm{K}_2\sigma_\theta^2)} \tag{A.V.1.10}$$

Substituting the right-hand sides of (A.V.1.4) and (A.V.1.8) into (A.V.1.10) and simplifying, gives the result reported as equation (37a) of Chapter V:

$$\lim_{n\to 1} x_{NE} = x^* \tag{A.V.1.11}$$

The limit as $n \to \infty$ of $x_{NE,1}$ and its counterpart expression for $x_{NE,2}$ are straightforward to derive:

$$\lim_{n\to\infty} x_{NE,1} = 1 - \frac{\gamma\varepsilon\,\sigma_\theta^2}{[\gamma^2(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\sigma_\phi^2 + \{\varepsilon + c_u[\alpha+\varepsilon(1-\alpha)]\}\sigma_\theta^2]} \tag{A.V.1.12a}$$

$$\lim_{n\to\infty} x_{NE,2} = \frac{\alpha + \gamma(1-\alpha)}{\alpha} \tag{A.V.1.12b}$$

Since the right-hand side of (A.V.1.12a) is identical to $x_{NE,\,atomistic\,case}$, as given by equation (35) of Chapter V, it is clear that $x_{NE,1}$ satisfies the second of our criteria for determining whether or not a solution to (A.V.1.2) is a legitimate Nash equilibrium. Since the right-hand side of (A.V.1.12b) differs from that of (A.V.1.12a), and, furthermore, is known to be an inadmissible value for x under the simple rule, it is also clear that $x_{NE,2}$ does not meet this requirement.

Turning to the limit as $\varepsilon \to \infty$ of $x_{NE,1}$ and the corresponding limit for $x_{NE,2}$, these are found to be as follows:

$$\lim_{\varepsilon \to \infty} x_{NE,1} = x^* \tag{A.V.1.13a}$$

$$\lim_{\varepsilon \to \infty} x_{NE,2} = 1 + \frac{n\gamma(1-\alpha)}{(n-1)\alpha} \tag{A.V.1.13b}$$

Of our two candidates for the status of symmetric Nash equilibrium, therefore, only $x_{NE,1}$ has the desirable property that its value in the limit as $\varepsilon \to \infty$ is equal to the efficient degree of indexation. Hence $x_{NE,2}$ does not satisfy any of our three criteria for deciding which solutions to (A.V.1.2) can be considered valid in economic terms. Furthermore, $x_{NE,2}$ is also unsatisfactory in two further respects. Firstly, unlike $x_{NE,1}$, $x_{NE,2}$ does not equal the efficient degree of indexation when c_u is assigned (or approaches in the limit) either one of the two extremes of its set of admissible values. Economic reasoning suggests that in such circumstances the externality should not be present, but only $x_{NE,1}$ is satisfactory in the sense that it confirms this intuition:

$$x_{NE,1}\Big|_{c_u=0} = 1 - \frac{\gamma\sigma_\theta^2}{[\gamma^2(1-\alpha)\sigma_\phi^2 + \sigma_\theta^2]} = x^*\Big|_{c_u=0} \tag{A.V.1.14a}$$

$$x_{NE,2}\Big|_{c_u=0} = 1 + \frac{\gamma[\alpha + n\varepsilon(1-\alpha)]}{(n-1)\varepsilon\alpha} \tag{A.V.1.14b}$$

$$\lim_{c_u \to \infty} x_{NE,1} = \lim_{c_u \to \infty} x^* = 1 \tag{A.V.1.15a}$$

$$\lim_{c_u \to \infty} x_{NE,2} = 1 + \frac{n\gamma(1-\alpha)}{(n-1)\alpha} \tag{A.V.1.15b}$$

Secondly, $x_{NE,2}$, unlike $x_{NE,1}$, does not exhibit the desirable property that in the extreme case in which productivity shocks are absent, i.e. $\sigma_\theta^2 = 0$, its value should be unity.

$$x_{NE,1}\Big|_{\sigma_\theta^2=0} = 1 \tag{A.V.1.16a}$$

$$x_{NE,2}\Big|_{\sigma_\theta^2=0} = \frac{\gamma\alpha + \Lambda\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}}{(n-1)\alpha\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}} \qquad \text{(A.V.1.16b)}$$

These findings regarding $x_{NE,2}$ imply that only $x_{NE,1}$ can be considered an economically valid solution to the model for cases in which n is such that $1 < n < \infty$, and consequently $x_{NE,1}$ is the solution presented and discussed as x_{NE} in Section 3 of Chapter V.

Our sole remaining task in this Appendix is to prove that $x_{NE,1}$ is increasing in n and decreasing in ε as intuition suggests it should be. As mentioned in the main text, rigorous proofs that $\partial x_{NE,1}/\partial n > 0$ and $\partial x_{NE,1}/\partial \varepsilon < 0$ hold for all possible combinations of admissible parameter values have not been found. However, proofs that these inequalities do hold in the absence of velocity shocks are rather easier to devise, and we therefore confine ourselves here to demonstrating that $\partial(x_{NE,1}|_{\sigma_\phi^2=0})/\partial n > 0$ and $\partial(x_{NE,1}|_{\sigma_\phi^2=0})/\partial \varepsilon < 0$ are true. The method of proof makes use of the implicit function theorem, which when applied to (A.V.1.2), the equation implicitly defining $x_{NE,1}$ (and $x_{NE,2}$), implies the following:

$$\frac{dx}{dn} = -\frac{\partial f/\partial n}{\partial f/\partial x} \qquad \text{(A.V.1.17)}$$

$$\frac{dx}{d\varepsilon} = -\frac{\partial f/\partial \varepsilon}{\partial f/\partial x} \qquad \text{(A.V.1.18)}$$

For notational simplicity, f has been used in these expressions as shorthand for $f(x,n,\varepsilon)$. It is apparent from (A.V.1.17) that if $\partial f/\partial n$ differs in sign from $\partial f/\partial x$, $dx/dn > 0$ will then be the case, implying $\partial x_{NE,1}/\partial n > 0$ as intuition suggests.[3]

[3] To avoid possibly confusing the reader, we stress that as items of notation dx/dn and $\partial x_{NE,1}/\partial n$ have essentially the same meaning: both denote the derivative of x with respect to n, with all other structural parameters treated as fixed, but taking into account any influence of n on x working via the composite parameter Λ which is a function of n. (Note, however, that unlike $\partial x_{NE,1}/\partial n$, dx/dn is not specific about which of the two x solutions to (A.V.1.2) is being differentiated.) The reason for using the notation dx/dn at this point is to emphasise the fact that it has been derived by means of the implicit function theorem.

($dx/dn > 0$ also indicates that $\partial x_{NE,2}/\partial n > 0$, of course, but we henceforth disregard what our findings in respect of $f(x,n,\varepsilon)$ imply about the economically invalid solution, $x_{NE,2}$.) Similarly, (A.V.1.18) tells us that if $\partial f/\partial\varepsilon$ and $\partial f/\partial x$ have the same sign, $dx/d\varepsilon < 0$ is implied, which would confirm our intuition that $\partial x_{NE,1}/\partial\varepsilon < 0$.

Proceeding therefore to apply this method of proof, the relevant partial derivatives are presented below:

$$\frac{\partial f}{\partial x} = Z_4\sigma_\phi^2 + Z_5\sigma_\theta^2 \qquad \text{(A.V.1.19)}$$

where:

$Z_4 \equiv \gamma^2(1-\alpha)\{Z_6 + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)][n\gamma(1-\alpha)+2(n-1)\alpha(1-x)]\}$

$Z_5 \equiv (n-1)\varepsilon[\gamma+2\alpha(1-\gamma-x)]+[\alpha+\varepsilon(1-\alpha)]\{\gamma+c_u[n\gamma(1-\alpha)+2(n-1)\alpha(1-x)]\}$

$Z_6 \equiv \gamma[\alpha+n\varepsilon(1-\alpha)]+2(n-1)\varepsilon\alpha(1-x)]$

$$\frac{\partial f}{\partial n} = [\gamma(1-\alpha)+\alpha(1-x)](Z_7\sigma_\phi^2 + Z_8\sigma_\theta^2) \qquad \text{(A.V.1.20)}$$

where:

$Z_7 \equiv -\gamma^2(1-\alpha)(1-x)\{\varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}$

$Z_8 \equiv \gamma\varepsilon - (1-x)\{\varepsilon + c_u[\alpha+\varepsilon(1-\alpha)]\}$

$$\frac{\partial f}{\partial \varepsilon} = -[n\gamma(1-\alpha)+(n-1)\alpha(1-x)](Z_9\sigma_\phi^2 + Z_{10}\sigma_\theta^2) \qquad \text{(A.V.1.21)}$$

where:

$Z_9 \equiv \gamma^2(1-\alpha)(1-x)[1+c_u(1-\alpha)^2]$

$Z_{10} \equiv (1-x)[1+c_u(1-\alpha)]-\gamma$

In perusing the foregoing expressions, it should be borne in mind that while our $x_{NE,1}$ and $x_{NE,2}$ expressions ((A.V.1.3a) and (A.V.1.3b) respectively) have been derived on the assumption that n is such that $1 < n < \infty$, the partial derivatives

(A.V.1.19) to (A.V.1.21) can be evaluated for $n = 1$.[4] Before making use of these derivatives (with σ_ϕ^2 set to zero, of course), we must first of all demonstrate that $x_{NE,1}$ is a continuous function of *n* and ε for all admissible values of these parameters (i.e. for $1 < n < \infty$ and $1 < \varepsilon < \infty$). It is necessary to do so since we have yet to show that the inequality $x^* < x_{NE,1} < x_{NE,\text{atomistic case}}$ holds for $1 < n < \infty$. Note that since it has already been established that $x^* < x_{NE,\text{atomistic case}}$, and that $\lim_{n \to 1} x_{NE,1} = x^*$, all that is required to prove rigorously that $x^* < x_{NE,1} < x_{NE,\text{atomistic case}}$ is a demonstration that $x_{NE,1}$ is a continuous function of *n*. Perusal of (A.V.1.3a) reveals that K_1, K_2, K_5 and its denominator are all polynomials in *n* and ε, which implies that all these component terms must be continuous in *n* and ε. Since the denominator is strictly positive for all admissible *n* (i.e. for $1 < n < \infty$), it follows that a necessary and sufficient condition for $x_{NE,1}$ to be continuous in *n* and ε is for K_5 to be non-negative. Thankfully, it is straightforward to demonstrate that $K_5 > 0$ is the case. Expansion of the constituent terms of K_5, together with subsequent rearrangement, reveals that K_5 can alternatively be expressed as the sum of three terms, each of which is unambiguously positive:

$$K_5 \equiv H_1\sigma_\phi^4 + H_2\sigma_\theta^4 + H_3\sigma_\phi^2\sigma_\theta^2 \qquad \text{(A.V.1.22)}$$

where:

$$H_1 \equiv \gamma^6(1-\alpha)^2[\alpha + n(1-\alpha)\delta_1]^2$$

$$H_2 \equiv \gamma^2\left\{ \alpha^2[1-\varepsilon(n-1)]^2 + \delta_2\left[\delta_2 n^2(1-\alpha)^2 + 2\alpha\{2(n-1)\alpha + n(1-\alpha)[1+\varepsilon(n-1)]\}\right] \right\}$$

$$H_3 \equiv 2\gamma^4(1-\alpha)\left\{ \alpha^2\{1+(n-1)c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\} + \alpha\delta_1[n-\alpha+(n-1)n\varepsilon(1-\alpha)] + n(1-\alpha)\delta_2[\alpha+n(1-\alpha)\delta_1] \right\}$$

$$\delta_1 \equiv \varepsilon + c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]$$

$$\delta_2 \equiv \varepsilon + c_u[\alpha+\varepsilon(1-\alpha)]$$

[4] This is because the derivation of $x_{NE,1}$ and $x_{NE,2}$ involves dividing both sides of an equation by $n-1$, and this step is performed after the preliminary derivation of $f(x,n,\varepsilon) = 0$, as given by (A.V.1.2).

Equipped with the knowledge that $x^* < x_{NE,1} < x_{NE,\text{atomistic case}}$, we now proceed to implement our strategy of examining the conditions which must hold for $\partial f/\partial x$ and $\partial f/\partial n$ to differ in sign, and hence imply, via (A.V.1.17) that $dx/dn > 0$ is the case. Note that the known facts regarding $x_{NE,1}$ imply that, with $x = x_{NE,1}\big|_{\sigma_\phi^2=0}$, $1-x$ will be positive wherever it occurs in expressions (A.V.1.19) to (A.V.1.21).[5] Furthermore, coefficients Z_4, Z_7 and Z_9 need not be considered, since our attention here is directed to the $\sigma_\phi^2 = 0$ case. It is clear that $Z_5 > 0$ is necessary and sufficient to ensure $(\partial f/\partial x)\big|_{\sigma_\phi^2=0} > 0$, and that similarly $Z_8 < 0$ is necessary and sufficient for $(\partial f/\partial n)\big|_{\sigma_\phi^2=0} < 0$. Manipulating Z_5, we find that $(\partial f/\partial x)\big|_{\sigma_\phi^2=0} > 0$ if (and only if):

$$\gamma\{(n-1)\varepsilon + [\alpha + \varepsilon(1-\alpha)][1 + nc_u(1-\alpha)]\} > 2(n-1)\alpha[\gamma\varepsilon - \{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}(1 - x_{NE,1})] \tag{A.V.1.23}$$

where $x_{NE,1}$ has been substituted for x to emphasise the fact that it is this solution that is of interest to us. The term $1 - x_{NE,1}$ is evidently at its smallest, and the right-hand side of (A.V.1.23) consequently at its highest possible numerical value in the limiting case in which $n \to \infty$. Obtaining $\lim_{n\to\infty}(x_{NE,1}\big|_{\sigma_\phi^2=0})$ by setting σ_ϕ^2 to zero in (A.V.1.12a), and substituting this expression into (A.V.1.23) yields the following sufficient condition for $(\partial f/\partial x)\big|_{\sigma_\phi^2=0} > 0$ for all n such that $1 < n < \infty$:

$$\gamma\{(n-1)\varepsilon + [\alpha + \varepsilon(1-\alpha)][1 + nc_u(1-\alpha)]\} > 0 \tag{A.V.1.24}$$

It is clear that this condition is unambiguously satisfied.

We now turn to the aforementioned necessary and sufficient condition for $(\partial f/\partial n)\big|_{\sigma_\phi^2=0} < 0$, namely $Z_8 < 0$. Manipulating Z_8, we find that $(\partial f/\partial n)\big|_{\sigma_\phi^2=0} < 0$ holds if and only if:

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[5] The fact that $x^* < x_{NE,1} < x_{NE,\text{atomistic case}}$ also implies that both $\gamma(1-\alpha) + \alpha(1-x)$ in (A.V.1.20) and $n\gamma(1-\alpha) + (n-1)\alpha(1-x)$ in (A.V.1.21) are strictly positive.

$$\gamma\varepsilon < \{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}(1 - x_{NE,1})\big|_{\sigma_\phi^2=0} \qquad \text{(A.V.1.25)}$$

$(1 - x_{NE,1})\big|_{\sigma_\phi^2=0}$ is at its smallest in the limit as $n \to \infty$. It therefore follows that if (A.V.1.25) holds for this extreme case it must also hold for $1 < n < \infty$. Noting from (A.V.1.12a) that $\lim_{n\to\infty}(1 - x_{NE,1})\big|_{\sigma_\phi^2=0} = \gamma\varepsilon/\{\varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]\}$, and that $(1 - x_{NE,1})\big|_{\sigma_\phi^2=0,1<n<\infty} > \lim_{n\to\infty}(1 - x_{NE,1})\big|_{\sigma_\phi^2=0}$, it is evident that (A.V.1.25) must hold for $1 < n < \infty$, and therefore that $(\partial f/\partial n)\big|_{\sigma_\phi^2=0} < 0$.

This finding, together with our earlier finding that $(\partial f/\partial x)\big|_{\sigma_\phi^2=0} > 0$, therefore implies the following result:

$$\frac{\partial(x_{NE,1}\big|_{\sigma_\phi^2=0})}{\partial n} > 0 \qquad \text{(A.V.1.26)}$$

Our next task is to establish the sign of $\partial(x_{NE,1}\big|_{\sigma_\phi^2=0})\big/\partial\varepsilon$. Equation (A.V.1.21) implies that a necessary and sufficient condition for $(\partial f/\partial\varepsilon)\big|_{\sigma_\phi^2=0} > 0$ is $Z_{10} < 0$. This condition can alternatively be written as follows:

$$x_{NE,1} > \frac{[1 - \gamma + c_u(1-\alpha)]}{[1 + c_u(1-\alpha)]} \qquad \text{(A.V.1.27)}$$

Since the right-hand side of (A.V.1.27) is equal to $x^*\big|_{\sigma_\phi^2=0}$, it follows that (A.V.1.27) is true, and hence that $(\partial f/\partial\varepsilon)\big|_{\sigma_\phi^2=0} > 0$. This result, in conjunction with our earlier finding that $(\partial f/\partial x)\big|_{\sigma_\phi^2=0} > 0$, in turn implies the following:

$$\frac{\partial(x_{NE,1}\big|_{\sigma_\phi^2=0})}{\partial\varepsilon} < 0 \qquad \text{(A.V.1.28)}$$

For the version of the model without velocity shocks, we have therefore obtained two results, namely (A.V.1.26) and (A.V.1.28), which confirm our intuitions regarding the nature of the relationships between $x_{NE,1}$ and the parameters n and ε.

Appendix V.2: The Slope of the Individual Union's Locus of Possible Outcomes under the Simple Rule

Union *j*'s real wage variance and employment variance, as functions of x_j and x, are given by expressions (19a) and (19b) of Chapter V. The partial derivatives of these expressions with respect to x_j are respectively as follows:

$$\frac{\partial E(w_j - p)^2}{\partial x_j} = \frac{-2(1-x_j)[\gamma^2(1-\alpha)^2\sigma_\phi^2 + \sigma_\theta^2]}{[\gamma(1-\alpha)+\alpha(1-x)]^2} \qquad \text{(A.V.2.1)}$$

$$\frac{\partial El_j^2}{\partial x_j} = \frac{-2(\xi_1\sigma_\phi^2 + \xi_2\sigma_\theta^2)}{[\alpha+\varepsilon(1-\alpha)]^2[\gamma(1-\alpha)+\alpha(1-x)]^2} \qquad \text{(A.V.2.2)}$$

where:

$\xi_1 \equiv \gamma^2\varepsilon(1-\alpha)[\alpha(1-x)+\varepsilon(1-\alpha)(1-x_j)]$

$\xi_2 \equiv \varepsilon\{\gamma - \varepsilon(1-x_j)+(\varepsilon-1)[\gamma(1-\alpha)+\alpha(1-x)]\}$

The slope of the atomistic union's locus of possible outcomes is given by the ratio of the above partial derivatives:

$$\frac{dEl_j^2}{dE(w_j - p)^2} = \frac{\partial El_j^2/\partial x_j}{\partial E(w_j-p)^2/\partial x_j} \qquad \text{(A.V.2.3)}$$

Substituting (A.V.2.1) and (A.V.2.2) into (A.V.2.3) yields:

$$\frac{dEl_j^2}{dE(w_j - p)^2} = \frac{(\xi_1\sigma_\phi^2 - \xi_2\sigma_\theta^2)}{(1-x_j)[\alpha+\varepsilon(1-\alpha)]^2[\gamma^2(1-\alpha)^2\sigma_\phi^2+\sigma_\theta^2]} \qquad \text{(A.V.2.4)}$$

By substituting x^* and x_{NE}, as given by equations (22) and (35) of Chapter V, for x in (A.V.2.4), the slope of atomistic union *j*'s locus of possible outcomes given efficient and equilibrium indexation at the aggregate level may be obtained. We omit these expressions on account of their length, and because our principal interest resides in any case with the individually optimal choice of indexation, denoted x_j^{**}, implied by each of these slopes. The optimal choice, of course, is that which ensures union j's locus slope, (A.V.2.4), is equal to the slope of each of its isoloss curves, $-c_u$. Given efficient indexation at the aggregate level, the x_j which solves the following equation:

$$\left.\frac{dEl_j^2}{dE(w_j-p)^2}\right|_{x=x^*}=-c_u \qquad \text{(A.V.2.5)}$$

is found in general to exceed x^*:

$$\left.x_j^{**}\right|_{x=x^*}=x^*+\frac{c_u\gamma\alpha[\alpha+\varepsilon(1-\alpha)]\sigma_\theta^2}{\{\varepsilon^2+c_u[\alpha+\varepsilon(1-\alpha)]^2\}\{\gamma^2(1-\alpha)[1+c_u(1-\alpha)^2]\sigma_\phi^2+[1+c_u(1-\alpha)]\sigma_\theta^2\}}$$

(A.V.2.6)

It is clear that the second term on the right-hand side of (A.V.2.6) is zero only when certain parameters take extreme values which are known to prevent the externality from arising, namely $c_u=0$, the limit cases in which $c_u\to\infty$ and/or $\varepsilon\to\infty$, and the cases for which $\sigma_\theta^2/\sigma_\phi^2=0$.

Given equilibrium indexation at the aggregate level, however, we find that the point on union j's resulting locus of individual possible outcomes which is brought about by setting $x_j=x_{NE}$ is the tangency point between the locus and the isoloss map. In other words, the x_j which solves the following equation:

$$\left.\frac{dEl_j^2}{dE(w_j-p)^2}\right|_{x=x_{NE}}=-c_u \qquad \text{(A.V.2.7)}$$

is the Nash equilibrium degree of indexation itself:

$$\left.x_j^{**}\right|_{x=x_{NE}}=x_{NE} \qquad \text{(A.V.2.8)}$$

Appendix V.3: Non-Equilibrium Solutions to the Equation Implicitly Defining the Symmetric Nash Equilibrium under Discretion

As stated in Section 3.4 of Chapter V, union *j*'s first-order condition for its individually optimal degree of indexation, with other unions' indexation choices taken as given, is obtained by combining (33) with the derivatives with respect to x_j of (54a) and (54b). Note that this first-order condition is derived on the assumption that $n > 1$. Imposing on this first-order condition the restriction that indexation be symmetric, $x_j = x \forall j$, and simplifying[6], the equation which implicitly defines x_{NE} is found to be:

$$2c_s^2\alpha(1-\alpha)^2 + \{c_s\varepsilon(1-\alpha) - (1-x)^2 c_u[\alpha + \varepsilon(1-\alpha)]\}[(n-1)(1-x)^2 + (n+1)c_s(1-\alpha)^2] = 0 \tag{A.V.3.1}$$

There are four solutions to this equation, two of which are the equilibria, $x_{NE,1}$ and $x_{NE,2}$, presented as expressions (63a) and (63b) in Section 3.4 of Chapter V. The two non-real solutions are:[7]

$$x_3 = 1 - \left\{ \frac{c_s(1-\alpha)\{(n-1)\varepsilon - (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} - \chi_1^{1/2}}{2(n-1)c_u[\alpha + \varepsilon(1-\alpha)]} \right\}^{1/2} \tag{A.V.3.2a}$$

$$x_4 = 1 + \left\{ \frac{c_s(1-\alpha)\{(n-1)\varepsilon - (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} - \chi_1^{1/2}}{2(n-1)c_u[\alpha + \varepsilon(1-\alpha)]} \right\}^{1/2} \tag{A.V.3.2b}$$

where:

$$\chi_1 \equiv c_s^2(1-\alpha)^2 \left\{ \{(n-1)\varepsilon + (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}^2 + 8(n-1)c_u\alpha[\alpha + \varepsilon(1-\alpha)] \right\}$$

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[6] The simplifying step is to multiply both sides by $n[\alpha + \varepsilon(1-\alpha)][(1-x)^2 + c_s(1-\alpha)^2]^3 / 2(1-x)\sigma_\theta^2$. Note that this step loses the fifth solution to the equation which results when $x_j = x$ is imposed on union *j*'s first-order condition with respect to x_j, namely $x = 1$.

[7] To eliminate entirely the potential for confusion, we emphasise that throughout this appendix $\chi_1^{1/2}$ is to be understood as denoting only the positive square root of χ_1, with $-\chi_1^{1/2}$ denoting the negative square root of χ_1.

The necessary and sufficient condition for x_3 and x_4 to be non-real is :

$$\chi_1^{1/2} > c_s(1-\alpha)\{(n-1)\varepsilon - (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} \qquad \text{(A.V.3.3)}$$

or, after dividing both sides by $c_s(1-\alpha)$,

$$[(a+b)^2 + c]^{1/2} > a - b \qquad \text{(A.V.3.4)}$$

where: $a \equiv (n-1)\varepsilon$

$b \equiv (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]$

$c \equiv 8(n-1)c_u\alpha[\alpha + \varepsilon(1-\alpha)]$

Since $n > 1$ has been assumed in deriving (A.V.3.1), *a*, *b* and *c* are all strictly positive. It follows immediately from this fact that the following inequality must hold:

$$(a+b)^2 + c > (a-b)^2 \qquad \text{(A.V.3.5)}$$

The final step in the proof that x_3 and x_4 are non-real makes use of the fact that for any positive real number *r*, the derivative of its square root, $r^{1/2}$, with respect to *r* is positive: $dr^{1/2}/dr = r^{-1/2}/2 > 0$. Combining this fact with our knowledge that (A.V.3.5) holds necessarily, it follows directly that (A.V.3.4) must also hold necessarily, and hence it must be the case that x_3 and x_4 are both non-real.

There is a third real solution to the equation which results when $x_j = x$ is imposed on union *j*'s first-order condition, namely $x = 1$. In other words, given that every other union sets its indexation parameter at unity (i.e. given $x_k = 1 \forall k \neq j$), union *j* can satisfy its first-order condition for a minimum by also choosing to index fully (i.e. by setting $x_j = 1$). The question arises, of course, whether setting $x_j = 1$ when $x_k = 1 \forall k \neq j$ satisfies union *j*'s second-order condition for a minimum. To answer this we need to ascertain the sign of the second derivative, with respect to x_j, of union *j*'s expected loss given $x_k = 1 \forall k \neq j$. Taking into account the fact that, with non-atomistic unions, $x = (1/n)\sum_{k=1}^{n} x_k = (1/n)\left(x_j + \sum_{\substack{k=1 \\ k \neq j}}^{n} x_k \right)$, it follows that, given $x_k = 1 \forall k \neq j$, the aggregate degree of indexation is:

$$x\big|_{x_k = 1 \forall k \neq j} = \frac{x_j + n - 1}{n} \qquad \text{(A.V.3.6)}$$

Setting x equal to this expression in equation (60) of Chapter V yields union j's expected loss under discretion, given $x_k = 1 \forall k \neq j$:

$$E(\Omega_j^u\big|_{x_k = 1 \forall k \neq j}) = \frac{\sigma_\theta^2}{[\alpha + \varepsilon(1-\alpha)]^2[(1-x_j)^2 + n^2 c_s (1-\alpha)^2]^2}\Big[\{(n-1)\varepsilon(1-x_j)^2 - n^2 c_s (1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}^2 + n^2 c_u [\alpha + \varepsilon(1-\alpha)]^2 (1-x_j)^4 \Big] \qquad \text{(A.V.3.7)}$$

We shall refrain from setting out the first and second derivatives of (A.V.3.7) with respect to x_j, since it suffices for our purpose here merely to state the second derivative of (A.V.3.7) with respect to x_j evaluated for $x_j = 1$:

$$\left.\frac{d^2 E(\Omega_j^u\big|_{x_k = 1 \forall k \neq j})}{dx_j^2}\right|_{x_j = 1} = \frac{-4[\alpha + n\varepsilon(1-\alpha)]\sigma_\theta^2}{n^2 c_s (1-\alpha)^4 [\alpha + \varepsilon(1-\alpha)]} \qquad \text{(A.V.3.8)}$$

This expression is unambiguously negative for all $n \geq 1$, indicating that when $x_k = 1 \forall k \neq j$, setting $x_j = 1$ locally maximises union j's expected loss. An immediate implication of this finding is that $x = 1$ cannot be a symmetric Nash equilibrium.

Appendix V.4: The Relationship between the Symmetric Nash Equilibrium Degrees of Indexation under Discretion and the Parameters n and ε

The symmetric Nash equilibria for cases such that $1 < n < \infty$ are given by equations (63a) and (63b) of Chapter V, and these expressions are stated again at this point for convenience:

$$x_{NE,1} = 1 - \Xi^{1/2} \qquad \text{(A.V.4.1a)}$$

$$x_{NE,2} = 1 + \Xi^{1/2} \qquad \text{(A.V.4.1b)}$$

where $\Xi \equiv \dfrac{c_s(1-\alpha)\{(n-1)\varepsilon - (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} + \chi_1^{1/2}}{2(n-1)c_u[\alpha + \varepsilon(1-\alpha)]}$,

and where χ_1 is as stated following (A.V.3.2b) in Appendix V.3.

Our first task is to determine the limits of $x_{NE,1}$ and $x_{NE,2}$ as $n \to 1$. Since $\lim_{n\to 1} \chi_1^{1/2} = 2c_s c_u (1-\alpha)^2[\alpha + \varepsilon(1-\alpha)]$, it is clear that $\lim_{n\to 1} \Xi = 0/0$, and hence that the limits of $x_{NE,1}$ and $x_{NE,2}$ as $n \to 1$ are undefined. We are therefore compelled to have recourse to L'Hôpital's rule. The derivative of the numerator of Ξ with respect to n is:

$$\frac{\partial(\Xi\,\text{Numerator})}{\partial n} = c_s(1-\alpha)\{\varepsilon - c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} + \frac{\chi_1^{-1/2}}{2}\left(\frac{\partial \chi_1}{\partial n}\right) \qquad \text{(A.V.4.2)}$$

where:

$$\frac{\partial \chi_1}{\partial n} = 2c_s^2(1-\alpha)^2\Big[\ \{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\}\{(n-1)\varepsilon + (n+1)c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} + 4c_u\alpha[\alpha + \varepsilon(1-\alpha)]\ \Big] \qquad \text{(A.V.4.3)}$$

It therefore follows that:

$$\lim_{n\to 1}\frac{\partial(\Xi\,\text{Numerator})}{\partial n} = 2c_s[\alpha + \varepsilon(1-\alpha)] \qquad \text{(A.V.4.4)}$$

The derivative of the denominator of Ξ with respect to n is independent of n:

$$\frac{\partial(\Xi\,\text{Denominator})}{\partial n} = 2c_u[\alpha + \varepsilon(1-\alpha)] \qquad \text{(A.V.4.5)}$$

Applying L'Hôpital's rule, we have:

$$\lim_{n\to 1}\Xi = \frac{\lim_{n\to 1}\partial(\Xi\,\text{Numerator})/\partial n}{\lim_{n\to 1}\partial(\Xi\,\text{Denominator})/\partial n} = \frac{c_s}{c_u} \qquad \text{(A.V.4.6)}$$

It follows directly from this result that the values of $x_{NE,1}$ and $x_{NE,2}$ in the limit as $n \to 1$ are the two efficient solutions:

$$\lim_{n\to 1} x_{NE,1} = 1 - \left(\frac{c_s}{c_u}\right)^{1/2} \qquad \text{(A.V.4.7a)}$$

$$\lim_{n\to 1} x_{NE,2} = 1 + \left(\frac{c_s}{c_u}\right)^{1/2} \qquad \text{(A.V.4.7b)}$$

Our second task in this appendix is to provide a proof of results (66a) and (66b) reported in Chapter V, namely that $\partial x_{NE,1}/\partial n > 0\,\forall\, n > 1$ and $\partial x_{NE,2}/\partial n < 0\,\forall\, n > 1$. In what follows, we focus on proving the former of these two results, since if proven it will imply that the counterpart result for $x_{NE,2}$ must also be true. It is evident from (A.V.4.1a) that the derivative of $x_{NE,1}$ with respect to n can be written as follows:

$$\frac{\partial x_{NE,1}}{\partial n} = -\frac{1}{2}\Xi^{-1/2}\left(\frac{\partial \Xi}{\partial n}\right) \qquad \text{(A.V.4.8)}$$

Note that it is known from result (A.V.4.6) above that $\lim_{n \to 1} \Xi > 0$, while equation (64a) of Chapter V implies that $\lim_{n \to \infty} \Xi > 0$. It has already been established, therefore, that Ξ is strictly positive for the two extremes of the set of admissible values for *n*. (We remind the reader at this point that this set consists of all real numbers in excess of unity, but does not include unity itself since Ξ is undefined when $n = 1$.) These facts, together with the fact that Ξ is a continuous function of *n* for all $n > 1$, implies that in order to prove that $\Xi^{1/2} > 0$ is the case for all $n > 1$, it is necessary and sufficient to demonstrate that Ξ is a monotonic function of *n* for all admissible *n*. It then follows immediately that a necessary and sufficient condition for $\partial x_{NE,1}/\partial n > (<) 0 \forall n > 1$ is that the inequality $\partial \Xi/\partial n < (>) 0$ hold for all $n > 1$. The method of proof therefore requires us to examine the derivative $\partial \Xi/\partial n$:

$$\frac{\partial \Xi}{\partial n} = \frac{c_s(1-\alpha)^2}{(n-1)^2 \chi_1^{1/2}} \left[\chi_1^{1/2} - c_s\{(n-1)[\varepsilon(1-\alpha)+2\alpha]+(n+1)c_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]\} \right]$$

(A.V.4.9)

Since $\chi_1 > 0$ and $\chi_1^{1/2} > 0$ for all $n > 1$, it follows that the next expression is a necessary and sufficient condition for $\partial \Xi/\partial n < 0$:

$$c_s\{(n-1)[\varepsilon(1-\alpha)+2\alpha]+(n+1)c_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]\} > \chi_1^{1/2} \qquad \text{(A.V.4.10)}$$

Since the left-hand side of (A.V.4.10) is positive, and since $\chi_1 > 0$, squaring both sides of (A.V.4.10) will not cause the inequality sign to reverse direction. After performing this step and simplifying, this inequality reduces to:

$$(n-1)[1+c_u(1-\alpha)^2] > 0 \qquad \text{(A.V.4.11)}$$

This expression holds for all admissible values of *n* (i.e. for all $n > 1$), and hence it follows that Ξ is a monotonic decreasing function of *n* for all admissible *n*. An immediate corollary of this finding that $\partial \Xi/\partial n < 0 \forall n > 1$ is that $\partial x_{NE,1}/\partial n > 0 \forall n > 1$ is indeed the case. Furthermore, since $x_{NE,2} = 2 - x_{NE,1}$, a further implication is that

$\partial x_{NE,2}/\partial n < 0 \, \forall \, n > 1$. Hence for both equilibria, an increase in the number of unions exacerbates the departure of the equilibrium degree of indexation from the efficient degree.

Our third and final task in this appendix is to provide a proof of expressions (67a) and (67b) of Chapter V, namely that $\partial x_{NE,1}/\partial \varepsilon < 0 \, \forall \, n > 1$ and $\partial x_{NE,2}/\partial \varepsilon > 0 \, \forall \, n > 1$. Since $x_{NE,2} = 2 - x_{NE,1}$, a proof of the former result implies the latter is also true. It is convenient to write the derivative of $x_{NE,1}$ with respect to ε as follows:

$$\frac{\partial x_{NE,1}}{\partial \varepsilon} = -\frac{1}{2}\Xi^{-1/2}\left(\frac{\partial \Xi}{\partial \varepsilon}\right) \tag{A.V.4.8}$$

It has already been established earlier in the appendix that Ξ is positive for all admissible values of *n*, and it therefore follows that it is also positive for all admissible values of ε. This implies that $\partial \Xi/\partial \varepsilon > 0$ is a necessary and sufficient condition for $\partial x_{NE,1}/\partial \varepsilon < 0$. The expression for $\partial \Xi/\partial \varepsilon$ is:

$$\frac{\partial \Xi}{\partial \varepsilon} = \frac{c_s(1-\alpha)}{2c_u[\alpha + \varepsilon(1-\alpha)]^2}\left[\alpha + \chi_1^{-1/2} c_s(1-\alpha)\{(n-1)\varepsilon\alpha + (n-3)c_u\alpha(1-\alpha)[\alpha + \varepsilon(1-\alpha)\}\right] \tag{A.V.4.9}$$

Every term on the right-hand side is positive, implying that $\partial \Xi/\partial \varepsilon > 0$ for all admissible values of the parameters, and hence that expressions (67a) and (67b) of Chapter V must be valid. This amounts to a proof that for both equilibria, an increase in the degree of goods-market competition mitigates the departure of the equilibrium degree of indexation from the efficient degree. One final point of interest here is that we would expect to find that $\lim_{n\to 1} \partial \Xi/\partial \varepsilon = 0$, since in the case of a single union the externality is known to be absent and the equilibrium degree of indexation coincides with the efficient degree. Sure enough, when note is taken of the fact that $\lim_{n\to 1} \chi_1^{1/2} = 2c_s c_u(1-\alpha)^2[\alpha + \varepsilon(1-\alpha)]$, it is clear from (A.V.4.9) that $\lim_{n\to 1} \partial \Xi/\partial \varepsilon = 0$ is indeed the case.

Appendices to Chapter VI

Appendix VI.1: Solutions to the Equations Implicitly Defining the Symmetric Nash Equilibrium under Karni Indexation

The purpose of this appendix is twofold. First, it sets out the two values for the degree of symmetric indexation to aggregate output which, together with a symmetric degree of indexation to the price level of unity, satisfy the pair of simultaneous equations which implicitly define (x_{NE}, b'_{NE}) solution pairs. Secondly, it demonstrates that these solutions have certain properties which imply that only one of them can be considered a valid symmetric Nash equilibrium on economic grounds.

The two solution values in question are:[1]

$$b'_{NE,1} = \frac{(n-1)\alpha(\varepsilon+1) - n[1+c_u(1-\alpha)][\alpha+\varepsilon(1-\alpha)] + \chi_2^{1/2}}{2(n-1)\alpha\{\varepsilon+c_u[\alpha+\varepsilon(1-\alpha)]\}} \qquad \text{(A.VI.1.1a)}$$

$$b'_{NE,2} = \frac{(n-1)\alpha(\varepsilon+1) - n[1+c_u(1-\alpha)][\alpha+\varepsilon(1-\alpha)] - \chi_2^{1/2}}{2(n-1)\alpha\{\varepsilon+c_u[\alpha+\varepsilon(1-\alpha)]\}} \qquad \text{(A.VI.1.1b)}$$

where: $\chi_2 \equiv \{(n-1)\alpha(\varepsilon+1) - n[1+c_u(1-\alpha)][\alpha+\varepsilon(1-\alpha)]\}^2 +$
$4(n-1)\alpha[\alpha+n\varepsilon(1-\alpha)]\{\varepsilon+c_u[\alpha+\varepsilon(1-\alpha)]\}$

$b'_{NE,1}$ is the satisfactory solution, in the sense that its value in various special cases of the model which involve a structural parameter taking an extreme value, is consistent with what economic reasoning suggests its value should be. For this reason it is $b'_{NE,1}$ which is presented in Chapter VI as the symmetric Nash equilibrium value of b', where it is denoted more simply as b'_{NE}. Our sole task in this appendix with regard to $b'_{NE,1}$ is to demonstrate that, as mentioned in the main text, the limit as $n \to 1$ of $b'_{NE,1}$ is indeed the efficient solution, namely b'^*, as given by equation (56b) of Chapter VI. It is evident that the limits as $n \to 1$ of both the numerator and the denominator of

[1] Throughout these appendices to Chapter VI, $r^{1/2}$ denotes the positive square root of expression r, where r is equal to a positive real number.

$b'_{NE,1}$ are zero, and consequently that L'Hôpital's rule must be deployed to evaluate the limit as $n \to 1$ of $b'_{NE,1}$. As a preliminary step to doing so, however, we state the derivatives with respect to *n* of the numerator of $b'_{NE,1}$ and of χ_2:

$$\frac{\partial(b'_{NE,1}\ \text{Numerator})}{\partial n} = \alpha(\varepsilon+1) - \delta_3[\alpha+\varepsilon(1-\alpha)] + \left(\frac{1}{2\chi_2^{1/2}}\right)\left(\frac{\partial\chi_2}{\partial n}\right) \qquad \text{(A.VI.1.2a)}$$

$$\frac{\partial\chi_2}{\partial n} = 2\{\delta_3[\alpha+\varepsilon(1-\alpha)] - \alpha(\varepsilon+1)\}\{n\delta_3[\alpha+\varepsilon(1-\alpha)] - (n-1)\alpha(\varepsilon+1)\}$$
$$+ 4\alpha\delta_2[\alpha+(2n-1)\varepsilon(1-\alpha)] \qquad \text{(A.VI.1.2b)}$$

where: $\delta_2 \equiv \varepsilon + c_u[\alpha+\varepsilon(1-\alpha)]$

$\delta_3 \equiv 1 + c_u(1-\alpha)$

At this point we take note of three facts. Firstly, the derivative of the denominator of $b'_{NE,1}$ is simply $2\alpha\delta_2$. Secondly, it is straightforward to show that:

$$\lim_{n\to 1}\chi_2 = \delta_3^2[\alpha+\varepsilon(1-\alpha)]^2 \qquad \text{(A.VI.1.3a)}$$

Combining (A.VI.1.3a) with the limit, as $n \to 1$, of (A.VI.1.2b) gives us our third useful fact, namely:

$$\lim_{n\to 1}\left[\frac{(\partial\chi_2/\partial n)}{2\chi_2^{1/2}}\right] = -\{\alpha(\varepsilon+1) - \delta_3[\alpha+\varepsilon(1-\alpha)]\} + \frac{2\alpha\delta_2}{\delta_3} \qquad \text{(A.VI.1.3b)}$$

Making use of these facts, together with (A.VI.1.2a), in the application of L'Hôpital's rule, gives us the following result:

$$\lim_{n\to 1} b'_{NE,1} = \frac{\lim_{n\to 1}[\partial(b'_{NE,1}\ \text{Numerator})/\partial n]}{\lim_{n\to 1}[\partial(b'_{NE,1}\ \text{Denominator})/\partial n]} = b'^{*} \qquad \text{(A.VI.1.4)}$$

where $b'^{*} = 1/[1 + c_u(1-\alpha)]$. This result indicates that $b'_{NE,1}$ is a legitimate solution in economic terms, since it tells us that in the limit as $n \to 1$, $b'_{NE,1}$ reduces to the setting of b' that would be chosen by a single economy-wide union.

Turning to the second solution, we find that the limit as $n \to 1$ of $b'_{NE,2}$ is not equal to the efficient solution:

$$\lim_{n \to 1} b'_{NE,2} = \frac{-2\delta_3[\alpha + \varepsilon(1-\alpha)]}{0} = \infty \qquad \text{(A.VI.1.5a)}$$

This result would by itself compel us to regard $b'_{NE,2}$ as an economically spurious solution to the equations which implicitly define the $(x_{NE} = 1, b'_{NE})$ solution pair. However, we also find that $b'_{NE,2}$ can be justifiably rejected on economic grounds on account of the value it takes in certain other special cases of the model, which we set out below:

$$\lim_{n \to \infty} b'_{NE,2} = \frac{-(1-\alpha)}{\alpha} \neq b'_{NE,\,atomistic\,case} \qquad \text{(A.VI.1.5b)}$$

$$\lim_{\varepsilon \to \infty} b'_{NE,2} = -\frac{n(1-\alpha)}{(n-1)\alpha} \neq b'^{*} \qquad \text{(A.VI.1.5c)}$$

$$\lim_{c_u \to \infty} b'_{NE,2} = -\frac{n(1-\alpha)}{(n-1)\alpha} \neq \lim_{c_u \to \infty} b'^{*} \qquad \text{(A.VI.1.5d)}$$

$$b'_{NE,2}\Big|_{c_u = 0} = -\frac{[\alpha + n\varepsilon(1-\alpha)]}{(n-1)\varepsilon\alpha} \neq b'^{*}\Big|_{c_u = 0} \qquad \text{(A.VI.1.5e)}$$

Appendix VI.2: Proof that $\partial b'_{NE}/\partial n < 0$ provided $0 < c_u < \infty$

For convenience we reproduce here our expression for b'_{NE}, namely equation (58b) of Chapter VI:[2]

$$b'_{NE} = \frac{(n-1)\alpha(\varepsilon+1) - n[1+c_u(1-\alpha)][\alpha+\varepsilon(1-\alpha)] + \chi_2^{1/2}}{2(n-1)\alpha\{\varepsilon + c_u[\alpha+\varepsilon(1-\alpha)]\}} \qquad \text{(A.VI.2.1)}$$

where: $\chi_2 \equiv \{(n-1)\alpha(\varepsilon+1) - n[1+c_u(1-\alpha)][\alpha+\varepsilon(1-\alpha)]\}^2 +$

$$4(n-1)\alpha[\alpha+n\varepsilon(1-\alpha)]\{\varepsilon+c_u[\alpha+\varepsilon(1-\alpha)]\}$$

As mentioned in Chapter VI.2, it is readily apparent that in the atomistic case b'_{NE}, as given by equation (57b), is in general below the efficient setting of this indexation parameter, b'^*, as given by (56b), the exceptions being the special cases in which $c_u = 0$ (as well as the two limiting cases in which $c_u \to \infty$ and $\varepsilon \to \infty$). In what follows we assume that all parameters are finite and that $c_u > 0$, so that $b'_{NE,\,atomistic\,case} < b'^*$ is indeed the case. On the basis of our results reported elsewhere in this work, we surmise on intuitive grounds that the externality under Karni indexation is stronger, and hence that the departure of b'_{NE} from b'^* is greater, the larger is *n*. Clearly, to prove this conjecture formally we must show that the derivative $\partial b'_{NE}/\partial n$ is unambiguously negative for all $n > 1$. Note that we have yet to show that $b'_{NE,\,atomistic\,case} < b'_{NE,1<n<\infty} < b'^*$. However, since χ_2 is polynomial in *n*, and is strictly positive for all $n > 1$, b'_{NE} is clearly a continuous function of *n* for all $n > 1$. Therefore, a proof that $\partial b'_{NE}/\partial n < 0$ is indeed the case would immediately imply that $b'_{NE,\,atomistic\,case} < b'_{NE,1<n<\infty} < b'^*$ is true.

The following expression must therefore be the focus of our attention:

[2] b'_{NE} is identical to the expression denoted $b'_{NE,1}$ in the previous appendix. Note that b'_{NE} has been derived on the assumption that $n > 1$.

$$\frac{\partial b'_{NE}}{\partial n} = \frac{1}{2(n-1)^2 \alpha \delta_2} \left\{ \delta_3[\alpha + \varepsilon(1-\alpha)] + \left(\frac{1}{\chi_2^{1/2}} \right) \left[\left(\frac{(n-1)}{2} \right) \left(\frac{\partial \chi_2}{\partial n} \right) - \chi_2 \right] \right\} \qquad \text{(A.VI.2.2)}$$

where: $\delta_2 \equiv \varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]$,

$\delta_3 \equiv 1 + c_u(1-\alpha)$

$$\frac{\partial \chi_2}{\partial n} = 2\{\delta_3[\alpha + \varepsilon(1-\alpha)] - \alpha(\varepsilon+1)\}\{n\delta_3[\alpha + \varepsilon(1-\alpha)] - (n-1)\alpha(\varepsilon+1)\} + 4\alpha\delta_2[\alpha + (2n-1)\varepsilon(1-\alpha)]$$

The necessary and sufficient condition for $\partial b'_{NE} / \partial n < 0$ is the following:

$$\delta_3[\alpha + \varepsilon(1-\alpha)] + \left(\frac{1}{\chi_2^{1/2}} \right) \left[\left(\frac{(n-1)}{2} \right) \left(\frac{\partial \chi_2}{\partial n} \right) - \chi_2 \right] < 0 \qquad \text{(A.VI.2.3)}$$

Using the above expressions:

$$\left(\frac{(n-1)}{2} \right) \left(\frac{\partial \chi_2}{\partial n} \right) - \chi_2 = -[\alpha + \varepsilon(1-\alpha)] \left[\delta_3\{(n-1)\alpha(\varepsilon-1) + n\delta_3[\alpha + \varepsilon(1-\alpha)]\} + 2(n-1)c_u\alpha^2 \right] \qquad \text{(A.VI.2.4)}$$

Equations (A.VI.2.3) and (A.VI.2.4) together imply that $\partial b'_{NE} / \partial n < 0$ if (and only if):

$$(n-1)\alpha(\varepsilon-1) + n\delta_3[\alpha + \varepsilon(1-\alpha)] + \frac{2(n-1)c_u\alpha^2}{\delta_3} > \chi_2^{1/2} \qquad \text{(A.VI.2.5)}$$

Since both sides of (A.VI.2.5) are strictly positive, squaring both sides will not cause the inequality sign to reverse direction.[3] Performing this step, cancelling terms common to both sides of the resulting expression, and simplifying, the necessary and sufficient condition for $\partial b'_{NE} / \partial n < 0$ is found to reduce to:

[3] Note that it is helpful (but not essential) to add and subtract the term $(n-1)\alpha(\varepsilon+1)$ to the left-hand side, so that after squaring, the term $\{n\delta_3[\alpha + \varepsilon(1-\alpha)] - (n-1)\alpha(\varepsilon+1)\}^2$ is common to both sides and hence can immediately be cancelled.

$$c_u\alpha[1+c_u(1-\alpha)^2]n > c_u\alpha[1+c_u(1-\alpha)^2] \qquad \text{(A.VI.2.6)}$$

The coefficient in n on the left-hand side of (A.VI.2.6) is strictly positive provided $c_u > 0$, an assumption made at the outset of this appendix. Dividing through by this coefficient then completes the proof of our original conjecture, namely that $\partial b'_{NE}/\partial n < 0$, provided $c_u > 0$. (Note that the proof fails in the $c_u = 0$ case, which is unsurprising since we know that $b'_{NE}\big|_{c_u=0} = b'^*\big|_{c_u=0}$, and hence that $b'_{NE}\big|_{c_u=0}$ is independent of n.)

Appendix VI.3: Proof that $\partial b'_{NE}/\partial\varepsilon > 0$ *provided* $0 < c_u < \infty$

Throughout this appendix it us assumed that all parameters are finite and that $n > 1$ and $c_u > 0$, so that the externality is present. The derivative with respect to ε of b'_{NE}, as given by equation (58b) of Chapter VI, or equation (A.VI.2.1) of the previous appendix, is as follows:

$$\frac{\partial b'_{NE}}{\partial\varepsilon} = \frac{1}{2(n-1)\alpha\delta_2^2}\left\{\alpha[(n-1)c_u\alpha+\delta_3] + \left(\frac{1}{\chi_2^{1/2}}\right)\left[\left(\frac{\delta_2}{2}\right)\left(\frac{\partial\chi_2}{\partial\varepsilon}\right) - \delta_3\chi_2\right]\right\} \qquad \text{(A.VI.3.1)}$$

where: $\delta_2 \equiv \varepsilon + c_u[\alpha + \varepsilon(1-\alpha)]$

$$\delta_3 \equiv 1 + c_u(1-\alpha)$$

$$\frac{\partial\chi_2}{\partial\varepsilon} = 2[(n-1)\alpha - n(1-\alpha)\delta_3]\{(n-1)\alpha(\varepsilon+1) - n\delta_3[\alpha+\varepsilon(1-\alpha)]\}$$
$$+ 4(n-1)\alpha\{\alpha[\delta_3 + nc_u(1-\alpha)] + 2n\varepsilon(1-\alpha)\delta_3\}$$

(A.VI.3.1) implies that the necessary and sufficient condition for $\partial b'_{NE}/\partial\varepsilon > 0$ is the following:

$$\left(\frac{1}{\chi_2^{1/2}}\right)\left[\left(\frac{\delta_2}{2}\right)\left(\frac{\partial\chi_2}{\partial\varepsilon}\right) - \delta_3\chi_2\right] > -\alpha[(n-1)c_u\alpha+\delta_3] \qquad \text{(A.VI.3.2)}$$

Noting that $\chi_2 > 0$, it is convenient to rearrange this as follows:

$$\frac{1}{\alpha[(n-1)c_u\alpha+\delta_3]}\left[\delta_3\chi_2 - \left(\frac{\delta_2}{2}\right)\left(\frac{\partial\chi_2}{\partial\varepsilon}\right)\right] < \chi_2^{1/2} \qquad \text{(A.VI.3.3)}$$

At this point we make use of the following fact:

$$\delta_3\chi_2 - \left(\frac{\delta_2}{2}\right)\left(\frac{\partial\chi_2}{\partial\varepsilon}\right) \equiv \alpha[(n-1)c_u\alpha+\delta_3]\{n\delta_3[\alpha+\varepsilon(1-\alpha)] - (n-1)\alpha(\varepsilon+1)\}$$
$$+ 2(n-1)\alpha^2\delta_2[\delta_3 - nc_u(1-\alpha)] \qquad \text{(A.VI.3.4)}$$

Hence, (A.VI.3.3) and (A.VI.3.4) together imply that $\partial b'_{NE}/\partial\varepsilon > 0$ if, and only if, the following condition holds:

$$\delta_4 < \chi_2^{1/2} \qquad \text{(A.VI.3.5)}$$

where: $\delta_4 \equiv n\delta_3[\alpha + \varepsilon(1-\alpha)] - (n-1)\alpha(\varepsilon+1) + \dfrac{2(n-1)\alpha\delta_2[\delta_3 - nc_u(1-\alpha)]}{[(n-1)c_u\alpha + \delta_3]}$

The composite parameter δ_4 is not necessarily non-negative for all possible combinations of the structural parameters. Since (A.VI.3.5) holds necessarily when $\delta_4 < 0$, our sole concern is to show that it also holds when $\delta_4 > 0$. Clearly, for the $\delta_4 > 0$ case a necessary and sufficient condition for $\partial b'_{NE}/\partial\varepsilon > 0$ equivalent to (A.VI.3.5) is that $\delta_4^2 < \chi_2$ be true. It follows, therefore, that a demonstration that $\delta_4^2 < \chi_2$ holds will amount to a proof that $\partial b'_{NE}/\partial\varepsilon > 0$ for all $n > 1$. Our next step, therefore, is to obtain an expression for $\delta_4^2 - \chi_2$:

$$\delta_4^2 - \chi_2 \equiv \frac{4(n-1)\alpha\delta_2\delta_5}{[(n-1)c_u\alpha + \delta_3]^2} \qquad \text{(A.VI.3.6)}$$

where: $\delta_5 \equiv (n-1)\alpha\delta_2[nc_u(1-\alpha) - \delta_3]^2$

$$-[(n-1)c_u\alpha + \delta_3][nc_u(1-\alpha) - \delta_3]\{n\delta_3[\alpha + \varepsilon(1-\alpha)] - (n-1)\alpha(\varepsilon+1)\}$$

$$-[(n-1)c_u\alpha + \delta_3]^2[\alpha + n\varepsilon(1-\alpha)]$$

Hence our necessary and sufficient condition for $\partial b'_{NE}/\partial\varepsilon > 0$ to be the case boils down to the requirement that $\delta_5 < 0$ hold for all $n > 1$. δ_5 is quadratic in *n* and its first and second derivatives with respect to *n* are as follows:

$$\frac{\partial\delta_5}{\partial n} = (1 + \alpha - 2n)c_u[1 + c_u(1-\alpha)^2]\delta_2 \qquad \text{(A.VI.3.7a)}$$

$$\frac{\partial^2 \delta_5}{\partial n^2} = -2c_u[1 + c_u(1-\alpha)^2]\delta_2 \quad \text{(A.VI.3.7b)}$$

It is evident from these two expressions that δ_5 is at a maximum when $n = (1+\alpha)/2$. Since $(1+\alpha)/2 < 1$, and since $\delta_5|_{n=1} = 0$, it follows that $\delta_5 < 0 \, \forall \, n > 1$, and hence that $\delta_4^2 < \chi_2$ for all $n > 1$. Hence (A.VI.3.5) also holds, and this completes the proof that $\partial b'_{NE}/\partial \varepsilon > 0$ for all $n > 1$ (provided $c_u > 0$).

Appendix VI.4: Solutions to the Equations Implicitly Defining the Symmetric Nash Equilibrium under Indexation to the Price Level and Aggregate Employment

This appendix is the analogue of Appendix VI.1 for the model in which wages are indexed to the price level and aggregate employment, and in respect of which the only known b''_{NE} solutions for $1 < n < \infty$ are the following:

$$b''_{NE,1} = \frac{\varepsilon(n-1) - nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)] + \chi_3^{1/2}}{2(n-1)c_u[\alpha+\varepsilon(1-\alpha)]} \tag{A.VI.4.1a}$$

$$b''_{NE,2} = \frac{\varepsilon(n-1) - nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)] - \chi_3^{1/2}}{2(n-1)c_u[\alpha+\varepsilon(1-\alpha)]} \tag{A.VI.4.1a}$$

where:

$$\chi_3 \equiv \{\varepsilon(n-1) - nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2 + 4(n-1)c_u[\alpha+\varepsilon(1-\alpha)][\alpha+n\varepsilon(1-\alpha)]$$

We wish to derive $\lim_{n\to1} b''_{NE,1}$ and $\lim_{n\to1} b''_{NE,2}$, and a relevant fact to this end is $\lim_{n\to1} \chi_3 = c_u^2(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]^2$. It is clear that the limits, as $n \to 1$ of both the numerator and denominator of $b''_{NE,1}$ are zero. We therefore proceed to obtain:

$$\frac{\partial(b''_{NE,1}\text{ Numerator})}{\partial n} = \varepsilon - c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)] + \left(\frac{1}{2\chi_3^{1/2}}\right)\left(\frac{\partial\chi_3}{\partial n}\right) \tag{A.VI.4.2a}$$

$$\begin{aligned}\frac{\partial\chi_3}{\partial n} &= 2\{\varepsilon - c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}\{\varepsilon(n-1) - nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\} \\ &\quad + 4c_u[\alpha+\varepsilon(1-\alpha)][\alpha+(2n-1)\varepsilon(1-\alpha)]\end{aligned} \tag{A.VI.4.2b}$$

Hence we have:

$$\lim_{n\to1}\left(\frac{1}{2\chi_3^{1/2}}\right)\left(\frac{\partial\chi_3}{\partial n}\right) = \frac{[2+c_u(1-\alpha)^2][\alpha+\varepsilon(1-\alpha)] - \varepsilon(1-\alpha)}{(1-\alpha)} \tag{A.VI.4.3a}$$

Applying L'Hôpital's rule, equations (A.VI.4.2a) and (A.VI.4.3a), together with the fact that $\partial(b''_{NE,1}\text{ Denominator})/\partial n = 2c_u[\alpha+\varepsilon(1-\alpha)]$, imply that:

$$\lim_{n\to 1} b''_{NE,1} = \frac{\lim_{n\to 1}[\partial(b''_{NE,1}\,\text{Denominator})/\partial n]}{\lim_{n\to 1}[\partial(b''_{NE,1}\,\text{Numerator})/\partial n]} = b''^{*} \qquad \text{(A.VI.4.3b)}$$

where $b''^{*} = 1/c_u(1-\alpha)$.

So far as the second solution, $b''_{NE,2}$, is concerned, this can be disregarded since for the following special cases which each involve a structural parameter being assigned an extreme admissible value, the value taken by $b''_{NE,2}$ is inconsistent with the economic theory on which the model's formulation is based:[4]

$$\lim_{n\to 1} b''_{NE,2} = \infty \neq b''^{*} \qquad \text{(A.VI.4.3c)}$$

$$\lim_{n\to\infty} b''_{NE,2} = -(1-\alpha) \neq b''_{NE,\,atomistic\ case} \qquad \text{(A.VI.4.3d)}$$

$$\lim_{\varepsilon\to\infty} b''_{NE,2} = \frac{-n(1-\alpha)}{(n-1)} \neq b''^{*} \qquad \text{(A.VI.4.3e)}$$

$$\lim_{c_u\to 0} b''_{NE,2} = \frac{-[\alpha + n\varepsilon(1-\alpha)]}{(n-1)\varepsilon} \neq \lim_{c_u\to 0} b''^{*} \qquad \text{(A.VI.4.3f)}$$

$$\lim_{c_u\to\infty} b''_{NE,2} = \frac{-n(1-\alpha)]}{(n-1)} \neq \lim_{c_u\to\infty} b''^{*} \qquad \text{(A.VI.4.3g)}$$

[4] Note that equation (A.VI.4.3f) for $\lim_{c_u\to 0} b''_{NE,2}$ has been derived using L'Hôpital's rule.

Appendix VI.5: Proof that $\partial b''_{NE}/\partial n < 0$

b''_{NE} is given by equation (67b) of Chapter VI, and is reproduced here for ease of reference:[5]

$$b''_{NE} = \frac{(n-1)\varepsilon - nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)] + \chi_3^{1/2}}{2(n-1)c_u[\alpha+\varepsilon(1-\alpha)]} \tag{A.VI.5.1}$$

where:

$$\chi_3 \equiv \{(n-1)\varepsilon - nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2 + 4(n-1)c_u[\alpha+\varepsilon(1-\alpha)][\alpha+n\varepsilon(1-\alpha)]$$

Since it has been assumed in deriving b''_{NE} that $n > 1$ and that $c_u > 0$, these assumptions underpin the entirety of this appendix. We also assume throughout that all structural parameters are finite, so that the externality is present. The derivative of b''_{NE} with respect to n may be written as follows:

$$\frac{\partial b''_{NE}}{\partial n} = \frac{1}{2}\left[\frac{(1-\alpha)}{(n-1)^2} + \frac{1}{c_u[\alpha+\varepsilon(1-\alpha)]}\left\{\frac{\partial[\chi_3/(n-1)^2]^{1/2}}{\partial n}\right\}\right] \tag{A.VI.5.2a}$$

where:

$$\frac{\partial[\chi_3/(n-1)^2]^{1/2}}{\partial n} = \frac{(n-1)}{2\chi_3^{1/2}}\left\{\frac{\partial[\chi_3/(n-1)^2]}{\partial n}\right\} \tag{A.VI.5.2b}$$

$$\frac{\partial[\chi_3/(n-1)^2]}{\partial n} = \frac{-2c_u[\alpha+\varepsilon(1-\alpha)]}{(n-1)^3}\{nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)] + (n-1)[2\alpha+\varepsilon(1-\alpha)]\} \tag{A.VI.5.2c}$$

(A.VI.5.2a) implies that $\partial b''_{NE}/\partial n < 0$ if (and only if):

$$\frac{(1-\alpha)}{(n-1)^2} < \frac{-1}{c_u[\alpha+\varepsilon(1-\alpha)]}\left\{\frac{\partial[\chi_3/(n-1)^2]^{1/2}}{\partial n}\right\} \tag{A.VI.5.3}$$

[5] b''_{NE} is identical to the solution denoted $b''_{NE,1}$ in the previous appendix.

It is evident from (A.VI.5.2c) that $\partial[\chi_3/(n-1)^2]/\partial n < 0$, and hence (A.VI.5.2b) implies that $\partial[\chi_3/(n-1)^2]^{1/2}/\partial n < 0$ also. This in turn implies that the right-hand side of (A.VI.5.3) is strictly positive. After squaring both sides of (A.VI.5.3), the necessary and sufficient condition for $\partial b''_{NE}/\partial n < 0$ reduces to:

$$\chi_3(1-\alpha)^2 < \{nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)]+(n-1)[2\alpha+\varepsilon(1-\alpha)]\}^2 \qquad \text{(A.VI.5.4)}$$

Substituting for χ_3 in this expression, expanding both sides and dividing through by the common factor $4[\alpha+\varepsilon(1-\alpha)]$, yields another inequality which can be rearranged so that the term $nc_u(1-\alpha)^2[\alpha+\varepsilon(1-\alpha)][n-1+nc_u(1-\alpha)^2]$ is common to both sides and can therefore be cancelled. After a few further simplifying steps, the necessary and sufficient condition for $\partial b''_{NE}/\partial n < 0$ is found to boil down to $n > 1$, which is true since it is one of the implicit assumptions made in deriving b''_{NE}.

Appendix VI.6: Proof that $\partial b''_{NE}/\partial\varepsilon > 0$

As in the previous appendix, it is important to bear in mind that b''_{NE} has been derived on the assumption that $n > 1$ and $c_u > 0$. The derivative of b''_{NE} with respect to ε may be written as follows:

$$\frac{\partial b''_{NE}}{\partial\varepsilon} = \frac{1}{2(n-1)c_u}\left[\frac{(n-1)\alpha}{[\alpha+\varepsilon(1-\alpha)]^2} + \frac{\partial\{\chi_3/[\alpha+\varepsilon(1-\alpha)]^2\}^{1/2}}{\partial\varepsilon}\right] \qquad \text{(A.VI.6.1a)}$$

where:

$$\frac{\partial\{\chi_3/[\alpha+\varepsilon(1-\alpha)]^2\}^{1/2}}{\partial\varepsilon} = \frac{[\alpha+\varepsilon(1-\alpha)]}{2\chi_3^{1/2}}\left[\frac{\partial\{\chi_3/[\alpha+\varepsilon(1-\alpha)]^2\}}{\partial\varepsilon}\right] \qquad \text{(A.VI.6.1b)}$$

$$\frac{\partial\{\chi_3/[\alpha+\varepsilon(1-\alpha)]^2\}}{\partial\varepsilon} = \frac{2(n-1)\alpha}{[\alpha+\varepsilon(1-\alpha)]^3}\{(n-1)\varepsilon+(n-2)c_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}$$

(A.VI.6.1c)

(A.VI.6.1a) implies that $\partial b''_{NE}/\partial\varepsilon > 0$ if (and only if):

$$\frac{2(n-1)\alpha\chi_3^{1/2}}{[\alpha+\varepsilon(1-\alpha)]^3} > -\frac{\partial\{\chi_3/[\alpha+\varepsilon(1-\alpha)]^2\}}{\partial\varepsilon} \qquad \text{(A.VI.6.2)}$$

The left-hand side of (A.VI.6.2) is strictly positive, but its right-hand side is of ambiguous sign. Therefore, the inequality sign will not reverse direction when both sides are squared if $2(n-1)\alpha\chi_3^{1/2}/[\alpha+\varepsilon(1-\alpha)]^3 > \left|-\partial\{\chi_3/[\alpha+\varepsilon(1-\alpha)]^2\}/\partial\varepsilon\right|$. However, this latter condition must hold if the square of the left-hand side of (A.VI.6.2) is greater than the square of its right-hand side. Consequently the necessary and sufficient condition for $\partial b''_{NE}/\partial\varepsilon > 0$ becomes:

$$\frac{4(n-1)^2\alpha^2\chi_3}{[\alpha+\varepsilon(1-\alpha)]^6} > \left[\frac{\partial\{\chi_3/[\alpha+\varepsilon(1-\alpha)]^2\}}{\partial\varepsilon}\right]^2 \qquad \text{(A.VI.6.3)}$$

Substituting our expression for χ_3 into the left-hand side, and (A.VI.6.1c) into the right-hand side of (A.VI.6.3), expanding the right-hand side of the resulting inequality, and performing various simplifying steps,[6] the necessary and sufficient condition for $\partial b''_{NE}/\partial\varepsilon > 0$ reduces to the following:

$$\alpha > -(1-\alpha)\{\varepsilon + c_u(1-\alpha)[\alpha + \varepsilon(1-\alpha)]\} \qquad \text{(A.VI.6.4)}$$

This condition holds necessarily, and this completes the proof that $\partial b''_{NE}/\partial\varepsilon > 0$.

[6] The steps involve multiplying both sides of the inequality by $[\alpha+\varepsilon(1-\alpha)]^6/4(n-1)^2\alpha^2$, rewriting the right-hand side so that the term $\{(n-1)\varepsilon - nc_u(1-\alpha)[\alpha+\varepsilon(1-\alpha)]\}^2$ is common to both sides and can be cancelled, and then dividing through by the common factor $4(n-1)c_u[\alpha+\varepsilon(1-\alpha)]$.

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