Coded cooperative diversity with low complexity encoding and decoding algorithms.

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Coded Cooperative Diversity with Low Complexity Encoding and Decoding Algorithms

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Abstract

One of the main concerns in designing the wireless communication systems is to provide sufficiently large data rates while considering the different aspects of the implementation complexity that is often constrained by limited battery power and signal processing capability of the devices. Thus, in this thesis, a low complexity encoding and decoding algorithms are investigated for systems with the transmission diversity, particularly the receiver diversity and the cooperative diversity. Design guidelines for such systems are provided to provide a good trade-off between the implementation complexity and the performance.

The order statistics based list decoding techniques for linear binary block codes of small to medium block length are investigated to reduce the complexity of coded systems. The original order statistics decoding (OSD) is generalized by assuming segmentation of the most reliable independent positions of the received bits. The segmentation is shown to overcome several drawbacks of the original order statistics decoding. The complexity of the OSD is further reduced by assuming a partial ordering of the received bits in order to avoid the highly complex Gauss elimination. The bit error rate performance and the decoding complexity trade-off of the proposed decoding algorithms are studied by computer simulations. Numerical examples show that, in some cases, the proposed decoding schemes are superior to the original order statistics decoding in terms of both the bit error rate performance as well as the decoding complexity. The complexity of the order statistics based list decoding algorithms for linear block codes and binary block turbo codes (BTC) is further reduced by employing highly reliable cyclic redundancy check (CRC) bits. The results show that sending CRC bits for many segments is the most effective tecnhique in reducing the complexity.

The coded cooperative diversity is compared with the conventional receiver coded diversity in terms of the pairwise error probability and the overall bit error rate (BER). The expressions for the pairwise error probabilities are obtained analytically and verified by computer simulations. The performance of the cooperative diversity is found to be strongly relay location dependent. Using the analytical as well as extensive numerical results, the geographical areas of the relay locations are obtained for small to medium signal-to-noise ratio values, such that the cooperative coded diversity outperforms the receiver coded diversity. However, for sufficiently large signal-to-noise ratio (SNR) values, or if the path-loss attenuations are not considered, then the receiver coded diversity always outperforms the cooperative coded diversity. The obtained results have important implications on the deployment of the next generation cellular systems supporting the cooperative as well as the receiver diversity.

The coded cooperative diversity is then studied as the two problems of the distributed encoding and decoding. Specifically, in the first problem, the source and the relay employ the same channel encoding, and the objective is to find how to distribute the decoding complexity between the relay and the destination. The objective of the second problem is to distribute the encoding complexity between the source and the relay while using the same channel decoding at the relay and at the destination. Extensive simulation results are obtained assuming binary linear block codes and block turbo codes and the order statistics based decoding. It is found that the decoding complexity distribution is strongly relay location dependent, and, for the case without the path-loss, the distribution of the minimum Hamming distances of the channel sub-codes are more important than the distribution of the code rates. On the other hand, regardless of the relay location, the code rates of the sub-codes are more important than the minimum Hamming distance of the sub-codes.

Declarations and statements

DECLARATION

This work has not previously been accepted in substance for any degree and is not being concurrently submitted in candidature for any degree.

Signed.. ..(candidate)
Date........23/06/2011....

STATEMENT 1

This thesis is the result of my own investigations, except where otherwise stated. Where correction services have been used, the extent and nature of the correction is clearly marked in a footnote(s).

Other sources are acknowledged by footnotes giving explicit references. A bibliography is appended.

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STATEMENT 2

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Contents

Acknowledgment

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List of abbreviations

AF	amplify and forward
ALMT	a list of the most a priori likely tests algorithm
ARQ	automatic repeat request
AWGN	additive white Gaussian noise
BER	bit error rate
BMA	box and match miller algorithm
BOPS	binary operations
BPSK	binary phase shift keying
BPC	binary product code
BTC	block turbo code
BW	bandwidth
CB	control band
CF	compress and forward
CRC	cyclic redundancy check
CSI	channel state information
DF	decode and forward
DTC	distributed turbo code
EGC	equal gain combining
FEC	forward error correction
FLOPS	floating point operations
GMD	general minimum distance decoding
iid	independent identically distributed
ISD	inner sphere decoding
LDPC	low density parity check code
LLR	log likelihood ratio
LRP	least reliable position
MAP	maximum a posteriori probability
MIMO	multiple-input multiple-output
ML	maximum likelihood
MRC	maximal ratio combining
MRIP	most reliable independent position

OSD	order statistics decoding
PEP	pairwise error probability
PDF	probability density function
POSD	partial order statistics decoding
PSK	phase shift keying
QAM	quadrature amplitude modulation
R.V	random variable
SF	select and forward
SNR	signal-to-noise ratio
TEP	test error pattern

List of notations

$\mathcal{C}$	block code
N	number of codeword bits
K	number of information bits
c	binary codeword
Z_2	binary feild
u	information vector
G	generator matrix
P	parity check matrix
$\mathcal{M}()$	mapping process
x	modulated sequence
$\mathrm{E}[\cdot]$	expected value
$\lvert \cdot \rvert$	absolute value
$\oplus$	modulo 2 addition
h_i	channel fadding coefficient
w_i	additive white gaussian noise sample
R	code rate
$f(\cdot)$	probability density function
$\mathrm{sign}(\cdot)$	sign of a real number
$\cdot$	dot-product of vectors
$\mathrm{Pr}(\cdot)$	probability of error
λ'	permutation
$\mathbf{e}$	test error patterns
$\mathcal{E}_L$	coding list size
$\mathrm{P_e}$	probability of codeword error
$\mathbf{c}_{\mathrm{Tx}}$	the transmitted codeword
$d_{\min}$	minimum Hamming distance
$\binom{X}{Y}$	binomial coefficients
$\lceil \cdot \rceil$	the ceiling function
$w_{\mathrm{H}}[]$	Hamming weight
$\sum_{l=0}^{N}$	summation from i to N
N_0	one-sided power spectral density
α	path-loss coefficient
β_{AF}	amplification factor
γ_b	energy per bit
σ_{w}^2	variance
$\Lambda(\lvert)$	LLR values

1

Introduction

1.1 Motivations

The quality of service in wireless communication systems is affected by propogation channel impairments such as noise, interference, and fading. In addition, the wireless communications are currently experiencing a rapid deployment of embedded wireless devices. Since these devices are often limited to a single chip and are battery-powered, their signal processing capabilities (i.e., the processing speed and the available on-board memory) can severely limit the use of many conventional physical layer signaling techniques. The signal processing techniques are also significantly limited in wireless networks containing very high data rate links (of the order of Gigabits per second). In such cases, complexity reduction and diversity become important to remedy such challenges. The main objective of diversity is to improve the reliability of the transmitted messages. The diversity can be realized as the time diversity, frequency diversity, polarization diversity, spatial diversity, cooperative diversity. Some of the well known techniques to exploit the diversity are channel coding and multi-input multi-output (MIMO) communication systems.

One of the important diversity methods considered in this thesis is the forward error correction (FEC) coding that is often used as the channel coding. A major difficulty in employing the FEC is the implementation complexity especially for the decoding at the receiver and the associated decoding latency for long codewords. Correspondingly, the FEC coding is often designed to trade-off the bit error rate (BER) with the decoding complexity and latency. Many universal decoding algorithms have been proposed for the decoding of linear binary block codes [1]. The decoding algorithms in [2]–[3] are based on the testing and re-encoding of the information bits as initially considered by Dorsch in [4]. In particular, a list of the likely transmitted codewords is generated using the reliabilities of the received bits, and then, the most likely codeword is selected from this list. The list of the likely transmitted codewords can be constructed from a set of the test error patterns. The test error patterns can be predefined as in [2] and [5], predefined and optimized for the channel statistics as in [6], or defined adaptively for a particular received sequence as suggested in [7]. The complexity of the list decoding can be further reduced by the skipping and stopping rules as shown, for example, in[2] and [5]. Among numerous variants of the list decoding techniques, the order statistics decoding (OSD) is well-known [2], [5]. The structural properties of the FEC code are utilized to reduce the OSD complexity in [8]. The achievable coding gain of the OSD is improved by considering the multiple information sets in [9]. An alternative approach to the soft-decision decoding of linear binary block codes relies on the sphere decoding techniques [10, 11].

The problem of the decoding complexity has motivated the research community how to most efficiently apply the order statistics based list decoding techniques for linear binary block codes of small to medium block length. The original OSD is generalized in this thesis by assuming segmentation of the most reliable independent positions of the received bits. Such segmentation is shown to overcome several drawbacks of the original OSD decoding algorithm. The complexity of the order statistics based decoding is further reduced by assuming a partial ordering of the received bits in order to avoid the complexity of the Gaussian elimination. For comparison purposes, the input sphere decoder (ISD) is also considered in this thesis as a trivial sphere decoding algorithm. Even further reduction in complexity of the OSD decoding algorithm and block turbo codes (BTCs) is achieved by employing highly reliable CRC bits (free error bits) and by using single generator matrix for the BTCs.

Another diversity method considered in this thesis is the cooperative communication diversity. In the cooperative diversity, signal processing is distributed among network nodes in order to exploit spatial diversity and combat the multipath fading

[12, 13]. Hence, communications between a source and a destination with the help of a single relay is a well-studied example of the distributed signal processing. Particularly, from the source point of view, the relay and the destination cooperate to recover the transmitted information, and the relay represents the distributed receiver antenna. On the other hand, from the destination point of view, the source and the relay cooperate to deliver information to the destination, so that the relay acts as the distributed transmitter antenna. Different relaying strategies can be used to perform cooperative diversity such as amplify-and-forward (AF), decode-and-forward (DF), and compress-and-forward (CF)[14]. The cooperative communication diversity has many advantages such as the design flexibility, the number of nodes (relays) that can be utilized, improved coverage, and the ability to be readily integrated with channel coding. Because of these advantages, the cooperative diversity is adopted and used in different areas of wireless communications, for example, in wireless networks (IEEE 802.15.4) and wireless LANs (80.2.11/a/b/g/n).

On the other hand, the cooperative diversity faces many challenges. The first and the main challenge is that the end-to-end performance is dominated by the detection reliability at the relay which may cause as an error propagation. The second challenge is that the network throughput is smaller than for MIMO diversity systems. However, the cooperative diversity has other advantages over the MIMO systems such as smaller channel correlation and the complexity reduction. The third challenge is to find an efficient way of cooperation among the different relays in the system. In this thesis our main aim is to investigate different solutions to reduce the complexity of the cooperative system and to reduce the error propagation in such systems in order to improve the reliability of detecting the received signal in the destination node. The error propagation and the complexity reduction in cooperative systems have been addressed in this thesis in a number of different ways: 1) Relay restrictions and positions, 2) Distributed decoding and distributed complexity requirements of BTCs, and 3) Distributed encoding of BTCs.

For the relay positions, identical block codes at the source node and at the relay node with AF and DF relaying schemes at the relay with different combining schemes at the destination are considered. The performances of the cooperative systems with different assumptions are investigated. Moreover, a comparison of the cooperative and receiver diversity with channel coding in terms of the BER performance and implementation complexity is obtained. We will show by mathematical analysis and numerical results, that the coded cooperative system is highly dependent on the relay positions, the relaying schemes and the combining schemes employed at the destination.

The distribution complexity of the decoding between the relay and the destination is investigated. The same block codes are used at both source and at the relay. However, the decoding process at the relay and at the destination either different or the same but with different decoding parameters. Moreover the cases where the relay is on the only link between the source and the destination link are investigated. The OSD decoding algorithms that are developed in this thesis are shown to achieve flexibility and good trade-off between the performance and the complexity.

In case of the distributed BTC, the channel code at the destination is a BTC having a repetition code as one of its component codes. For example, generalization of the distributed product code to the uplink and downlink scenarios to facilitate cooperation among a group of mobile terminals using single parity check codes is considered in [15, 16, 17]. However, cooperative system design principles for such algorithms which are rarely considered motivate our investigatation of design principles in this thesis. Also, the relations between parameters in such systems are not considered in the literature, so more investigation in depth was carried out in this thesis.

1.2 Contributions

The OSD-based decoding strategies for linear binary block codes are investigated. Our aim is to obtain low-complexity decoding schemes that provide sufficiently large or valuable coding gains, and most importantly, that are well-suited for implementation in communication systems with limited hardware resources, e.g., at nodes of the wireless sensor network. We modify the original OSD by considering the disjoint segments of the most reliable independent positions (MRIPs). The segmentation of the MRIPs creates flexibility that can be exploited to fine tune a trade-off between the BER performance and the decoding complexity. Thus, the original OSD can be considered to be a special case of the segmentation-based OSD having only one segment corresponding to the MRIPs. Since the complexity of obtaining a row echelon form of the generator matrix for every received codeword represents a significant part of the overall decoding complexity, we examine a partial-order statistics decoding (POSD) when only the systematic part of the received codeword is ordered.

Further reduction in complexity for OSD decoding algorithm and block turbo code (BTC) is achieved by employing highly reliable CRC bits and a design of single generator matrix for BTC.

In order to reduce the error propagation at the relay in coded cooperative system, we investigated the suitable relay positions and restriction. We compare the BER versus signal-to-noise ratio (SNR) of two communication systems assuming uncoded transmissions as well as assuming encoding of the packets using a simple channel coding prior to their transmission. The first system represents a cooperative diversity consisting of a single source S, a relay R and a destination D. The relay performs either AF or DF processing of the received packets. In case of the DF relaying we assume that the relay uses the same encoder as the source and the same decoder as the destination. The second system considered in Chapter 6 represents a simple receiver diversity with two receiver antennas. In both systems, the destination coherently combines the received signals using maximum ratio combining (MRC) or equal gain combining (EGC) which is followed by the demodulation and the channel decoding . Thus, in both systems, diversity order of at most two can be achieved at the combiner output. More importantly, we compare the BER performances of the two systems assuming the path-loss attenuation between the communicating nodes. Intuitively, assuming that all the channels between nodes are independent, one can expect that the receiver diversity outperforms the cooperative diversity. However, our mathematical and numerical results indicate that the BER performance of the system with cooperative diversity is strongly dependent on the relay location. Assuming channel coding and non-binary modulations, to the best of the authors' knowledge, such comparison has not been done previously.

The main two problems of the DF cooperation are investigated assuming simple-to-implement encoding and decoding techniques. The simplicity of the encoding and decoding is important in applications where the cost of implementation cannot be neglected (e.g., in large scale deployments), or where the implementation complexity is limited by other factors such as high data rate or small latency. On the other hand, the simple encoding and decoding techniques can rarely approach the channel capacity, and thus, the asymptotic mathematical theoretic analysis must be often replaced by computer simulations.

In the first problem, the source and the relay employ the same channel encoder, however, the type of the channel decoder used at the relay and at the destination is either different or the same but its parameters may be different. We consider the order statistics decoding (OSD) developed in [2] and refined in Chapter 4 in this thesis for its inherent flexibility to trade-off the decoding complexity and the bit error rate (BER) performance. The main objective of the first problem is to investigate how to distribute the (overall) decoding complexity of the OSD between the relay and the destination. The decoding complexity of the OSD can be measured as the total number of the test error patterns (TEPs) searched in the decoding process.

In the second problem, the relay and the destination use the OSD with the same parameters, however, the channel encodings at the source and at the relay are different. The main objective of this problem is to investigate how to distribute the channel code rates and the minimum Hamming distances of the codes between the source and the relay since the code rate can be used as a measure of the encoding complexity. Note also that, in general, the modulation constellations used at the source and at the relay can be different, however, this case and the associated distribution of the implementation complexity are not considered in this thesis.

1.3 Organization of thesis

In Chapter 2 and Chapter 3, we represent an over view for some fundamentals and related literature for coded systems in general and coded cooperative systems in particular.

In Chapter 4, the order statistics based list decoding techniques for linear binary block codes of small to medium block length are investigated. System model is described in Section 4.2. Construction of the list of test error patterns is investigated in Section 4.3. The list decoding algorithms are developed in Section 4.4. The performance analysis is considered in Section Five. Numerical examples to compare the

BER performance and the decoding complexity of the proposed decoding schemes are presented in Section 4.6. Finally, conclusions are given in Section 4.7.

In Chapter 5, two different techniques are proposed to reduce the complexity of the OSD and the POSD decoding algorithms for linear binary block codes and binary block turbo codes (BTC). System model is described in Section 5.2. In Section 5.3, the OSD decoding algorithm and CRC techniques are proposed with numerical examples. The design of a single genrator matrix of binary BTC is discussed in Section 5.4. Conclusions are given in Section 5.5.

In Chapter 6, a Comparison is carried between the transmission reliabilities of a cooperative diversity system employing a single relay and a system employing the conventional receiver diversity with the two receiver antennas. Section 6.2 describes the system models including the modulation and channel coding and decoding for two systems employing the receiver and the cooperative diversity, respectively. The PEP as a key measure of the transmission reliability for the two systems under consideration is analyzed in Section 6.3. The performance of the two systems are compared in Section 6.4 the optimum relay locations for the system with the cooperative diversity are determined, so that it outperforms the system with the receiver diversity. Finally, conclusions are given in Section 6.5.

In Chapter 7, two problems of the DF cooperation assuming simple-to-implement encoding and decoding techniques are investigated. The system model is adopted from Chapter 6 and is briefly described in Section 7.2. The distributed decoding operations and its pefromance for the first problem considered is also described in Section 7.3. The distributed encoding operations and its pefromance for the second problem considered is also described in Section 7.4. Conclusions are given in Section 7.5.

Chapter 8 is the final chapter in this thesis, it summarises the conclusions of the thesis and highlights numerous areas for further research in future.

Parts of this work has been presented in the following journal papers and conference proceedings:

- S. E. A. Alnawayseh and P. Loskot, "Low-Complexity Soft-Decision Decoding Techniques For Linear Binary Block Codes," in *Proc.IEEE WCSP 2009*, Nanjing, China, Nov. 13-15, 2009.

- S. E. A. Alnawayseh and P.Loskot, "Cooperative Versus Receiver Coded Diversity With Low-Complexity Encoding and Decoding," in *Proc. IEEE VTC'10 Spring*, May 2010. pp. 1–5.

- S. E. A. Alnawayseh and P. Loskot, "Decode-and-forward cooperation as the distributed encoding and decoding," in *Proc.IEEE ISWCS'10*, Sept. 2010. pp. 656-660.

- S. E. A. Alnawayseh and P. Loskot, "Order Statistics Based List Decoding Techniques for Linear Binary Block Codes, "*IEEE Trans. Inform. Theory*, submitted, Jan. 2011.

- S. E. A. Alnawayseh and P. Loskot, "Error Rate Performance of Cooperative Versus Receiver Coded Diversity,"*IEEE Trans. Vehicular. Technology*, submitted in March. 2011.

- S. E. A. Alnawayseh and P. Loskot, "Complexity Reduction of Order Statistics Decoding Based on High Reliable CRCs," *IEEE Comm. Letters* , submitted, May. 2011.

2

Fundamentals of Coded Systems

2.1 Channel Coding

The main goal of channel coding is to protect transmitted information during transmission over a communication channel against the channel impairments such as noise, fading, interference, and attenuation. Thus, the aim is to reduce the probability of receiving the information in error. However, the actual way how to protect information against impairments of the transmission medium is not obvious [18]. The presence of channel impairments limit information flow through the channel but not the quality with which the message can be reconstructed. This is the main idea of information theory which created channel coding concept. In channel coding, the error control codes techniques are utilized to enable reliable delivery of information from a source to a destination. In error control codes, additional information (parity) is added by the code so the receiver can use it to recover original data. These additional or redundant bits which are added to the original data by the encoder at the transmitter side limit the flow of information to a value that is smaller than channel capacity. Thus, the channel coding aims to deliver reliable information to a destination at information flow rates close to the channel capacity. Shannon described an important concept of channel coding and information theory [19], which states " by proper encoding of the information, errors caused by noise channel and channel impairments can be reduced to any desired level without sacrificing the information flow rate ". Since then, an extensive work has been carried out by research comity to design efficient encoding and decoding algorithms to achieve reliable communications even for high speed transmissions of these days standars in 3G and 4G cellular systems[20]. There are two main functions of the error control coding.

1. Error detection: ability to detect errors caused by channel impairments.

2. Error correction: correction of errors to recover the original data . The error correction can be classified into two main categories:

   A) Automatic repeat request (ARQ): which is an error control scheme that uses acknowledgements to provide reliable data transmission in two way channels. The transmitter retransmits information until either it receives acknowledgment from the destination that the information was received correctly or the number of allowable transmissions runs out.

   B) Forward error correction (FEC): an error control scheme where the source employs an encoder to encode the transmitted information by adding redundant bits in a systematic way, so the decoder at the receiver side can detect and correct errors. In FEC, no feedback channel is used to request transmission of data as in the ARQ. According to a manner in which redundancy is added to information error control coding, the FEC can be divided into two classes:

   1) Block codes [18, 20, 21].

   2) Convolutional codes [22].

In this thesis, we limit our focus to binary linear block codes.

2.1.1 Binary Linear Block Codes

Corresponding to 2^K possible messages, there are 2^K codewords where K is a number of information bits. The set of 2^K codewords is called a block code. In binary linear block codes, all codewords form a K dimensional subspace of the vector space of all binary N bits. Hence and importantly, a modulo 2 sum of any two codewords is another codeword. For binary linear block codes, any group of linear independent codewords can form generator matrix G of the code and any codeword is a linear combination of such codewords. Let $u_i = 0$ or 1, $1 < i < K$, be infromation bits to be encoded and let $(g_1, g_2..., g_K)$ be K linear independent codewords arranged in a matrix G,i.e.,

$$\mathbf{G} = \begin{bmatrix} g_{(1,1)} & \cdots & g_{(1,N)} \\ g_{(2,1)} & \cdots & g_{(2,N)} \\ \vdots & \cdots & \vdots \\ g_{(K,1)} & \cdots & g_{(K,N)} \end{bmatrix}$$

Then the encoded bits in a codeword c are calculated as

$$\mathbf{c} = \mathbf{uG}$$

The matrix G is called the generator matrix of a linear block code C and is of size $(K \times N)$ where N is the number of codeword bits and K is the number of information bits only. In many cases, it is desirable if the linear block code is systematic. For any G, there is $(N - K) \times N$ matrix called a parity check matrix H with linearly independent rows in which any vector space in G is orthogonal to the rows of H.The information bits appear in the first K positions, the remaining $(N - K)$ bits are redundant bits and they are a function of the information bits. For a given code C, the minimum Hamming distance d_{min} among all possible codewords in C is given as,

$$d_{min} = min\,(d_H(\bar{c_1}, \bar{c_2})\bar{c_1} \neq \bar{c_1}, \bar{c_1}, \bar{c_2} \in C) \tag{2.1}$$

where $d_H(\bar{c_1}, \bar{c_2})$ is the Hamming distance between $\bar{c_1}$ and $\bar{c_2}$. The most important property of C is the error correction Capability t, which is the largest Hamming sphere about all codewords $c \in C$. The value of t is defined in terms of d_{min} of C as,

$$\lceil t = (d_{min} - 1)/2 \rceil \tag{2.2}$$

where $\lceil \cdot \rceil$ is the floor function. The block code having the minimum Hamming distance d_{min} is capable of detecting all error patterns of $(d_{min} - 1)$ as this number of errors or fewer errors corresponds to the received message which is not a codeword. However, in case of the errors with Hamming weight of d_{min} or larger not all errors can be detected as there exist at least two pairs of codewords that differ in d_{min} positions, and there is an error pattern of Hamming weight of d_{min} that converts a codeword to another codeword. Thus, the error detection capability of C is $(d_{min} - 1)$. In general, for any linear block code, the number of error patterns cabable of error detection is 2^{N-K}, since there are $2^K - 1$ error patterns that are identical to the codewords. For large block code, there is a small fraction of errors that can pass through the decoder without detection. The probability of error detection can be calculated using a weight distribution of the code [21],i.e.,

$$P_d = \sum_{i=1}^{N} A_i \overset{i}{\mathrm{Pr}}(1 - \mathrm{Pr})^{(N-i)} \tag{2.3}$$

where A_i and P is the number of codewords with weight (i) and the transition probability, respectively. The probability of decoding error for block code C with the error correction cabability t is bounded as,

$$\mathrm{P_e} \leq \sum_{i=1}^{N} \binom{N}{i} \overset{i}{\mathrm{Pr}}(1 - \mathrm{Pr})^{(N-i)}. \tag{2.4}$$

2.1.2 Block Turbo Codes

Block turbo code (BTC) are serially concatenated codes and such codes are suitable for efficient construction of long codes from two short codes. The BTC is one example for such codes. The BTC is formed by the product of two systematic codes $C_1 = (N_1, K_1, d_{min1})$ of code rate R_1 and $C_2 = (N_2, K_2, d_{min})$ of code rate R_2. Thus, the product code $P = C_1 \times C_2$ is constructed as follows [23] and shown in Fig. 2.1.

1. Placing K_1, K_2 information bits in an array of K_1 rows and K_2 columns.
2. Coding the K_1 rows using C_2 and appending $(N_2 - K_2)$ parity bits to each row.
3. Coding the N_2 columns using C_1 and appending $(N_1 - K_1)$ parity bits to each column.

The parameters of BTC $P = (N, K, d_{min})$ are $N = N_1 \times N_2$, $K = K_1 \times K_2$, $d_{min} = d_{min1} \times d_{min2}$ and the code rate is $R = R_1 \times R_2$. This procedure allow to

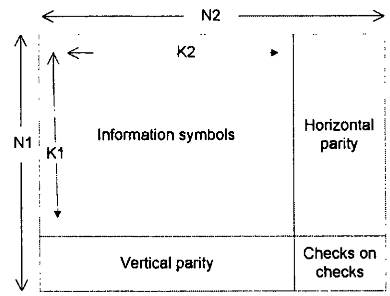

Figure 2.1: Construction of BTC $P = C1 \otimes C2$.

build long codes from short codes. Thus, all columns of matrix the P, where P is the block turbo code, are the codewords of C_2 while all rows of P are the codewords of C_1. The decoding process of the BTC can be done sequentially,i.e., first for the rows and then for columns or vise versa. However, inorder to obtain an optimum bit error rate performance, soft-input soft-output decoding algorithms have to be employed for decoding of each sub-code.

2.1.3 ML Decoding and Union Bound

In the soft decision decoding, the maximum liklihood (ML)principle is an important concept that should be explained as follows. The (ML) decoding finds the closest transmitted codeword to the received sequence. The brute force approach to the ML of linear (N, K) block code requires computation of the Euclidian distance between the codewords in a coding list and the received sequence. The codeword from the coding list with the minimum Euclidian distance to the received sequence is then chosen. The goal of the ML decoding is to achieve the inequality in (4.1),where c_0 is the closest codeword from the coding list to the received sequence r and c any other codeword in the coding list [20, 22, 24].

$$\Pr(c_0|r) \geq \Pr(c|r) \tag{2.5}$$

If all code words in the code have equal probability of being transmitted, then maximization $\Pr(c_0|r)$ is equivalent to maximize $\Pr(r|c)$,where $\Pr(c_0|r)$ is the conditional probability of chosing c_0 as the best codeword when r is the received sequnce, and

$\Pr(r|c)$ is the conditional probability that r is the received sequence when c is the transmitted codeword.

$$\Pr(c|r) = \Pr(r|c)/\Pr(r). \tag{2.6}$$

The $\Pr(c|r)$is called the maximum a posteriori (MAP), and the conditional probability $\Pr(r|c)$ is the ML function. To minimize the error probability, the ML decoder decodes a received sequence to a codeword such that $\Pr(r|c_0) \geq \Pr(r|c)$ for all codewords in the coding list. Thus, the ML decoding is concerned in finding the suitable codeword c_0 from the coding list that can satisfies the inequalities

$$\prod_{i=1}^{N} \Pr(r_i|c_{0i}) \geq \prod_{i=1}^{N} \Pr(r_i|c_i) \tag{2.7}$$

$$\sum_{i=1}^{N} ln(\Pr(r_i|c_{0i})/\Pr(r_i|c_i)) \geq 0. \tag{2.8}$$

One of the important parameters in the soft decision decoding is the reliability of the received bit or a bit log-likli hood ratio ϕ_i that can be written for AWGN as,

$$\begin{aligned} \phi_i =& \frac{\Pr(r_i|0)}{\Pr(r_i|1)} \\ =& 4(\sqrt{E_b}/N_0) \bullet r_i \end{aligned}$$

where E_b and N_0 are the energy per bit and nosie spectral density respectively. The objective function of the ML can be expressed as,

$$\sum_{i=1}^{N} (\phi_i - (-1)^{c_{0i}})^2 \geq \sum_{i} (\phi_i - (-1)^{c_i})^2 \tag{2.9}$$

where $\sum_{i=1}^{N}(\phi_i - (-1)^{c_{0i}})^2$ represent the Euclidian distance between the received sequence r and the optimum codeword c_0. Since the ML represents the best performance of the soft decision decoding, it is the reference for any decoding algorithm, but this operation becomes too complex for long codes. In order to have a reference for newly developed decoding algorithms, the upper bounds are usually derived for long block codes such as the soft union (SU) bound. In the SU bound, an all zero codeword is transmitted and let Γ be the event that the distance between received vector and any codeword of non-zero Hamming weight is smaller than the distance between the received vector and the all zero codeword.

$$\Pr(\Gamma|A_0) = \Pr(d_E(\phi, c) < d_E(\phi, 0)|A_0) \tag{2.10}$$

where $\Pr(\Gamma|A_0)$ is the conditional probability of event Γ when A_0 is the transmitted codeword and $d_E(\phi, c)$ is the Euclidian distance between the received sequence and any codeword c in the code. The union bound states that the sum of probabilities of individual events is greater or equal than the probability of union of the events,i.e.,

$$\Pr(E_1) + \Pr(E_2) + \ldots + \Pr(E_N) \geq \Pr(E_1 \cup E_2 \cup \ldots E_N) \tag{2.11}$$

where E_i is an event.The word error probability $\mathrm{P_e}$ of the ML decoder when all zeros code A_0 is selected for transmission is (**??**):

$$\mathrm{P_e} \leq \sum_{w=d_{min}}^{w} A_w \Pr(\Gamma|A_0) \tag{2.12}$$

where A_w is the number of codewords with Hamming weight w and the probability,

$$\begin{aligned} \Pr(\Gamma|A_0) =& \sum_{i=0}^{w} \Pr(r_i < 0|A_0) \\ =& Q(\sqrt{2wE_s/N_0}) \end{aligned} \tag{2.13}$$

Then, the bit error probability $\mathrm{P}(i)$ as

$$\mathrm{P}(i) \leq \sum_{w=d_{min}}^{w} \frac{\delta_w}{K} A_w Q(\sqrt{2wRE_b/N_0}) \tag{2.14}$$

where δ_w is the number of information words associated with the codeword of the Hamming weight w. For moderate and high SNR the first term in (2.14) is the most significant. The number of codewords of the minimum Hamming weight is the main factor which determine the BER. However, computing (2.2) can be very demanding task because it requires the Hamming weight distribution of all codewords. For long codes, creating a list of 2^K information words is impractical. In this case, a truncated union bound is used. Thus, the all information words up to a certain Hamming weight are considered. Then the corresponding coding list is created and the weight distribution is computed from the list. The upper limit of the Hamming weight is chosen empirically until the union bound converges. In this thesis, we have used this truncated union bound to calculate the soft union bound for the BCH codes in Chapter 4.

2.1.4 Soft-Decision-Decoding

There are two methods of decoding error correcting codes based on the real values of the received sequences [18, 20, 21]:

1. Hard decision decoding (HDD): the main idea of the HDD is to correct errors produced during the hard quantization process. The detector produces binary values after quantization of the received real values r_i into two levels,i.e., ones and zeros.

$$z_i = \frac{1 - \text{sign}(\bar{r}_i)}{2} \tag{2.15}$$

   Then, the hard decision values are processed using some decoding algorithm. Where z_i is the HD value, and $\text{sign}(\cdot)$ denotes the sign function.

2. Soft Decision Decoding (SDD): the real values received from the channel are processed directly un-quantized or quantized in more than two levels to find the most likely transmitted codeword. The important parameters are considered in SDD such as : the Ecludian Distance, and the correlation. The SDD can be classified into two main categories.

   A) A code structure based decoding such as the Viterbi algorithm. which is based on a trellis representation of the code and can achieve the ML decoding performance.

   B) A reliabilities based decoding: utilizes reliability measures of the received sequence in the decoding process. In such decoding algorithms, symbols of the received sequence are categorized into two groups:

   1) Least reliable positions (LRPs).

   2) Most reliable positions (MRPs).

Based on this classification, the decoding is implemented for processing either on the LRPs or the MRPs of the received sequence after ordering as

$$\tilde{\mathbf{r}}' = \lambda'[\mathbf{r}] = (\tilde{r}'_1, \cdots, \tilde{r}'_N). \tag{2.16}$$

General minimum distance (GMD)decoding [25, 26] : The GMD is a decoding algorithm that utilizes reliability of information for the LRPs of the received sequence to improve the algabric decoding.

Chase decoder [27]: is a generalizing of GMD algorithm that utilize reliability information for for LRP's of received sequence to improve algabric decoding. Chase is divides into three types.

Order Statistics Decoding Algorithm (OSD) [2, 5]: OSD is a decoding algorithm that utilize reliability information for MRP's of received sequence in order to reduce the complexity of the ML decoding, order statistics decoding (OSD) has been proposed in [2].

The operation of these algorithms is summarized as follows.

1. Generate HDs z of received sequence r as using (4.10).
2. Construct a list of error patterns e based either on LRPs in case of GMD and Chase algorithms or based on MRPs in case of the OSD algorithm.
3. Generate a list of sequences by modifying the z sequence assuming with list of error patterns e as $z + e$.
4. Decode $z + e$ list using algabric decoder in case of GMD and Chase or encode the $z + e$ list in case of OSD and then the codeword with minimum Euclidean distance to the received sequence.

The detailed describtion of these decoding algorithms are represented in the following chapters.

2.2 Diversity in Wireless Communications

Diversity is a method to improve reliability of transmitted messages by exploiting the random nature of radio propagation. The main idea behind the diversity is to obtain independent signal paths that experience different levels of fading and interference so that multiple versions of the signal can be transmitted or received. Moreover, the channel coding techniques can be added to the transmitted message to improve its immunity against the channel impairments.

2.2.1 Types of Diversity

The diversity schemes can be classified as [24, 28]:

- The time diversity: this type of diversity can be achieved in two ways, i.e., either by transmitting multiple versions of the message at different time slots or by using FEC techniques and spread the message in time by interleaving techniques.
- The frequency diversity: the diversity is achieved either by transmitting multiple versions of the message over different frequency channels or by spreading the message over wide spectrum of frequencies.
- The space diversity: the message is transmitted over different propagation paths. The diversity in these schemes for wireless systems can be also achieved by antenna diversity, where multiple antennas are utilised at the receiver side to receive multiple versions of the transmitted message which is known as the receiver diversity. Multiple antennas can be also used at the transmitter side to send multiple versions of the message through different propagation paths.
- The polarization diversity: multiple versions of the messages are transmitted and received with different polarization.
- The cooperative diversity [24, 29, 30, 31]: this diversity utilises relays to improve the reliability of the transmitted message. Such systems consist of a source, a relay, and a destination. The source broadcasts the signal to both the relay and

the destination, and then the relay forward the signal to the destination. These diversity schemes rely on two principles:

- Due to a broadcast nature of the wireless communication medium, most of transmissions can be heard by multiple nodes in the network with no additional transmission power and bandwidth (BW).
- Different nodes in the network have independent channel fading statistics towards given destination nodes and the destinations can listen, store, and then combine signals received from the different nodes.

At the relay, different protocols of cooperative diversity can be used to achieve full diversity such as the amplify-and=forward (AF) , the decode- and-forward (DF), the compress-and-forward (CF), and the select-and-forward (SF) protocols. These protocols are explained next.

- The amplify-and-forward (AF) relaying is a diversity scheme that allows the relay to amplify the received signal from source and then forward it to the destination. The amplifying corresponds to a linear transformation at the relay.
- The compress-and-forward (CF): diversity scheme allows the relay station to compress the received signal from the source node and forward it to the destination without decoding the signal.
- The decode-and-forward(DF): diversity scheme allows the relay station to decode the received signal from the source node, re-encode it and forward it to the destination station.

2.2.2 Diversity Combining Schemes

The diversity combining schemes allow the destination to combine multiple received signals.

- The maximal-ratio-combining (MRC):is often used in large phased array systems. Thus, the received signals from all paths are weighted according to their SNR and then summed up.
- The equal-gain-combining: The received signals are summed up coherently. In such case, the weight of signals from all paths are set to unity.
- The selection combining: The strongest signal among all the received signals from all paths is selected.

2.2.3 Summary

The previous sections gave a brief of general concepts the will be investigated later in thesis in depth. The two main functions of the error control coding were proposed. Shannon theorem was proposed and the idea behind channel coding was discussed. A brief of main concepts about linear block codes were proposed such as generator matrix, d_{min}, error correction and detection of block codes, and some general expressions for error decoding probability of block codes. Moreover, a general idea about BTC was presented and how to do the encoding process for such codes. Also general description of error performance bounds were explained such as Union bounds. Finally, basic principles about diversity in wireless communication system were summarized such as diversity types, diversity combining schemes.

3

Overview of Relevant Literature

3.1 Reliability Based Decoding Algorithms

One of the most important features of digital communication systems is reliability of digital data transmission. A major concern of communication system designers is the error control techniques to reduce the effects of noise over the communication channel in order to recover the transmitted data correctly. Shannon has showen in his theory using the noisy channel coding theorem that coding of information using codes of rate R less than the channel capacity C and utilizing suitable decoding techniques can reduce the effects of noise over information during transmission [19]. Based on Shannon theory, designing optimal coded systems became essential to achieve a low error probability and better performance of communication systems. Two major problems are encountered for designing coded systems and to achieve good performance [20]. Particularly, the first problem is to construct good codes that satisfy Shannon theorem. The second problem is to find optimum decoding techniques that can achieve performance close to the maximum likelihood (ML) decoding which is the best performance that can be theoretically achieved assuming word-errors and equally likely codewords.

The decoding algorithms are often based on the channel noise characteristics and the type of code which is used in channel coding. Generally, decoding techniques can be classified into two main categories [24]: the hard decision decoding and the soft decision decoding. In the hard decision decoding, the output of the modulator in the receiver is quantized into two levels, 0's and 1's, which represent the hard decisions. The hard decision decoder executes a certain decoding method to decode the received sequence to the closest codeword in the sense of the Hamming distance. It should be noted that the hard decisions cause loose of information which increases the probability of incorrect decisions.

In the soft decision decoding, the demodulator outputs are un-quantized or quantized to more than two levels. The soft decisions are processed by suitable decoding techniques to find the closest codeword to the received sequence. The soft decision decoding offers better performance than the hard decision decoding, by typically $2-3$ dB. On the other hand, the soft decision decoder is much more complex to implement than the hard decision decoder.

Because of a good performance of the soft decision decoding techniques, many methods and algorithms were developed to obtain the optimum decoding performance and provide a desirable trade-off between the complexity and performance. The soft decision decoding can be classified into the following two main catagories [20].

1. The reliability based decoding algorithms
   - **A)** Algebraic decoding
     - **i)** Generalized minimum distance decoding (GMD) [25]
     - **ii)** Chase algorithm [27]
   - **B)** Reliability based reordering algorithms
     - **i)** Dorsh decoding [4]
     - **ii)** Order statistics decoding [2]
2. The algorithms based on code structure such as Vetirbi algorithm

In the reliability based decoding algorithms, the hard decisions of the received sequence are ordered in decreasing order according to the reliability of the received sequence. This way, fewer errors in the most reliable positions (MRP) are much more likely than among the least reliable positions (LRP). Thus, the decoding based on reliabilities can be classified into two groups. The first group processes the LRPs, and the second processes the MRPs. The GMD algorithm in [25] utilizes the LRPs after re-ordering to improve the algebraic decoding. In this algorithm, after re-ordering the hard decisions of the received sequence according to the reliabilities, the list of $(d_{min} + 1/2)$ candidates codeword is generated by modifying the hard decisions of the received sequence using an error and erasure algebraic decoding; e.g. by erasing the least reliable bit, three least reliable bits, $\cdots$,$((d_{min} - 1)/2)$. Then, the candidate codeword from the list is chosen. One of the main disadvantages of this algorithm is that algebraic decoder is too complex to implement and moreover may not succeed in generating candidate codeword for some error patterns.

The GMD was generalized and developed by Chase in [27, 33]. Chase proposed three algorithms for decoding linear binary block codes. The Chase algorithm is similar to the GMD except that the operation of modifying the hard decisions after re-ordering to create the coding list is developed by complementing. In addition the error correction only algebraic decoder is used instead of the error and erasure algebraic decoder. The Chase algorithm 3 achieves the trade-off between the complexity and the error performance of the GMD , but it suffers from the same drawbacks regarding the probability that the algebraic decoder may fail to produce a candidate codeword.

The Chase algorithm 2 is an improvement of Chase algorithm 3. This improvement is the size of the candidate codewords list that can be generated by taking all possible combinations of 0's and 1's of error patterns for the $(d_{min}/2)$ LRPs. The number of error patterns $2^{d_{min}/2}$ exponentially increases with d_{min} of the code, and thus the list of candidate codewords increases dramatically. As a result, the Chase algorithm 2 achieves better performance than the Chase algorithm 3, but with greater complexity. The Chase algorithm 1 uses a list of candidate codewords of size $\begin{pmatrix} n \\ d_{min}/2 \end{pmatrix}$ generated by complementing all possible combinations of $(d_{min}/2)$ LRPs. Thus, compared with the other two Chase algorithms, the Chase algorithm 3 is considered to be too complex.

Another types of the soft decoding algorithms for binary linear block codes based on processing the MRPs were developed. One of these algorithms is the Dorsh algorithm [4]. Dorsh used the reliability values of the received sequence to rework the generator matrix or the parity check matrix, so that the less reliable bits are treated as a parity check set and the most reliable bits are treated as an information set which determines the values of the parity set. After re-ordering columns of the generator matrix or the parity check matrix according to reliability of the received sequence, the new generator or parity check matrix is reduced to echelon canonical form by a Gaussian elimination [34]. Then a list of error patterns of increasing Hamming weight is generated, and the most reliable independent K information bits of the received sequence are modified by error patterns. Finally, re-encoding process is done to generate list of candidate codewords and the best codeword with the minimum Euclidian distance from the received sequence is chosen as a transmitted word.

The main advantage of the Dorsh algorithm over the Chase and the GMD algorithms is that the Dorsh agorithm is less complex to implement. The Chase and the GMD algorithms require an algebraic decoder whereas the Dorsh algorithm only needs to re-order received sequence according to reliabilities values, and re-encode the MRIP K information bits after some modifications. One more advantage of the Dorsh algorithm is that each new re-encoded word yields a new codeword while the Chase and GMD algebraic decoders may fail to produce a new codeword. In addition, the Dorsh algorithm provides a long coding list with more many candidates than the Chase algorithm. Another advantage of the Dorsh algorithm over the Chase is that the Dorsh algorithm is applicable to any linear block code while the Chase algorithms can be applied only to codes for which the algebraic decoder is available. Generally, exact comparison between the Chase algorithms and the Dorsh algorithm is quite difficult because the differences in operations. However, the complexity of the Chase algorithms is proportional to the number of decoding attempts while the Dorsh algorithm only requires sorting a low growth in sorting complexity for each decoding attempt [35]. Another algorithm for MRIP soft decoding were developed in [2, 5]. These algorithms process MRIPs of the received sequence in stages based on the joint statistics the noise after ordering of the received bits. Following the same basic idea of the Dorsh algorithm, in the OSD algorithm, the soft decision information is used to rank reliability of the received symbols, and then hard decisions are derived using bit by bit detection. Lower reliability bits are then erased and the generator matrix is sorted according to reliabilities and reduced to a echelon canonical form. The main difference between the Dorsh algorithm and the OSD algoritthm is the generation of error patterns. The generated error patterns in the Dorsh algorithm are listed according to the increasing Hamming weights, while in the OSD algorithm, the error patterns are generated in stages. For instance, for stage (I) all possible changes from zero to I of K MRIPs are made to generate stage or order(I) error patterns. Finally, the corresponding candidate codewords list is created after re-encoding. The OSD of order (I) requires $\sum_{j=0}^{I} \binom{K}{j}$ error patterns where K is the number of information bits to make the decoding decision.

It should be emphasized that the OSD algorithm processes the MRIP information bits which contain small number of errors, and makes all possible changes of small number of positions to produce the most likely ML codeword. For short codes of length less than 32 bits, or medium codes of length between 32 and 64 bits with code rate $R \geq 0.6$, near optimum performance is achieved within the first two stages of processing. However, longer codes, three or more stages are required to achieve near optimum performance. Eventhough most of the coding gain is achieved in the first two stages of processing.

One of the most important drawbacks of the OSD algorithm is that most of the candidate codewords are processed at phase j of order I reprocessing. To overcome this problem and to reduce the length of coding list, a solution was proposed in [8, 46]. The main idea of this technique in [8] is to reduce the coding list of the OSD order (I) re-processing by discarding the less likely error patterns. This is done by discarding a set of positions with the largest cardinality from the reprocessing algorithm. Furthermore, in [8, 36, 37], the structural properties of the code and the optimal cost decoding were utilized to reduce computation complexity and to speed up the decoding process. To achieve the same previous goal, a new technique was developed in [38] to accelerate the search for the most likely error patterns in the OSD algorithm. Test vectors are sorted in an increasing order of their weight which corresponds to the Hamming weight.

One more drawback of the OSD algorithm is that LRPs are not used, [9, 39, 40]. However, it is possible to obtain the performance of several information sets using the reliabilities by replacing positions in the MRPs by positions outside the MRP. As a result, smaller order of reprocessing of the MRPs can be considered. Thus the number of candidate codewords is reduced with near ML performance especially for codes with $R < 0.5$. However, since less reliable information bits are considered a large number of computations is needed by this technique as the sufficient condition of optimality which is derived in OSD is effective for the first set of information decoding. One more disadvantage of this technique in [9, 39, 40] is that only one change between two sets of positions is possible and the decoding is successful only if the new information sets are within the error correcting capability of the MRP reprocessing type algorithms.

A new approach was demonstrated in [41] to generate multiple information sets in more simple and efficient way than those in [9, 39, 40]. This approach allows multiple combinations when exchanging positions between two sets. It repeatedly uses new soft decision decodings by adding randomly generated biases to the received sequence. One of the most important advantage of [41] is that it can be applied to any MRP reprocessing type such as OSD or Box -Millar algorithm (BMA). Another technique is presented as a sort and match aglorithm in [43] to provide efficient decoding that reduces complexity of the ML decoding for general linear block code over memoryless channels. The sort and match algorithm reduces complexity of searching through the codebook or its trellis without degradation in the error performance. The algorithm splits the received sequence into two halves in number of different ways. Number of error patterns are generated for each half, and then, the two generated lists for each half are sorted and matched to form the coding list, and the best codeword can be chosen.

Another drawback of the OSD algorithm is that no intermediary error performance can be achieved between order $(I-1)$ and (I) reprocessing . The work in [9] demonstrated a new approach in trying to achieve intermediary error performance and utilize reliability values outside the MRIP. This approach reprocess $(K+P)$ MRP for some $0 < P < min\,(K, N-K)$, where K and N are the number of information bits and the length of the codeword respectively. The error patterns are divided into two classes: class 1, which contains all error patterns with at most $(I-1)$ errors in the K MRIPs, and class 2, which contains all error patterns with exactly (I) errors in the $K+P$ MRIP and error-free outside the MRPs. Class 1 error patterns are corrected by the conventional OSD $(I-1)$ reprocessing.

Another algorithm was developed in [3] which is known as box and match technique. The aim of this algorithm is to improve the error performance of the OSD algorithm and to achieve practically optimum decoding of long codes of rate $R = 1/2$. This approach utilizes conventional OSD algorithm to reprocess the MRIPs until the order (I) reprocessing to create the first part of the coding list. Then the matching process is completed for this coding list. The matching process is done by choosing several bits after the K-MRP called the control band (CB) in each codeword in the coding list. Sorting process is carried for the codeword in the coding list according to the values of the CB to divide codewords into groups or boxes. The codewords in each box are added to give new codewords added to original OSD algorithm coding list. As a result, the error performance is improved by increasing the number of error patterns which is about $(2I)$,where (I) is the order of convention OSD.

It should be stated that the BMA in [3], generally acheives lower time complexity compared with the sort and match technique in [43] due to the savings of exponential size sorting. On the other hand in [3], time complexity reduction is achieved at the cost of space complexity.

To achieve better performance than the BMA, the larger orders of I of OSD are required but the number of candidates in the coding list and memory size are increased simultaneously. Thus, it is desired to increase the decoding capability by increasing additional sets of candidates in the coding list without the $(i + 1)$ processing as investigated [44, 45]. The received blocks in [44] after re-ordering MRIPs are divided into K parts and the LRPs are divided into $N - K$ parts. The parity check matrix after re-ordering according to the reliability and reduced to echelon form is divided into the MRPs and the LRPs. The error patterns list is generated for each part and the syndrome list is calculated for each list. New syndrome lists are sorted and symmetrical syndromes are matched to form a list corresponding to valid codewords, and finally the most probable word is chosen.

The exponentially increasing of decoding complexity of the OSD with order (I) is still a weak point. An improved technique for the BMA was proposed in [33] by constructing a CB which is error-free with high probability. The matching capability of BMA is improved and the performance of BMA of order $(I + 1)$ is achieved without increase in memory but at the cost of linear increase in complexity. Some methods of reduction computations were proposed in [8, 36, 37], and the stopping criteria and conditions of optimality are utilized in [50].

The error patterns in the OSD algorithm may be tested so that probable error pattern are proposed before the less probable error patterns. This observation is used by many studies on error performance analysis for reliability based decoding such as in [48] and motivated the decoding algorithm in [6] named ALMT. The main idea of [6] is to generate a list of most priory likely tests with weigh function defined as in [38]. The test list is generated once based on the order statistics distribution of long block codes at given SNR, since for long codes the arranged vector of bit reliabilities can be approximated by mean values of the sorted reliabilities. Then, the received sequence is decoded using the arranged list of tests vectors. As a result, the ALMT algorithm achieves better performance than the OSD order (2) with lower number of error patterns.

The dorsch decoder was extended in [52] to produce a decoder that is capable of maximum-likelihood decoding. This technique ensures that for any linear code, that the $(N - K)$ LRPs soft decisions of the received sequence can be treated as erasures to determine candidate codewords. These codewords are derived from low information weight codewords and it is shown that an upper bound of the information weight may be calculated from each received vector in order to guarantee that the decoder will achieve the ML decoding performance. Using the cross-correlation function, it is shown that the most likely codeword may be derived from a partial correlation function of these low information weight codewords, which leads to an efficient fast decoder.

3.2 Coded Cooperative Diversity

In wireless communications, the signal is transmitted over detrimental channel conditions such as noise, fading, and interference. To reduce the effects of such conditions, different kinds of diversity can be employed. Antenna diversity is one of the traditional approaches to achieve diversity [53]. Antenna diversity offers effective resistance against multiple path fading effects and noise. Also, it offers better coverage for high data rate services due to feasibility of utilizing multiple antennas [12]. However, such approach of diversity is not very desirable in wireless systems due to the additional hardware and the integration that is required compared to a single antenna system. A more practical diversity scheme is the cooperative diversity using relays.

Such system consists of a source, a relay, and a destination. The source broadcasts the signal to both the relay and the destination, and then the relay forwards the signal to the destination. At the relay, different protocols of cooperative diversity achieving full diversity can be applied such as amplify-and-forward (AF) , decode-and-forward (DF), compress-and-forward (CF), and select-and-forward (SF) [14]. The destination combines both signals by one of the combining schemes such as equal gain combining (EGC), maximum ratio combining (MRC), and selection diversity [54].

In [14], a low complexity diversity protocols are developed to reduce the channel impairments due to fading and multipath propagation by employing cooperative terminals with single antenna in each terminal. This protocol achieves full diversity, however, it requires half duplex cooperation and thus, twice the bandwidth of direct transmission for a given rate. Moreover, extra hardware is needed to relay signal from the source to the relay . Such relaying is useful for cellular systems that utilize frequency division multiplexing which is not the case in ad-hoc and multiple hop networks. Another type of diversity is the transmitter diversity. In the transmitter diversity, two or more independent separated sources transmit copies of the signal through independent fading paths to a base station. This kind of diversity improves the performance of systems by overcoming the channel impairments. The user cooperation is considered as a transmitter diversity in [30, 31]. In there, traditional user cooperation is presented in which partners re-transmit received bits from each other by forwarding. The user cooperation was modified and further developed as coded cooperation in [17, 55, 56, 57, 58]. In the coded cooperation, codewords of the users are partitioned into two parts. One part is transmitted by the user and the other one by his partner. The main idea of the coded cooperation is to achieve coding gain and to keep the same overall rate for coding and transmission. In other words, no more system resources are used. The performance analysis of coded diversity is derived in [55]. Thus, the two users cooperate by dividing the transmitted codeword into two segments. Both users exchange segments by sending one and receiving the other. Each user decodes the received segment, then, if the decoding process succeeds by checking the CRC, the user computes additional parity bits for the partner's received data segment, and relays it to the destination. In case of the decoding failure, the user sends its own parity bits in the second segment. For such cooperative protocol, the error performance is analysed. However, such analysis is based on certain assumptions that the errors occurring in codewords are uniformly distributed among the segments, which is not always the case. A more complicated analysis of this system, which complexity grows significantly with the number of errors, is presented in [55].

The coded cooperation based on splitting a mother codeword is proposed in [17]. The puncture pattern of the codeword is fixed regardless of the channel realisations. Equal puncturing of a rate compatible punctured convolution code (RCPC) is applied. The performance analysis in [17] shows that this scheme achieves impressive gain in slow Rayleigh fading channels despite bad conditions of the inter- user channels. Some extensions to [17] are proposed in [56] where coded cooperation is implemented using turbo codes. In [56], space time coding principles are used to improve the performance in fast fading channels. In this scheme, both users send their own parity bits as well as their partner's parity bits in the second frame. However, using partner's channel as in [17] is not practical for this scheme assuming fast fading, as the user uplink channels see independent fading between the first frame and the second frame. Moreover, the second frame in space-time cooperation is taking advantage of a path diversity as each user transmits both user's parities.

The coded cooperation is implemented using turbo codes as space-time turbo coded cooperation. The available power in the second frame is distributed over each user's own transmitted bits and his partner's parity bits. In the case the first frame is not decoded successfully, the user will transmit his own second set of parity bits. As mentioned before, equal puncturing is considered in [17] and the code is split equally. In [59], different puncturing patterns are investigated for the RCPC to test the effect on the performance over different channel realizations. The main idea in [59] is to calculate the BER for all puncture patterns based on the knowledge of instantaneous SNR of cooperative system channels by the source, so the source chooses a puncture pattern corresponding to the minimum BER. Then, the conventional coded cooperation can be used.

Many publications analysed cooperative communications and applied channel coding such as LDPC [60, 61]. Utilizing LDPC in cooperative relay systems has a good coding gain and acheives good performance but with the large encoder and decoder complexity. Different network architectures have been studied recently to support high requirements of next generation wireless systems. Future wireless networks are expected to provide high data rates necessary to apply new applications and high quality of service for these applications. The required data rates will be much higher than what is provided by the $3G$ systems and beyond. Hence, it is obvious that recent cellular system architectures are not suitable for new standards. The concept of multiple relays is very attractive for many reasons. In such scenarios, many relays are placed in the same cell and each relay in the cell will serve a small area with a small amount of the transmit power. Hence, the signal transmission distance is reduced resulting in lower channel impairments as well as the interference and propagation loss are lowered and the SNR for users are increased. Therefore, the system performance can be improved. The user cooperation can be considered as an example of network coding where modulo 2 addition of the user's transmission is used [62]. The main idea of network coding in a two user relay system is that the relay encodes the received bits from both users, and then the relay sends the modulo 2 addition of the users packets to both users. The user 1 can decode the user 2 data by performing modulo 2 addition of the received packet and its own bits. In [62], the network coding and channel coding are performed jointly. The redundancy of the channel code protects the transmitted signal against the channel impairments, and the network redundancy support the channel code for a better protection. The turbo code is used as a channel code of one user and the network code forms distributed turbo code that can be decoded iteratively by the other user.

A new coded cooperation is proposed in [63]. Thus, the source broadcasts a codeword to the relay and the destination. The relay decodes and interleaves the received codeword, and then encodes the message again before retransmitting it to the destination. The two received signals are combined at the destination by one of the combining schemes such as MRC or EGC, and then a standard turbo decoder performs the decoding process. The extra coding gain over the diversity gain is achieved in [63] due to interleaving gain of the turbo code construction and the turbo decoder gain. In [64], algorithms proposed in [14] are extended and developed to provide immunity against fading in large networks. As mentioned before cooperative algorithms presented in [14] show inefficient utilization of the bandwidth, as each relay needs its own subchannel to retransmit information received from the source. Alternative approach to what is presented in [14] is suggested in [54]. Thus, the space-time coding is utilized jointly with cooperative protocols to allow relays to transmit in the same sub-channel. Furthermore, space-time cooperative protocols in [65] offer a full spatial diversity for a number of cooperating terminals and not just for the relays.

One important comment should be made, i.e., coded schemes and networks should generally be investigated and designed for cooperative protocols. Design of a channel code for cooperation use over slow fading channels is presented in [64]. Where the same cooperation schemes as in [14] are adopted. The main idea in [64] is to design a good channel code for the inter-user channel and add parity bits to obtain a good cooperative code for block fading cooperative system. Block fading model of [85] is used to find channel codes that are suitable for cooperation. However, certain design criteria are considered for cooperative communications which do not exist in traditional block fading. The usefullness of cooperative block fading is dependent on successful decoding in the partner's side. Thus, the channel code used by the inter-user should provide enough immunity against the channel impairments to increase the probability of successful decoding at the partner's side. The two levels of diversity is a result of exploiting a large channel code gain in the inter-user channel. In case of a partner decoder failure, the user continues its transmission. The channel codes for inter-user channels are suggested in [85] and it is shown that the cooperative coding can affect the packet routing in wireless networks. The soft decision distributed decoding is presented in [12]. The main idea in [12] is to utilize a SISO decoder at the relay to combat the main disadvantage of the DF in which hard decisions are applied to the received codeword in the source. The relay generates the LLRs using a SISO decoder, and then the LLR values are scaled, compressed, and quantized to 3 bits symbols for each source bit. In this distributed decoding scheme, the source is using soft information received from the relay as an extrinsic information for the turbo decoder. In some cases, this scheme performs better than the conventional DF scheme; however, more network resources are used such as the bandwidth because of quantization process for representing the soft values. However, most of the previous distributed turbo coded schemes are based on certain assumptions that the relay can perform error free decoding. This assumption is not practical unless automatic repeat request (ARQ) scenario is used in the link between the source and the relay which reduces the system transmission throughput. A distributed turbo code (DTC) with the soft information relaying schemes is proposed in [67]. The main benefit of this scheme is its ability to achieve error-free decoding at the relay without utilizing ARQ, thus, improving the system transmission throughput. The DTC in [67] uses soft information relaying when imperfect decisions occur rather than making decisions at the relay. This scheme can be summarized into two steps. First, the decoder at the relay employs maximum a posteriori (MAP) to calculate the aposteroiri probabilities (APPs) of information symbols received at the relay. Second, the APPs derived in the first step are used to calculate the soft estimation of parity bits of the information symbols.

The DTC was improved in [65] reducing the effects of the error free decoding assumption of the DTC in [12, 67]. The main idea of [14] is to calculate the LLR values of the received bits at the relay, and then transmit only highly reliable hard-decided bits to the destination while discard less reliable bits. The issue of reliability is decided by a threshold at the relay chosen to reduce the end-to-end BER of the system. The destination combines both signals received form the relay and the source and the decoding is performed. Unlike the conventional DF cooperation systems using the cyclic redundancy check (CRC), the threshold is set after the CRC to prevent correct bits from being blocked or wasted in the case of the CRC failure.

A multiple source cooperation diversity is suggested in [68]. This solution has the advantage when slow fading event damages small portion of the codeword unlike [12, 31, 58]. The users in [68] transmit their data to the destination and the other users. The other users receive this data and encode it by certain error correction codes and transmit the generated parity bits to the destination. The CRC that is used at the relay reduces the effects of errors over inter-user channels, and non-reliable data bits are discarded from this process of parity bits calculations at the relay. The performance of [68] can be improved if each user creates its bits so that the iterative decoding can be used at the destination. The distributed product code is proposed in [69] over AWGN. The coding is achieved jointly between the source and the relay in cooperative manner. The source uses sub-row code to encode information and sends it to the relay. The relay adds redundancy using systematic column code. The destination combines the received codewords from the source and relay to construct a product code. However, in [69], neither the criteria of choosing the sub-codes between the source and the relay are discussed nor the effects upon the desirable diversity level and the end-to-end performance of the system are investigated.

3.3 Summary

The signal processing techniques are significantly limited in wireless networks containing very high data rate links (of the order of Gigabits per second). In such cases, complexity reduction and diversity become important to remedy such challenge The problem of the decoding complexity has motivated the research community how to most efficiently apply the order statistics based list decoding techniques for linear binary block codes of small to medium block length. The original OSD, Chase, and GMD algorithms and all modifications that have been achieved on them are still suffering from several drawbacks. The complexity of original OSD decoding algorithm grows exponentially as shown in the literature. Thus no clear algorithm or solution has been proposed yet to reduce complexity significantly and keep the same performance. One of the main draw backs of the classical OSD algorithms is that the MRIP is considered as one segment assuming that all error are equally distributed along this segment, which is not correct .Moreover, the complexity of obtaining a row echelon form of the generator matrix for every received codeword represents a significant part of the overall decoding complexity, however, the solution of such problem has not been investigated in the literature. Also a deep analysis for the optimum list construction and the probability of selection most likely errors patterns in the coding list has not been carried out clearly in the literature.

Regarding coded diversity, all user cooperation schemes proposed in [17, 55, 56, 57], the diversity is achieved by dividing the transmitted codeword into segments, and then exchange the segments and forward it to destination. A comprehensive design guide lines for such process are not considered or clarified clearly. Also for the distributed decoding process in [12, 14, 67, 69] , neither the criteria of choosing the subcodes between the source and the relay such as the code rate and Hamming distance , nor the relay positions and the decoding complexity distribution and their effects upon the desirable diversity level and the end-to-end performance of the system are investigated.

4

List Decoding Techniques for Linear Binary Block Codes

4.1 Introduction

In this Chapter, the OSD-based decoding strategies are investigated for linear binary block codes. The main aim is to obtain low-complexity decoding schemes that provide sufficiently large or valuable coding gains, and most importantly, that are well-suited for implementation in communication systems with limited hardware resources, e.g., at nodes of the wireless sensor network. The original OSD was modified by considering the disjoint segments of the most reliable independent positions (MRIPs). The segmentation of the MRIPs creates flexibility that can be exploited to fine tune a trade-off between the BER performance and the decoding complexity. Thus, the original OSD can be considered to be a special case of the segmentation-based OSD having only one segment corresponding to the MRIPs. Since the complexity of obtaining a row echelon form of the generator matrix for every received codeword represents a significant part of the overall decoding complexity, we examine a partial-order statistics decoding (POSD) when only the systematic part of the received codeword is ordered.

This Chapter is organized as follows. System model is described in Section 4.2. Construction of the list of test error patterns is investigated in Section 4.3. The list decoding algorithms are developed in Section 4.4. The performance analysis is considered in Section 4.5. Numerical examples to compare the BER performance and the decoding complexity of the proposed decoding schemes are presented in Section 4.6. Finally, conclusions are given in Section 4.7.

4.2 System Model

Consider transmission of codewords of a linear binary block code $\mathcal{C}$ over an additive white Gaussian noise (AWGN) channel with Rayleigh fading. The code $\mathcal{C}$, denoted as $(N, K, d_{\min})$, has block length N, dimension K, and the minimum Hamming distance between any two codewords $d_{\min}$. Binary codewords $\mathbf{c} \in \mathbb{Z}_2^N$ where $\mathbb{Z}_2 = \{0, 1\}$ are generated from a vector of information bits $\mathbf{u} \in \mathbb{Z}_2^K$ using the generator matrix $\mathbf{G} \in \mathbb{Z}_2^{K \times N}$, i.e., $\mathbf{c} = \mathbf{u}\mathbf{G}$, and all binary operations are considered over a Galois field GF(2). If the code $\mathcal{C}$ is systematic, the generator matrix has the form, $\mathbf{G} = [\mathbf{I}\ \mathbf{P}]$, where $\mathbf{I}$ is the $K \times K$ identity matrix, and $\mathbf{P} \in \mathbb{Z}_2^{K \times (N-K)}$ is the matrix of parity checks. The codeword $\mathbf{c}$ is mapped to a binary phase shift keying (BPSK) sequence $\mathbf{x} \in \{+1, -1\}^N$ before transmission using a mapping, $x_i = \mathcal{M}(c_i) = (-1)^{c_i}$, for $i = 1, 2, \cdots, N$. Assuming bits u_i and u_j, the mapping $\mathcal{M}$ has the property,

$$\mathcal{M}(u_i \oplus u_j) = \mathcal{M}(u_i)\,\mathcal{M}(u_j) \tag{4.1}$$

where $\oplus$ denotes the modulo 2 addition. The encoded bit c_i can be recovered from the symbol x_i using the inverse mapping, $c_i = \mathcal{M}^{-1}(x_i) = (1 - x_i)/2$. For brevity, we also use the notation, $\mathbf{x} = \mathcal{M}(\mathbf{c})$ and $\mathbf{c} = \mathcal{M}^{-1}(\mathbf{x})$, to denote the component-wise modulation mapping and de-mapping, respectively. The code $\mathcal{C}$ is assumed to have equally probable values of the encoded bits, i.e., the probability, $\Pr\{c_i = 0\} = \Pr\{c_i = 1\} = 1/2$, for $i = 1, 2, \cdots, N$. Consequently, all the codewords are transmitted with the equal probability, i.e., $\Pr\{\mathbf{c}\} = 2^{-K}$ for $\forall \mathbf{c} \in \mathcal{C}$.

The signal at the output of the matched filter at the receiver can be written as,

$$y_i = h_i x_i + w_i$$

where the frequency non-selective channel fading coefficients h_i as well as the AWGN samples w_i are mutually uncorrelated zero-mean circularly symmetric complex Gaussian random variables. The variance of h_i is unity, i.e., $\mathrm{E}[|h_i|^2] = 1$ where $\mathrm{E}[\cdot]$

is expectation, and $|\cdot|$ denotes the absolute value. The samples w_i have the variance, $\mathrm{E}[|w_i|^2] = (R\gamma_c)^{-1}$, where $R = K/N$ is the coding rate of $\mathcal{C}$, and γ_c is the signal-to-noise ratio (SNR) per transmitted encoded binary symbol. The covariance, $\mathrm{E}\big[h_i h_j^*\big] = 0$ for $i \neq j$, where $(\cdot)^*$ denotes the complex conjugate, corresponds to the case of a fast fading channel with ideal interleaving and deinterleaving. For a slowly block-fading channel, the covariance, $\mathrm{E}\big[h_i h_j^*\big] = 1$ for $\forall i, j = 1, 2, \cdots, N$, and the fading coefficients are uncorrelated between transmissions of adjacent codewords.

In general, denote as $f(\cdot)$ the probability density function (PDF) of a random variable. The reliability r_i of the received signal y_i corresponds to a ratio of the conditional PDFs of y_i [70], i.e.,

$$\frac{f(y_i|x_i = +1, h_i)}{f(y_i|x_i = -1, h_i)} \propto \mathrm{Re}\{h_i^* y_i\} = r_i$$

since the PDF $f(y_i|x_i, h_i)$ is conditionally Gaussian. Thus, the reliability r_i can be written as,

$$r_i = \mathrm{Re}\{h_i\}\,\mathrm{Re}\{y_i\} + \mathrm{Im}\{h_i\}\,\mathrm{Im}\{y_i\} = |h_i|^2 x_i + |h_i| w_i.$$

The bit-by-bit quantized (i.e., hard) decisions are then defined as,

$$\hat{c}_i = \mathcal{M}^{-1}\left(\mathrm{sign}(r_i)\right)$$

where $\mathrm{sign}(\cdot)$ denotes the sign of a real number.

More importantly, even though the primary metric of our interest is the BER performance of the code $\mathcal{C}$, it is mathematically more convenient to obtain and analyze the list decoding algorithms assuming the probability of codeword error. Thus, we assume that the list decoding with a given decoding complexity obtained for the probability of codeword error will also have a good BER performance. The maximum likelihood (ML) decoder minimizing the probability of codeword error provides the decision $\hat{\mathbf{c}}_{\mathrm{ML}}$ on the most likely transmitted codeword, i.e.,

$$\begin{aligned} \hat{\mathbf{c}}_{\mathrm{ML}} &= \underset{\mathbf{c}\in\mathcal{C}:\ \mathbf{x}=\mathcal{M}(\mathbf{c})}{\mathrm{argmin}} \ \|\mathbf{y} - \mathbf{h} \odot \mathbf{x}\|^2 \\ &= \underset{\mathbf{c}\in\mathcal{C}}{\mathrm{argmax}} \sum_{i=1}^{N} \mathrm{Re}\{y_i h_i^* x_i^*\} \overset{\mathrm{BPSK}}{=} \underset{\mathbf{c}\in\mathcal{C}:\ \mathbf{x}=\mathcal{M}(\mathbf{c})}{\mathrm{argmax}} \ \mathbf{r} \cdot \mathbf{x} \end{aligned} \tag{4.2}$$

where **y**, **h**, **x**, and **r** denote the N-dimensional row vectors of the received signals y_i, the channel coefficients h_i, the transmitted symbols x_i, and the reliabilities r_i within one codeword, respectively, $\|\cdot\|$ is the Euclidean norm of a vector, $\odot$ is the component-wise (Hadamard) product of vectors, and the binary operator $\cdot$ is used to denote the dot-product of vectors. The codewords $\mathbf{c} \in \mathcal{C}$ used in (4.2) to find the maximum or the minimum value of the ML metric are often referred to as the test codewords. In the following subsection, the soft-decision decoding algorithms are investigated with low implementation complexity to replace the computationally demanding ML decoding (4.2).

For the case of 16 QAM modulation the codewords are interleaved and mapped to either binary to 16 quadrature amplitude modulation (QAM) symbols. We assume natural mapping of the consecutive sequences of 4 encoded bits (c_1, c_2, c_3, c_4) to 16QAM modulation symbols $x = x_I + \mathrm{j}x_Q$ such that the encoded bits (c_1, c_3) are mapped to $x_I \in \{\pm 1, \pm 3\}$, and the encoded bits (c_2, c_4) are mapped to $x_Q \in \{\pm 1, \pm 3\}$, [71].

In general,at the receiver, the soft-decision value for each encoded bit c is obtained from the received symbol y using the log-likelihood ratio (LLR) [70],

$$\Lambda(c|y) = \log \frac{\mathrm{P_r}(c = 0|y)}{\mathrm{P_r}(c = 1|y)}$$

where $\Pr(\cdot)$ denotes the probability. The LLR values are deinterleaved before the channel decoding is performed. Denote the received symbol $y = y_I + \mathrm{j}y_Q =$, and let $y = g\sqrt{E_s}x + w$. Assuming natural mapping of the encoded bits to 16QAM symbols, we have modified the LLR expressions in [72] to include the channel fading and path-loss attenuation. Thus, the approximate LLR values are computed as [72],

$$\Lambda(c_1|y_I) = \begin{cases} \frac{8gy_I\sqrt{E_s}}{N_0} + \frac{8g^2E_s}{N_0} & y_I \leq -2g\sqrt{E_s} \\ \frac{4gy_I\sqrt{E_s}}{N_0} & -2g\sqrt{E_s} < y_I < 2g\sqrt{E_s} \\ \frac{8gy_I\sqrt{E_s}}{N_0} - \frac{8g^2E_s}{N_0} & y_I \geq 2g\sqrt{E_s} \end{cases}$$

$$\Lambda(c_3|y_I) = \begin{cases} \frac{4gy_I\sqrt{E_s}}{N_0} + \frac{8g^2E_s}{N_0} & y_I \leq 0 \\ \frac{-4gy_I\sqrt{E_s}}{N_0} + \frac{8g^2E_s}{N_0} & y_I > 0. \end{cases}$$

Similar expressions are obtained for the LLRs $\Lambda(c_2|y_Q)$ and $\Lambda(c_4|y_Q)$. The LLR values are used by the soft-decision decoders that are described before in this chapter. In particular, the received bits are first ordered according to their reliabilities given as the absolute value of their LLRs. The decoder then searches a list of very likely error patterns in order to find the codeword having the minimum Euclidean distance.

4.3 List Decoding

The list-based decoding algorithms are investigated. For simplicity, we assume binary block codes that are linear and systematic [20]. It is noticed that whereas the extension of the list-based decoding algorithms to non-systematic codes is straightforward, the list based decoding of non-linear codes is complicated by the fact that the list of the test codewords is, in general, dependent on the received sequence. The decoding (time) complexity O of the list decoding algorithms can be measured as the list size given by the number of the test codewords that are examined in the decoding process. Thus, the ML decoding (4.2) has the complexity, $O_{\mathrm{ML}} = 2^K$, which is prohibitive for larger values of K. Among the practical list-based decoding algorithms with the acceptable decoding complexity, we investigate the order statistics decoding (OSD) [2] based list decoding algorithms for soft-decision decoding of linear binary block codes.

The OSD decoding resumes by reordering the received sequence of reliabilities as,

$$|\tilde{r}'_1| \geq |\tilde{r}'_2| \geq \cdots |\tilde{r}'_N| \tag{4.3}$$

where the tilde is used to denote the ordering. This ordering of the reliabilities defines a permutation, λ', i.e.,

$$\tilde{\mathbf{r}}' = \lambda'[\mathbf{r}] = (\tilde{r}'_1, \cdots, \tilde{r}'_N).$$

The permutation λ' corresponds to the generator matrix $\tilde{\mathbf{G}}' = \lambda'[\mathbf{G}]$ having the reordered columns. In order to obtain the most reliable independent positions (MRIPs) for the first K bits in the codeword, additional swapping of the columns of $\tilde{\mathbf{G}}'$ may have to be used which corresponds to the permutation λ'', and the generator matrix $\tilde{\mathbf{G}}'' = \lambda''\left[\tilde{\mathbf{G}}'\right]$. The matrix $\tilde{\mathbf{G}}''$ can be manipulated into a row (or a reduced row) echelon form using the Gauss (or the Gauss-Jordan) elimination. To simplify the notation, let $\tilde{\mathbf{r}}$ and $\tilde{\mathbf{G}}$ denote the reordered sequence of the reliabilities $\mathbf{r}$ and the reordered generator matrix $\tilde{\mathbf{G}}$ in a row (or a reduced row) echelon form, respectively, after employing the permutations λ' and λ'', to decode the received sequence $\mathbf{y}$. Thus, for $i \geq j$, the reordered sequence $\tilde{\mathbf{r}}$ has elements, $|\tilde{r}_i| \geq |\tilde{r}_j|$, for $i, j = 1, \cdots, K$, and for $i, j = K+1, \cdots, N$.

The complexity of the ML decoding (4.2) of the received sequence $\mathbf{y}$ can be reduced by assuming a list of the L test codewords, so that $L \ll 2^K$. Denote such a list of the test codewords of cardinality L generated by the matrix $\tilde{\mathbf{G}}$ as, $\mathcal{E}_L = \{\mathbf{e}_0, \mathbf{e}_2, \cdots, \mathbf{e}_{L-1}\}$, and let $\mathbf{e}_0 = \mathbf{0}$ be the all-zero codeword. Then, the list decoding of $\mathbf{y}$ is defined as,

$$\hat{\mathbf{c}} = \underset{\mathbf{e} \in \mathcal{E}_L:\ \mathbf{x} = \mathcal{M}(\hat{\mathbf{c}}_0 \oplus \mathbf{e})}{\operatorname{argmax}} \ \tilde{\mathbf{r}} \cdot \mathbf{x} \tag{4.4}$$

where the systematic part of the codeword $\hat{\mathbf{c}}_0$ is given by the hard-decision decoded bits at the MRIPs. The decoding step to obtain the decision $\hat{\mathbf{c}}_0$ is referred to as the 0-th order OSD reprocessing in [2]. In addition, due to linearity of $\mathcal{C}$, we have that $(\mathbf{c}_0 \oplus \mathbf{e}) \in \mathcal{C}$, and thus, the test codewords $\mathbf{e} \in \mathcal{E}_L$ can be also referred to as the test error patterns in the decoding (4.4). Using the property (4.1), we can rewrite the decoding (4.4) as,

$$\hat{\mathbf{c}} = \underset{\mathbf{e} \in \mathcal{E}_L}{\operatorname{argmax}} \ \tilde{\mathbf{r}} \cdot \hat{\mathbf{x}}_0 \cdot \mathcal{M}(\mathbf{e}) = \underset{\mathbf{e} \in \mathcal{E}_L}{\operatorname{argmax}} \ \tilde{\mathbf{r}}_0 \cdot \mathcal{M}(\mathbf{e}) \tag{4.5}$$

where we denoted $\hat{\mathbf{x}}_0 = \mathcal{M}(\hat{\mathbf{c}}_0)$ and $\tilde{\mathbf{r}}_0 = \tilde{\mathbf{r}} \odot \hat{\mathbf{x}}_0$. The system model employing the list decoding (4.5) is illustrated in Fig. 4.1. More importantly, as indicated in Fig. 4.1, the system model can be represented as an equivalent channel with the binary vector input $\mathbf{c}$ and the vector soft-output $\tilde{\mathbf{r}}_0$.

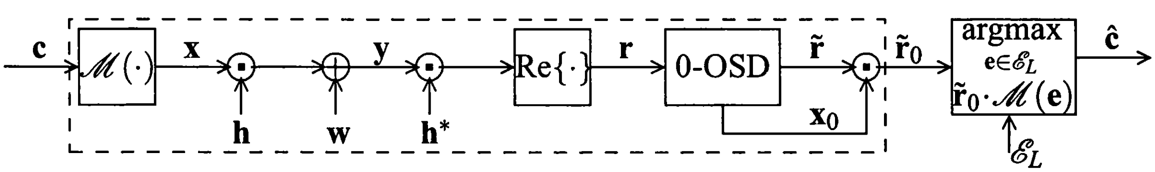

Figure 4.1: The system model and an equivalent vector channel with the binary vector input $\mathbf{c}$ and the vector soft-output $\tilde{\mathbf{r}}_0$.

4.4 List Selection

The selection of the test error patterns $\mathbf{e}$ to the list $\mathcal{E}_L$ as well as the list size L have a dominant effect upon the probability of incorrect codeword decision by the list decoding. Denote such probability of codeword error as $\mathrm{P_e}$, and let $\mathbf{c}_{\mathrm{Tx}}$ be the transmitted codeword. In [10], the probability $\mathrm{P_e}$ is expanded as,

$$\begin{aligned}\mathrm{P_e} &= \Pr\{\hat{\mathbf{c}} \neq \mathbf{c}_{\mathrm{Tx}} | \hat{\mathbf{c}}_{\mathrm{ML}} \neq \mathbf{c}_{\mathrm{Tx}}\} \Pr\{\hat{\mathbf{c}}_{\mathrm{ML}} \neq \mathbf{c}_{\mathrm{Tx}}\} \\ &\quad + \Pr\{\hat{\mathbf{c}} \neq \mathbf{c}_{\mathrm{Tx}} | \hat{\mathbf{c}}_{\mathrm{ML}} = \mathbf{c}_{\mathrm{Tx}}\} \Pr\{\hat{\mathbf{c}}_{\mathrm{ML}} = \mathbf{c}_{\mathrm{Tx}}\}\end{aligned}$$

where the decision $\hat{\mathbf{c}}$ is obtained by the decoding (4.5), and the condition, $\hat{\mathbf{c}}_{\mathrm{ML}} \neq \mathbf{c}_{\mathrm{Tx}}$, is true provided that the vectors $\hat{\mathbf{c}}_{\mathrm{ML}}$ and $\mathbf{c}_{\mathrm{Tx}}$ differ in at least one component, i.e., $\hat{\mathbf{c}}_{\mathrm{ML}} = \mathbf{c}_{\mathrm{Tx}}$ if and only if all the components of the vectors are equal. Since, for any list $\mathcal{E}_L$, the probability, $\Pr\{\hat{\mathbf{c}} \neq \mathbf{c}_{\mathrm{Tx}} | \hat{\mathbf{c}}_{\mathrm{ML}} \neq \mathbf{c}_{\mathrm{Tx}}\} = 1$, and usually, the probability, $\Pr\{\hat{\mathbf{c}}_{\mathrm{ML}} = \mathbf{c}_{\mathrm{Tx}}\}$ is close to 1, $\mathrm{P_e}$ can be tightly upper-bounded as,

$$\mathrm{P_e} \leq \Pr\{\hat{\mathbf{c}}_{\mathrm{ML}} \neq \mathbf{c}_{\mathrm{Tx}}\} + \Pr\{\hat{\mathbf{c}} \neq \mathbf{c}_{\mathrm{Tx}} | \hat{\mathbf{c}}_{\mathrm{ML}} = \mathbf{c}_{\mathrm{Tx}}\}. \tag{4.6}$$

The first term on the right hand side of (4.6) is the codeword error probability of the ML decoding, and the second term is the conditional codeword error probability of the list decoding. The probability, $\Pr\{\hat{\mathbf{c}} \neq \mathbf{c}_{\mathrm{Tx}} | \hat{\mathbf{c}}_{\mathrm{ML}} = \mathbf{c}_{\mathrm{Tx}}\}$, is decreasing with the list size. In the limit of the maximum list size when the list decoding becomes the ML decoding, the bound (4.6) becomes, $\mathrm{P_e} = \Pr\{\hat{\mathbf{c}}_{\mathrm{ML}} \neq \mathbf{c}_{\mathrm{Tx}}\}$. The bound (4.6) is particularly useful to analyze the performance of the list decoding (4.5). However, in order to construct the list of the test error patterns, we consider the following expansion of the probability $\mathrm{P_e}$, i.e.,

$$\begin{aligned}\mathrm{P_e} &= \Pr\{\hat{\mathbf{c}} \neq \mathbf{c}_{\mathrm{Tx}} | (\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0) \in \mathcal{E}_L\} \Pr\{(\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0) \in \mathcal{E}_L\} \\ &\quad + \Pr\{\hat{\mathbf{c}} \neq \mathbf{c}_{\mathrm{Tx}} | (\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0) \notin \mathcal{E}_L\} \Pr\{(\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0) \notin \mathcal{E}_L\} \\ &= 1 - \underbrace{\Pr\{\hat{\mathbf{c}} = \mathbf{c}_{\mathrm{Tx}} | (\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0) \in \mathcal{E}_L\}}_{\mathrm{P_I}} \underbrace{\Pr\{(\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0) \in \mathcal{E}_L\}}_{\mathrm{P_{II}}}.\end{aligned}$$

Using (4.4) and (4.5), the probability $\mathrm{P_I}$ that the list decoding (4.5) selects the transmitted codeword provided that such codeword is in the list (more precisely, provided that the error pattern $\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0$ is in the list) can be expressed as,

$$\mathrm{P_I} = \Pr\{\tilde{\mathbf{r}} \cdot \mathcal{M}(\mathbf{c}_{\mathrm{Tx}} \oplus \hat{\mathbf{c}}_0) \geq \tilde{\mathbf{r}}_0 \cdot \mathcal{M}(\mathbf{e}), \forall \mathbf{e} \in \mathcal{E}_L\}. \tag{4.7}$$

The probability (4.7) decreases with the list size, and, in the limit of the maximum list size $L = 2^K$, $\mathrm{P_I} = 1 - \mathrm{P_e}$. On the other hand, the probability $\mathrm{P_{II}}$ that the transmitted codeword is in the decoding list increases with the list size, and $\mathrm{P_{II}} = 1$, for $L = 2^K$.

Since the coding $\mathcal{C}$ and the communication channel are linear, then, without loss of generality, it can be assumed that the all-zero codeword, $\mathbf{c}_{\mathrm{Tx}} = \mathbf{0}$, is transmitted. Consequently, given the list decoding complexity L, the optimum list $\mathcal{E}_L^*$ minimizing the probability $\mathrm{P_e}$ is constructed as,

$$\mathcal{E}_L^* = \operatorname*{argmax}_{\mathcal{E}:\ |\mathcal{E}|=L} \Pr\{\hat{\mathbf{c}} = \mathbf{0} | \hat{\mathbf{c}}_0 \in \mathcal{E}\} \Pr\{\hat{\mathbf{c}}_0 \in \mathcal{E}\} \tag{4.8}$$

where $|\mathcal{E}|$ is the cardinality of the test list $\mathcal{E}$, and the hard-decision codeword $\hat{\mathbf{c}}_0 \in \mathcal{C}$ represents the error pattern observed at the receiver after transmission of the codeword $\mathbf{c}_{\mathrm{Tx}} = \mathbf{0}$. For a given list of the error patterns $\mathcal{E}$ in (4.8), and for the system model in Section 4.2 with asymptotically large SNR, the probability $\mathrm{P_I} = \Pr\{\hat{\mathbf{c}} = \mathbf{0} | \hat{\mathbf{c}}_0 \in \mathcal{E}\}$ is dominated by the error events corresponding to the error patterns with the smallest Hamming distances. Since the error patterns are also codewords of $\mathcal{C}$, the smallest Hamming distance between any two error patterns in the list $\mathcal{E}$ is at least $d_{\min}$. Assuming that the search in (4.8) is constrained to the lists $\mathcal{E}$ having the minimum Hamming distance between any two error patterns given by $d_{\min}$, the probability $\mathrm{P_I}$ is approximately constant for all the lists $\mathcal{E}$, and we can consider the suboptimum list construction,

$$\mathcal{E}_L = \operatorname*{argmax}_{\mathcal{E}:\ |\mathcal{E}|=L} \Pr\{\hat{\mathbf{c}}_0 \in \mathcal{E}\}. \tag{4.9}$$

The list construction (4.9) is recursive in its nature, since the list $\mathcal{E}$ maximizing (4.9) consists of the L most probable error patterns. However, in order to achieve a small probability of decoding error $\mathrm{P_e}$ and approach the probability of decoding error, $\Pr\{\hat{\mathbf{c}}_{\mathrm{ML}} \neq \mathbf{c}_{\mathrm{Tx}}\}$, of the ML decoding, the list size L must be large. We can obtain a practical list construction by assuming the L sufficiently probable error patterns rather than assuming the L most likely error patterns. Theorem 1 and Theorem 2 are restated in [2] to obtain the likely error patterns and to define the practical list decoding algorithms.

Denote as $\mathrm{P}(i_1, i_2, \cdots, i_n)$ the n-th order joint probability of bit errors at bit positions $1 \leq i_1 < i_2 < \cdots < i_n \leq N$ in the received codeword after the ordering

λ' and λ'' and before the decoding. Since the test error pattern $\mathbf{e}$ is a codeword of $\mathcal{C}$, the probability $\mathrm{P}(i_1, i_2, \cdots, i_n)$, for $i_n \le K$, is equal to the probability $\Pr\{\mathbf{e} = \hat{\mathbf{c}}_0\}$ assuming that the n bit errors occurred during the transmission corresponding to the positions (after the ordering) $i_1, i_2, \cdots, i_n$. We have the following lemma.

Lemma 1 *For any bit positions* $\mathcal{I}_1 \subseteq \mathcal{I} \subseteq \{1, 2, \cdots, N\}$,

$$\mathrm{P}(\mathcal{I}) \le \mathrm{P}(\mathcal{I}_1).$$

The lemma is proved by noting that

$$\mathrm{P}(\mathcal{I}) = \mathrm{P}(\mathcal{I}_1, \mathcal{I}\backslash\mathcal{I}_1) = \mathrm{P}(\mathcal{I}\backslash\mathcal{I}_1|\mathcal{I}_1)\mathrm{P}(\mathcal{I}_1) \le \min\{\mathrm{P}(\mathcal{I}_1), \mathrm{P}(\mathcal{I}\backslash\mathcal{I}_1|\mathcal{I}_1)\} \le \mathrm{P}(\mathcal{I}_1)$$

where $\mathcal{I}\backslash\mathcal{I}_1$ denotes the difference of the two sets. Using Lemma 1, it can be shown, for example, that, $\mathrm{P}(i, j) \le \mathrm{P}(i)$ and $\mathrm{P}(i, j) \le \mathrm{P}(j)$. We can now restate Theorem 1 and Theorem 2 in [2] as follows. Assume bit positions $1 \le i < j < k \le N$, and let the corresponding reliabilities be $|\tilde{r}_i| \ge |\tilde{r}_j| \ge |\tilde{r}_k|$. Then, the bit error probabilities,

$$\begin{aligned} \mathrm{P}(i) &\le \mathrm{P}(j) \\ \mathrm{P}(i, j) &\le \mathrm{P}(i, k). \end{aligned}$$

Without loss of generality, it is assumed that the symbols $x_i = -1$, $x_j = -1$ and $x_k = -1$ have been transmitted. Then, before the decoding, the received bits would be decided erroneously if the reliabilities $\tilde{r}_i > 0$, $\tilde{r}_j > 0$, and $\tilde{r}_k > 0$. Conditioned on the transmitted symbols, let $f(\cdot)$ denote the conditional PDF of the ordered reliabilities $\tilde{r}_i$, $\tilde{r}_j$ and $\tilde{r}_k$.

Consider first the inequality $\mathrm{P}(i) \le \mathrm{P}(j)$. Since, for $\tilde{r}_i > 0$, $f(\tilde{r}_i) < f(-\tilde{r}_i)$, using $f(\tilde{r}_i, \tilde{r}_j) = f(\tilde{r}_i|\tilde{r}_j)f(\tilde{r}_j)$, it can shown that, for $\tilde{r}_i > 0$ and any $\tilde{r}_j$, $f(\tilde{r}_i, \tilde{r}_j) < f(-\tilde{r}_i, \tilde{r}_j)$. Similarly, using $f(-\tilde{r}_i, \tilde{r}_j) = f(\tilde{r}_j| - \tilde{r}_i)f(-\tilde{r}_i)$, it can be shown that, for $\tilde{r}_j > 0$ and any $\tilde{r}_i$, $f(-\tilde{r}_i, \tilde{r}_j) < f(-\tilde{r}_i, -\tilde{r}_j)$. Then, the probability of error for bits i

and j, respectively, is,

$$\begin{aligned}
\mathrm{P}(i) &= \int_0^\infty \int_{-\tilde{r}_i}^{\tilde{r}_i} f(\tilde{r}_i, \tilde{r}_j) \mathrm{d}\tilde{r}_j \mathrm{d}\tilde{r}_i \\
&= \int_0^\infty \int_0^{\tilde{r}_i} f(\tilde{r}_i, \tilde{r}_j) \mathrm{d}\tilde{r}_j \mathrm{d}\tilde{r}_i + \int_0^\infty \int_{-\tilde{r}_i}^0 f(\tilde{r}_i, \tilde{r}_j) \mathrm{d}\tilde{r}_j \mathrm{d}\tilde{r}_i \\
\mathrm{P}(j) &= \int_{-\infty}^\infty \int_0^{|\tilde{r}_i|} f(\tilde{r}_i, \tilde{r}_j) \mathrm{d}\tilde{r}_j \mathrm{d}\tilde{r}_i \\
&= \int_0^\infty \int_0^{\tilde{r}_i} f(\tilde{r}_i, \tilde{r}_j) \mathrm{d}\tilde{r}_j \mathrm{d}\tilde{r}_i + \int_0^\infty \int_{-\tilde{r}_i}^0 f(-\tilde{r}_i, -\tilde{r}_j) \mathrm{d}\tilde{r}_j \mathrm{d}\tilde{r}_i
\end{aligned}$$

and thus, $\mathrm{P}(i) \leq \mathrm{P}(j)$.

The second inequality, $\mathrm{P}(i, j) \leq \mathrm{P}(i, k)$, can be proved by assuming conditioning, $\mathrm{P}(i, j) = \mathrm{P}(j|i)\mathrm{P}(i)$, $\mathrm{P}(i, k) = \mathrm{P}(k|i)\mathrm{P}(i)$, and $f(\tilde{r}_i, \tilde{r}_j, \tilde{r}_k) = f(\tilde{r}_j, \tilde{r}_k|\tilde{r}_i)f(\tilde{r}_i)$, and by using inequality $\mathrm{P}(i) \leq \mathrm{P}(j)$, and following the steps in the first part of the theorem proof.

4.5 List Decoding Algorithms

Using Theorem 1 and Theorem 2 in [2], the original OSD assumes the following list of error patterns,

$$\mathcal{E}_L = \{\mathbf{eG} : 0 \leq w_{\mathrm{H}}[\mathbf{e}] \leq I, \ \mathbf{e} \in \mathbb{Z}_2^K\} \tag{4.10}$$

where I is the so-called reprocessing order of the OSD, and $w_{\mathrm{H}}[\mathbf{e}]$ is the Hamming weight of the vector $\mathbf{e}$. The list (4.10) uses a K-dimensional sphere of radius I defined about the origin $\mathbf{0} = (0, \cdots, 0)$ in $\mathbb{Z}_2^K$. The decoding complexity for the list (4.10) is $L = \sum_{l=0}^{I} \binom{K}{l}$ where l is referred to as the phase of the order I reprocessing in [2]. Assuming an AWGN channel, the recommended reprocessing order is $I = \lceil d_{\min}/4 \rceil \ll K$ where $\lceil \cdot \rceil$ is the ceiling function. Since the OSD algorithm may become too complex for larger values of I and K, a stopping criterion for searching the list $\mathcal{E}_L$ was developed in [8].

The following inefficiencies of the original OSD algorithm are indentified. First, provided that no stopping nor skipping rules for searching the list of the test error patterns are used, once the MRIPs are found, the ordering of bits within the MRIPs according to their reliabilities becomes irrelevant. Second, whereas the BER performance of the OSD is modestly improving with the reprocessing order I, the complexity of the OSD increases rapidly with I [8]. Thus, for given K, the maximum value of I is limited by the allowable OSD complexity to achieve a certain target BER performance. It can be addressed that the inefficiencies of the original OSD by more carefully exploiting the properties of the probability of bit errors given by Lemma 1 and Theorem 4.1. Hence, the aim is to construct a well-defined list of the test error patterns *without* considering the stopping and the skipping criteria to search this list.

Recall that the error patterns can be uniquely specified by bits in the MRIPs whereas the bits of the error patterns outside the MRIPs are obtained using the parity check matrix. In order to design a list of the test error patterns independently of the particular generator matrix of the code as well as independently of the particular received sequence, only the bit errors within the MRIPs are considered. Thus, it can be assumed that, for all error patterns, the bit errors outside the MRIPs affect the value of the metric in (4.5) equally. More importantly, in order to improve the list decoding complexity and the BER performance trade-off, the partitioning of the MRIPs is considered into disjoint segments. This decoding strategy employing the segments of the MRIPs is investigated next.

4.5.1 Segmentation-Based OSD

Assuming Q disjoint segments of the MRIPs, the error pattern e corresponding to the K MRIPs can be expressed as a concatenation of the Q error patterns $\mathrm{e}^{(q)}$ of length K_q bits, $q = 1, \cdots, Q$, i.e.,

$$\mathrm{e} = (\mathrm{e}^{(1)}, \cdots, \mathrm{e}^{(Q)}) \in \mathbb{Z}_2^K$$

so that $\sum_{q=1}^{Q} K_q = K$, and $w_{\mathrm{H}}[\mathrm{e}] = w_{\mathrm{H}}[\mathrm{e}^{(1)}] + \cdots + w_{\mathrm{H}}[\mathrm{e}^{(Q)}]$. As indicated by Lemma 1 and Theorem 4.1, more likely error patterns have smaller Hamming weights and they correct the bit positions with smaller reliabilities. In addition, the decoding complexity given by the total number of error patterns in the list should grow linearly with the number of segments Q. Consequently, for a small number of segments Q, it is expected that a good decoding strategy is to decode each segment independently, and then, the final decision is obtained by selecting the best error (correcting) pattern from each of the segments decodings. In this thesis, we refine this strategy for $Q = 2$ segments as a generalization of the conventional OSD having only $Q = 1$ segment.

Assuming that the two segments of the MRIPs are decoded independently, the list of error patterns can be written as,

$$\mathcal{E}_L = \mathcal{E}_{L_1}^{(1)} \cup \mathcal{E}_{L_2}^{(2)} \tag{4.11}$$

where $\mathcal{E}_{L_1}^{(1)}$ and $\mathcal{E}_{L_2}^{(2)}$ are the lists of error patterns corresponding to the list decoding of the first segment and of the second segment, respectively, and $L = L_1 + L_2$. Obviously, fewer errors, and thus, fewer error patterns can be assumed in the shorter segments with larger reliabilities of the received bits. Similarly to the conventional OSD having one segment, for both MRIPs segments, assuming all the error patterns up to the maximum Hamming weight I_q, $q = 1, 2$. Then, the lists of error patterns in (4.11) can be defined as,

$$\begin{aligned} \mathcal{E}_{L_1}^{(1)} &= \{(\mathrm{e}, \mathbf{0})\mathrm{G} : \ 0 \leq w_{\mathrm{H}}[\mathrm{e}] \leq I_1, \ \mathrm{e} \in \mathbb{Z}_2^{K_1}\} \\ \mathcal{E}_{L_2}^{(2)} &= \{(\mathbf{0}, \mathrm{e})\mathrm{G} : \ 0 \leq w_{\mathrm{H}}[\mathrm{e}] \leq I_2, \ \mathrm{e} \in \mathbb{Z}_2^{K_2}\}. \end{aligned} \tag{4.12}$$

The decoding complexity of the segmentation-based OSD with the lists of error patterns defined in (4.12) is,

$$L = \sum_{l=0}^{I_1} \binom{K_1}{l} + \sum_{l=0}^{I_2} \binom{K_2}{l}$$

where $K = K_1 + K_2$, and we assume $I_1 \ll K_1$ and $I_2 \ll K_2$.

Recall that the original OSD, denoted as OSD(I), has one segment of length K bits, and that the maximum number of bit errors assumed in this segment is I. The segmentation-based OSD is denoted as OSD(I_1, I_2), and it is parameterized by the segment length K_1, and K_2, and the maximum number of errors I_1 and I_2, respectively. The segment sizes K_1 and K_2 are chosen empirically to minimize the BER for a given decoding complexity and for a class of codes under consideration. In particular, for systematic block codes of block length $N < 128$ and of rate $R \geq 1/2$, it is found that the recommended length of the first segment is,

$$K_1 \approx 0.35K$$

so that the second segment length is $K_2 = K - K_1$. The maximum number of bit errors I_1 and I_2 in the two segments are selected to fine-tune the BER performance and the decoding complexity trade-off. For instance, the list decoding schemes can be obtained having the BER performance as well as the decoding complexity between those corresponding to the original decoding schemes $\mathrm{OSD}(I)$ and $\mathrm{OSD}(I+1)$.

Finally, it can be noticed that it is straightforward to develop the skipping criteria for efficient searching of the list of error patterns in the OSD-based decoding schemes. In particular, one can consider the Hamming distances for one or more segments of the MRIPs between the received hard decisions (before the decoding) and the temporary decisions obtained using the test error patterns from the list. If any or all of the Hamming distances are above given thresholds, the test error pattern can be discarded without re-encoding and calculating the corresponding Euclidean distance. For the $Q = 2$ segments OSD, our empirical results indicate that the thresholds for the first and the second segments should be $0.35\, d_{\min}$ and $d_{\min}$, respectively.

4.5.2 Partial-Order Statistics Decoding

The Gauss (or the Gauss-Jordan) elimination employed in the OSD-based decoding algorithms represents a significant portion of the overall implementation complexity. A new row (or a reduced row) echelon form of the generator matrix must be obtained after every permutation λ'' until the MRIPs are found. Hence, a partial-order statistics decoding (POSD)is advised that completely avoids the Gauss elimination, and thus, it further reduces the decoding complexity of the OSD-based decoding. The main idea of the POSD is to order only the first K received bits according to their reliabilities, so that the generator matrix remains in its systematic form. The ordering of the first K received bits in the descending order can improve the coding gain of the segmentation-based OSD. Assuming $Q = 2$ segments, we use the notation $\mathrm{POSD}(I_1, I_2)$. The parameters K_1, K_2, I_1 and I_2 of $\mathrm{POSD}(I_1, I_2)$ can be optimized similarly as in the case of $\mathrm{OSD}(I_1, I_2)$ to fine-tune the BER performance versus the implementation complexity. On the other hand, it will be shown in Section 4.6 that the partial ordering (i.e., the ordering of the first K out of N received bits) is irrelevant for the OSD decoding having one segment of the MRIPs and using the list of error patterns (4.10). In this case, the $\mathrm{POSD}(I)$ decoding can be referred to as the input-sphere decoding $\mathrm{ISD}(I)$.

Table 4.1: Implementation Complexity of the OSD and the POSD

OSD(I_1) and OSD(I_1, I_2)	
operation	complexity
$\mathbf{r}$	$2N$ FLOPS
$\tilde{\mathbf{r}}'$	$N\log_2(N)$ FLOPS
Gauss el. $\mathbf{G}'$	$N\min(K, N-K)^2$ BOPS
$\tilde{\mathbf{r}}''$	$K + K(N-K)$ BOPS

POSD(I_1) $\equiv$ ISD(I_1)	
operation	complexity
$\mathbf{r}$	$2N$ FLOPS
$\tilde{\mathbf{r}}'$	0 BOPS

POSD(I_1, I_2)	
operation	complexity
$\mathbf{r}$	$2N$ FLOPS
$\tilde{\mathbf{r}}'$	$K\log_2(K)$ FLOPS

4.5.3 Implementation Complexity

A comparison is shown for the number of binary operations (BOPS) and the number of floating point operations (FLOPS) required to execute the decoding algorithms proposed in this paper. Assuming a $(N, K, d_{\min})$ code, the complexity of the OSD and the POSD are given in Table 4.1 and Table 4.2. The implementation complexity expressions in Table 4.1 for OSD(I) are from the reference [2]. For example, the OSD decoding of the BCH code $(128, 64, 22)$ requires at least 1152 FLOPS and 528448 BOPS to find the MRIPs and to obtain the corresponding equivalent generator matrix in a row echelon form. All this complexity can be completely avoided by assuming the partial ordering in the POSD decoding. The number of the test error patterns is $L = 2080$ for OSD(2), and $L = 1177$ for OSD(2, 2) with $K_1 = 21$ and $K_2 = 43$ whereas the coding gain of OSD(2) can be only slightly better than the coding gain of OSD(2, 2); see, for example, Fig. ??. Hence, the overall complexity of the OSD-based schemes can be substantially reduced by avoiding the Gauss (Gauss-Jordan) elimination.

Table 4.2: Decoding List Sizes for the OSD and the POSD

OSD(I)	$\sum_{l=0}^{I}\binom{K}{l}$
OSD(I_1, I_2)	$\sum_{l=0}^{I_1}\binom{K_1}{l}+\sum_{l=0}^{I_2}\binom{K_2}{l}$
POSD(I) $\equiv$ ISD(I)	$\sum_{l=0}^{I}\binom{K}{l}$
POSD(I_1, I_2)	$\sum_{l=0}^{I_1}\binom{K_1}{l}+\sum_{l=0}^{I_2}\binom{K_2}{l}$

4.5.4 Skipping Criteria and Threshold Test

By simulation we find a skipping criteria and threshold that reduce and improve the speed of decoding for OSD and POSD. This skipping criteria works as follows: After re-encoding, check Hamming distances in all segments against the initial hard-decisions (including parity bits). If the Hamming distance in all or any of the segments is above threshold, do not compute Euclidean distance and move on to another test codeword.The skipping threshold is determined as the following:

For the fist segment threshold for $K1$ is: $d_{max} = C * d_{min} + d_{min}$ where d_{min} is the Hamming distance of the code. C is the same the suitable segmentation parameter which was derived in section 4.5 $K1 = 0.35 * K$.

For the second segment: (K_2) $d_{max} = d_{min}$.

This skipping criteria is very significant espically for the second coding list,it gives a significant reduction in the coding list length.

Another threshold test as in [8] is applicable for full order segmenatation and POSD to reduce compuations according to the fact that code whose weight distribution approaches a binomial distribution,on average $(N-K)/2$ least reliable parity check bits are in error when ever MRIP bit is wrong. In genral , $NQ(2\sqrt{2/N_0}) << (N - K/2)$, which suggest threshold T to distinguish between two cases,where $NQ(2\sqrt{2/N_0})$ is the average number of erros per received block in BSC associated with AWGN. If D represents number of positions for which hard decsisons of the ordered received sequence differ from order(0) reprocessing codeword .The threshold test becomes :

- if $D \leq T$,accept the ordered(0) codeword as the decoded codeword.

- Else,carry on decoding algorithm.

4.6 Performance Analysis

Recall that assuming a memoryless communication channel as described in Section 4.2. The probability $\Pr\{\hat{\mathbf{c}}_0 \in \mathcal{E}_L\}$ is derived in (4.9) that the error pattern $\hat{\mathbf{c}}_0$ observed at the receiver after transmission of the codeword $\mathbf{c}_{\mathrm{Tx}} = \mathbf{0}$ is an element of the chosen decoding list $\mathcal{E}_L$. The derivation relies on the following generalization of Lemma 3 in [2].

Lemma 2 *For any ordering of the N received bits, consider the I bit positions $\mathcal{I} \subseteq \{1, 2, \cdots, N\}$, and the $\binom{I}{I_1}$ subsets $\mathcal{I}_1 \subseteq \mathcal{I}$ of $I_1 \leq I \leq N$ bit positions. Then, the total probability of the I_1 bit errors within the I bits can be calculated as,*

$$\sum_{\mathcal{I}_1:\, |\mathcal{I}_1| = I_1} \mathrm{P}(\mathcal{I}_1) = \binom{I}{I_1} p_0^{I_1}$$

where p_0 is the probability of bit error corresponding to the bit positions $\mathcal{I}$.

The ordering of the chosen I bits given by the set $\mathcal{I}$ is irrelevant since *all* subsets $\mathcal{I}_1$ of I_1 errors within the I bits $\mathcal{I}$ are considered. Consequently, the bit errors in the set $\mathcal{I}$ can be considered to be independent having the equal probability denoted as p_0. Using Lemma 2, it can be obsereved that the lists of error patterns (4.10) and (4.12) are constructed, so that the ordering of bits within the segments is irrelevant. Then, the bit errors in a given segment can be considered to be conditionally independent. This observation is formulated as the following corollary of Lemma 2.

Corollary 1 *For the* $\mathrm{OSD}(I)$ *and the list of error patterns (4.10), the bit errors in the MRIPs can be considered as conditionally independent. Similarly, for the* $\mathrm{POSD}(I_1, I_2)$ *and the list of error patterns (4.12), the bit errors in the two segments can be considered as conditionally independent.*

Thus, the bit errors in Corollary 1 are independent conditioned on the particular segment being considered as shown next(see appendix 4.9).

Let P_0 be the bit error probability of the MRIPs for the OSD(I) decoding. Similarly, let P_1 and P_2 be the bit error probabilities in the first and the second segments of the OSD(I_1, I_2) decoding, respectively. Denote the auxiliary variables, $v_1 = |\tilde{r}_{K_1}|$, $v_2 = |\tilde{r}_{K_1+1}|$, and $v_3 = |\tilde{r}_{K+1}|$ of the order statistics (4.3), and let $u \equiv |r_i|$, $i = 1, 2, \cdots, K$. Hence, always, $v_1 \geq v_2$, and, for simplicity, ignoring the second permutation λ'', also, $v_2 \geq v_3$. The probability of bit error P_0 for the MRIPs is calculated as,

$$\mathrm{P}_0 = \mathrm{E}_u\left[\int_0^u \frac{f_{v_3}(v)}{1 - F_u(v)}\mathrm{d}v\right]$$

where $\mathrm{E}_u[\cdot]$ denotes the expectation w.r.t. (with respect to) u, $f_{v_3}(v)$ is the PDF of the $(K+1)$-th order statistic in (4.3), and $F_u(v)$ is the cumulative distribution function (CDF) of the magnitude (the absolute value) of the reliability of the received bits (before ordering). Similarly, the probability of bit error P_1 for the first segment is calculated as,

$$\mathrm{P}_1 = \mathrm{E}_u\left[\int_0^u \frac{f_{v_2}(v)}{1 - F_u(v)}\mathrm{d}v\right]$$

where $f_{v_2}(v)$ is the PDF of the $(K_1 + 1)$-th order statistic in (4.3). The probability of bit error P_2 for the second segment is calculated as,

$$\mathrm{P}_2 = \mathrm{E}_u\left[\int_0^u \int_u^\infty \frac{f_{v_1}(v) f_{v_3}(v')}{(F_u(v) - F_u(v'))(1 - F_{v_1}(v'))}\mathrm{d}v\mathrm{d}v'\right]$$

where $f_{v_1}(v)$ and $F_{v_1}(v')$ is the PDF and the CDF of the K_1-th order statistic in (4.3), respectively. The values of the probabilities P_0, P_1 and P_2 have to be evaluated by numerical integration. Finally, we use Lemma 2 and substitute the probabilities P_0, P_1 and P_2 for p_0 to calculate the probability $\Pr\{\hat{\mathrm{c}}_0 \in \mathcal{E}_L\}$ of the error patterns in the list $\mathcal{E}_L$.

4.7 Numerical Examples

We use computer simulations to compare the BER performances of the proposed soft-decision decoding schemes. All the block codes considered are linear and systematic.

4.7.1 Bit Error Performance Of Decoding Algorithms For BPSK and 16QAM over AWGN

The BER of the $(31, 16, 7)$ BCH code over an AWGN channel is shown in Fig. 4.2 assuming $\text{ISD}(2)$ and $\text{ISD}(3)$ with $K = 16$ having 137 and 697 test error patterns, respectively, and assuming $\text{POSD}(1, 3)$ and $\text{POSD}(2, 3)$ with $K_1 = 6$ and $K_2 = 10$ having 183 and 198 test error patterns, respectively. We observe that $\text{POSD}(1, 3)$ achieves the same BER as $\text{ISD}(3)$ while using much less error patterns which represents the gain of the ordering of the received information bits into two segments. At the BER of 10^{-4}, $\text{POSD}(1, 3)$ outperforms $\text{ISD}(2)$ by 1.1 dB using approximately 50% more test error patterns. Thus, the $\text{POSD}(1, 3)$ decoding provides 2.3 dB coding gain in performance with the small implementation complexity at the expense of 2 dB loss compared to the ML decoding.

Fig. 4.3 shows the BER of the $(63, 45, 7)$ BCH code over an AWGN channel. The number of test error patterns for the $\text{ISD}(2)$, $\text{ISD}(3)$, $\text{POSD}(1, 3)$ and $\text{OSD}(2)$ decodings are 1036, 15226, 5503 and 1036, respectively. We observe from Fig. 4.3 that $\text{ISD}(3)$ has the same BER as $\text{POSD}(1, 3)$ with two segments of $K_1 = 13$ and $K_2 = 32$ bits. However, especially for the high rate codes (i.e. of rates greater than $1/2$), one has to also consider the complexity of the Gauss elimination to obtain the row echelon form of the generator matrix for the OSD. For example, the Gauss elimination for the $(63, 45, 7)$ code requires approximately $20,400$ BOPS; cf. Table 4.1.

The BER of the $(128, 64, 22)$ BCH code over an AWGN channel is shown in Fig. 4.4 assuming $\text{OSD}(1)$ and $\text{OSD}(2)$ with $K = 64$, and assuming $\text{OSD}(2, 2)$ with $K_1 = 21$ and $K_2 = 43$. The number of test error patterns for the $\text{OSD}(1)$, $\text{OSD}(2)$ and $\text{OSD}(2, 2)$ decodings are 64, 2081 and 1179. A truncated union bound of the BER in Fig. 4.2 is used to indicate the ML performance [70, Ch. 10]. We observe

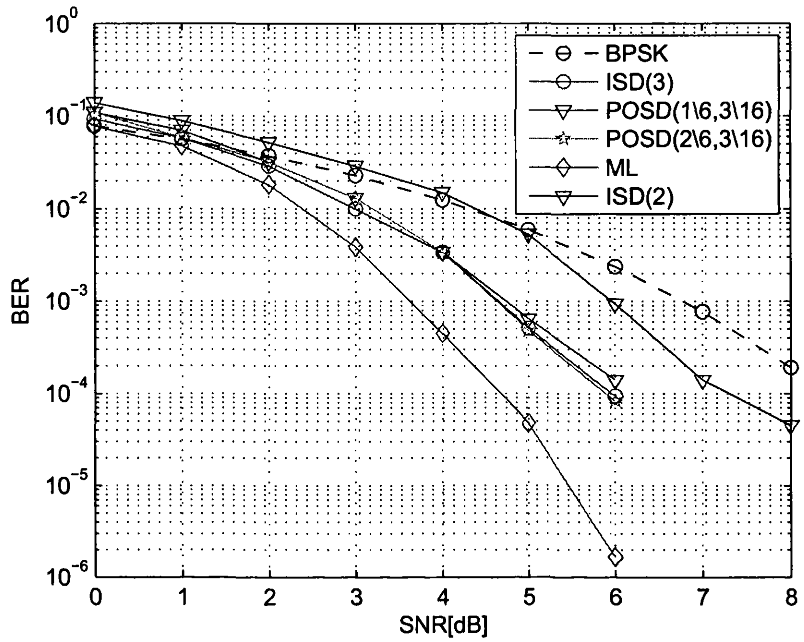

Figure 4.2: The BER of the (31, 16, 7) BCH code over an AWGN channel.

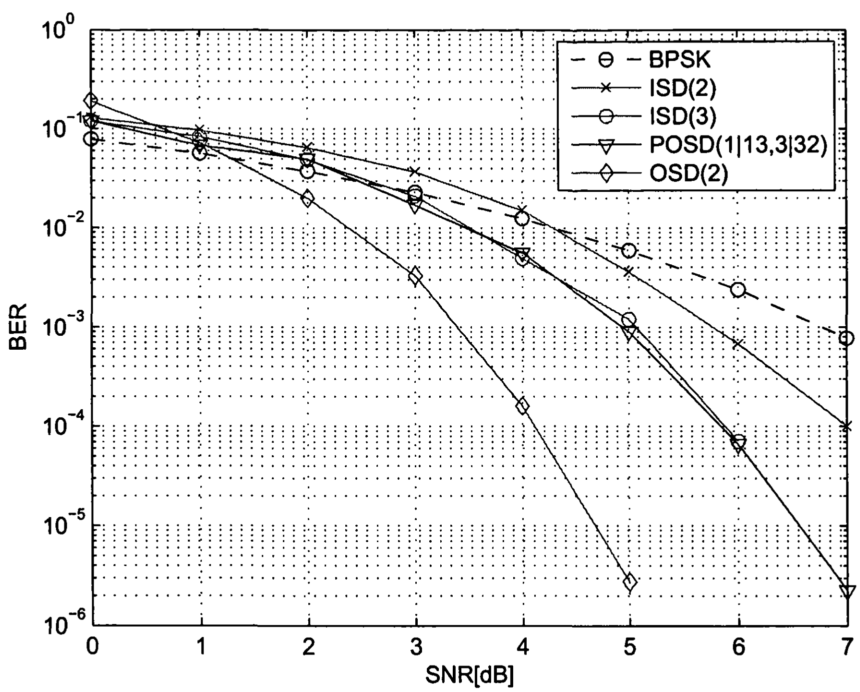

Figure 4.3: The BER of the (63, 45, 7) BCH code over an AWGN channel.

that both OSD(2) and OSD(2, 2) have the same BER performance for the BER values larger than 10^{-3}, and $OSD(2)$ outperforms OSD(2, 2) by at most 0.5 dB for the small values of the SNR. Our numerical results indicate that, in general, OSD(2, 2) decoding can achieve approximately the same BER as OSD(2) for small to medium SNR while using about 50% less error patterns. Thus, a slightly smaller coding gain (less than 0.5dB) of OSD(2, 2) in comparison with OSD(2) at larger values of the SNR is well-compensated for by the reduced decoding complexity. More importantly, OSD(2, 2) can trade-off the BER performance and the decoding complexity between those provided by OSD(1) and OSD(2), especially at larger values of SNR.The BER performance of OSD and POSD employing 16QAM over AWGN is shown in Fig. 4.5. It is observed from Fig. 4.5 that for non-binary modulation the ISD(3) also has the same BER as POSD(1, 3) with two segments of $K_1 = 6$ and $K_2 = 10$ bits.Last observation shows that POSD algoithm is also effecient for non-binary modulation schemes over AWGN.

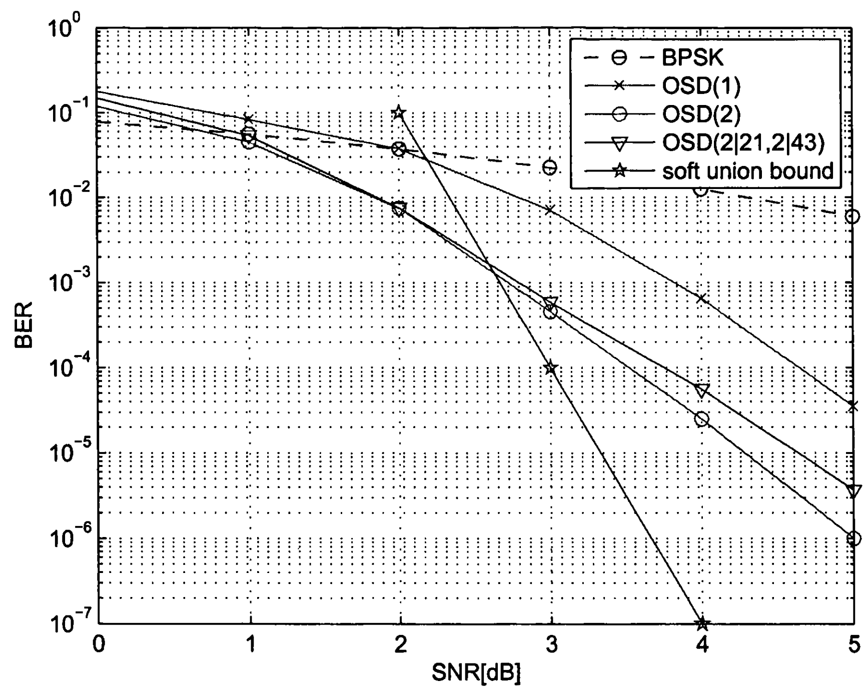

Figure 4.4: The BER of the (128, 64, 22) BCH code over an AWGN channel.

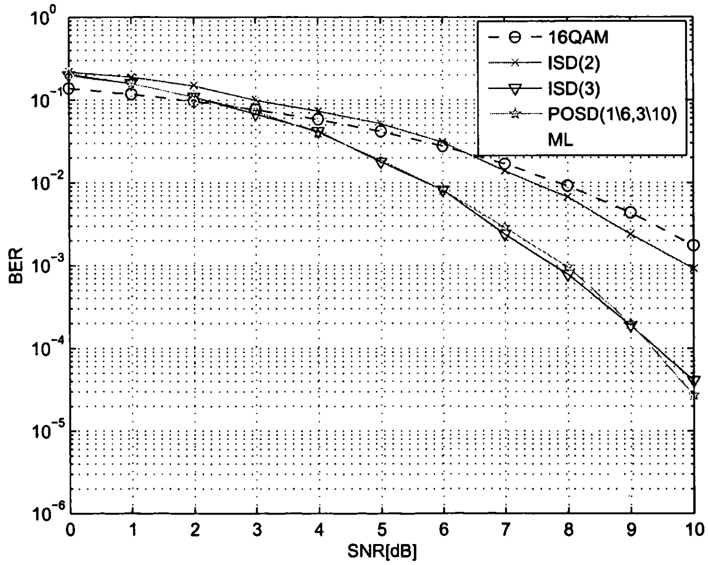

Figure 4.5: The BER of the $(31, 16, 7)$ Golay coded 16QAM over an AWGN channel.

4.7.2 Bit Error Performance Of Decoding Algorithms For BPSK and 16QAM Over Rayleigh Fading Channel.

The BER of the $(31, 16, 7)$ BCH code over a fast Rayleigh fading channel is shown in Fig. 4.6. We assume the same decoding schemes as in Fig. 4.3. The POSD$(1, 3)$ decoding with 183 error patterns achieves the coding gain of 17 dB over an uncoded system, the coding gain of 4 dB over ISD(2) with 137 error patterns, and it has the same BER as OSD(3) with 697 error patterns.

The BER of the high rate BCH code $(64, 57, 4)$ over a fast Rayleigh channel is shown in Fig. 4.7. In this case, the number of test error patterns for the ISD(2), ISD(3), POSD$(2, 3)$ and OSD(2) decoding is 1654, 30914, 8685 and 1654, respectively. We observe that, for small to medium SNR, POSD$(2, 3)$ which does not require the Gauss elimination (corresponding to approximately $3,000$ BOPS) outperforms OSD(2) by 1dB whereas, for large SNR values, these two decoding schemes achieve approximately the same BER performance.The BER performance of OSD and POSD employing 16QAM over a fast Rayleigh fading channel is shown in Fig. 4.8. It is observed from Fig. 4.8 that for non-binary modulation the ISD(3) also has the same BER as

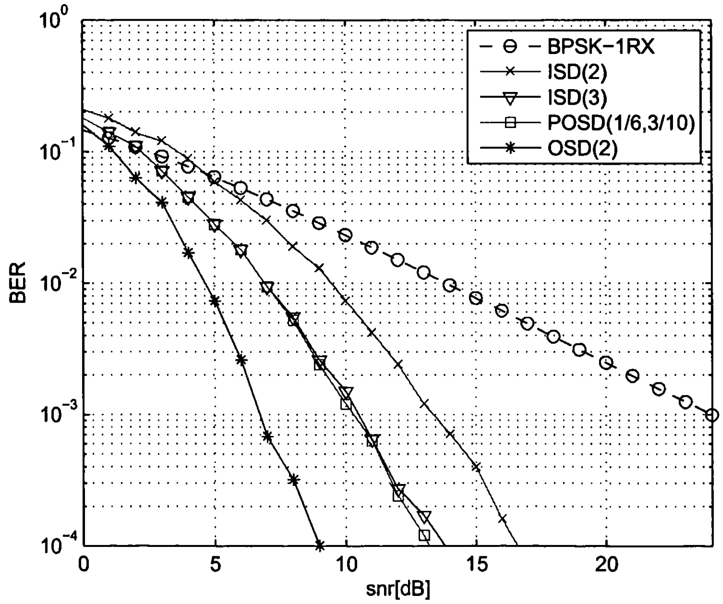

Figure 4.6: The BER of the $(31, 16, 7)$ BCH code over a Rayleigh fading channel.

$\mathrm{POSD}(1, 3)$ with two segments of $K_1 = 4$ and $K_2 = 8$ bits.Last observation shows that POSD algoithm is also effecient for non-binary modulation schemes over rayleigh fading channel.

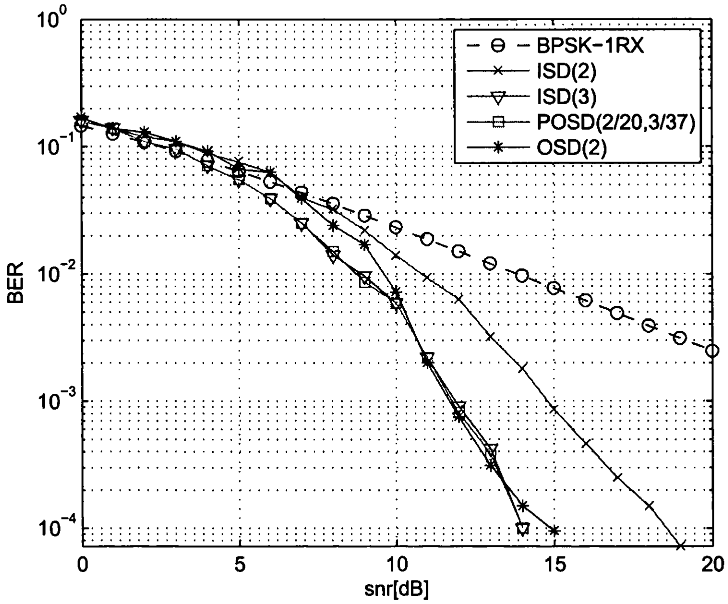

Figure 4.7: The BER of the $(64, 57, 4)$ BCH coded over a Rayleigh fading channel.

4.8 Conclusions

Low-complexity soft-decision decoding techniques employing a list of the test error patterns for linear binary block codes of small to medium block length were investigated. The optimum and suboptimum construction of the list of error patterns was developed. Some properties of the joint probability of error of the received bits after ordering were derived. The original OSD algorithm was generalized by assuming a segmentation of the MRIPs. The segmentation of the MRIPs was shown to overcome several drawbacks of the original OSD and to enable flexibility for devising new decoding strategies. The decoding complexity of the OSD-based decoding algorithms was reduced further by avoiding the Gauss (or the Gauss Jordan) elimination using the partial ordering of the received bits in the POSD decoding. The performance analysis was concerned with the problem of finding the probability of the test error patterns contained in the decoding list. The BER performance and the decoding complexity of the proposed decoding techniques were compared by extensive computer simulations. Numerical examples demonstrated excellent flexibility of the proposed decoding schemes to trade-off the BER performance and the decoding complexity. In some cases, both

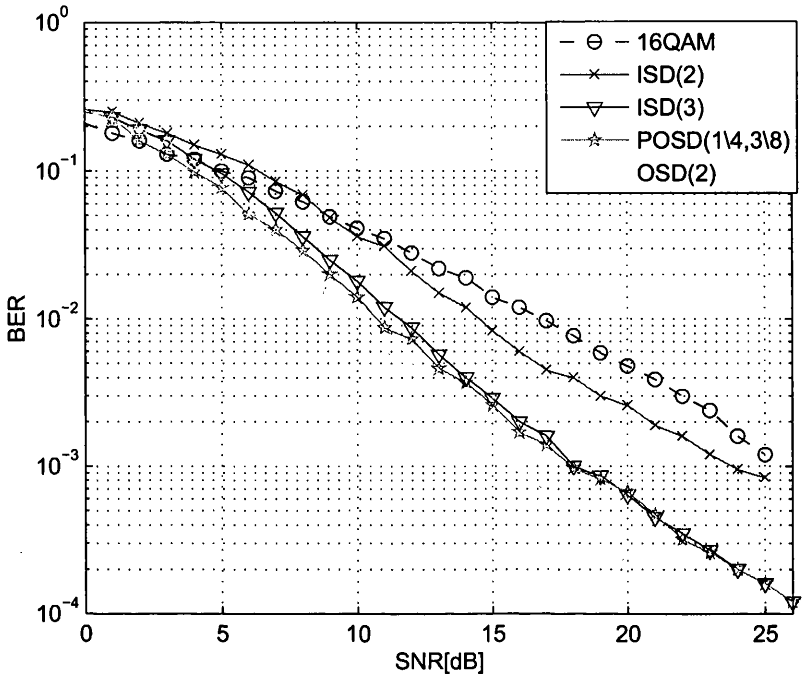

Figure 4.8: The BER of the $(24, 12, 8)$ Golay coded 16QAM over fading channel.

the BER performance as well as the decoding complexity of the segmentation-based OSD were found to be improved compared to the original OSD.

4.9 Appendix

The probabilities P_0, P_1 and P_2 are derived in Section 4.6. Without loss of generality, we assume that the all-ones codeword was transmitted, i.e., $x_i = -1$ for $\forall i$. Then, after ordering, the i-th received bit, $i = 1, 2, \cdots, N$, is in error, provided that $\tilde{r}_i > 0$. The probability of bit error P_0 for the MRIPs is obtained as,

$$\mathrm{P}_0 = \int_0^\infty \int_0^\infty f_u(u|u \geq v_3) f_{v_3}(v_3) \mathrm{d}v_3 \mathrm{d}u$$

where the conditional PDF [86],

$$f_u(u|u \geq v_3) = \begin{cases} \frac{f_u(u)}{1-F_u(v_3)} & u \geq v_3 \\ 0 & u < v_3 \end{cases}$$

and $f_u(u)$ and $F_u(v_3)$ are the PDF and the CDF of the reliability of the received bits, respectively, so that,

$$\mathrm{P}_0 = \int_0^\infty f_u(u) \int_0^u \frac{f_{v_3}(v_3)}{1 - F_u(v_3)} \mathrm{d}v_3 \mathrm{d}u.$$

Similarly, the probability of bit error P_1 for the first segment is calculated as,

$$\begin{aligned} \mathrm{P}_1 &= \int_0^\infty \int_0^\infty f_u(u|u \geq v_2) f_{v_2}(v_2) \mathrm{d}v_2 \mathrm{d}u \\ &= \int_0^\infty f_u(u) \int_0^u \frac{f_{v_2}(v_2)}{1 - F_u(v_2)} \mathrm{d}v_2 \mathrm{d}u. \end{aligned}$$

The probability of bit error P_2 for the second segment is calculated as,

$$\mathrm{P}_2 = \int_0^\infty \int_0^\infty \int_0^\infty f_u(u|v_1 \geq u \geq v_3) f_{v_1,v_3}(v_1, v_3) \mathrm{d}v_1 \mathrm{d}v_3 \mathrm{d}u$$

where the conditional PDF,

$$f_u(u|v_1 \geq u \geq v_3) = \begin{cases} \frac{f_u(u)}{F_u(v_1)-F_u(v_3)} & v_1 \geq u \geq v_3 \\ 0 & \text{otherwise} \end{cases}$$

and the joint PDF of the order statistics $v_1 \geq v_3$ is,

$$f_{v_1,v_3}(v_1, v_3) = \begin{cases} \frac{f_{v_1}(v_1)}{1-F_{v_1}(v_3)} f_{v_3}(v_3) & v_1 \geq v_3 \\ 0 & v_1 < v_3 \end{cases}$$

and thus,

$$\mathrm{P}_2 = \int_0^\infty f_u(u) \int_0^u \int_u^\infty \frac{f_{v_1}(v) f_{v_3}(v')}{(F_u(v) - F_u(v'))(1 - F_{v_1}(v'))} \mathrm{d}v \mathrm{d}v' \mathrm{d}u.$$

5

Complexity Reduction of BTC and OSD Decoding Using Highly Reliable CRC Bits

5.1 Introduction

In this chapter, two different techniques are proposed to reduce the complexity of the OSD and the POSD decoding algorithms for linear binary block codes and binary block turbo codes (BTC).

In the first technique, the aim is to reduce the complexity of the OSD decoding algorithm by reducing the size of the coding list by using highly reliable CRC bits within the transmitted codeword. The size of the coding list is considered to be the main measure of the complexity for such decoding techniques. Many schemes were developed based on the OSD decoding techniques to acheive a good trade-off between the bit-error rate (BER) performance and the implementation complexity [3, 9, 41, 43]. The CRC bits are used as data verification and add no information to the message. They are used because of the implementation simplicity since most of the CRC bits assume the field GF(2). Also, the CRC bits may give some indication about the Hamming weight of the transmitted codeword [21, 73]. The CRC bits were used in the litreature either to reduce the complexity or improve the performance [74, 75]. In [74], the CRC bits are used jointly with the turbo decoder, where iterations of the turbo decoding are stopped when the CRC determines that there are no errors in the decoded codeword. The technique in [74], shows that the decoding complexity is reduced by decreasing the number of iterations in the trubo decoder. For performnace improvement purposes, the CRC bits are utilized in [75] jointly with the single parity check (SPC) product code to detect unrecoverable errors. In general, three different types of the CRC bits can be applied depending on the segmentation considered for the transmitted codeword. The first CRC type, sends highly reliable CRC bits for different segments along the whole codeword. The second type, sends the CRC bits for different segments over the information bits only. The third type, sends highly reliable CRC bits for segments along the parity bits only.

In the second technique, a design of single generator matrix for the BTCs is considered in order to enable the OSD based decoding of segments as well as to apply the POSD algorithm directly for the whole codeword without the need of utlizing the SISO decoding based on the OSD for BTCs. The BTC achieves high performance with a simple block structure. Different decoding algorithms were proposed in the literature [76]. Most of these decoding algorithms are based on iterative principles. Such schemes improve the performance and with a decreased complexity. In [76], the iterative decoding of any product code using linear block code is proposed. The main idea is to use the SISO decoder to decode components of the product codes. The Chase decoder is used as a SISO decoder for high rate block codes after certain modifications. The Chase decoder in [27] delivers binary decisions as its outputs which are modified inoredr to produce the soft outputs corresponding to the log likelihood ratios (LLRs) of binary decisions. The soft outputs of the horizontal decoding process are delivered to the vertical decoding and vice versa. However, due to the use of the Chase algorithm, the method used in [76] can not be extended to other classes of codes. The SISO decoding of the product code based on the OSD is proposed in [45]. The OSD is modified to convert binary decisions of the classical OSD to soft output values. The main idea of algorithms in [45, 76] is to produce a list of codewords. Based on the fact that, for each symbol in the received sequence there should be at least one codeword with the value '1' in that position and at least another codeword with the value '0' value in the same position. In some cases and due to a limited size of the coding list, the Chase decoder extrinsic information is estimated for positions where the codewords with 0 and 1 in those positions are not available. However, this is not the case for the decoding in [45] where the list is designed to contain a sufficient number of codewords. The SISO decoding algorithm proposed in [45] is further developed in [77] to produce the soft outputs for the OSD with much less complexity than an order I processing and it is performed once rather than $(K+1)$ times as in [45].

This chapter is organized as follows. System model is described in Section 5.2. The OSD decoding algorithm and CRC techniques are proposed in Section 5.3. The performance is illustrated by numerical examples in Section 5.4. The design of a single genrator matrix of binary BTC is discussed in Section 5.4. Conclusions are given in Section 5.5.

5.2 System Model

The transmission of codewords is considered for a linear binary block code $\mathcal{C}$ over a AWGN. The code $\mathcal{C}$, denoted as $(N, K, d_{\min})$, has dimension K, block length N, and the minimum Hamming distance between any two codewords $d_{\min}$. Binary codewords $\mathbf{c} \in \mathbb{Z}_2^N$ where $\mathbb{Z}_2 = \{0, 1\}$ are generated from a vector of information bits $\mathbf{u} \in \mathbb{Z}_2^K$ using the generator matrix $\mathbf{G} \in \mathbb{Z}_2^{K\times N}$, i.e., $\mathbf{c} = \mathbf{u}\mathbf{G}$, and all binary operations are considered over a Galois field GF(2). If the code $\mathcal{C}$ is systematic, the generator matrix has the form, $\mathbf{G} = [\mathbf{I}\ \mathbf{P}]$, where $\mathbf{I}$ is the $K \times K$ identity matrix, and $\mathbf{P} \in \mathbb{Z}_2^{K\times(N-K)}$ is the matrix of parity checks. The codewords are interleaved and mapped to binary phase shift keying (BPSK) sequences $\mathbf{x} \in \{+1, -1\}^N$ before transmission, i.e., $\mathbf{x}_i = (-1).^{\mathbf{c}_i}$ where $\mathbf{x}_i$ denotes the i-th component of the vector $\mathbf{x}$, and $i = 1, 2, \cdots, N$. Assuming ideal coherent detection (i.e., channel phases are perfectly known at the receiver), the received signals are written as,

$$r_i = x_i + w_i \tag{5.1}$$

where w_i are uncorrelated and identically distributed samples of a zero-mean additive white Gaussian noise (AWGN) with variance σ_w^2 per dimension. The noise variance is computed as, $\sigma_w^2 = (2R\gamma_b)^{-1}$, where $R = K/N$ is coding rate of the channel code

$\mathcal{C}$, and the SNR per transmitted binary symbol is $\gamma_b = E_b/N_0$ where N_0 denotes the double-sided power spectral density of AWGN.

5.3 OSD Based Decoding With High Reliable CRC Bits

The OSD and the POSD decoding algorithms have been discussed in Chapter 4. The OSD decoding has complexity $O(|\mathcal{E}|)$. The original OSD suggests the list of error patterns,

$$\mathcal{E} = \{\mathbf{e} \in \mathbb{Z}_2^K : 0 < w_{\mathrm{H}}[\mathbf{e}] \leq I\} \tag{5.2}$$

where $I \approx \lceil d_{\min}/4 \rceil \ll K$ is the reprocessing order of the OSD, $w_{\mathrm{H}}[\mathbf{e}]$ is the Hamming weight of the vector e, and $\lceil \cdot \rceil$ is the ceiling function. The list $\mathcal{E}$ in (5.2) has cardinality $|\mathcal{E}| = \sum_{J=1}^{I} \binom{K}{J}$ where J is the phase of order I reprocessing, and $\binom{K}{J}$ is a binomial coefficient. Hence, the OSD algorithm may become excessively complex for larger values of I and K.

Three different types of CRC bits can be applied depending on the segmentation of the transmitted codeword. The first type sends highly reliable CRC bits for different segments along the whole codeword. In this case, we assume a single parity,i.e., highly reliable CRC bit is sent for the whole codeword to indicate about the Hamming weight whether it is even or odd. Another possibility is to divide the codeword into many segments and then send highly reliable CRC bits for each segment. The OSD decoder will perfrom ordinary decoding process to obtain the $\mathcal{E}$ as in (5.2). However, in this case, the list $\mathcal{E}$ must be reversed as:

$$\mathcal{E}_{new} = \lambda''\lambda'[\mathcal{E}]$$

Where λ' and λ'' are the permutation for MRIP ordering and MRP ordering repspectively. Then, the Hamming weight is calculated for each codeword in $\mathcal{E}_{new}$ and tested against to the received CRC bits. The failed codewords that do not satisfy the CRC test are discarded from $\mathcal{E}_{new}$. In the second method, the CRC bits are sent for different segments along the information bits only. In the third method, highly reliable CRC bits are sent for different segments along the parity bits only.

5.3.1 Numerical Examples and Discussion

We pefrom soft-decision decoding based on the OSD decoding algorithm for some BCH codes. Fig. 5.1–Fig. 5.3 compare the complexity reduction ratios of the coding list size $\mathcal{E}$ for different CRC methods using computer simulations.

The ratios of the coding list complexity reduction for the whole codeword for different BCH codes with different R over an AWGN channel are shown in Fig. 5.1 assuming OSD(2) decoding. We observe that as the number of highly reliable CRC bits for segments along the whole codeword increases, the coding list size decreases sharply. For example, for BCH $(128, 64, 22)$ and 3 CRC bits, the codeword is partioned into three segments. For each segment, 1 CRC bit is sent as the indication whether each segment contains even or odd number of ones. The codewords that do not satisfy the CRC constraints are discareded from the coding list, so the size decreases to about 28% of the orginal size (i.e., about 582 codewords out of 2081). When the number of segments increases up to 6, and 1 CRC bit is sent for each segment, the list size reaches about 0.06 of original size which is about 41 codewords. Moreover, it is shown in Fig. 5.1 that sending information about the exact number of ones in the whole codeword reduces the coding list size but less effectively than when sending CRC bits for each segment. For example, for BCH $(128, 64, 22)$, 7 CRC bits are required to represent the exact number of ones in the whole codeword and to reduce the complexity to about 0.28 of the orginal coding list. However, the 6 CRC bits for 6 segments reduce the size of the coding list to about 0.06 of the original size. The ratio of the coding list complexity reduction over information part for BCH codes with different R over an AWGN channel is shown in Fig. 5.2 assuming OSD(2) decoding. It is shown that sending the CRC bits for a number of segments along information part is very effective for different code rates R. For BCH $(64, 36, 12)$ using 3 CRC bits reduces the coding list size to about 0.12 of the orginal size. In Fig. 5.3, the same assumptions of Fig. 5.2 are used except that CRC bits are over parity bits only. The main observation we have from Fig. 7.3 is that using CRC bits over the parity bits only is not effective for high code rates. Moreover, comparing Fig. 5.1,Fig. 5.2, and Fig. 5.3, we can conclude that using CRC bits among the information bits only or the parity bits only is more effective in the complexity reduction than using CRC bits along the whole codeword.

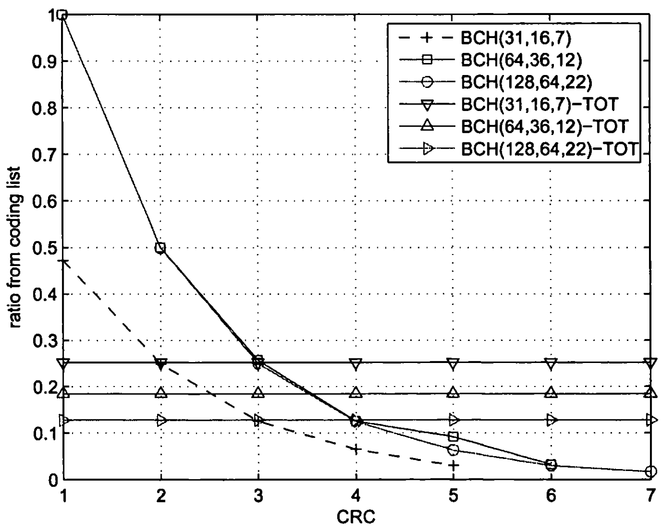

Figure 5.1: The complexity versus the number of CRC bits used for all bits in a codeword for some BCH codes over an AWGN channel.

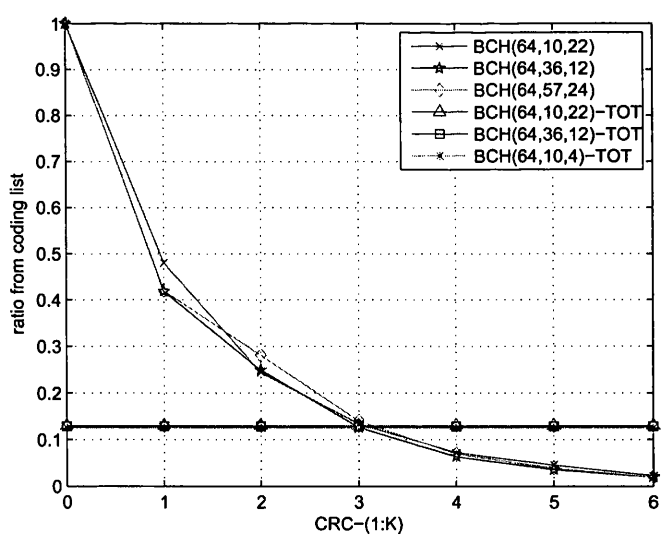

Figure 5.2: The complexity versus the number of CRC bits used for the information bits for some BCH codes over an AWGN channel.

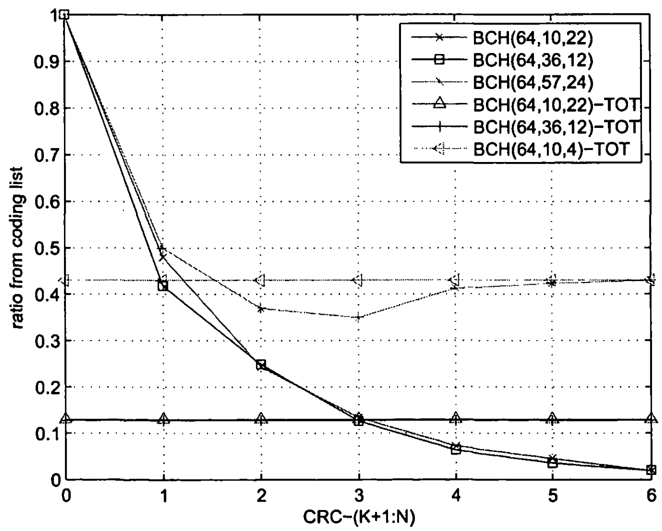

Figure 5.3: The complexity versus the number of CRC bits used for parity bits for some BCH codes over an AWGN channel.

5.4 OSD Based Decoding of BTC Codes

As mentioned in Chapter 2, the product codes are serially concatenated codes and this kind of codes are simple for constructing long codes by using two short codes. A binary block product code (BPC) is a good example of such codes. The BPC is formed by the product of two systematic codes C_1 with (N_1, K_1, d_{min1}) and code rate R_1 and C_2 with (N_2, K_2, d_{min}) and the code rate R_2. The product code $P = C_1 \times C_2$. In this section, a single generator matrix is derived for the BPC rather than using two generator matrices. Designing a single generator matrix enables to get the same properties of the BPC and represents the parity bits separately. Based on this idea, there is no need to decode sequentially first the rows and then the columns or vice versa. Moreover, there is no need to obtain optimum performance soft-input soft-output (SISO) decoding algorithms since the soft-input and hard-output decoding algorithms are enough. However, the SISO decoding algorithm can be modified and integrated to work with a single generator matrix. In this way, the iterative decoding can be applied with a single generator matrix.

Hence, a single generator matrix of a systematic binary linear product code is derived. Assume the set $\mathbb{Z}_2 = \{0, 1\}$. Denote the all-zero matrix as $\mathbf{0}$, the binary identity matrix as $\mathbf{I}_{(K)} \in \mathbb{Z}_2^{K \times K}$, and define the $(K \times K)$ permutation matrix as,

$$\mathbf{J}_{(K)} = \begin{bmatrix} 0 & 0 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ 0 & 0 & \cdots & 1\,0 \end{bmatrix}_{(K \times K)} .$$

Let the systematic binary linear block codes (N_1, K_1, b_1) and (N_2, K_2, b_2) be generated by the matrices $\mathbf{G}_1 = [\mathbf{I}_{(K_1)} | \mathbf{P}_1] \in \mathbb{Z}_2^{K_1 \times (K_1 + M_1)}$ and $\mathbf{G}_2 = [\mathbf{I}_{(K_2)} | \mathbf{P}_2] \in \mathbb{Z}_2^{K_2 \times (K_2 + M_2)}$, respectively, where the number of parity bits $M_1 = N_1 - K_1$ and $M_2 = N_2 - K_2$. Then, the codewords of the systematic product code $(N_1, K_1, b_1) \times (N_2, K_2, b_2)$ can be written as,

$$\mathbf{uG} = \mathbf{u} \left[\mathbf{I}_{(K_1 K_2)} | \mathbf{Q}_1 | \mathbf{Q}_2 | \mathbf{Q}_3 \right]$$

where $\mathbf{u} \in \mathbb{Z}_2^{K_1 K_2}$ is a row vector of information bits, and the matrices,

$$\mathbf{Q}_1 = \begin{bmatrix} \mathbf{P}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{P}_1 \end{bmatrix}_{(K_1 K_2 \times K_2 M_1)}$$

$$\mathbf{Q}_2 = \begin{bmatrix} \mathbf{J}_{(K_1)}^0 \mathbf{A}_1 & \mathbf{J}_{(K_1)}^1 \mathbf{A}_1 & \cdots & \mathbf{J}_{(K_1)}^{K_1 - 1} \mathbf{A}_1 \\ \mathbf{J}_{(K_1)}^0 \mathbf{A}_2 & \mathbf{J}_{(K_1)}^1 \mathbf{A}_2 & \cdots & \mathbf{J}_{(K_1)}^{K_1 - 1} \mathbf{A}_2 \\ \vdots & \vdots & & \vdots \\ \mathbf{J}_{(K_1)}^0 \mathbf{A}_{K_2} & \mathbf{J}_{(K_1)}^1 \mathbf{A}_{K_2} & \cdots & \mathbf{J}_{(K_1)}^{K_1 - 1} \mathbf{A}_{K_2} \end{bmatrix}_{(K_1 K_2 \times K_1 M_2)}$$

$$\mathbf{A}_i = \begin{bmatrix} \mathrm{row}_i(\mathbf{P}_2) \\ \mathbf{0} \end{bmatrix}_{(K_1 \times M_2)}$$

$$\mathbf{Q}_3 = \mathbf{Q}_1 \begin{bmatrix} \mathbf{J}_{(M_1)}^0 \mathbf{B}_1 & \cdots & \mathbf{J}_{(M_1)}^{M_1 - 1} \mathbf{B}_1 \\ \mathbf{J}_{(M_1)}^0 \mathbf{B}_2 & \cdots & \mathbf{J}_{(M_1)}^{M_1 - 1} \mathbf{B}_2 \\ \vdots & & \vdots \\ \mathbf{J}_{(M_1)}^0 \mathbf{B}_{K_2} & \cdots & \mathbf{J}_{(M_1)}^{M_1 - 1} \mathbf{B}_{K_2} \end{bmatrix}_{(M_1 K_2 \times M_1 M_2)}$$

$$\mathbf{B}_i = \begin{bmatrix} \mathrm{row}_i(\mathbf{P}_2) \\ \mathbf{0} \end{bmatrix}_{(M_1 \times M_2)} .$$

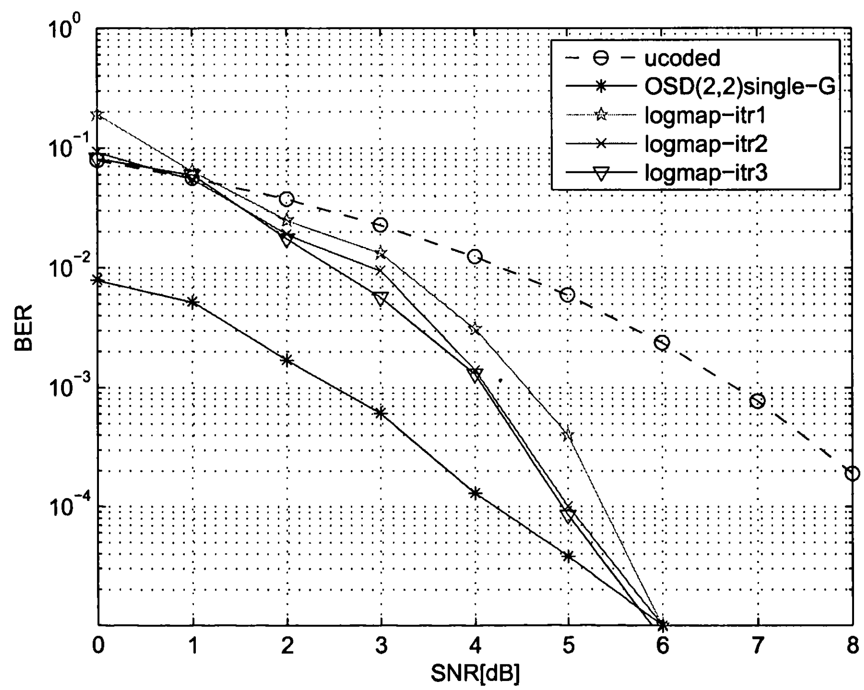

Figure 5.4: The BER of BPC code using log-map decoder and OSD using a single generator matrix.

5.4.1 Complexity and Performance Evaluation

It is shown in Table 5.1 that complexity of the decoding when presenting a product code as a single gnerator matrix is much less than the decoding complexity for classical BPC using SISO based on the OSD in [45]. For example consider a $(7,4,3)\times(7,4,3)$ BPC. The number of real operations using SISO based on the the OSD(I) is equal to about $(17 \times 33 \times 16 = 8976)$ operations where using OSD(I) for a single generator matrix $(49,16,9)$ requires $(33\times 137 = 4521)$ operations which is about 50% reduction. Using OSD(2/5,2/11), the number of real operations will be $(33 \times (11+79) = 2970)$ which is further reduction compared to a SISO based on the OSD.

Moreover, it is shown in Fig. 5.4 that the performnace of OSD based on segmentation is the same for log-map decoder and even SISO based on OSD decoder.

Table 5.1: Comparison of the number of real operations for SISO based on OSD and a single generator matrix using OSD to decode BTC.

$\text{OSD}(I) - 1G$	$(N-K)\sum_{l=0}^{I}\binom{K}{l}$
$\text{OSD}(I_1, I_2) - 1G$	$(N-K)\sum_{l=0}^{I_1}\binom{K_1}{l} + \sum_{l=0}^{I_2}\binom{K_2}{l}$
$SISO - \text{OSD}(I)$	$(K+1)(N-K)\sum_{l=0}^{I}\binom{K}{l}$

5.5 Conclusions

The size of coding list for the OSD decoding algorithm is reduced by sending high reliable CRC bits. Three different types of CRC bits can be applied: the first type is to send high reliable CRC bits for different segments along the whole codeword$(1:N)$, the second type is to send of CRC bits for different segments along the information bits only $(1:K)$, and the third type is to send high reliable CRC bits for segments along the parity bits only $(K+1:N)$. The comparison between these different types of CRC bits shows that sending high reliable CRC bits for many segments over the information bits only is the most effective way to reduce the complexity of the coding list. Also sending CRC bits for many segements is more effective in complexity reduction than sending the exact information about the number of ones even for the whole codeword or the information bits and the parity bits only. A design of single generator matrix for binary BTC is also represented to apply the OSD based on segmentation and the POSD algorithms directly to the codeword, without the need of utlizing the SISO based on OSD decoding algorithm for BPC.

6

Error Rate Performance of Cooperative Versus Receiver Coded Diversity

6.1 Introduction

The roll-out of the 4G cellular systems is expected to commence in the near future. Various forms of the transmission diversity are one of the key technical enablers of the 4G systems. The relays deployed about the 4G base stations will provide the improved coverage and enable higher data rates services by realizing the distributed transmission diversity. The existence of relays, however, also significantly complicates the deployment of the 4G networks, for example, due to the increased capital and operational expenditures, and the need to allocate additional communication channels within the cell. It is therefore vital to investigate the conditions when the cooperative diversity realized by the relays can bring the antennas closer to the user terminals, and thus, outperform the conventional receiver diversity realized by the multiple antennas at the receiver. Such comparison can be done in terms of the transmission reliabilities represented by the pairwise error probabilities (PEPs) and the bit error rates (BERs).

The uncoded cooperative diversity techniques were studied in [14] and in [78]. The multiuser cooperative protocols are proposed in [79, 80]. An overview of the coded cooperation schemes is given in [16]. The performance of conventional coded antenna diversity techniques is investigated in [81]. The performance of coded systems over block fading channels is analyzed in [85]. An upper-bound of the transmission error probability for binary block codes over slow and fast fading channels is obtained in [82]. A specific two-user coded cooperative scheme is proposed and analyzed [17, 55]. General analytical expressions for the error performance of the amplify-and-forward (AF) and the decode-and-forward (DF) relaying employing the turbo codes are obtained in [13]. The coded cooperation is also studies in [92].

In this chapter, a comparison is carried out between the transmission reliabilities of a cooperative diversity system employing a single relay and a system employing the conventional receiver diversity with the two receiver antennas. Thus, both systems can achieve the diversity order of at most two. We formulate the research problem such that the source and the destination are stationary, and the task is to find the relay locations, so that the cooperative diversity can outperform the receiver diversity. This is a dual problem to the scenario where the destination (source) and the relay are stationary, and the task is to find the source (destination) locations, so that the cooperative diversity can outperform the receiver diversity. The locations of network nodes are taken into account through the path-loss attenuations. The results indicate that, if the path-loss attenuations, and thus, the mutual nodes locations are not considered, then the conventional receiver diversity always outperforms the cooperative diversity. On the other hand, the path-loss attenuations may cause the system with the cooperative diversity to outperform the system with the receiver diversity, particularly at smaller values of the signal-to-noise ratio (SNR). All the channels between network nodes are assumed to be independent. In both systems, the destination coherently combines the received signals using the maximum ratio combining (MRC) or the equal gain combining (EGC) [24].

More importantly, we assume encoding of the packets using a simple binary linear block coding and mapping to non-binary linear modulation constellations prior to their transmission. For the cooperative diversity, assuming that time division channel orthogonalization and a usual two time-slot relaying protocol are used in order to avoid the interference of the transmitted packets. In case of the DF relaying, assuming that the relay uses the same encoder as the source and the same decoder as the destination.

For the decoding of short length binary linear block codes, we employ the soft-decision decoding techniques developed in Chapter 4 that are referred to as the partial-order statistics decoding (POSD). These techniques achieve a good BER performance versus the implementation complexity trade-off, and, in some cases, the POSD techniques can even closely approach the performance of the maximum-likelihood (ML) decoder [8, 84].

The rest of this chapter is organized as follows. Section 6.2 describes the system models including the modulation and channel coding and decoding for two systems employing the receiver and the cooperative diversity, respectively. The PEP as a key measure of the transmission reliability for the two systems under consideration is analyzed in Section 6.3. The performance of the two systems are compared in Section 6.4 the optimum relay locations for the system with the cooperative diversity are determined, so that it outperforms the system with the receiver diversity. Finally, conclusions are given in Section 6.5.

6.2 System Model

We compare the BER performance of two communication systems. System I uses a single relay 'R' to realize a distributed diversity in order to improve the transmission reliability from a source 'S' to a destination 'D'. All nodes in System I are equipped with a single transmitting and a single receiving antenna. On the other hand, System II achieves the transmission reliability by exploiting the receiver diversity. In System II, a source 'S' with one transmitting antenna transmits information to a destination 'D' having two receiving antennas. Hence, both systems can achieve the transmission diversity of order at most two. We assume a flat fading channel model with an additive white Gaussian noise (AWGN) between any pair of network nodes, and also, that all channels are mutually independent. Without any loss of generality, we omit symbol-time indices in the expressions.

For System I using the cooperative diversity, we use the following notation to describe the transmission from a node $\mathrm{X} \in \{\mathrm{S}, \mathrm{R}\}$ to a node $\mathrm{Y} \in \{\mathrm{R}, \mathrm{D}\}$, i.e.,

$d_{\mathrm{XY}} > 0$	distance between X and Y
$\alpha_{\mathrm{XY}} > 0$	path-loss coefficient
$h_{\mathrm{XY}} \in \mathcal{C}$	channel fading coefficient
$\gamma_{\mathrm{XY}} \geq 0$	instantaneous SNR at node Y
$w_{\mathrm{XY}} \in \mathcal{C}$	AWGN
$y_{\mathrm{XY}} \in \mathcal{C}$	received signal at node Y

where $\mathcal{C}$ denotes the set of complex numbers. For System II using the receiver diversity with the receiver antenna $i = 1, 2$, we use the notation,

$d > 0$	distance between S and D
$\alpha > 0$	path-loss coefficient
$h_{(i)} \in \mathcal{C}$	channel fading coefficient
$\gamma_{(i)} \geq 0$	instantaneous SNR at node D
$w_{(i)} \in \mathcal{C}$	AWGN
$y_{(i)} \in \mathcal{C}$	received signal at node D.

Furthermore, we make the following assumptions common to both systems. The channel fading coefficients h are complex-valued wide-sense stationary jointly Gaussian random processes having zero-mean and unit-variance. Thus, the channel fading amplitudes $|h|$ are Rayleigh distributed, and $\mathrm{E}[h] = 0$ and $\mathrm{E}[|h|^2] = 1$, where $\mathrm{E}[\cdot]$ is expectation, and $|\cdot|$ is the absolute value. The channel fading coefficients are either assumed to be constant and then change independently during the transmission of one codeword (corresponding to a slow block fading channel model), or they change independently for every transmitted symbol (i.e., a fast fading channel model with ideal interleaving and deinterleaving of symbols). All coefficients of AWGNs w are uncorrelated zero-mean complex-valued jointly Gaussian random processes having the *equal* variance $\sigma_{\mathrm{w}}^2 = \mathrm{E}[|w|^2] = N_0$ where N_0 is a constant one-sided power spectral density of the AWGNs.

In general, the signal amplitude attenuation due to a path-loss at distance d from the transmitter antenna is proportional to const $\times\, d^{-\mu/2}$ where the constant is a function of the carrier frequency, and $\mu > 0$ is the path-loss exponent. Let d_0 be the reference distance at which the path-loss is equal to unity. Then, the path-loss coefficient α_{XY} and α at the distance d_{XY} and d, respectively, from the transmitter antenna can be expressed as,

$$\alpha_{\mathrm{XY}} = \left(\frac{d_{\mathrm{XY}}}{d_0}\right)^{-\mu/2} \qquad \alpha = \left(\frac{d}{d_0}\right)^{-\mu/2}.$$

Since the nodes S and D are common to both systems under consideration, in the sequel, we assume that the path-loss between S and D in both systems is unity, i.e., $d_0 = d_{\text{SD}} = d$. Hence, the path-loss coefficients at distances greater (smaller) than the reference distance d_0 are smaller (larger) than unity. Note that the choice of the reference distance shifts the SNR values of all links equally. Thus, one can choose an arbitrary common reference distance d_0 without biasing the BER comparisons of the two systems.

Let x denote a modulation symbol in the transmitted codeword. The modulation symbols have zero-mean and are normalized, so that the average energy per symbol $\text{E}[|x|^2]$ is equal to a constant $E_s > 0$. For the cooperative diversity system with the AF relaying, the received signals at two consecutive time slots corresponding to the transmitted symbol x can be written as,

$$\begin{aligned} y_{\text{SD}} &= \alpha_{\text{SD}} h_{\text{SD}}\, x + w_{\text{SD}} \\ y_{\text{SR}} &= \alpha_{\text{SR}} h_{\text{SR}}\, x + w_{\text{SR}} \\ y_{\text{RD}} &= \beta_{\text{AF}} \alpha_{\text{RD}} h_{\text{RD}} y_{\text{SR}} + w_{\text{RD}} \end{aligned}$$

where β_{AF} is the amplification factor used at the relay. The amplification factor β_{AF} normalizes the average energy of the signal transmitted from the relay to be equal to E_s, i.e., [13, 29],

$$\beta_{\text{AF}} = \frac{\sqrt{E_s}}{\sqrt{\text{E}[|y_{\text{SR}}|^2]}} = \sqrt{\frac{E_s}{\alpha_{\text{SR}}^2 |h_{\text{SR}}|^2 E_s + \sigma_{\text{w}}^2}}$$

where expectation in the denominator is conditioned on the amplitude $|h_{\text{SR}}|$. For the cooperative diversity system with the DF relaying, the received signal at the destination at the second time slot corresponding to the transmitted symbol x can be written as,

$$y_{\text{RD}} = \beta_{\text{DF}} \alpha_{\text{RD}} h_{\text{RD}}\, \hat{x} + w_{\text{RD}}$$

where the relay amplification factor $\beta_{\text{DF}} = 1$ and $\hat{x}$ is a re-encoded symbol at the relay. We assume that the symbol $\hat{x}$ is from the same modulation constellation as the symbol x; if $\hat{x} \neq x$, then a decoding error occurred at the relay.

For the receiver diversity system, the received signals at the two receiver antennas corresponding to the transmitted symbol x can be written as,

$$\begin{aligned} y_{(1)} &= \alpha\, h_{(1)}\, x + w_{(1)} \\ y_{(2)} &= \alpha\, h_{(2)}\, x + w_{(2)}. \end{aligned}$$

At the destination, the received signals are coherently combined using MRC or EGC. In particular, the MRC output signals are written as,

$$\begin{aligned} y &\overset{\text{System I}}{=} \frac{\beta\, \alpha_{\mathrm{RD}}\alpha_{\mathrm{SR}} h^*_{\mathrm{RD}} h^*_{\mathrm{SR}}}{\beta^2\alpha^2_{\mathrm{RD}}|h_{\mathrm{RD}}|^2+1} y_{\mathrm{RD}} + \alpha_{\mathrm{SD}} h^*_{\mathrm{SD}} y_{\mathrm{SD}} \\ y &\overset{\text{System II}}{=} h^*_{(1)} y_{(1)} + h^*_{(2)} y_{(2)} \end{aligned}$$

and for EGC, the output signals are written as,

$$\begin{aligned} y &\overset{\text{System I}}{=} \frac{\mathrm{e}^{-\mathrm{j}(\angle h_{\mathrm{RD}} + \angle h_{\mathrm{SR}})}}{\sqrt{\beta^2\alpha^2_{\mathrm{RD}}|h_{\mathrm{RD}}|^2+1}} y_{\mathrm{RD}} + \mathrm{e}^{-\mathrm{j}h_{\mathrm{SD}}} y_{\mathrm{SD}} \\ y &\overset{\text{System II}}{=} \mathrm{e}^{-\mathrm{j}\angle h_{(1)}} y_{(1)} + \mathrm{e}^{-\mathrm{j}\angle h_{(2)}} y_{(2)} \end{aligned}$$

where $\mathrm{j} = \sqrt{-1}$ is the imaginary unit, and $\angle(\cdot)$ denotes the phase of a complex number. Note that, since the path-loss coefficients are time-invariant, they can be used as the weighting factors of the EGC; however, in this chapter, only the phase-compensating weighting factors are considered in the EGC combiner.

Recall that all the receivers in the network are assumed to have the identical time-invariant power spectral densities of the background AWGNs. The instantaneous SNR of the communication link between a pair of nodes for the system with the cooperative and the receiver diversity, respectively, is defined as,

$$\gamma_{\mathrm{XY}} = \alpha^2_{\mathrm{XY}}\, |h_{\mathrm{XY}}|^2 \gamma_b \qquad \gamma_{(i)} = \alpha^2\, |h_{(i)}|^2\, \gamma_b$$

where $\gamma_b = E_s/(N_0 \log_2 M)$ is the SNR per transmitted bit assuming an M-ary modulation constellation. In this paper, we assume that all links are subject to independent and identically distributed Rayleigh fading, and thus, the SNR of each link is exponentially distributed [24]. Provided that a channel coding of rate $R < 1$ is used at the source, the AWGNs at the relay and destination receivers have the equal variance

$\sigma_{\mathrm{w}}^2 = \mathrm{E}[|w|^2] = N_0 = E_s/(R\gamma_b \log_2 M)$. Then, the instantaneous SNR at the output of the MRC combiner at the destination for the system with the cooperative and the receiver diversity, respectively, can be expressed as,

$$\gamma \overset{\text{System I}}{=} \gamma_{\text{SD}} + \frac{\gamma_{\text{SR}}\gamma_{\text{RD}}}{\gamma_{\text{SR}} + \gamma_{\text{RD}} + 1}$$

$$\gamma \overset{\text{System II}}{=} \gamma_{(1)} + \gamma_{(2)}.$$

In general, depending on the relay location, the average SNR at the combiner output at the destination can be larger or smaller for the cooperative diversity than for the case of the receiver diversity. However and importantly, if the path-loss is not considered (i.e., the average SNR values are location-invariant), then the average SNR of the receiver diversity is always larger than the average SNR of the cooperative diversity. In addition, note that, for a fair comparison, we assume that both the source and the relay transmits with the average energy per symbol E_s, so that the total average energy per transmitted symbol is $2E_s$ over the two time-slots whereas the total average energy per transmitted symbol for the system with the receiver diversity is E_s.

6.2.1 Modulation and Channel Coding and Decoding

We assume that the transmissions between nodes are realized using a linear memoryless modulation and using a linear binary block code of short block length. The encoding of information bits by a binary channel code is performed by multiplying the vector of K information bits by a binary generator matrix in order to produce a binary codeword of N encoded bits. The binary channel coding C is denoted as a triplet $(N, K, d_{\min})$ where $d_{\min}$ is the minimum Hamming distance between any two codewords, and $R = K/N$ is the code rate. The codewords are possibly interleaved and mapped to either binary phase shift keying (BPSK) symbols or to 16 quadrature amplitude modulation (QAM) symbols. For the 16QAM modulation, we assume a natural mapping of the consecutive sequences of 4 encoded bits (c_1, c_2, c_3, c_4) to the modulation symbols $x = x_I + \mathrm{j}x_Q$ such that the encoded bits (c_1, c_3) are mapped to $x_I \in \{\pm 1, \pm 3\}$, and the encoded bits (c_2, c_4) are mapped to $x_Q \in \{\pm 1, \pm 3\}$, as in Chapter 4 and [71].

6.3 Analysis of Transmission Reliability

The theoretical analysis is mathematically tractable provided that we assume a block fading channel model, i.e., the channel fading coefficients are generated independently and held constant for the transmission of each codeword. Recall that the channel fading coefficients between the network nodes are assumed to be mutually independent, and they are perfectly known at the receivers. For notational simplicity, the path-loss coefficients α are merged into the channel fading coefficients h, so that the variances $\mathrm{E}[|h|^2]$ are scaled by α^2. We denote as $g = |h|$ the amplitudes of the channel fading coefficients h. In our analysis, we consider the performance of the EGC at the destination receiver for the case of BPSK modulation. For BPSK signaling, we denote the codewords $\mathbf{0} = (0, \cdots, 0)$, $\mathbf{a} = (a_1, \cdots, a_N)$ and $\mathbf{b} = (b_1, \cdots, b_N)$ corresponding to the transmitted sequences $\mathbf{x}^{(0)} = (1, \cdots, 1)$, $\mathbf{x}^{(a)} = (x_1^{(a)}, \cdots, x_N^{(a)})$ and $\mathbf{x}^{(b)} = (x_1^{(b)}, \cdots, x_N^{(b)})$, respectively. We assume that all codewords are equally likely to be transmitted and that an all-zero codeword has been transmitted. Note that the latter assumption may slightly bias the analysis for System II due to non-linearity of the DF relaying. The ML detector at the destination receiver selects the most likely codeword corresponding to the transmitted sequence with the smallest Euclidean distance from the received sequence $\mathbf{y}$.

In general, the probability of transmission error for coded systems can be upper-bounded using a union-bound [87]. Thus, the BER of coded systems can be upper-bounded as [88],

$$\mathrm{BER} \leq \sum_{\substack{\mathbf{a} \in C \\ \mathbf{a} \neq \mathbf{0}}} \frac{w_{\mathrm{H}}[\mathbf{u}]}{K} \Pr\{\mathbf{0} \rightarrow \mathbf{a}\} \tag{6.1}$$

where $w_{\mathrm{H}}[\mathbf{u}]$ is the Hamming weight of the information vector $\mathbf{u}$ corresponding to the codeword $\mathbf{a}$ of a binary linear block code $C = (N, K, d_{\min})$. The PEP $\Pr\{\mathbf{0} \rightarrow \mathbf{a}\}$ is the probability that the all-zero codeword $\mathbf{0}$ was transmitted, and the receiver decides between the codewords $\mathbf{0}$ and $\mathbf{a}$ that $\mathbf{a}$ has been transmitted. Provided that the PEP $\Pr\{\mathbf{0} \rightarrow \mathbf{a}\}$ can be expressed as a function of the Hamming weight $w_{\mathrm{H}}[\mathbf{a}]$, the union bound (6.1) can be evaluated more effectively using a weight enumerator of the code C [70]. More importantly, note that the union bound (6.1) is dominated by the largest PEP $\Pr\{\mathbf{0} \rightarrow \mathbf{a}\}$. Thus, in the sequel, we evaluate the PEP $\Pr\{\mathbf{0} \rightarrow \mathbf{a}\}$ rather than the overall union bound (6.1) as a key measure of the transmission reliability for the coded communication systems.

6.3.1 System I with the AF Diversity

Assuming System I with the AF relaying, the output signal of the EGC at the destination receiver can be written as,

$$\begin{aligned} y_i &= \mathrm{Re}\left\{ \left(g_{\mathrm{SD}} + \frac{\beta_{\mathrm{AF}} g_{\mathrm{RD}} g_{\mathrm{SR}}}{\sqrt{\beta_{\mathrm{AF}}^2 g_{\mathrm{RD}}^2 + 1}} \right) x_i^{(0)} + \frac{\beta_{\mathrm{AF}} g_{\mathrm{RD}} w_{\mathrm{SR}i} + w_{\mathrm{RD}i}}{\sqrt{\beta_{\mathrm{AF}}^2 g_{\mathrm{RD}}^2 + 1}} + w_{\mathrm{SD}i} \right\} \\ &= \left(g_{\mathrm{SD}} + \frac{g_{\mathrm{RD}} g_{\mathrm{SR}}}{\sqrt{g_{\mathrm{RD}}^2 + g_{\mathrm{SR}}^2 + c_1}} \right) x_i^{(0)} + w_{\mathrm{AF}i} \\ &= g_{\mathrm{AF}} x_i^{(0)} + w_{\mathrm{AF}i} \end{aligned}$$

where $i = 1, 2, \cdots, N$ is the symbol index in the transmitted codeword, $\mathrm{Re}\{\cdot\}$ is the real part of a complex number, $w_{\mathrm{AF}i}$ is an equivalent zero-mean AWGN having the variance $\mathrm{E}[|w_{\mathrm{AF}i}|^2] = \sigma_{\mathrm{w}}^2 = N_0$, and $c_1 = N_0/E_s$ is the inverse of the SNR per transmitted symbol. Note that the signal received from the relay is normalized by the factor $\sqrt{\beta_{\mathrm{AF}}^2 g_{\mathrm{RD}}^2 + 1}$ in order to make the AWGN variances of the two diversity signals before combining equal. Given the value of g_{AF}, the conditional PEP of System II with the AF relaying is calculated as the probability that the Euclidean distance $w_{\mathrm{E},0}$ from the received sequence $\mathbf{y}$ for the codeword $\mathbf{0}$ is greater than the Euclidean distance from $\mathbf{y}$ for the codeword $\mathbf{a}$, i.e.,

$$\begin{aligned} \Pr\{\mathbf{0} \to \mathbf{a} | g_{\mathrm{AF}}\} &= \Pr\{w_{\mathrm{E},0}^2 > w_{\mathrm{E},a}^2\} \\ &= \Pr\left\{ \sum_{i=1}^{N} \left(y_i - g_{\mathrm{AF}} x_i^{(0)} \right)^2 > \sum_{i=1}^{N} \left(y_i - g_{\mathrm{AF}} x_i^{(a)} \right)^2 \right\} \\ &= \Pr\left\{ \sum_{i=1}^{N} -g_{\mathrm{AF}}^2 (x_i^{(a)} - x_i^{(0)})^2 + 2 g_{\mathrm{AF}} (x_i^{(a)} - x_i^{(0)}) w_{\mathrm{AF}i} > 0 \right\}. \end{aligned}$$

$$f_{\tilde{g}_{\mathrm{AF}}}(z) = \frac{(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)}{2c_2^{5/2}c_3^{3/2}} \mathrm{e}^{-(\sigma_{\mathrm{RD}}^{-2}+\sigma_{\mathrm{SD}}^{-2}+\sigma_{\mathrm{SR}}^{-2})z^2/2} \Big(-2c_2^2\sigma_{\mathrm{SR}}^2\sigma_{\mathrm{SD}}^2\sqrt{c_3}\, z\mathrm{e}^{\frac{(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)}{2\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2}z^2} -$$
$$2c_2^2\sqrt{c_3}\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2 z\mathrm{e}^{\frac{1}{2\sigma_{\mathrm{SD}}^2}z^2} - 2c_2^2\sqrt{c_3}\sigma_{\mathrm{SD}}^2\sigma_{\mathrm{SR}}^2 z\mathrm{e}^{\frac{(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)}{2\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2}z^2} +$$
$$\sqrt{2\pi}\sigma_{\mathrm{SD}}\sigma_{\mathrm{SR}}\sigma_{\mathrm{RD}}\mathrm{e}^{\left(\frac{1}{\sigma_{\mathrm{RD}}^2}+\frac{1}{\sigma_{\mathrm{SR}}^2}+\frac{\sigma_{\mathrm{RD}}^2\sigma_{\mathrm{SR}}^2}{\sigma_{\mathrm{SD}}^2 c_2}\right)\frac{z^2}{2}} \times$$
$$(\sigma_{\mathrm{SD}}^2\sigma_{\mathrm{SR}}^2 + \sigma_{\mathrm{RD}}^2(\sigma_{\mathrm{SD}}^2+\sigma_{\mathrm{SR}}^2)\, c_3 z^2) \left(\mathrm{erf}\left(\frac{\sigma_{\mathrm{RD}}\sigma_{\mathrm{SR}}\sqrt{c_3}z}{4\sigma_{\mathrm{SD}}v}\right) + \mathrm{erf}\left(\frac{\sigma_{\mathrm{SD}}(\sigma_{\mathrm{RD}}^2+\sigma_{\mathrm{SR}}^2)\sqrt{c_3}z}{4\sigma_{\mathrm{RD}}\sigma_{\mathrm{SR}}\sqrt{c_2}}\right)\right)$$

Since, for a zero mean unit variance Gaussian random variable W, the probability $\Pr\{W > w\} = Q(w)$ where $Q(\cdot)$ is the Q-function [24], we have that,

$$\begin{aligned} \Pr\{\mathbf{0} \to \mathbf{a} | g_{\mathrm{AF}}\} &= \Pr\left\{ W > g_{\mathrm{AF}} \frac{\sqrt{\sum_{i=1}^{N}(x_i^{(a)} - x_i^{(0)})^2}}{2\sqrt{N_0}} \right\} \\ &= Q\left(g_{\mathrm{AF}} \frac{w_{\mathrm{E}}\left[\mathbf{x}^{(a)}, \mathbf{x}^{(0)}\right]}{2\sqrt{N_0}} \right) = Q\left(g_{\mathrm{AF}}\sqrt{w_{\mathrm{H}}[\mathbf{a}]\,\gamma_b} \right) \end{aligned}$$

where $w_{\mathrm{E}}\left[\mathbf{x}^{(a)}, \mathbf{x}^{(0)}\right]$ is the Euclidean distance between the vectors $\mathbf{x}^{(a)}$ and $\mathbf{x}^{(0)}$. Then, the PEP is evaluated as,

$$\Pr\{\mathbf{0} \to \mathbf{a}\} = \int_0^\infty \Pr\{\mathbf{0} \to \mathbf{a}|z\}\, f_{g_{\mathrm{AF}}}(z)\mathrm{d}z \tag{6.2}$$

where $f_{g_{\mathrm{AF}}}(z)$ is the probability density function (PDF) of g_{AF}. In general, a closed form expression for $f_{g_{\mathrm{AF}}}(z)$ is difficult to obtain. However, since, always, $g_{\mathrm{AF}} \leq g_{\mathrm{SD}} + \min(g_{\mathrm{RD}}, g_{\mathrm{SR}}) = \tilde{g}_{\mathrm{AF}}$, and the channel fading amplitudes g_{SD}, g_{SR} and g_{RD} are independent and have the variances σ_{SD}^2, σ_{SR}^2 and σ_{RD}^2, respectively, we can lower-bound the PEP (6.2), i.e.,

$$\Pr\{\mathbf{0} \to \mathbf{a}\} \geq \int_0^\infty \Pr\{\mathbf{0} \to \mathbf{a}|z\}\, f_{\tilde{g}_{\mathrm{AF}}}(z)\mathrm{d}z$$

where, after lengthy manipulations, the closed form expression of the PDF $f_{\tilde{g}_{\mathrm{AF}}}(z)$ is shown at the top of this page, $c_2 = \sigma_{\mathrm{SD}}^2\sigma_{\mathrm{SR}}^2 + \sigma_{\mathrm{RD}}^2(\sigma_{\mathrm{SD}}^2 + \sigma_{\mathrm{SR}}^2)$, $c_3 = \log(\mathrm{e})$ and the function $\mathrm{erf}(x) = 1 - 2Q\big(\sqrt{2}x\big)$.

6.3.2 System I with the DF Diversity

In order to analyze the PEP of the DF relaying, we assume that the source transmits the all-zero codeword **0**, however, the relay decodes and forwards a codeword **b**. In this case, the EGC output signal at the destination receiver is written as,

$$\begin{aligned} y_i &= \mathrm{Re}\Big\{ g_{\mathrm{SD}} x_i^{(0)} + w_{\mathrm{SD}i} + g_{\mathrm{RD}} x_i^{(b)} + w_{\mathrm{RD}i} \Big\} \\ &= g_{\mathrm{SD}} x_i^{(0)} + g_{\mathrm{RD}} x_i^{(b)} + w_{\mathrm{DF}i} \end{aligned}$$

where $w_{\mathrm{DF}i}$ is an equivalent zero-mean AWGN with the variance $\mathrm{E}[|w_{\mathrm{DF}i}|^2] = \sigma_{\mathrm{w}}^2 = N_0$. The PEP of the destination receiver conditioned on the values of the channel fading amplitudes g_{SD}, g_{SR} and g_{RD} is then calculated as,

$$\Pr\{\mathbf{0} \to \mathbf{a} | g_{\mathrm{SD}}, g_{\mathrm{SR}}, g_{\mathrm{RD}}\} = \sum_{\mathbf{b} \in C} \Pr\{\mathbf{0} \to \mathbf{a} | \mathbf{b}, g_{\mathrm{SD}}, g_{\mathrm{RD}}\} \Pr\{\mathbf{0} \to \mathbf{b} | g_{\mathrm{SR}}\} \tag{6.3}$$

where $\Pr\{\mathbf{0} \to \mathbf{b} | g_{\mathrm{SR}}\}$ is the conditional PEP that the relay decodes the codeword **b**. The first conditional PEP in (6.3) is again equal to the probability that the Euclidean distance $w_{\mathrm{E},0}$ from the received sequence **y** for the all-zero codeword is greater than the Euclidean distance $w_{\mathrm{E},a}$ corresponding to the codeword **a**, i.e.,

$$\begin{aligned} &\Pr\{\mathbf{0} \to \mathbf{a} | \mathbf{b}, g_{\mathrm{SD}}, g_{\mathrm{RD}}\} = \Pr\{w_{\mathrm{E},0}^2 > w_{\mathrm{E},a}^2\} \\ &= \Pr\left\{ \sum_{i=1}^{N} \left(y_i - (g_{\mathrm{SD}} + g_{\mathrm{RD}}) x_i^{(0)} \right)^2 > \sum_{i=1}^{N} \left(y_i - (g_{\mathrm{SD}} + g_{\mathrm{RD}}) x_i^{(a)} \right)^2 \right\} \\ &= \Pr\left\{ \sum_{i=1}^{N} (g_{\mathrm{RD}} t_i - g_{\mathrm{SD}} s_i) s_i + 2 s_i w_{\mathrm{DF}i} > 0 \right\} \\ &= Q\left(\frac{\sum_{i=1}^{N} (g_{\mathrm{SD}} s_i - g_{\mathrm{RD}} t_i) s_i}{2\sqrt{N_0 \sum_{i=1}^{N} s_i^2}} \right), \end{aligned} \tag{6.4}$$

where we defined, $s_i = x_i^{(a)} - x_i^{(0)}$ and $t_i = 2x_i^{(b)} - x_i^{(a)} - x_i^{(0)}$. Assuming that $x_i^{(0)} = 1$ for $\forall i$, we can show that, for any values of g_{SD} and g_{RD}, the argument of the Q-function in (6.4) is, in general, increasing with the Hamming distance between the codewords **a** and **b**. The argument of the Q-function in (6.4) is minimized for $\mathbf{a} = \mathbf{b}$ (i.e., the vectors are component-wise identical) while $\mathbf{0} \neq \mathbf{a}$ which corresponds to the worst case scenario when the value of the PEP defined in (6.4) is maximized. On the other hand, we can show that, for any values of g_{SD} and g_{RD}, the value of the PEP (6.4) is minimized provided that $\mathbf{b} = \mathbf{0}$ (i.e., the relay correctly decodes the codeword transmitted from the source). This also indicate that the ability of the relay to correctly decode the transmitted codeword from the source has a major effect upon the overall probability of transmission error of the cooperative system.

Denote as $w_{\mathrm{E},b}$ the Euclidean distance from the received sequence $\mathbf{y}_{\mathrm{SR}}$ for the codeword $\mathbf{b}$ at the relay receiver. Then, the PEP $\Pr\{\mathbf{0} \to \mathbf{b}|g_{\mathrm{SR}}\}$ for the link from the source to the relay can be expressed as [85],

$$\begin{aligned}
\Pr\{\mathbf{0} \to \mathbf{b}|g_{\mathrm{SR}}\} &= \Pr\{w_{\mathrm{E},0}^2 > w_{\mathrm{E},b}^2\} \\
&= \Pr\left\{\sum_{i=1}^{N}\left(y_{\mathrm{SR},i} - g_{\mathrm{SR}}x_i^{(0)}\right)^2 > \sum_{i=1}^{N}\left(y_{\mathrm{SR},i} - g_{\mathrm{SR}}x_i^{(b)}\right)^2\right\} \\
&= Q\left(\frac{g_{\mathrm{SR}}w_{\mathrm{E}}\left[\mathbf{x}^{(0)}, \mathbf{x}^{(b)}\right]}{2\sqrt{N_0}}\right) = Q\left(g_{\mathrm{SR}}\sqrt{w_{\mathrm{H}}[\mathbf{b}]\,\gamma_b}\right)
\end{aligned}$$

where $y_{\mathrm{SR},i}$ is the received signal at the relay, $w_{\mathrm{H}}[\mathbf{b}]$ is the Hamming weight of the codeword $\mathbf{b}$, and $w_{\mathrm{E}}\left[\mathbf{x}^{(0)}, \mathbf{x}^{(b)}\right]$ is the Euclidean distance between the modulated sequences corresponding to the vectors $\mathbf{0}$ and $\mathbf{b}$.

Using (6.3), the PEP averaged over the independent Rayleigh distributed channel fading amplitudes g_{SD}, g_{SR} and g_{RD} is expressed as,

$$\Pr\{\mathbf{0} \to \mathbf{a}\} = \iiint_0^{\infty} \Pr\{\mathbf{0} \to \mathbf{a}|u, v, r\}\, f_{g_{\mathrm{SD}}}(u) f_{g_{\mathrm{SR}}}(v) f_{g_{\mathrm{RD}}}(r)\mathrm{d}u\mathrm{d}v\mathrm{d}r$$

$$= \sum_{\mathbf{b}\in C} \iint_0^{\infty} \Pr\{\mathbf{0} \to \mathbf{a}|\mathbf{b}, u, v\}\, f_{g_{\mathrm{SD}}}(u) f_{g_{\mathrm{SR}}}(v)\mathrm{d}u\mathrm{d}v \int_0^{\infty} \Pr\{\mathbf{0} \to \mathbf{b}|r\}\, f_{g_{\mathrm{RD}}}(r)\mathrm{d}r$$

$$= \sum_{\mathbf{b}\in C} \Pr\{\mathbf{0} \to \mathbf{a}|\mathbf{b}\} \Pr\{\mathbf{0} \to \mathbf{b}\}$$

Let the argument of the Q-function in (6.4) be a random variable,

$$Z = C_1 g_{\mathrm{SD}} - C_2 g_{\mathrm{RD}}$$

where the constants,

$$C_1 = \frac{\sqrt{\sum_{i=1}^{N} s_i^2}}{2\sqrt{N_0}} \quad \text{and} \quad C_2 = \frac{\sum_{i=1}^{N} s_i t_i}{2\sqrt{N_0 \sum_{i=1}^{N} s_i^2}}.$$

Then, the average PEP $\Pr\{\mathbf{0} \to \mathbf{a}|\mathbf{b}\}$ can be evaluated as,

$$\Pr\{\mathbf{0} \to \mathbf{a}|\mathbf{b}\} = \int_{-\infty}^{\infty} \Pr\{\mathbf{0} \to \mathbf{a}|\mathbf{b}, z\}\, f_Z(z)\mathrm{d}z.$$

The PDF of the random variable Z can be obtained by conditioning and integration [86], i.e.,

$$f_Z(z) = \begin{cases} \frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1/2}(k_1+k_2)^{-3} f_1(z) & z \geq 0 \\ \frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)^{-1/2}(k_1+k_2)^{-3}\left(\frac{k_1}{k_5+k_4}\right)^{-1/2} f_2(z) & z < 0 \end{cases}$$

where $k_1 = C_2^2\sigma_1^2$, $k_2 = C_2^2\sigma_2^2$, $k_3 = C_1^4\sigma_1^4$, $k_4 = C_2^4\sigma_2^4$, $k_5 = C_1^2C_2^2\sigma_1^2\sigma_2^2$, and,

$$\begin{aligned} f_1(z) = \mathrm{e}^{-(\frac{1}{k_1}+\frac{2}{k_2})\frac{z}{2}}\Bigg(2k_3\sqrt{\frac{1}{k_1}+\frac{1}{k_2}}\mathrm{e}^{\frac{z^2}{k_2}}z + 2k_5\sqrt{\frac{1}{k_1}+\frac{1}{k_2}}\mathrm{e}^{\frac{z^2}{k_2}}z \\ +\sqrt{2\pi}(k_1+k_2)(k_1+k_2-z^2)\mathrm{e}^{\frac{(2k_3+2k_5k_4)z^2}{2k_5(k_1+k_2)}} \\ -\sqrt{\frac{2\pi k_2}{k_1}+2\pi\sqrt{k_1^2+k_1k_2}}(k_1+k_2-z^2)\,\mathrm{erf}\left(\frac{\sqrt{k_2}z}{\sqrt{2k_3+2k_5}}\right)\Bigg) \end{aligned}$$

$$\begin{aligned} f_2(z) = \mathrm{e}^{\frac{-z^2}{k_2}}(k_1+k_2)\sqrt{\frac{k_1}{k_5+k_4}}\left(-2k_2\sqrt{\frac{1}{k_1}+\frac{1}{k_2}}z + \sqrt{2\pi}(k_1+k_2-z^2)\mathrm{e}^{\frac{(2k_1+k_2)z^2}{2k_5+2k_4}}\right) \\ +\sqrt{\frac{2\pi}{k_1}+\frac{\pi}{k_2}(k_1^2+k_1k_2-k_1z^2)}\mathrm{e}^{\frac{(2k_1+k_2)z^2}{2k_5+2k_4}}\mathrm{erf}\left(\frac{\sqrt{k_1}z}{\sqrt{2k_5+2k_4}}\right). \end{aligned}$$

The average PEP $\Pr\{\mathbf{0} \to \mathbf{b}\}$ can be obtained by using the Chernoff bound $Q(x) \leq \frac{1}{2}\mathrm{e}^{-x^2/2}$, for example, as in [82], or by using the Prony approximation $Q(x) \doteq 0.208\mathrm{e}^{-0.971x^2} + 0.147\mathrm{e}^{-0.525x^2}$ as in [88]. Assuming the latter expression, the average PEP is approximately equal to,

$$\Pr\{\mathbf{0} \to \mathbf{b}\} \doteq \frac{0.208}{1.942\, w_{\mathrm{H}}[\mathbf{b}]\, \sigma_{\mathrm{SR}}^2\gamma_b + 1} + \frac{0.147}{1.050\, w_{\mathrm{H}}[\mathbf{b}]\, \sigma_{\mathrm{SR}}^2\gamma_b + 1}$$

where γ_b is the SNR per encoded binary symbol.

6.3.3 System II with the Rx Diversity

Assuming the receiver diversity without relay, the output signal of the EGC at the destination receiver can be written as,

$$\begin{aligned} y_i &= \mathrm{Re}\Big\{ g_{(1)} x_i^{(0)} + g_{(2)} x_i^{(0)} + w_{(1)_i} + w_{(2)_i} \Big\} \\ &= (g_{(1)} + g_{(2)}) x_i^{(0)} + w_{\mathrm{Rx}i} = g_{\mathrm{Rx}} x_i^{(0)} + w_{\mathrm{Rx}i} \end{aligned}$$

where $w_{\mathrm{Rx}i}$ is an equivalent zero-mean AWGN with the variance $\mathrm{E}[|w_{\mathrm{Rx}i}|^2] = \sigma_{\mathrm{w}}^2 = N_0$. The PEP of System II is obtained similarly as for the source to relay link in System I. Thus, conditioned on the channel fading amplitude g_{Rx}, and BPSK signaling, the PEP is evaluated as,

$$\begin{aligned} \Pr\{\mathbf{0} \to \mathbf{a} | g_{\mathrm{Rx}}\} &= \Pr\{w_{\mathrm{E},0}^2 > w_{\mathrm{E},a}^2\} \\ &= \Pr\left\{ \sum_{i=1}^{N} \left(y_i - g_{\mathrm{Rx}} x_i^{(0)} \right)^2 > \sum_{i=1}^{N} \left(y_i - g_{\mathrm{Rx}} x_i^{(a)} \right)^2 \right\} \\ &= Q\Big(g_{\mathrm{Rx}} \sqrt{w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b} \Big). \end{aligned}$$

Consequently, the average PEP is calculated using the integration,

$$\Pr\{\mathbf{0} \to \mathbf{a}\} = \int_0^\infty \Pr\{\mathbf{0} \to \mathbf{a} | z\}\, f_{g_{\mathrm{Rx}}}(z) \mathrm{d}z.$$

The integration to obtain the average PEP $\Pr\{\mathbf{0} \to \mathbf{a}\}$ can be carried out using the Prony approximation method [88]. In particular, the conditional PEP is approximately equal to,

$$Q\Big(g_{\mathrm{Rx}} \sqrt{w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b} \Big) \doteq 0.208 \mathrm{e}^{-0.971 g_{\mathrm{Rx}}^2 w_{\mathrm{H}}[\mathbf{a}] \gamma_b} + 0.147 \mathrm{e}^{-0.525 g_{\mathrm{Rx}}^2 w_{\mathrm{H}}[\mathbf{a}] \gamma_b}$$

so that the average PEP is calculated as,

$$\Pr\{\mathbf{0} \to \mathbf{a}\} = 0.208 \int_0^\infty \mathrm{e}^{-A_1^2 z^2} f_{g_{\mathrm{Rx}}}(z) \mathrm{d}z + 0.147 \int_0^\infty \mathrm{e}^{-A_2^2 z^2} f_{g_{\mathrm{Rx}}}(z) \mathrm{d}z \tag{6.5}$$

where $A_1 = 0.971\, w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b$ and $A_2 = 0.525\, w_{\mathrm{H}}[\mathbf{a}]\, \gamma_b$.

The PDF $f_{g_{\mathrm{Rx}}}(z)$ of the channel fading amplitude g_{Rx} is again obtained by conditioning and integration. Thus, assuming the independent Rayleigh distributed channel fading amplitudes $g_{(1)}$ and $g_{(2)}$ of the variances $\sigma^2_{(1)}$ and $\sigma^2_{(2)}$, respectively, we obtain the PDF,

$$f_{g_{\mathrm{Rx}}}(z) = \frac{z\sigma^2_{(1)}}{V^2}\,\mathrm{e}^{-\frac{z^2}{2\sigma^2_{(1)}}} + \sqrt{\frac{\pi}{2}}\frac{z^2 - V}{V^{5/2}} r\sigma_{(1)}\mathrm{e}^{-\frac{z^2}{2V}}\left(1 + \mathrm{erf}\left(\frac{\sigma_{(2)} z}{\sqrt{2V}\sigma_{(1)}}\right)\right)$$

where $V = \sigma^2_{(1)} + \sigma^2_{(2)}$ is the variance of the EGC amplitude g_{Rx}. Finally, a closed form expression for the average PEP (6.5) based on the Prony approximation method is obtained using the following integration, i.e.,

$$\begin{aligned} I_{\sigma_{(1)},\sigma_{(2)}}(a) &= \int_0^\infty \mathrm{e}^{-a\,z^2} f_{g_{\mathrm{Rx}}}(z)\mathrm{d}z \\ &= \frac{2\sigma_{(1)}V^{3/2}}{1+2aV} + \frac{4a\sigma_{(2)}V^{5/2}}{(1+2aV)^{3/2}}\left(\arctan\left(\sqrt{1+2aV}\frac{\sigma_{(1)}}{\sigma_{(2)}}\right) - \pi\right) \end{aligned}$$

where $a > 0$ is a real constant, and V was defined previously. The PEP (6.5) is then computed as,

$$\Pr\{\mathbf{0} \to \mathbf{a}\} = 0.208\, I_{\sigma_{(1)},\sigma_{(2)}}(0.971\, w_{\mathrm{H}}[\mathrm{a}]\,\gamma_b) + 0.147\, I_{\sigma_{(1)},\sigma_{(2)}}(0.525\, w_{\mathrm{H}}[\mathrm{a}]\,\gamma_b)\,.$$

6.4 Performance Comparison of System I and System II

We use the PEP expressions obtained in the previous section to compare the error rate performances of System I and System II with the cooperative and the receiver diversity is investigated, respectively. In particular, the effect of the relay location on the performance of the cooperative diversity, and determine geographical areas for positioning the relay in which the relaying can outperform the conventional receiver diversity. Recall that the upper-bound of the BER (6.1) is dominated by the largest PEP $\Pr\{\mathbf{0} \to \mathbf{a}\}$, so that we can consider the PEP $\Pr\{\mathbf{0} \to \mathrm{a}\}$ to be the key performance metric of the system. More importantly, assuming our analysis in Section 6.3, it can be shown that, for System I as well as System II, the largest PEP $\Pr\{\mathbf{0} \to \mathrm{a}\}$ corresponds to the codeword a of the minimum Hamming weight $w_{\mathrm{H}}[\mathrm{a}] = d_{\min}$.

Denote as $\mathrm{PEP}_{\mathrm{AF}}$, $\mathrm{PEP}_{\mathrm{DF}}$ and $\mathrm{PEP}_{\mathrm{Rx}}$ the PEPs $\Pr\{\mathbf{0} \to \mathrm{a}\}$ of System I with the AF relaying, System I with the DF relaying and System II with the receiver diversity, respectively. The PEPs PEP_{AF} and PEP_{DF} are the relay location dependent. The relay location is denoted as a triplet $(d_{SR}/d_0, d_{RD}/d_0, d_{SD}/d_0)$ where d_0 is the reference distance. Recall that, without loss of generality, we assume $d_0 = d_{SD}$, i.e., the relay location is given by the triplet $(d_{SR}/d_{SD}, d_{RD}/d_{SD}, 1)$. For System I, the distance between the source and the destination is a scalar variable d; we assume that $d/d_{SD} = 1$. Thus, for System I as well as System II, the path-loss between the source and the destination is unity.

Fig. 6.1 shows an excellent agreement between the mathematical expressions obtained in Section 6.3 and the computer simulations for the PEP $\Pr\{\mathbf{0} \to \mathbf{a}\}$ of System I with the DF relaying assuming independent slow Rayleigh fading channels, BPSK modulation, and a codeword $\mathbf{a}$ of the Hamming weight $d_{\min}$ for the BCH codes $(31, 16, 7)$ and $(32, 26, 4)$. Fig. 6.2 compares the PEPs $\Pr\{\mathbf{0} \to \mathbf{a}\}$ of System II with the two receiver antennas and System I with the DF relaying assuming again independent slow Rayleigh fading channels, BPSK modulation, and a codeword $\mathbf{a}$ of the Hamming weight $d_{\min}$ for the BCH code $(31, 16, 7)$. Note that the distance between the source and the destination is normalized to 1. The relay location denoted as $(1, 1, 1)$ corresponds to the case when the path-loss is not considered. Provided that the path-loss is not considered, the receiver diversity always outperforms the DF diversity as one may intuitively expect. Relaying outperforms the receiver diversity, particularly at smaller values of the SNR. This is further confirmed by the PEP values in Fig. 6.3 versus the relay location $(d_{SR}/d_{SD}, 1 - d_{SR}/d_{SD}, 1)$ at a constant SNR $\gamma_b = 9$dB. More importantly, we observe from Fig. 6.3 that the relay located closer to the source achieves a better PEP performance than the relay located at the center between the source and the destination (cf. Fig. 6.5). Thus, the optimum relay location has to trade-off the error propagation due to the DF relaying and the path-loss attenuations between the nodes, and it is also influenced by the particular channel code used. Assuming the same parameters and settings as in Fig. 6.2 and Fig. 6.3, the PEP performance of System I with the AF relaying is shown in Fig. 6.4.

Also a numerical examples are presented for the overall BER performances of System I and System II. We consider uncoded as well as coded transmissions from the source to the destination using the BCH systematic codes $(31, 16, 7)$ and $(32, 16, 8)$ and BPSK and 16QAM modulations. We employ the POSD decoder at the destination

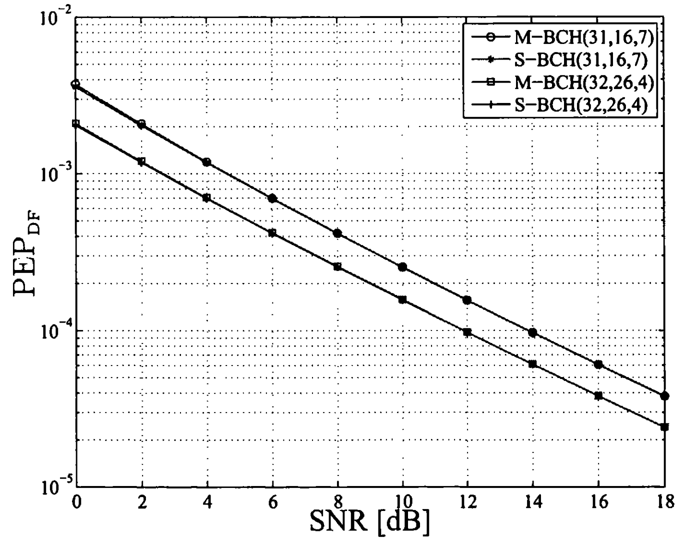

Figure 6.1: The PEP $\Pr\{\mathbf{0} \to \mathbf{a}\}$ for System I with the DF relaying, the BCH $(31, 16, 7)$ and $(32, 26, 4)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination (M-mathematical expression, S-simulation).

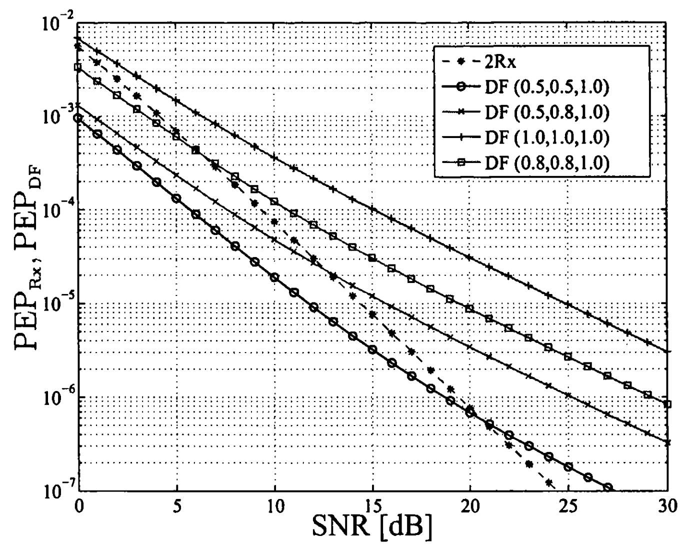

Figure 6.2: The PEP $\Pr\{\mathbf{0} \to \mathbf{a}\}$ for System I with the DF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination.

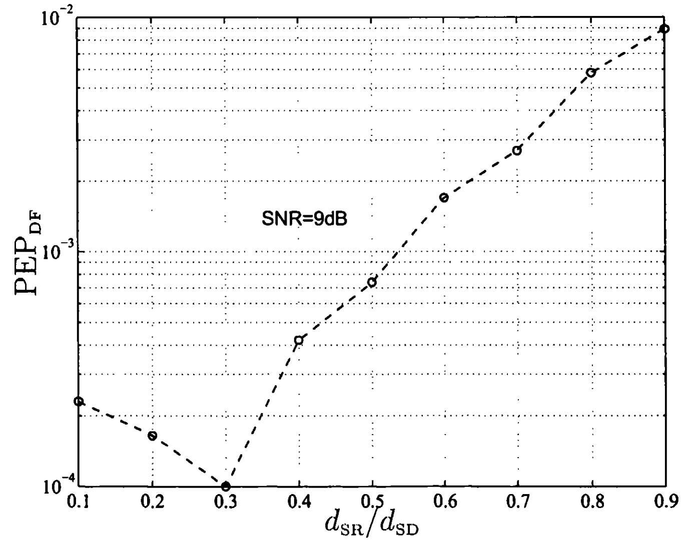

Figure 6.3: The PEP $\Pr\{\mathbf{0} \rightarrow \mathbf{a}\}$ for System I with the DF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, the EGC at the destination, the normalized distance $d_{\mathrm{RD}}/d_{\mathrm{SD}} = 1 - d_{\mathrm{SR}}/d_{\mathrm{SD}}$, and the SNR $\gamma_b = 9$dB.

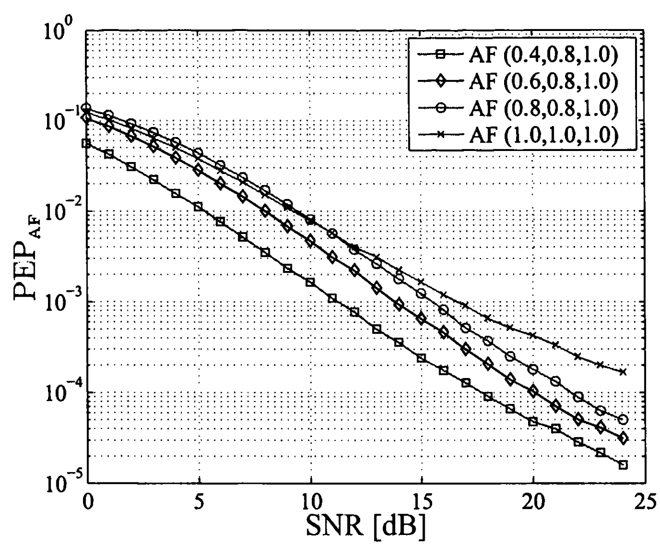

Figure 6.4: The PEP $\Pr\{\mathbf{0} \rightarrow \mathbf{a}\}$ for System I with the AF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination.

and also at the relay provided that the DF relaying is used. The POSD is optimized to achieve the best possible BER performance for the given decoding complexity [84]. In particular, for both BCH codes considered, the POSD searches two disjoint segments of 6 and 10 ordered information bits assuming at most 1 and 3 errors in each segment, respectively. We use the notation '2Rx' to denote the two antenna receiver diversity, and the notation '1Rx' to refer to the scenario where the destination is equipped with a single receiving antenna.

Fig. 6.5 compares the BER performances of System I with the AF relaying and the conventional receiver diversity assuming MRC at the destination. We observe that, for some relay locations, the AF relaying outperforms the conventional receiver diversity. The best BER performance of the AF relaying is achieved when the relay is located in the center between the source and the destination. On the other hand, as intuitively expected, the BER performance of the AF relaying deteriorates significantly when the relay is located at larger distances away from the source and the destination. In addition, we observe that the channel coding benefits significantly from the available diversity gain due to the relaying and all relay locations or due to the multiple receiver antennas.

The BER performance of the DF relaying is shown in Fig. 6.6 assuming the same parameters and relay locations as in Fig. 6.5. Unlike for the AF relaying in Fig. 6.6, we observe from Fig. 6.6 that the BER performance of the DF relaying is much more relay location dependent than the BER performance of the AF relaying, and such dependence is even more pronounced for higher order modulations. In addition, as already indicated in Fig. 6.3, the optimum relay location for the DF relaying is found, in general, closer to the source than to the destination in order to suppress the detrimental effect of error propagation due to erroneous decoding at the relay. Further examples of the BER for the DF relaying over fast and slow Rayleigh fading channels are shown in Fig. 6.7 and Fig. 6.8. We can again observe that there exist geographical areas of the relay locations where the conventional receiver diversity outperforms the DF relaying for all SNR values. On the other hand, also it is observed that, for sufficiently large SNR values, the conventional receiver diversity outperforms the DF relaying for all relay locations considered. Furthermore, we observe from Fig. 6.2 and Fig. 6.5–Fig. 6.8

that, particularly for higher order modulations and the DF relaying, System I does not achieve the diversity order of System II.

6.4.1 Optimum Relay Locations

The performance results in Fig. 6.1–Fig. 6.8 indicate that the relay location significantly affects the BER performance of System I with the cooperative diversity. We determine the optimum relay locations in the sense that System I with the cooperative diversity outperforms System II with the receiver diversity. In particular, we evaluate the PEP differences,

$$\Delta\text{PEP}_{\text{Rx-AF}} = \text{PEP}_{\text{Rx}} - \text{PEP}_{\text{AF}} \tag{6.6a}$$

$$\Delta\text{PEP}_{\text{Rx-DF}} = \text{PEP}_{\text{Rx}} - \text{PEP}_{\text{DF}}. \tag{6.6b}$$

Hence, if $\Delta\text{PEP}_{\text{Rx-AF}} > 0$ or $\Delta\text{PEP}_{\text{Rx-DF}} > 0$, then the cooperative diversity with the AF or the DF relaying, respectively, outperforms the second order receiver diversity. The relay positions for which the PEP differences (6.6a) and (6.6b) are greater than zero are obtained numerically by sampling the two-dimensional space of all possible relay locations. Examples of the PEP differences (6.6a) and (6.6b) versus the relay locations $(d_{\text{SR}}/d_{\text{SD}}, d_{\text{RD}}/d_{\text{SD}}, d_{\text{SD}})$ for the SNR $\gamma_b = 9$dB are shown in Fig. 6.9 and Fig. 6.10, respectively. More importantly, if the SNR exceeds a certain threshold value, then, for any relay location, the PEP differences (6.6a) and (6.6b) will always be negative, i.e., the receiver diversity will outperform the cooperative diversity.

In general, determination of the exact boundaries of the geographical areas of the relay locations where System I outperforms System II appears to be mathematically intractable, particularly, when the channel coding is employed. However, by evaluation of our extensive numerical results including those that are not presented in this chapter, we make the following proposition.

Proposition 1 *Assuming path-loss attenuations of the transmitted signals and independent channel fadings between the transmitter and the receiver antennas, the cooperative diversity with a single relay outperforms the two antenna receiver diversity*

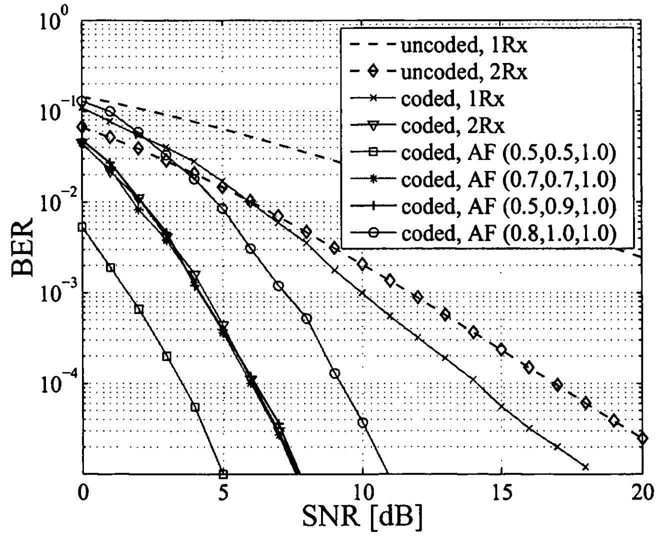

Figure 6.5: The BER of the BCH $(31,16,7)$ coded BPSK and the AF relaying and the receiver diversity with the MRC at the destination for several relay locations for fast Rayleigh fading channels.

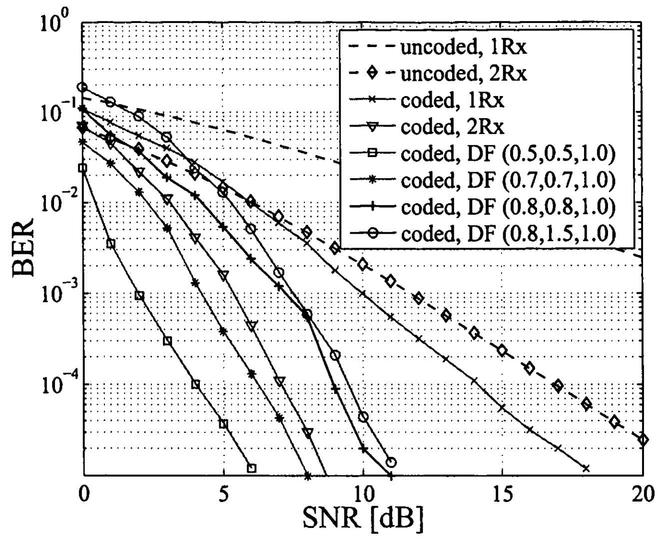

Figure 6.6: The BER of the BCH $(31,16,7)$ coded BPSK and the DF relaying and the receiver diversity with the MRC at the destination for several relay locations for fast Rayleigh fading channels.

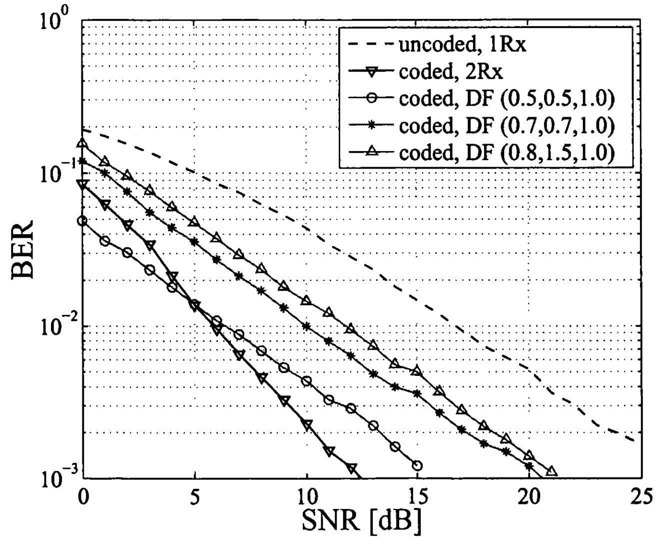

Figure 6.7: The BER of the BCH $(32, 16, 8)$ coded 16QAM and the DF relaying and the receiver diversity with the EGC at the destination for several relay locations for fast Rayleigh fading channels.

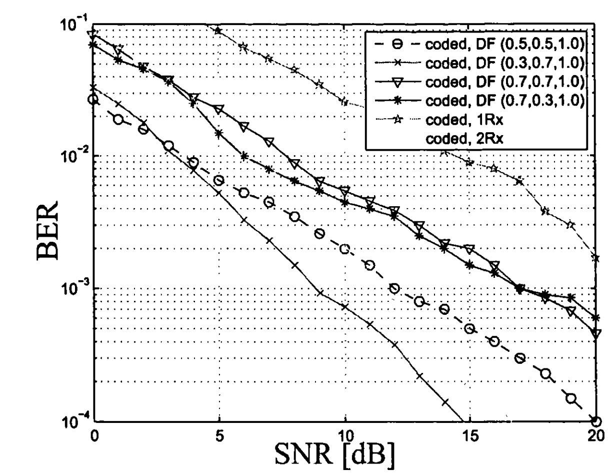

Figure 6.8: The BER of the BCH $(31, 16, 7)$ coded BPSK and the DF relaying and the receiver diversity with the EGC at the destination for several relay locations for slow Rayleigh fading channels.

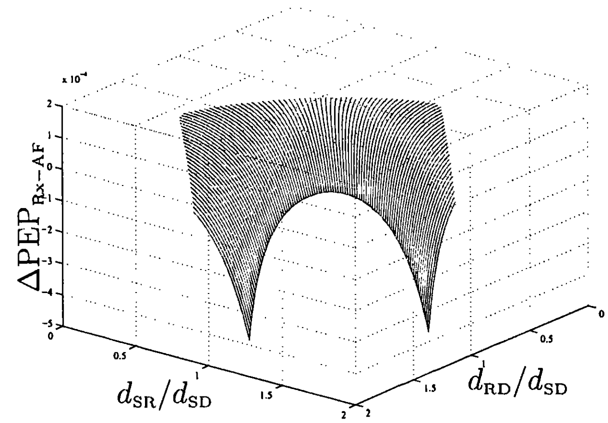

Figure 6.9: The PEPs difference $\Delta\text{PEP}_{\text{Rx-AF}}$ of System II and System I with the AF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination for the SNR $\gamma_b = 9$dB.

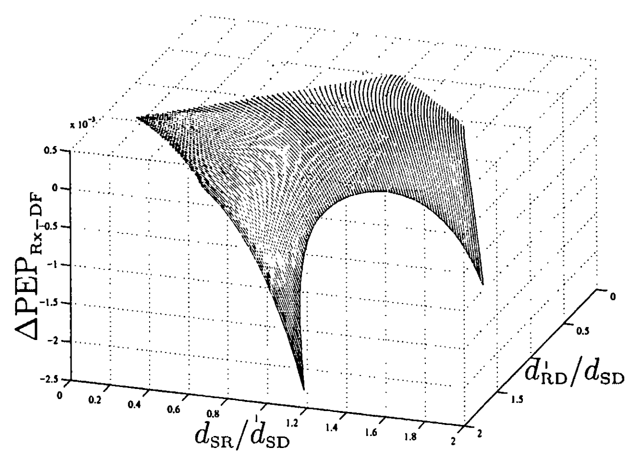

Figure 6.10: The PEPs difference $\Delta\text{PEP}_{\text{Rx-AF}}$ of System II and System I with the DF relaying and the BCH $(31, 16, 7)$ coded BPSK signaling over slowly Rayleigh fading channels, and the EGC at the destination for the SNR $\gamma_b = 9$dB.

provided that the relay location $(d_{\rm SR}/d_{\rm SD}, d_{\rm RD}/d_{\rm SD}, 1)$ *is constrained as,*

$$\begin{aligned} d_{\rm SR}/d_{\rm SD} &< 1.0 \\ d_{\rm RD}/d_{\rm SD} &< 1.0 \\ d_{\rm SR}/d_{\rm SD} + d_{\rm RD}/d_{\rm SD} &< \mathcal{A}_{\gamma,C} \end{aligned}$$

where the parameter $\mathcal{A}_{\gamma,C} > 0$ *upper-bounding the path-length from the source to the destination via the relay is a decreasing function of the SNR and a function of the channel coding C. Specifically, for small to medium SNR values and the path-loss exponent* $\mu = 2$, *and binary linear block codes of* $d_{\min} < 10$, $\mathcal{A}_{\gamma,C} \approx 2.0$ *for the AF relaying, and* $\mathcal{A}_{\gamma,C} \approx 1.5$ *for the DF relaying. In addition, for sufficiently large SNR or when the path-loss attenuations are not considered, the parameter* $\mathcal{A}_{\gamma,C} < 1.0$ *and the receiver diversity always outperforms the cooperative diversity.*

Note that Proposition 1 implicitly assumes the triangle inequality constraint, $d_{\rm SR}/d_{\rm SD} + d_{\rm RD}/d_{\rm SD} \geq 1.0$. Thus, if the parameter $\mathcal{A}_{\gamma,C}$ becomes smaller than 1, then, for no relay location can the cooperative diversity outperform the receiver diversity. A sub-optimum decoding scheme that is used in our numerical examples, and subsequently, used to formulate Proposition 1 appears to influence the threshold SNR value when the parameter $\mathcal{A}_{\gamma,C}$ becomes smaller than 1.0. Finally, it is straightforward to show that if the path-loss attenuations are not considered, then the receiver with K independent receiver antennas will always outperform a cooperative system with $(K-1)$ relays.

6.5 Conclusions

The transmission reliabilities of System I with the cooperative diversity and System II with the receiver diversity were investigated. Both systems can theoretically achieve the maximum diversity order of two. However, particularly the performance of System I suffers from the error propagation due to signal processing at the relay. Path-loss attenuations of the transmitted signals, independence of the channel fading coefficients and the use of channel coding with non-binary linear modulations were the main assumptions adopted in the system modeling. At the destination receiver, the diversity signals were combined using either MRC or EGC. A low-complexity soft-decision

POSD was extended for the decoding of binary linear block codes used with non-binary modulations.

The PEP was investigated as the key performance measure of the system transmission reliability. In particular, assuming channel coding and BPSK signaling, the PEP expressions were derived analytically for System I as well as for System II. The obtained PEP expressions were verified by computer simulations. The performance of System I was found to be strongly dependent on the relay location as expected. More importantly, it was found that, for some relay locations and SNR values, System I with the cooperative diversity may outperform System II with the receiver diversity. The approximate boundaries of such geographical areas of relay locations when System I outperforms System II were formulated in Proposition 1 using both the obtained mathematical analysis of the PEPs as well as using extensive computer simulations. The DF relaying was found to be more sensitive to and more restrictive about the relay location than the AF relaying. More importantly, if the path-loss attenuations are not considered, then the receiver diversity always outperform the cooperative diversity. These results have significant implications for the deployment and design of the current cellular systems supporting both the receiver as well as cooperative diversity.

7

Decode-and-Forward Cooperation as the Distributed Encoding and Decoding

7.1 Introduction

In this chapter, the two problems of the DF cooperation are investigated assuming simple-to-implement encoding and decoding techniques. The simplicity of the encoding and decoding is important in applications where the cost of implementation cannot be neglected (e.g., in large scale deployments), or where the implementation complexity is limited by other factors such as high data rate or small latency. On the other hand, the simple encoding and decoding techniques can rarely approach the channel capacity, and thus, the asymptotic mathematical theoretic analysis must be often replaced by computer simulations.

In the first problem, the main objective is to investigate how to distribute the (overall) decoding complexity of the OSD between the relay and the destination. The decoding complexity of the OSD can be measured as the total number of the test error patterns (TEPs) searched in the decoding process.

In the second problem, the relay and the destination use the OSD with the same parameters, however, the channel encodings at the source and at the relay are different.

The main objective of this problem is to investigate how to distribute the channel code rates and the minimum Hamming distances of the codes between the source and the relay since the code rate can be used as a measure of the encoding complexity. Note also that, in general, the modulation constellations used at the source and at the relay can be different, however, this case and the associated distribution of the implementation complexity are not considered in this chapter.

The rest of this chapter is organized as follows. The system model is adopted from Chapter 6 and is briefly described in Section 7.2. The distributed decoding operations and its pefromance for the first problem considered is also described in Section 7.3. The distributed encoding operations and its pefromance for the second problem considered is also described in Section 7.4. Conclusions are given in Section 7.5.

7.2 System Model

We assume transmission from a source S to a destination D using a single relay R. All nodes have one transmitting and receiving antenna, and the transmissions assume simple-to-implement channel encoding and decoding techniques. The propagation channels between nodes are impaired by path-loss due to physical separation of the transmitter and receiver antennas, mutually independent flat fadings due to scattering of the radio waves and due to ideal interleaving and deinterleaving at the transmitter and at the receiver, respectively, and additive white Gaussian noises (AWGNs) due to the receiver front-end stages. The following notation is used to describe the propagation channels from the node $\mathrm{X} \in \{\mathrm{S}, \mathrm{R}\}$ to the node $\mathrm{Y} \in \{\mathrm{R}, \mathrm{D}\}$, i.e.,

$$\begin{aligned} d_{\mathrm{XY}} &> 0 \quad \text{distance between X and Y} \\ \left(\frac{d_{\mathrm{XY}}}{d_{\mathrm{ref}}}\right)^{-\mu} &> 0 \quad \text{path-loss} \end{aligned}$$

where $\mu > 0$ is the path-loss exponent, and d_{ref} is the reference distance having the path-loss equal to unity. Note that the values $d_{\mathrm{XY}} < d_{\mathrm{ref}}$ can cause the path-loss to become amplification rather than attenuation. However, a given choice of d_{ref} shifts *all* the BER curves by the same signal-to-noise ratio (SNR) proportional to $10 \log_{10} d_{\mathrm{ref}}$ (in dB), so that the BER performance comparison of different systems remains unbiased. More importantly, as shown in Chapter 6, the relay location as well as normalization of the transmitter powers, and thus, the SNRs of each link between a pair of nodes have significant impact on the BER performance of different relaying schemes. In addition, one should consider a rate loss due to a two-time slot relaying since the source remains silent every second time slot, and also, we have to consider the loss of average energy per bit due to the parity bits of the channel coding. More detailed description of the system model investigated in this chapter is given in Chapter 6.

7.3 Distributed Decoding

The source and the relay employ the same channel encoder, however, the type of the channel decoder used at the relay and at the destination is either different or the same but its parameters may be different. We consider the order statistics decoding (OSD) developed in [2] and refined in [84] and in Chapter 4 for its inherent flexibility to trade-off the decoding complexity and the bit error rate (BER) performance. In general, the OSD is a list based decoding denoted as $\mathrm{OSD}(I_1, I_2, \cdots, I_Q)$ where Q is the number of non-overlapping segments of the most reliable information positions (MRIPs), and I_i are the maximum number of errors searched within the i-th segment. The efficient list generation for the case of $Q = 2$ segments is presented in [84].

The information bits at the source are encoded using a binary linear channel code (N_1, K_1, d_{min1}) where N_1 is the block length, K_1 is the code dimension (i.e., the number of information bits), and d_{min1} denotes the minimum Hamming distance of this code. The encoded bits are interleaved and mapped to modulation symbols that are broadcasted to the relay and to the destination. In the next time slot, the source stops the transmission, and the relay attempts to correct the transmission errors in the received codeword before forwarding this codeword to the destination. At the destination, the two copies of the codeword of the channel code (N_1, K_1, d_{min1}) received from the source and from the relay are coherently combined using the equal gain combining (EGC). For comparison, we also consider the case of direct transmission from the source to the destination without relaying, but employing the EGC of the uncorrelated dual branch receiver diversity. Note that, in both cases, the EGC corresponds to the sub-optimum decoding of the binary repetition code $(2, 1, 2)$.

7.3.1 Numerical Results

Extensive simulation results for decoding distribution are shown in the this section. Assuming binary linear block coding of short block length(N<32), binary modulation, fast Rayleigh fading, various relay locations modeled as the propagation path-loss, and the order statistics decoding is adopted due to its excellent flexibility to trade-off the decoding complexity and the bit error rate performance. The distances between nodes are described as triplets $(d_{\mathrm{SR}}/d_{\mathrm{ref}}, d_{\mathrm{RD}}/d_{\mathrm{ref}}, d_{\mathrm{SD}}/d_{\mathrm{ref}})$, i.e., the triplet $(1, 1, 1)$ represents the channels without the path-loss. If the link between the two nodes is not considered, then the corresponding value in the triplet is replaced with a symbol $\times$. The destination has one receiver antenna unless indicated otherwise. The OSDs considered have one or two segments. The BER values are denoted as $\mathrm{P_e}(\gamma_b)$ where γ_b is the average SNR per transmitted bit. The coding and diversity gains given in the examples assume the target BER 10^{-3}.

Numerical examples in Fig. 7.1–Fig. 7.4 are provided for the case of the systematic BCH code $(128, 64, 22)$ [70] in order to investigate the first problem formulated in Section 7.1. Fig. 7.1 compares the BER of the coded and uncoded BPSK over uncorrelated Rayleigh fading channels without the path-loss. Either OSD(1) or OSD(2) is used at the relay and at the destination. It is observed that the OSD(2) decoding at the destination with one receiver antenna outperforms the cooperative relaying employing OSD(1) at the relay and at the destination.

The BER curves in Fig. 7.2 indicate that, for the same path-loss factors, the OSD(1) decoding at the relay and the OSD(1) decoding at the destination outperforms the OSD(2) decoding at the destination with one receiving antenna by 2 dB, and provides the gain of 5.5 dB over an uncoded transmission without relaying and with two receiver antennas at the destination. Furthermore, the OSD(2) decoding at the destination with two receiver antennas and $d_{\mathrm{SD}}/d_{\mathrm{SR}} = 1.3$ outperforms by 2 dB the cooperative relaying with OSD(1) at the relay and at the destination. For the same path-loss assumptions, when there is no direct link between the source and the destination, the OSD(1) decodings at the relay and at the destination outperform by 2 dB the OSD(2) decoding at the destination with one receiving antenna without relay.

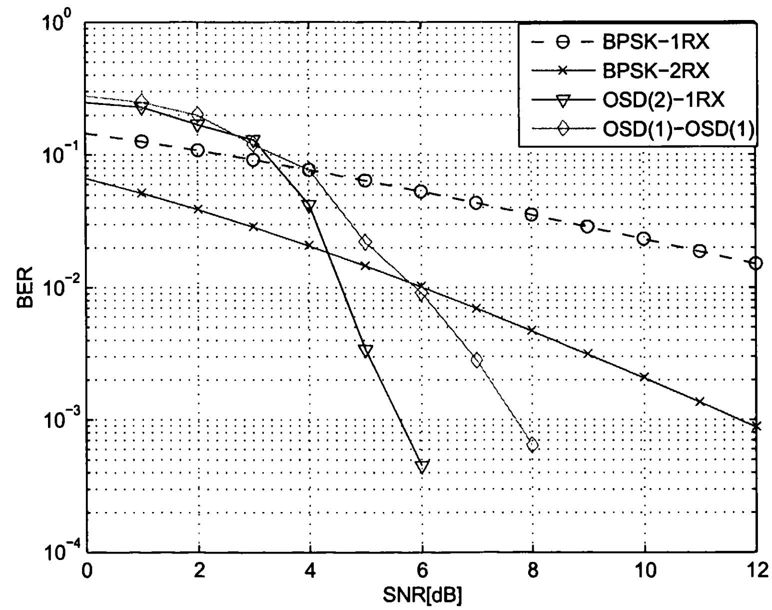

Figure 7.1: Examples of the BER of coded and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with-out the path-loss.

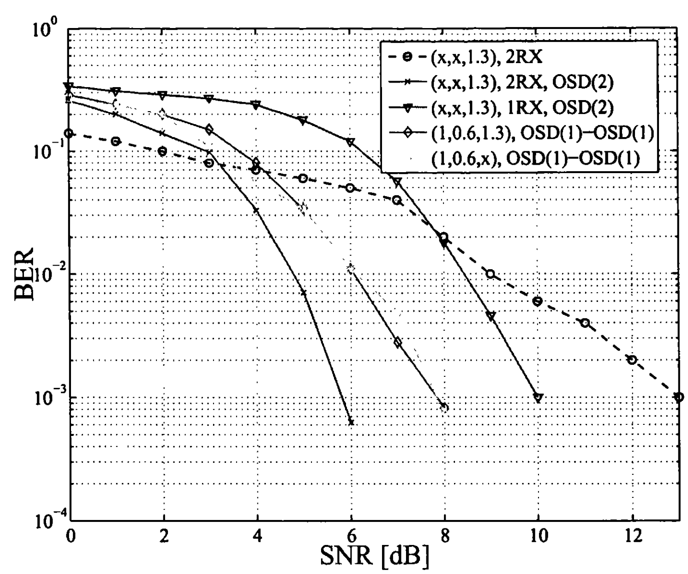

Figure 7.2: Examples of the BER of coded and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings.

The BER results in Fig. 7.3 assume that the relay is closer to the destination than to the source whereas the simulation results in Fig. 7.1 and Fig. 7.2 assume that the relay is closer to the source. The OSD(1) decoding at the relay and OSD(1) at the destination outperforms the OSD(2) decoding at the destination with one receiving antenna by 2dB and achieve the gain of 8 dB over the uncoded transmission with the same path-loss values even when employing two receiver antennas at the destination. Furthermore, OSD(2) with two receiver antennas at the destination and $d_{\mathrm{SD}}/d_{\mathrm{SR}} = 2.0$ outperforms by 2 dB the cooperative relaying with OSD(1) at the relay and at the destination. More generally, for the same path-loss assumptions, OSD(1) at the relay and at the destination, however, without the direct link between the source and the destination, outperforms OSD(2) at the destination with one receiving antenna by 2 dB. Similarly, we observe from Fig. 7.4 that using OSD(2) at the relay and OSD(1) at the destination gives 2 dB gain over OSD(1) at the relay and OSD(2) at the destination.

Table 7.1 summarizes the achievable gains over the uncoded BPSK over an Rayleigh fading channel without diversity for different system scenarios at the target BER of 10^{-3}. For all scenarios in this table, the diversity combining scheme that used at the destination is EGC. From the results presented in this chapter as well as other simulation results obtained, we can draw the following conclusions. First, approaching the maximum likelihood (ML) performance at the relay is more important than approaching the ML performance at the destination provided that the relay is closer to the destination. Second, using OSD(2, 2) at the relay and OSD(1) at the destination gives the same BER performance as the OSD(2) at the destination with two receiving antennas without relay. Third, using OSD(2, 2) at the relay to decode the first segment only, and using OSD(2, 2) at the destination to decode the second segment only gives the same BER performance as employing OSD(1) at the destination.

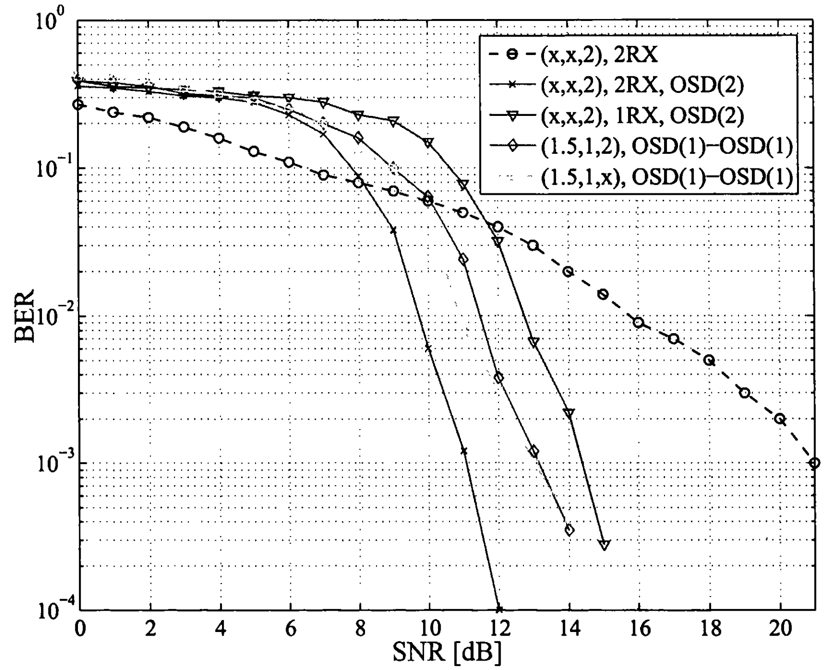

Figure 7.3: Examples of the BER of coded and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings.

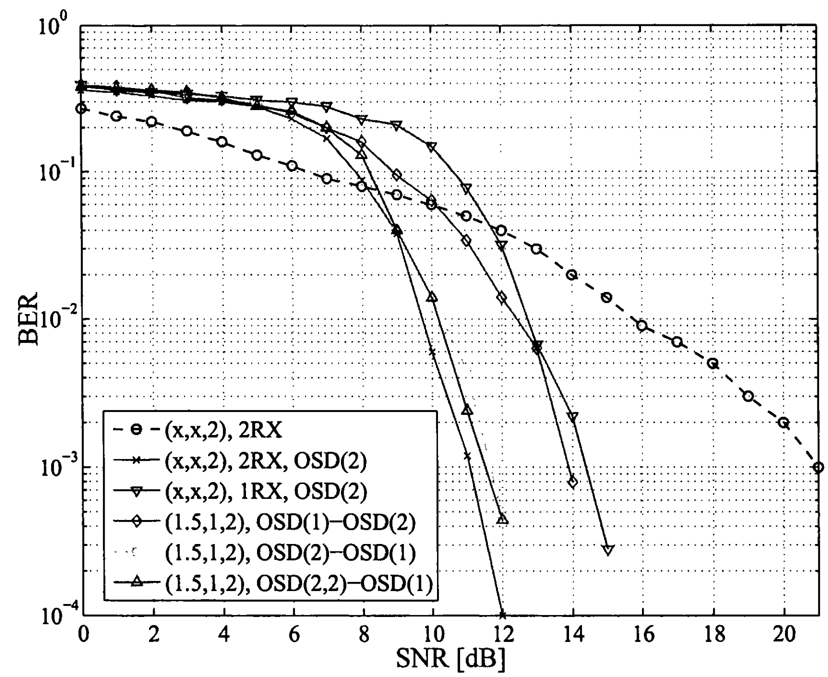

Figure 7.4: Examples of the BER of coded and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings.

Table 7.1: Distributed Decoding Gains for Different System Scenarios

decoding at relay	decoding at destination	d_{ref}	path-loss	direct link	antennas at destination	complexity (TEPs)	SNR [dB]
x	OSD(2)	d_{SR}	(x, x, 1)	yes	1	2081	5.5
OSD(1)	OSD(1)	d_{SR}	(1, 1, 1)	yes	1	130	7.3
OSD(1)	OSD(1)	d_{SR}	(1, 0.6, 1.3)	yes	1	130	7.3
x	OSD(2)	d_{SR}	(x, x, 1.3)	yes	1	2081	10.0
OSD(1)	OSD(1)	d_{SR}	(1, 0.6, x)	no	1	130	7.3
x	OSD(2)	d_{SR}	(x, x, 1.3)	yes	2	2081	5.5
x	x	d_{SR}	(1, 0.6, 1.3)	yes	2	1	13.0
OSD(1)	OSD(1)	d_{RD}	(1.5, 1, 2)	yes	1	130	13.0
x	OSD(2)	d_{RD}	(1.5, 1, 2)	yes	1	2081	15.3
OSD(1)	OSD(2)	d_{RD}	(1.5, 1, x)	no	1	2146	13.3
x	x	d_{RD}	(1.5, 1, 2)	yes	2	1	21.0
x	OSD(2)	d_{RD}	(x, x, 2)	yes	2	2081	11.0
OSD(1)	OSD(1, 2)	d_{RD}	(1.5, 1, 2)	yes	1	927	13.0
OSD(2, 2)	OSD(1)	d_{RD}	(1.5, 1, 2)	yes	1	1158	11.0
OSD(1)	OSD(2)	d_{RD}	(1.5, 1, 2)	yes	1	2146	13.0
OSD(2)	OSD(1)	d_{RD}	(1.5, 1, 2)	yes	1	2164	11.3

7.4 Distributed Encoding

The objective in this section is to distribute the encoding complexity between the source and the relay while using the same channel decoding at the relay and at the destination. Thus, the binary product codes are well-suited for distributed implementation in relay networks where each node probably uses different channel encoding to generate the parity bits. For our case of a single relay, the source broadcasts several codewords K_2 of a linear binary block code (N_1, K_1, d_{min1}) to the relay and to the destination. The relay attempts to correct the transmission errors in each of the received codewords. The decoding process in relay is a conventional OSD or OSD based on segmentation. The relay then generates the new parity bits by further encoding these codewords using a linear binary block code (N_2, K_2, d_{min2}) in order to create a codeword of a binary product code $(N_1, K_1, d_{min1}) \times (N_2, K_2, d_{min2})$. The parity bits generated at the source (at the relay) correspond to the horizontal (vertical) encoding of the binary product code. The relay sends the newly created parity bits towards the destination which performs the decoding of the received codewords of the binary product code. Hence, at the relay, the parity bits of the binary repetition code $(2, 1, 2)$ corresponding to the conventional DF relaying are now replaced by the parity bits of another channel code. The block diagram of such system is shown in Fig. 7.5

The decoding of the product code at the destination is obtained using the OSD, and, assuming binary phase shift keying (BPSK) modulation, the full decoding is carried out in one iteration only. In particular, the vertical codewords of the code (N_2, K_2, d_{min2}) are decoded first using the OSD. The decoded information bits are re-modulated using the BPSK modulation. Since these modulated symbols are proportional to the a priori values of information bits, these modulated symbols are added to the received BPSK symbols from the source representing the received reliabilities and corresponding to the channel code (N_1, K_1, d_{min1}).

In the following sub-sections, we study different parameters that play importnat role in designing such systems. These parameters include d_{min1}, d_{min2} and R_1, R_2, the relay position and the complexity of decoding at the relay. The results in the following subsection show the effects of such parameters where the path-loss is not considered.

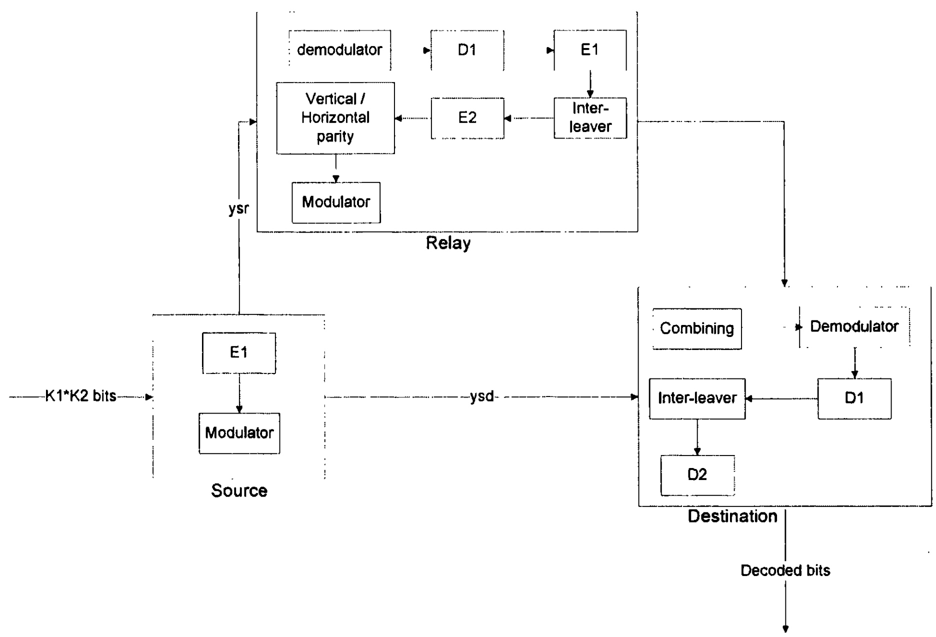

Figure 7.5: System model for distributed encoding.

7.4.1 Path-loss Not Considered

Examples of the simulation results to address the second problem formulated in Introduction are presented in Fig. 7.6 and Fig. 7.7. For simplicity, the path-loss is not considered, and the OSD(2) decoding is employed at the relay as well as at the destination. The distributed product code is denoted as $(N_1, K_1, d_{min1}) \times (N_2, K_2, d_{min2})$. We observe that, when the minimum Hamming distance of the constituent codes $d_{min1} = d_{min2}$, the BER performance is almost independent whether the higher rate encoding is used at the relay or at the destination. However, it is preferable to use the encoding with the smaller code rate $R_2 = K_2/N_2$ at the relay and use the encoding with the larger code rate $R_1 = K_1/N_1$ at the source in order to reduce the number of parity bits transmitted from the relay to the destination, and thus, to improve the overall relaying efficiency. Furthermore, from Fig. 7.7, we can conclude that using the code (N_1, K_1, d_{min1}) at the source with the minimum Hamming distance d_{min1} that is larger than the minimum Hamming distance d_{min2} of the code (N_2, K_2, d_{min2}) used at the relay is more efficient and also improves the BER performance. For example, the product code $(15, 5, 7) \times (31, 26, 3)$ outperforms the product code $(31, 26, 3) \times (15, 5, 7)$ by 4

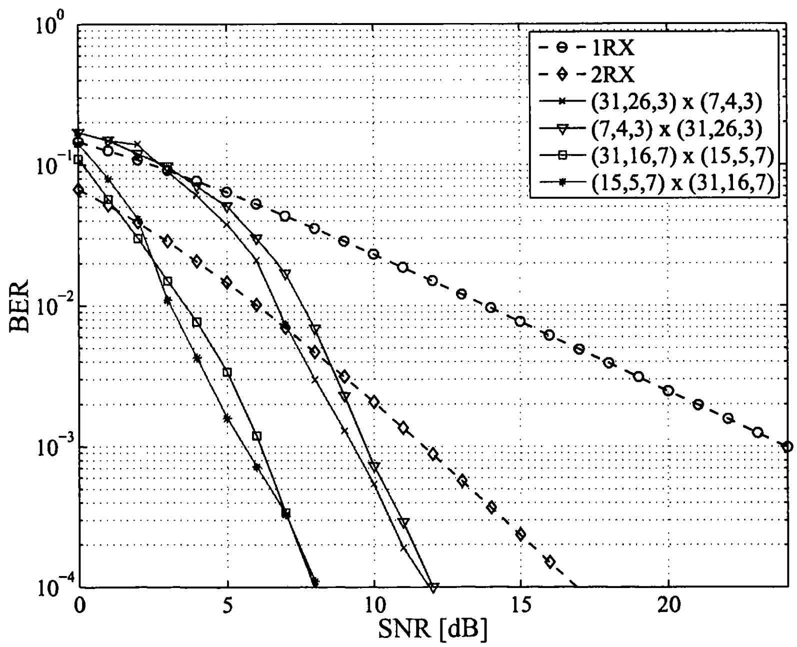

Figure 7.6: Examples of the BER of distributed product block coded and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings without the path-loss.

dB at the BER of 10^{-4}. In addition, we can again observe that the choice of the code rates at the source and at the relay is less important than the choice of the hamming distances for improving the BER performance, so it is preferable to choose $R_2 < R_1$ in order to improve the relaying efficiency. For example, when $d_{min1} = d_{min2}$, the SNR difference between $R_1 \ll R_2$ and $R_1 \gg R_2$ to obtain the BER of 10^{-4} is less than 1 dB.

For the case with the path-loss, the distributed encoding process is studied assuming three scenarios according to the relay position. These scenarios and the corresponding results are described in the following three sub-sections.

7.4.2 Relay in the Center

Recall that the distances between nodes are described as triplets $(d_{\text{SR}}/d_{\text{ref}}, d_{\text{RD}}/d_{\text{ref}}, d_{\text{SD}}/d_{\text{ref}})$. Thus, for this case the triplet $(0.5, 0.5, 1)$ represents the channels path-loss where d_{ref} corresponds to d_{SD}. Numerical examples in Fig. 7.8 are provided for the case of the

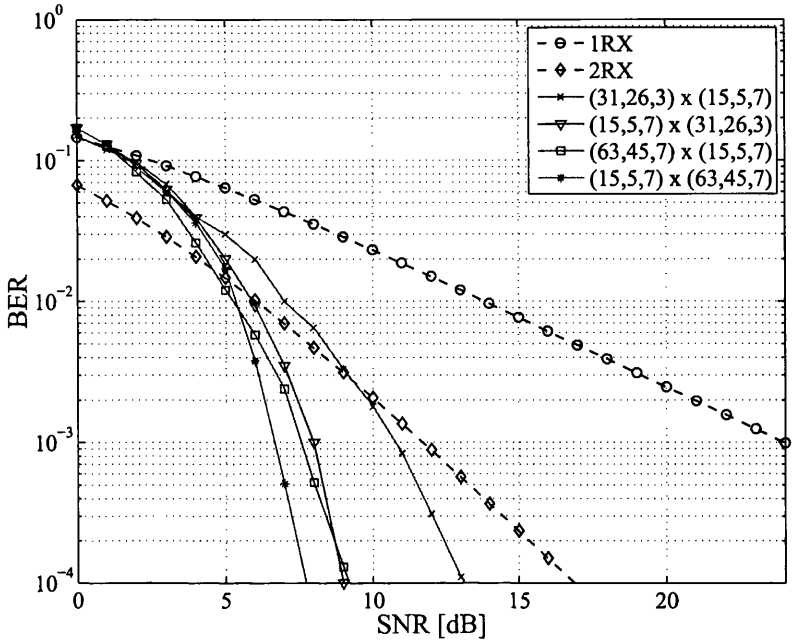

Figure 7.7: Examples of the BER of distributed product block coded and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings without the path-loss.

systematic BPC of $(63,45,7) \times (15,5,7)$ and $(15,5,7) \times (63,45,7)$ in order to study the effect of R_1 and R_2 assuming $d_{min1} = d_{min2}$. We use the OSD(2) decoding at the relay with one antenna. The BER curves in Fig. 7.8 indicate that, for the same path-loss factors, utilizing C_1 as the first sub-code of BPC at the source and C_2 is the second sub-code at the relay with $R_1 > R_2$and the OSD(2) decoding at the destination provides the gain of 6dB over the case where $R_1 < R_2$. Furthermore, in both scenarios, whether $R_1 < R_2$ or vice versa for the distributed encoding algorithm provides much larger coding gains over an uncoded transmission without relaying and with two receiver antennas at the destination. The explanation for such large difference of the performance for these two scenarios is that sending more reliable parity bits from the relay is more effective than sending more parity bits from the source.

The BER curves in Fig. 7.9 are shown for the case of a systematic BPC of $(32,16,8) \times (8,4,4)$ and $(8,4,4) \times (32,16,8)$ in order to study the effects of d_{min1} and d_{min2} for $R_1 = R_2$. The OSD(2) decoding is assumed at the relay with one antenna. The BER curves in Fig. 7.9 indicate that, for the same path-loss factors, utilizing C_1 as a

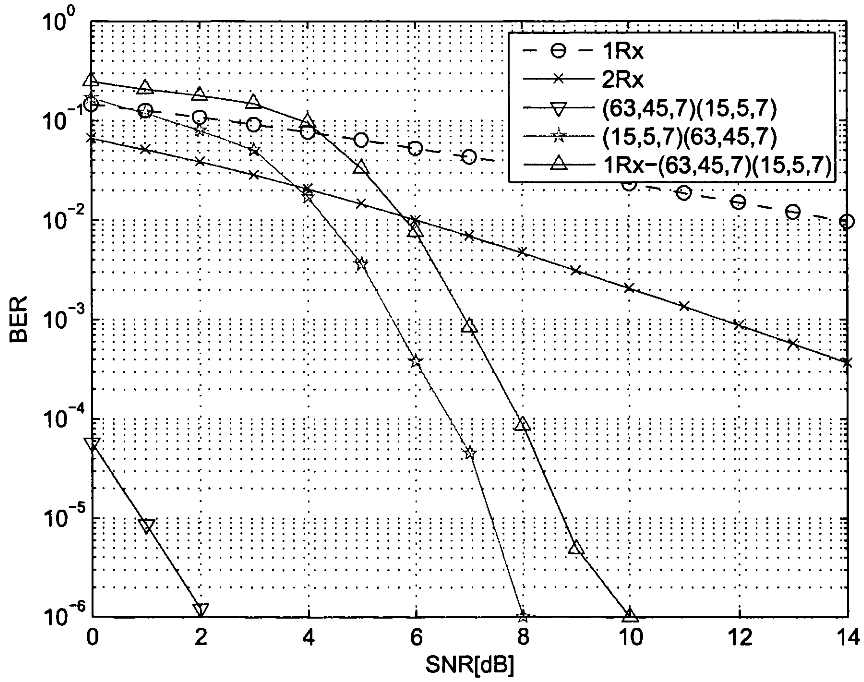

Figure 7.8: The BER of distributed product block coded and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.5,0.5,1).

sub-code at the source and C_2 at the relay with $d_{min1} > d_{min2}$ and the OSD(2) decoding at the destination provides the gain of 3dB over the case when $d_{min1} < d_{min2}$. Furthermore, in both scenarios, whether $d_{min1} > d_{min2}$ or vice versa for distributed encoding algorithm provides much larger coding gain over an uncoded transmission without relaying and with two receiver antennas at the destination. The explanation for this difference between the performances of these two scenarios for $d_{min1} > d_{min2}$ or the is that using stronger sub-code C_1 with larger d_{min1} from the source is more efficient to reduce the errors and the erros propagation in the SR link and the SD direct link. Moreover, the number of the reliable parity bits that the relay sends is the same whether $d_{min1} > d_{min2}$ and vice versa.

The BER curves in Fig. 7.10 are provided for the case of the systematic BPC $(32, 26, 4) \times (32, 16, 8)$ and $(63, 45, 7) \times (32, 26, 4)$ in order to investigate whether d_{min1} and d_{min2} or R_1 and R_2 of C_1 and C_2 are more effective in distributed encoding process. The $(32, 26, 4) \times (63, 45, 7)$ code performs better than the $(63, 45, 7) \times (32, 26, 4)$

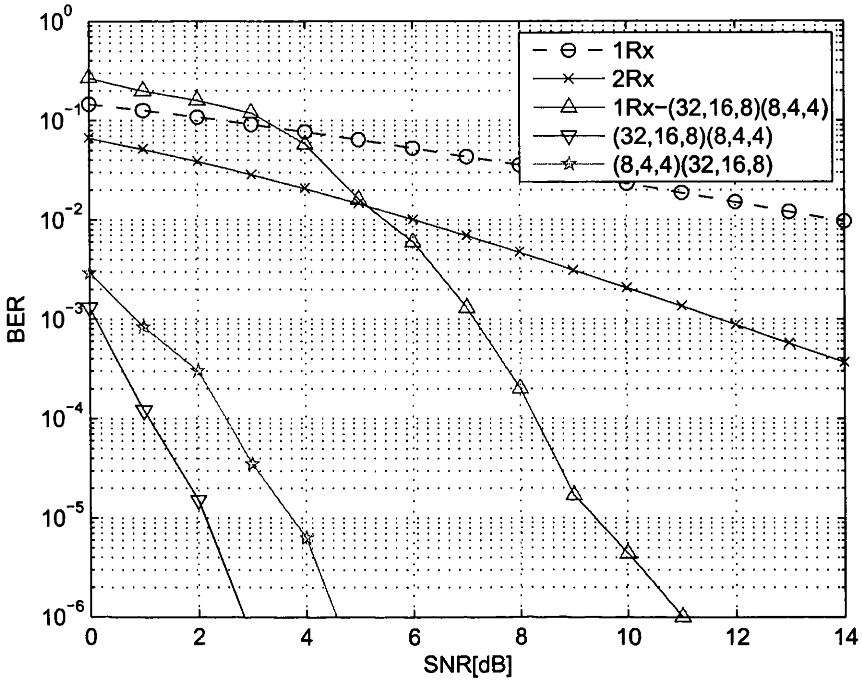

Figure 7.9: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.5,0.5,1).

code even though $d_{min1} < d_{min2}$; this indicated that code rates of sub-codes are more important than minimum hamming distance of the sub-codes. Furthermore, comparing $(32, 26, 4) \times (63, 45, 7)$ and $(32, 26, 4) \times (32, 16, 8)$ codes, it is shown that as the difference between the code rates of R_1 and R_2 increase, the difference of performances also increase. The main reason for this behaviour is that when $R_1 >> R_2$, the number of reliable parity bits that the relay sends is much larger than in the case when $R_1 << R_2$.

7.4.3 Relay Closer to the Source

Recall again that the distances between the nodes are described as triplets $(d_{\mathrm{SR}}/d_{\mathrm{ref}}, d_{\mathrm{RD}}/d_{\mathrm{ref}}, d_{\mathrm{SD}}/d_{\mathrm{ref}})$. In this subsection we assume the triplet $(0.3, 0.7, 1)$ representing the channels path-loss where d_{ref} is d_{SD}. Numerical examples in Fig. 7.11 are provided for the case of the systematic BPC $(63, 45, 7) \times (15, 5, 7)$ and $(15, 5, 7) \times (63, 45, 7)$ in order to study the effects of R_1 and R_2 when $d_{min1} = d_{min2}$. We use the

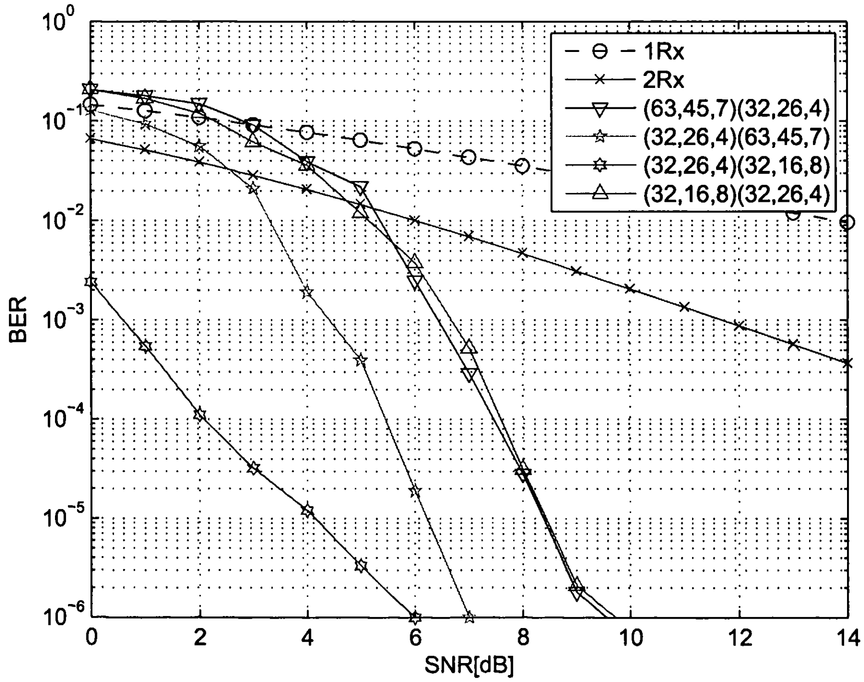

Figure 7.10: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.5,0.5,1).

OSD(2) decoding at the relay with one antenna. The BER curves in Fig. 7.11 indicate that, for the same path-loss factors, utilizing C_1 as sub-code at the source and C_2 at the relay with $R_1 > R_2$ and the OSD(2) decoding at the destination provides the gain of 6dB over the case where $R_1 < R_2$. Furthermore, in both scenarios, whether $R_1 < R_2$ or vice versa, the distributed encoding algorithm provides much larger coding gain over an uncoded transmission without relaying and with two receiver antennas at the destination. The explanation of such large difference of the performance of these two scenarios is that sending more reliable parity bits from the relay is more effective than sending more parity bits from the source.

The BER curves in Fig. 7.12 are provided for the case of the systematic BPC $(32, 16, 8) \times (8, 4, 4)$ and $(8, 4, 4) \times (32, 16, 8)$ in order to study the effects of d_{min1} and d_{min2} for $R_1 = R_2$. We employ the OSD(2) decoding at the relay with one antenna. The BER curves in Fig. 7.12 indicate that, for the same path-loss factors, utilizing C_1 as sub-code at the source and C_2 at the relay with $d_{min1} > d_{min2}$ and the OSD(2)

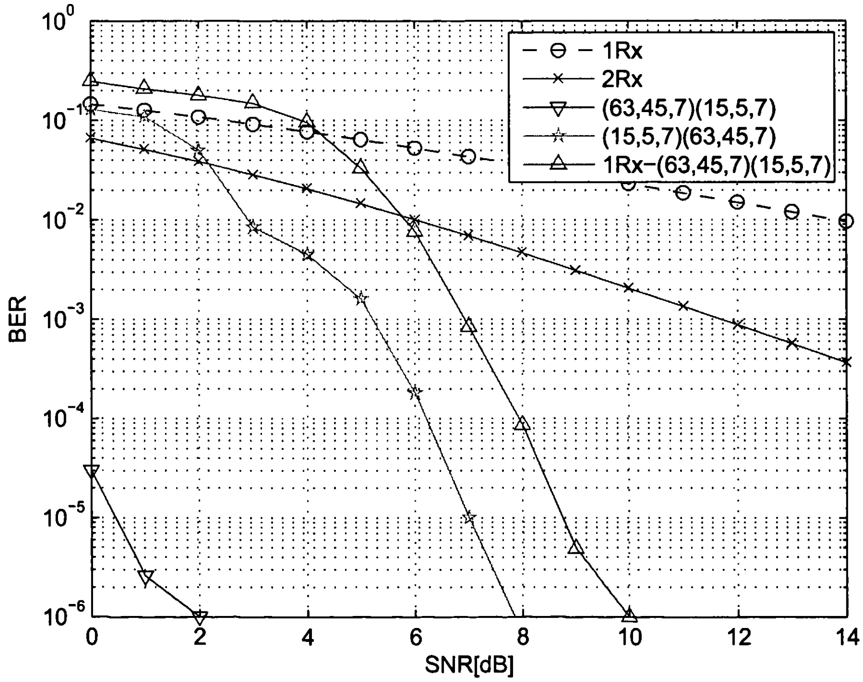

Figure 7.11: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.3,0.7,1).

decoding at the destination provides the gain of less than 1dB over the case where $d_{min1} < d_{min2}$. Furthermore, in both scenarios, whether $d_{min1} > d_{min2}$ or vice versa, the distributed encoding algorithm provides much larger coding gain over an uncoded transmission without relaying and with two receiver antennas at the destination. The explanation of such slight difference between the performance of these two scenarios where $R_1 = R_2$ and $d_{min1} > d_{min2}$ or the vice versa is that using stronger sub-code C_1 with larger d_{min} from the source is not so essential in the SR link since the relay is very close to the desination. Moreover, the number of the reliable parity bits that relay sends is the same whether $d_{min1} > d_{min2}$ or the vice versa. Thus the slight difference occurs because of using C_1 with larger d_{min} in the SD direct link.

The BER curves in Fig. 7.13 are provided for the case of the systematic BPC $(32, 26, 4) \times (32, 16, 8)$ and $(63, 45, 7) \times (32, 26, 4)$ in order to study whether d_{min1} and d_{min2} or R_1 and R_2 of C_1 and C_2 are more effective in the distributed encoding process. The $(32, 26, 4) \times (63, 45, 7)$ code outperforms the $(63, 45, 7) \times (32, 26, 4)$

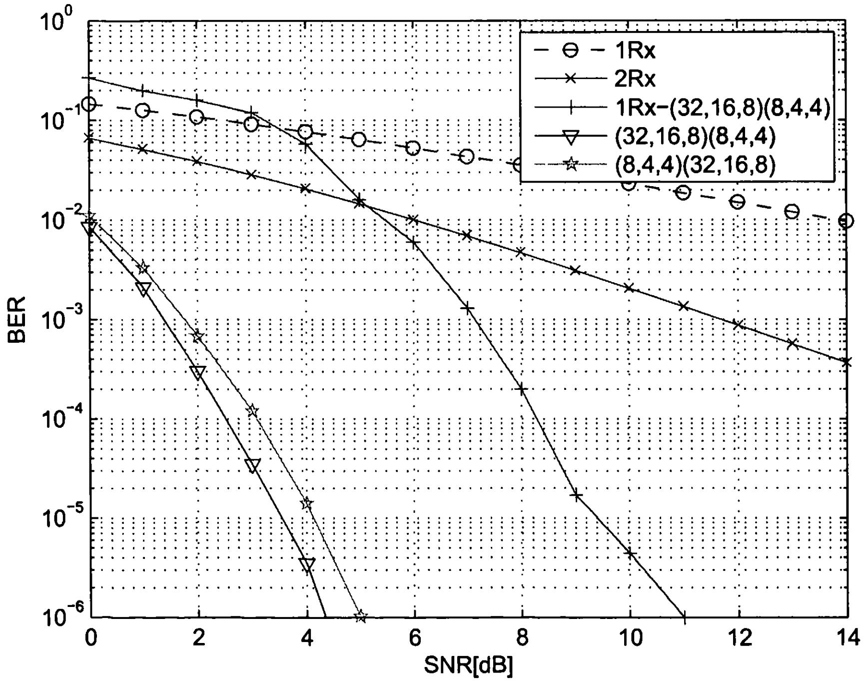

Figure 7.12: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.3,0.7,1).

code by about 5dB even though $d_{min1} < d_{min2}$. This indicates that the code rates of sub-codes are more important than minimum hamming distances of sub-codes even when the relay is closer to the source. Furthermore, comparing $(32, 26, 4) \times (63, 45, 7)$ and $(32, 26, 4) \times (32, 16, 8)$ codes, it is shown that as the difference between subcode rates R_1 and R_2 increase, the differences of performance increase and are almost the same when the code rates are equal. The main reason for this behaviour is that when $R_1 >> R_2$ the number of the reliable parity bits that the relay sends is much more than when $R_1 << R_2$.

7.4.4 Relay Closer to the Destination

Again the distances between nodes are described as triplets $(d_{\mathrm{SR}}/d_{\mathrm{ref}}, d_{\mathrm{RD}}/d_{\mathrm{ref}}, d_{\mathrm{SD}}/d_{\mathrm{ref}})$. In this case the triplet $(0.7, 0.3, 1)$ represents the channels path-loss where d_{ref} corresponds to d_{SD}. Numerical examples in Fig. 7.14 are provided for the case of systematic

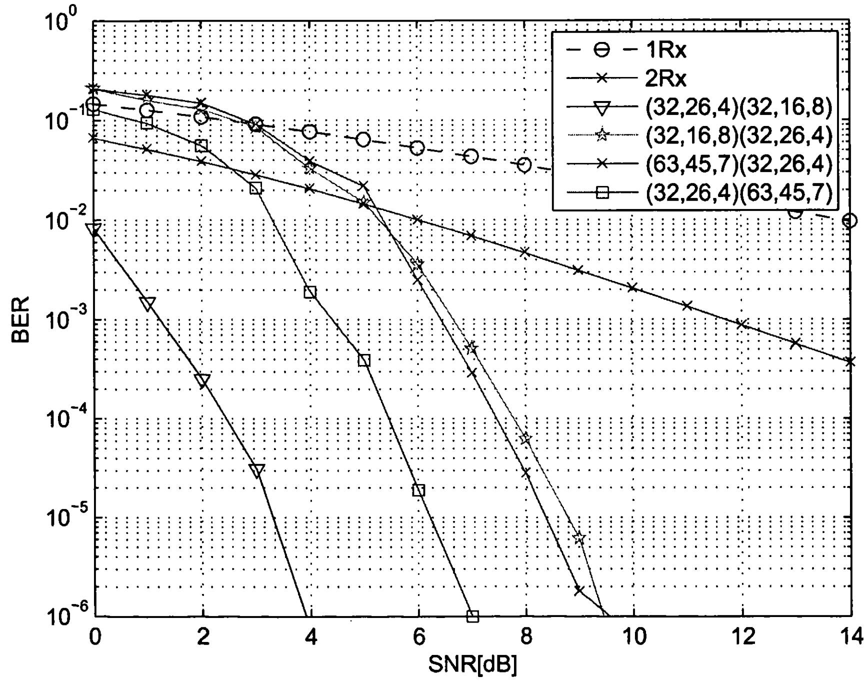

Figure 7.13: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.3,0.7,1).

BPCs of $(63,45,7) \times (15,5,7)$ and $(15,5,7) \times (63,45,7)$ in order to study the effects of R_1 and R_2 when $d_{min1} = d_{min2}$. We use the OSD(2) decoding at the relay with one antenna. The BER curves in Fig. 7.14 indicate that, for the same path-loss factors, utilizing C_1 as sub-code at the source and C_2 at the relay with $R_1 > R_2$ and the OSD(2) decoding at the destination provides the gain of 2dB over the case when $R_1 < R_2$. Furthermore, in both scenarios whether $R_1 < R_2$ or $R_1 > R_2$ for the distributed encoding algorithm provides much larger coding gain over an uncoded transmission without relaying and with two receiver antennas at the destination. We can conclude from these differences between the performance in these two scenarios is that sending more reliable parity bits from the relay is more effective than sending more parity bits from the source. Also it can be noticed that the differences in the performance because of the code rates are much smaller than in the case when relay is in the middle between the source and the destination. This can be explained that the reliability of parity bits that sent from the relay to the destination decreases because of the relay is closer to the

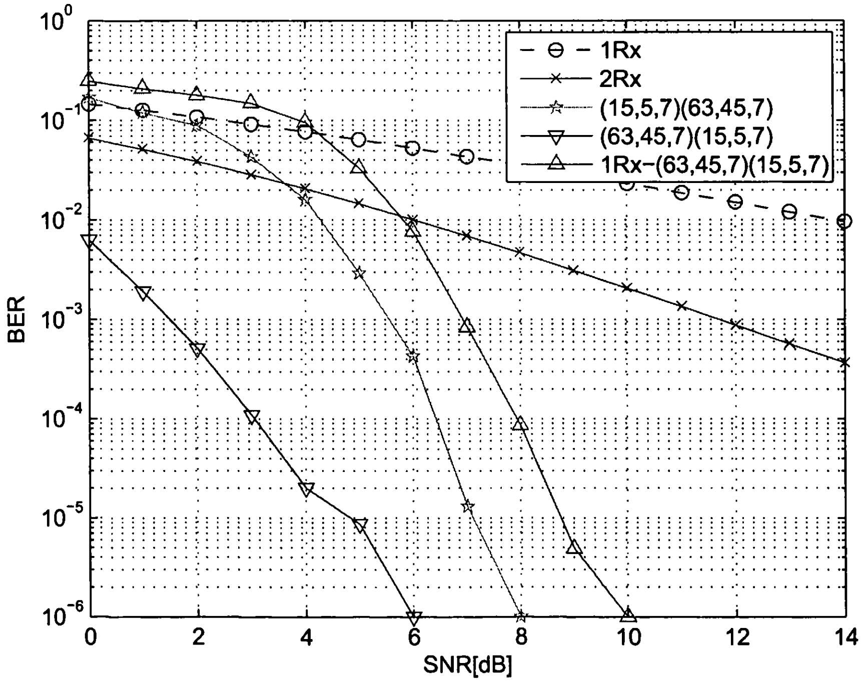

Figure 7.14: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.7,0.3,1).

destination. Thus, in case of $R_1 > R_2$, increasing the number of parity bits that are sent from the relay has minor effect to the performance compared to the other scenarios.

The BER curves in Fig. 7.15 are provided for the case of the systematic BPCs $(32, 16, 8) \times (8, 4, 4)$ and $(8, 4, 4) \times (32, 16, 8)$ in order to investigate the effects of d_{min1} and d_{min2} where $R_1 = R_2$. We employ the OSD(2) decoding at the relay with one antenna. The BER curves in Fig. 7.15 indicate that, for the same path-loss factors, utilizing C_1 as a sub-code at the source and C_2 at the relay with $d_{min1} > d_{min2}$ and the OSD(2) decoding at the destination provides the gain of 5dB over the case when $d_{min1} < d_{min2}$. Furthermore, both scenarios whether $d_{min1} > d_{min2}$ or vice versa for the distributed encoding algorithm provides much larger coding gain over an uncoded transmission without relaying and with two receiver antennas at the destination. The explanation of such difference between the performances of these two scenarios when $R_1 = R_2$ and $d_{min1} > d_{min2}$ or vice versa that using stronger sub-code C_1 with larger d_{min} from the source is more efficient to reduce errors and errors propogation in the

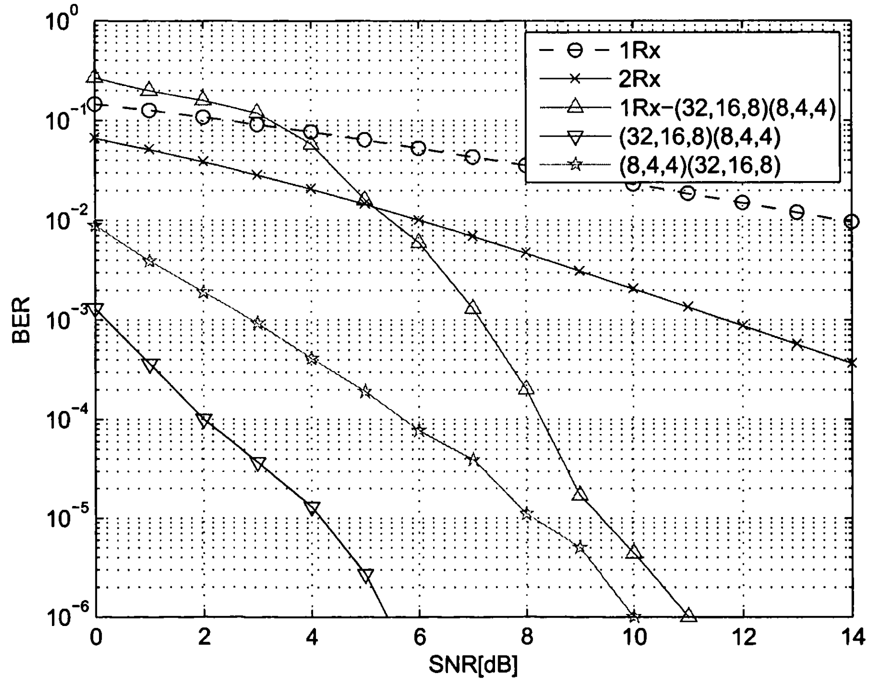

Figure 7.15: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.7,0.3,1).

SR link and the SD direct link . Moreover, the number of the reliable parity bits that relay send is the same whether $d_{min1} > d_{min2}$or vice versa.It should be noticed here that d_{min1} of C_1 is more effective here than other scenarios when the relay is in the center or closer to the destination. Also it is obvious that in case of $d_{min1} < d_{min2})$ the distributed encoding process is less efficient and the coding gain achieved over BPC $(32, 16, 8) \times (8, 4, 4)$ with 1-RX non cooperative scenario is just 1 dB.

The BER curves in Fig. 7.16 are provided for the case of the systematic BPCs $(32, 26, 4) \times (32, 16, 8)$ and $(63, 45, 7) \times (32, 26, 4)$ in order to study whether d_{min1} and d_{min2} or R_1 and R_2 are more effective in the distributed encoding process. The $(32, 26, 4) \times (63, 45, 7)$ codes performs better than$(63, 45, 7) \times (32, 26, 4)$ even though $d_{min1} < d_{min2}$, this indicates that code rates of sub-codes are more important than minimum hamming distance of sub-codes. Furthermore, comparing $(32, 26, 4) \times (63, 45, 7)$ and $(32, 26, 4) \times (32, 16, 8)$, it is shown that as the difference between code rates of R_1 and R_2 increases the performance increases. The main reason for this is that when

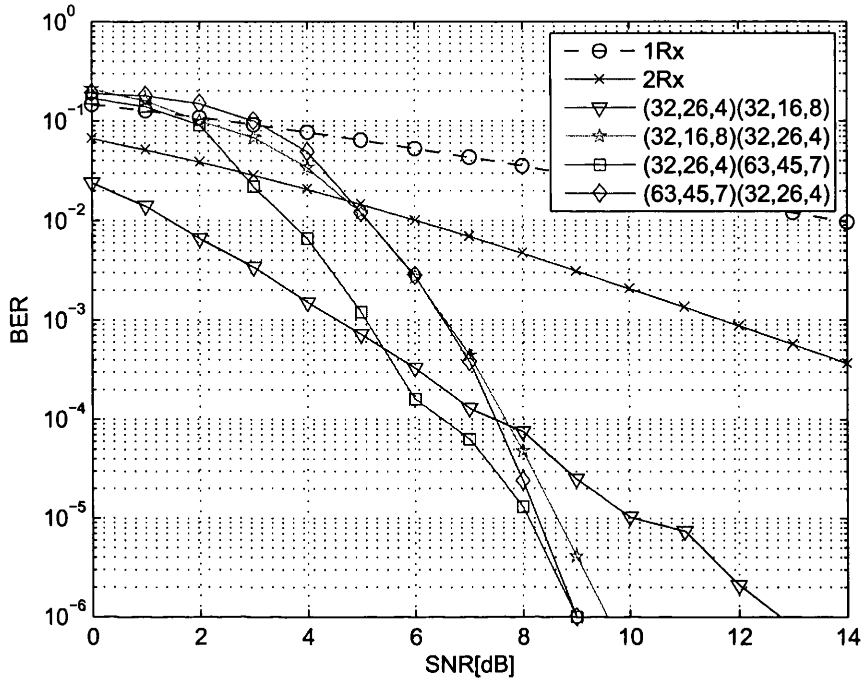

Figure 7.16: The BER of distributed product block codes and uncoded BPSK versus SNR over uncorrelated Rayleigh fadings with the path-loss (0.7,0.3,1).

$R_1 >> R_2$ the number of reliable parity bits that the relay sends is much more than the case when $R_1 << R_2$.

Table 7.3 summarizes the achievable gains over the uncoded BPSK over an Rayleigh fading channel without diversity for different system scenarios at the target BER of 10^{-6}. For all scenarios in this table, the diversity combining scheme that used at the destination is EGC. From the results presented in this chapter as well as other simulation results obtained, we can draw the following conclusions: When the relay is in the middle or closer to the source, the code rates of sub codes are more important than hamming distances of sub-codes and the main reason is that when $R_1 >> R_2$ the number of reliable parity bits that relay sends is much more than the case when $R_1 << R_2$. As sending more reliable parity bits from the relay is more effective than sending more parity bits from the source. However, this is not the case when the relay is closer to the destination as utilizing sub-code at the source with higher code rate than the sub-code in the relay or vice versa with the same d_{min}, since the performance

Table 7.3: Distributed Ecoding Gains for Different System Scenarios

Position of relay	R_1	R_2	d_{min1}	d_{min2}	Decoding at relay	SNR[dB]
$(x, x, 1)$	0.7	0.3	7	7	x	10
$(0.5, 0.5, 1)$	0.7	0.3	7	7	OSD(2)	2
$(0.5, 0.5, 1)$	0.3	0.7	7	7	OSD(2)	8
$(x, x, 1)$	0.5	0.5	8	4	x	11
$(0.5, 0.5, 1)$	0.5	0.5	8	4	OSD(2)	3
$(0.5, 0.5, 1)$	0.5	0.5	4	8	OSD(2)	4.5
$(0.5, 0.5, 1)$	0.7	0.8	7	4	OSD(2)	9.7
$(0.5, 0.5, 1)$	0.8	0.7	4	7	OSD(2)	9.7
$(0.5, 0.5, 1)$	0.8	0.5	4	8	OSD(2)	6
$(0.5, 0.5, 1)$	0.5	0.8	8	4	OSD(2)	7
$(0.3, 0.7, 1)$	0.7	0.3	7	7	OSD(2)	2
$(0.3, 0.7, 1)$	0.3	0.7	7	7	OSD(2)	7.8
$(0.3, 0.7, 1)$	0.5	0.5	8	4	OSD(2)	4.2
$(0.3, 0.7, 1)$	0.5	0.5	4	8	OSD(2)	5
$(0.3, 0.7, 1)$	0.7	0.8	7	4	OSD(2)	9.5
$(0.3, 0.7, 1)$	0.8	0.7	4	7	OSD(2)	7
$(0.3, 0.7, 1)$	0.8	0.5	4	8	OSD(2)	4
$(0.3, 0.7, 1)$	0.5	0.8	8	4	OSD(2)	9.5
$(0.7, 0.3, 1)$	0.7	0.3	7	7	OSD(2)	6
$(0.7, 0.3, 1)$	0.3	0.7	7	7	OSD(2)	8
$(0.7, 0.3, 1)$	0.5	0.5	8	4	OSD(2)	5
$(0.7, 0.3, 1)$	0.5	0.5	4	8	OSD(2)	10
$(0.7, 0.3, 1)$	0.7	0.8	7	4	OSD(2)	9
$(0.7, 0.3, 1)$	0.8	0.7	4	7	OSD(2)	9
$(0.7, 0.3, 1)$	0.8	0.5	4	8	OSD(2)	13
$(0.7, 0.3, 1)$	0.5	0.8	8	4	OSD(2)	9.5

is almost the same. This difference in performance can be explained that the relaibility of parity bits decrease as the relay is closer to destination.

7.5 Conclusions

The DF relaying employing the simple channel encoding and decoding techniques was investigated. The nodes locations were equivalently expressed as the propagation path-loss. In order to optimize the parameters of the DF relaying, two problems were formulated. In the first problem, the source and the relay employ the same channel code, however, the OSD used at the relay and at the destination can have different parameters. The simulation results confirmed that some of the decoding complexity at the destination can be moved to the relay provided that the sufficient cooperative diversity gain is available. Similarly, the decoding complexity at the destination can be reduced significantly if the diversity gain is available due to, e.g., multiple receiver antennas. In general, it was observed that achieving the ML decoding performance at the relay (at the destination) is more important than achieving the ML decoding performance at the destination (at the relay), provided that the relay is closer to the destination (to the source).

In the second problem, in case of no path-loss while the relay and the destination use the same decoding algorithm, however, the encodings at the source and at the relay can be different. For the case when the source generates the horizontal parity bits and the relay generates the vertical parity bits of the distributed product code, it was observed from the simulations that the minimum hamming distance of the constituent codes has much greater impact on the overall BER performance than the code rates.

In the case of path-loss when the relay is centred or closer to destination, utilizing sub-code at the source with higher code rate than the sub-code in the relay,i.e. $R_1 > R_2$ and the same d_{min} for both sub-codes and the same decoding at the destination provides large values of the coding gain over the case when $R_1 < R_2$. As sending more reliable parity bits from the relay is more effective than sending more parity bits from the source. However, this is not the case when the relay is closer to the destination as utilizing sub-code at the source with higher code rate than the sub-code in the relay or vice versa with the same d_{min}, since the performance is almost the same. The difference of the performance because of the code rate differences are much less than in the case when the relay is in the middle between the source and the destination. This can be explained that the reliability of parity bits sent from the relay to the destination decrease because of the relay is closer to the destination. Using stronger sub code C_1 with larger d_{min} from source more efficient to reduce errors either in the SR link or the SD direct link. The importance of using stronger sub-code with higher d_min at the source increases in the case when the relay is closer to the destination. Thus, in case of using sub-codes with the same code rates while the relay is centred or closer to the destination,$d_{min1} > d_{min2}$ performs better than the case when $d_{min1} < d_{min2}$. Finally, and regardless of the relay position, the code rates of the sub-codes are more important than minimum hamming distances and the main reason for this property is that when $R_1 >> R_2$ the number of reliable parity bits that the relay sends is much more than in the case when $R_1 << R_2$.

8

Conclusions and Future Work

This final chapter summarizes the conclusions of the thesis and highlights numerous areas for further research in future.

8.1 Conclusions

Achieving high data rate and reducing different aspects of the complexity are the main challenges for wireless communication systems that are investigated in this thesis. Thus, specific encoding and decoding algorithms are investigated for coded systems in general and coded cooperative systems in particular. The investigation concerns the implementation complexity reduction and characterization of system design guidelines under different scenarios.

In order to to achieve the main objectives in this thesis , the channel coding and the forward error corrections (FEC) coding are considered. In Chapter 4 and Chapter 5, the construction of the soft-decision decoding techniques is investigated with low complexity employing a list of the test error patterns for linear binary block codes of small to medium block length were investigated. The optimum and suboptimum construction of the list of error patterns was developed. Some properties of the joint probability of error of the received bits after ordering were derived. The original OSD algorithm was generalized by assuming a segmentation of the MRIPs, since the original OSD assumes only one segment and is a special case of the proposed segmentation decoding. The idea of segmentation is motivated by the fact that the distribution of errors in segments is non-uniform, and the segments of more reliable bits will contain less errors than the segments of less reliable bits. Hence, the distribution of errors in the segments can be better predicted, and the trial error patterns can be constructed more efficiently. The K MRP bits in the OSD algorithm were partitioned into several disjoint sets. Fewer errors were assumed in segments with higher reliability of bits. The error patterns in different segments were then combined to create the overall error patterns. The segmentation of the MRIPs was shown to overcome several drawbacks of the original OSD and to enable flexibility for devising new decoding strategies. The decoding complexity of the OSD-based decoding algorithms was reduced further by avoiding the Gauss (or the Gauss Jordan) elimination using the partial ordering of the received bits in the POSD decoding. The performance analysis was concerned with the problem of finding the probability of the test error patterns contained in the decoding list. The BER performance and the decoding complexity of the proposed decoding techniques were compared by extensive computer simulations. Numerical examples demonstrated excellent flexibility of the proposed decoding schemes to trade-off the BER performance and the decoding complexity. In some cases, both the BER performance as well as the decoding complexity of the segmentation-based OSD were found to be improved compared to the original OSD. The low-complexity soft-decision OSD and the POSD were extended for the decoding of binary linear block codes with non-binary modulations transmitted over the flat fading channels. The main complexity for such decoding techniques was assumed to be the size of the coding list. The size of the coding list for the OSD decoding algorithm can be reduced by sending highly reliable CRC bits. Three different types of the CRC can be applied. The first type is to send highly reliable CRC bits for different segments over the whole codeword, the second type is to send the CRC bits for different segments over the information bits only , and the third type is to send highly reliable CRC bits for segments over parity bits only. The comparison between these different types of the CRC bits showed that sending highly reliable CRC bits for many segments over information bits only is the most effective way to reduce the complexity of coding list. Also, sending the CRC bits for many segments is more effective in the complexity reduction than in sending the information about the exact Hamming weight e.g. for the whole codeword, the information bits or the parity bits only. The CRC bits can be used in distributed encoding and decoding schemes which were discussed in Chapter 5 and Chapter 7. The CRC bits are used in the litreature to prevent the relay from forwarding the codewords if the CRC fails. Such CRC scheme can be used also to reduce the complexity of the OSD and the POSD decoding algorithms at the relay to reduce the compexity of the decoding process as discussed in Chapter 5.

In Chapter 6, one of the main limitations of coded cooperative relay systems was investigated, which is the occurrence of detection errors at the relay. The relay positions and restrictions were investigated and proposed the system model for various cooperative diversity scenarios based upon different types of relay processing. The coded cooperative performance of the AF and the DF relaying and of the conventional receiver diversity were compared assuming the channel coding and non-binary linear modulations over the fast and slow fading channels with the path-loss attenuation. The low-complexity soft-decision POSD was extended for the decoding of binary linear block codes with non-binary modulations transmitted over the flat fading channels. The POSD was optimized for the decoding at the destination, and, provided that the DF relaying was considered, also for the decoding at the relay, confirming a good flexibility of the POSD that enables to trade-off the decoding complexity and the BER performance. The constraints on the relay location for the AF and the DF relaying to outperform the conventional receiver diversity were obtained by mathematical analysis and extensive computer simulations. Considering larger sensitivity of the BER performance of the DF relaying on the relay location different decoding algorithms were applied at the relay and at the destnation such as the OSD, POSD, and Log-Map decoding. The relay location conditions to provide the best performnace were obtained. These conditions can be satisfied more easily provided that there is more than one relay about the source, and for higher order modulations.

The examination of distributed BTC and the complexity requirements of decoding distribution between the relay and the destination are investigated in Chapter 7 and let to refined conclusions. The design guidelines for systems using coded cooperative diversity including design of distributed encoding and decoding techniques were investigated. The distribution of encoding and decoding complexity, the coding rates, the Hamming distance etc. were discussed. The nodes locations were equivalently expressed as the propagation path-loss. In order to optimize the parameters of the DF relaying, two problems were formulated. In the first problem, the source and the relay employ the same channel code, however, the OSD used at the relay and at the destination can have different parameters. The simulation results confirmed that some of the decoding complexity at the destination can be moved to the relay provided that the sufficient cooperative diversity gain is available. Similarly, the decoding complexity at the destination can be reduced significantly if the diversity gain is available due to, e.g., multiple receiver antennas. In general, it was observed that achieving the ML decoding performance at the relay (at the destination) is more important than achieving the ML decoding performance at the destination (at the relay), provided that the relay is closer to the destination (to the source).

In the second problem, in case of no path-loss, the relay and the destination use the same decoding algorithm, however, the encodings at the source and at the relay can be different. For the case when the source generates the horizontal parity bits and the relay generates the vertical parity bits of the distributed product code, it was observed from the simulations that the minimum Hamming distance of the constituent codes has much greater impact on the overall BER performance than the code rates in case of no path-loss. In the case when the path-loss was considered and when the relay is closer to the destination, utilizing a sub-code at the source with higher code rate R_1 than the sub-code in the relay of code rate $R_2 < R_1$, however with the same minimum Hamming distance of both sub-codes and the same decoding at the destination provides larger coding gains than the case where $R_1 < R_2$. This is due to the fact that parity bits from relay is more effective than sending more parity bits from the source. However, it is not the case when the relay is closer to the destination as utilizing sub-code at the source with higher code rate than the sub-code in the relay or vise versa assuming the same minimum Hamming distance of the sub-codes, the performance is almost the same. The difference in the performances of codes is due to the differences of the code rate and it is much less than the case when the relay is in the middle between the source and destination. This can be explained that the reliability of parity bits that are sent from the relay to the destination decrease because the relay is closer to the destination. For the case of equal code rates of sub-codes at the source and at the relay and the relay centered or closer to the destination with $d_{min1} > d_{min2}$ and the same decoding at the destination performs better than the case where $d_{min1} < d_{min2}$, since using stronger sub-code C_1 with larger d_{min} at the source is more efficient for reducing errors in the SR link and in the SD direct link. The importance of using stronger sub-code with higher d_{min} at the source increases in the case when the relay is closer to the destination. Finally, and regardless of the relay position, the code rates of sub-codes are more important than d_{min}s of sub-codes and the main reason for this is that when $R_1 >> R_2$ the number of reliable parity bits that the relay sends is much more than in the case when $R_1 << R_2$.

In conclusion, coded systems in general and coded cooperative systems specifically were investigated deeply and different solutions and system design principles were presented to achieve better trade-off between the performance and the complexity for the proposed systems.

8.2 Future Work

The results obtained in this thesis indicate that the OSD based on segmentation and the POSD are a promising approach to design practical decoding algorithms for the soft decision decoding of non-binary Galois fields in comparison with the algabric decoders. The OSD based on segmentation and the POSD algorithms can be modified to further improve their performance and to reduce their complexity by extending segmentation to the LRPs bits of the received sequence. In this scenario the POSD and the OSD based on segmentation can be integrated with the Chase decoding algorithm. After extensive computer simulations, it was noticed that the Monte Carlo simulations quickly become impractical due to excessive simulation run times, and thus future work may incorporate sample rejection with segmentation decoding to speed up the simulations and to improve the research work efficiently. The iterative soft decision decoding of the BTC based on the OSD can be further improved to reduce the complexity and to improve the performance especially for high rate BTCs.

Future work may consider comparison of the cooperative and the receiver diversity of orders more than two. In this case, the interference from other relays and cells have to be considered. For coded cooperative diversity, distributed decoding process can be investigated and utilized for multi-source multi-relay single-destination networks.

Then, the decoding complexity will be spread over many relays and the outputs from different relays will be combined at the destination. Moreover, the idea of distributed encoding BPC over multi-relay single destination networks is appealing especially for high rate BPCs. The threshold test can be applied for reliabilities of parity bits received at the destination from different relays and then the most reliable bits are selected and others bits are discarded to form a codeword from the most reliable bits to be decoded.

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