Paper:
Tucker, J. Richard Price and the History of Science. *Transactions of the Honourable Society of Cymmrodorion, 23*, 69-86.

RICHARD PRICE AND THE HISTORY OF SCIENCE

John V. Tucker

Abstract

Richard Price (1723–1791) was born in south Wales and practised as a minister of religion in London. He was also a keen scientist who wrote extensively about mathematics, astronomy, and electricity, and was elected a Fellow of the Royal Society. Written in support of a national history of science for Wales, this article explores the legacy of Richard Price and his considerable contribution to science and the intellectual history of Wales. The article argues that Price's real contribution to science was in the field of probability theory and actuarial calculations.

Introduction

Richard Price was born in Llangeinor, near Bridgend, in 1723. His life was that of a Dissenting Minister in Newington Green, London. He died in 1791. He is well remembered for his writings on politics and the affairs of his day – such as the American and French Revolutions. Liberal, republican, and deeply engaged with ideas and intellectuals, he is a major thinker of the eighteenth century. He is certainly pre-eminent in Welsh intellectual history.

Richard Price was also deeply engaged with science. He was elected a Fellow of the Royal Society for good reasons. He wrote about mathematics, astronomy and electricity. He had scientific equipment at home. He was consulted on scientific questions. He was a central figure in an eighteenth-century network of scientific people. He developed the mathematics and data needed to place pensions and insurance on a sound foundation – a contribution to applied mathematics that has led to huge computational, financial and social progress.

Richard Price was a polymath who made original and significant contributions to a number of fields. People who have come to know of him wonder why he isn't better known inside and outside intellectual circles, and especially in Wales. Perhaps polymaths have particular difficulties because of the depth of modern academic fields, which encourages narrowness. Perhaps being Welsh is also a handicap – it is very difficult to find out about our countrymen and even more so about our countrywomen. Wales has yet to remember or celebrate intellectual or scientific achievements.

In the case of a major figure like Price there are signs of change. *The Richard Price Society* was founded in 2014 to support study and focus interest. Paul Frame's superb new biography appeared in 2015.[1]

1 Paul Frame, *Liberty's Apostle. Richard Price. His Life and Times* (Cardiff: University of Wales Press, 2015).

In this paper I am going to explore the place of Richard Price FRS in the history of science.[2] I do this as a scientist interested in the scientific history and heritage of Wales, and one whose research connects to Price. I will begin by reflecting on the history of science and on the neglected area of Wales and science. After this prologue, which I hope will be of independent interest, I will introduce Price's science, his remarkable scientific circle, and his major contribution to science: probability theory (Bayes-Price Theorem) and the collection and analysis of data. As a computer scientist I find these topics irresistible.

Histories of science: international, national and local

There is a sort of orthodoxy about the development of modern science. Science's origins lie in the speculations, observations and practices of the ancient world.[3] However, the essential characteristics of modern science emerge over the last five centuries. In the sixteenth century, science is recognised as relevant for practical and commercial problems such as those of navigation and finance. In the seventeenth century there is the so-called Scientific Revolution, the days of Galileo and Newton, where science aligns with general culture and society. In the eighteenth century, science begins to transform industry and commerce, witnessed by the lives of Josiah Wedgwood and Richard Price. This century sees an Industrial Enlightenment prior to an Industrial Revolution of the nineteenth century, when science separates and becomes a self-governing enterprise. In the first half of the twentieth century, science is independent, living in the academic environment – professional conceptions of pure science. In the second half of the twentieth century, post-World War Two, science belongs in a nexus of technology, industry, and politics. It has become a pervasive agent influencing society, culture, identity… these are crude characterisations, of course, based on the prominence of topics in an enormous literature.

There are other orthodoxies about the history of science. One is that *national* histories of science are suspect and indeed a bad idea. National histories are too easily tainted by nationalistic and patriotic purposes. They encourage attention to second rank figures and secondary achievements; they inflate people and their achievement. Too often they are written or based upon antiquarian, narrow interests of little general significance. Many are surveys of local heroes justifiably unknown to the world at large.

I reject this latter view, of course. National histories of science are a very good idea. Antiquarian interests focus on detail, rescue the neglected and can expand on

2 Lecture delivered to the first joint meeting of the The Honourable Society of Cymmrodorion and the Learned Society of Wales, 3 December 2015, at Cardiff University. I am deeply grateful to Edwin Beggs, Michael Charlton, Paul Frame, Faron Moller and Tracey Rihll for valuable discussions relevant to the lecture and comments on this paper.

3 An excellent modern introduction to the origins is T. E. Rihll, *Greek Science* (Oxford: Oxford University Press, 2006); see also her *Technology and Society in the Ancient Greek and Roman Worlds* (Washington DC: American Historical Association, Society for the History of Technology, 2013).

what is of general significance. Attention to secondary figures and the amplification of their achievements brings us closer to science as it was – and is – rather than as it is often idealised in histories of international 'giants'. Obviously, the culture and government of a country offers a historically meaningful context within which to think, for example, about how science connects with general history.

Indeed, *regional* histories of science offer new perspectives, e.g., on social networks that manifest communication and recognition. They also easily integrate the history of science and scientists with society and place. Science is done locally but validated internationally.

What is History of Science? It is the history of problems, ideas, theories, methods, equipment, data, arguments, publications, social networks, institutions, companies, states, policies, industrial organisations, markets, events, *people* – and so many people! I think that a local history of science is able to encompass many of these notions *simultaneously* and offer an understanding of science that is intimate with social and economic conditions and is closer to the scientific life of individual scientists.

Which brings me to two audiences for the history of science: people interested in science and scientists, and people interested in societies, periods and places. The first come to history of science from the present state of the art. This is today's science, how did it arise, who contributed what? The Whiggish tendency is evident in the motivation: the bias to narrative paths that lead to a present end. The second audience come to history of science to find components for their narratives, though they tend to leave technicalities behind. Both motivations are natural and useful for extending our understanding of the past.

People change so science changes. Here we are, what is our history? Each generation has to rewrite history because each generation has different knowledge, experience, perspectives, curiosities, and *questions*. There are different contemporary agendas. Our primary sources change little, the secondary sources change a great deal, and the present is never intellectually still.

The history of science and Wales

What has Wales to offer the History of Science? First, there are the lives and scientific works of Welsh people and their significance. Many Welsh men and women have been educated and worked outside Wales, belonging to thinly spread scientific diasporas. Second, there is the science that has been developed in Wales. Wales, its land, fauna, flora – and people – is an important world class 'laboratory' for research and education for geology, biology, agriculture, geography and archaeology. Third, there is the creation and export of Welsh technological knowledge and skills around the world.

Shortly, I will speak of Price and so let me comment on the first offering. Who from Wales has contributed to the development of science? Like many interested in Welsh Science, I found my way to T. Iorwerth Jones' invaluable compendium of 93 biographies of scientists, based upon various sources, and published by the Honourable Society of Cymmrodorion in 1934.[4] T. Iorwerth Jones was an electronic engineer at the National Physical Laboratory, Teddington who published on radio electronics. One important tool was his use of an institution – The Royal Society. Jones studied the FRSs of Wales. Richard Price FRS is there (pp. 180–181), as are his scientifically talented nephews William Morgan FRS (pp. 154–56) and George Cadogan Morgan (pp. 153–54).

Jones' membership criteria are that both parents were of Welsh descent and born in Wales. In an appendix he used a category of *Anglo-Welsh*: 'merely born in Wales or to whom it became a land of adoption'; among the 13 examples are Christopher Rice Mansel Talbot and Alfred Russel Wallace. In another appendix, he included 26 'Welsh FRSs elected on political and other grounds': a prominent example is John Dillwyn Llewelyn. The compendium listed only men: no women were included.

Reflecting on his researches, Jones observed that Welsh scientists were 'isolated individuals rarely associated with one another and as unlinked as is conceivable with earlier or later prevailing thought in their native land. It therefore follows that little in the nature of a Welsh tradition in science emerges in these pages', and that Welsh science consisted of 'isolated, unrelated, personal contributions in diverse, scattered fields of science'. It is difficult to object to these observations, but there are exceptions. One exception is surely the community associated with the Royal Institution of South Wales, in early nineteenth-century Swansea.[5] Another is the community that created and sustained the South Wales Institute of Engineers in the second half of the century.[6] Less obviously, correspondence on natural philosophy can be found in eighteenth-century Wales.[7] Quite simply, research is needed.

T. Iorwerth Jones and the Cymmrodorion did Wales a service by creating this 192-page list of people who at the time (and even today?) could have been dismissed as a list of the justifiably forgotten; the list is a testimony to the value of antiquarianism.

More recently, others have been disturbed by the silence on Welsh scientists. Professor Phil Williams AM (1939–2003) regularly spoke of Wales as a nation of scientists and featured lists of men (again no women) who did something: for example, in the Short Debate *Science in Wales*, 24 May 2001, in the National Assembly for Wales he listed 15 major figures, many more recent but including Richard Price. I recall Neville Evans's two campaigns, two decades apart, that sent

4 T. Iorwerth Jones, 'The Contributions of Welshmen to Science', *Transactions of Honourable Society of Cymmrodorion*, 1932–33 (1934), 40–232.

5 Ronald Rees, *Heroic Science: Swansea and the Royal Institution of South Wales 1835–1865* (St. Athan: Glyndwr Publishing, 2005); see also Louise Miskell, *Intelligent Town. An Urban History of Swansea, 1780–1855* (Cardiff: University of Wales Press, 2006). The RISW continues its original mission and, in fact, is an outstanding local learned society.

6 The South Wales Institute of Engineers was founded in 1857, began its *Transactions* in 1859, received its Royal Charter in 1888, and became transformed into the South Wales Institute of Engineers Educational Trust in 2007.

7 For example, some correspondence on natural philosophy and manufactures is noted in R. T. Jenkins and Helen M. Ramage, *A History of the Honourable Society of Cymmrodorion and of the Gwyneddigion and Cymreigyddion Societies (1751–1951)* (London: Honourable Society of Cymmrodorion, 1951), pp. 243–44.

posters with images of *living* Welsh scientists to all schools in Wales. Steve Jones created a booklet of *Welsh Achievements* for the Welsh Development Agency. Most recently, there is the new *Scientists of Wales* series of books published by the University of Wales Press, with the support of the Learned Society of Wales. Three volumes have appeared, on Robert Recorde, William Grove, and Evan James Williams.

Conversely, what has the History of Science to offer Wales? For the people of a nation and a region, history and heritage are fundamental to identity. Who do we think we are? Who do we say we are? Answers depend upon: What do we know about ourselves? For Wales, these questions lead to lots of problems, but ones that are being addressed with fresh ideas. In his call for new thinking and action for our studies of Wales, M Wynn Thomas has observed:

> 'Welsh Studies' is the only term we have to describe the collective effort by scholars and writers in a multitude of different disciplines across the Sciences as well as the Arts to review every aspect of that which has made, and continues to make, us the people that we are. Without such intelligent, rigorous, reflective self-knowledge, Wales – like any other national community – would cease to exist.[8]

Local histories of science offer scientists a deeper understanding of the nature of science, and tools for science education and engagement. For historians, science and technology are drivers in the creation of Modern Wales, indeed of European Modernity, from the sixteenth century. For economists, science and technology are essential to economic understanding, especially competitiveness and growth. For politicians, public servants and citizens, science and technology are essential to innovation and change in contemporary Wales. For all of the above: *consult the historical record.*

Writing in 1934, T. Iorwerth Jones noted the lack of Welsh institutions. Klaas van Berkel, the Dutch historian of science, observed in his national history of science that 'institutions are the *terra firma* under the feet of those who investigate the history of science in a particular country'.[9] Institutions try to hold together the past, present and future using archives and collections; cultural traditions and living memories; quests for new thinking and activities; and the gathering together of people with some form of common purpose. The institutions of Wales are few in number. They are small, poor and weak by British standards. Few are old, some are *very* young. Tonight, of course, I am thinking of the Cymmrodorion (1751) and the Learned Society of Wales, founded in May 2010 and granted a Royal Charter in September 2015. The history of science for Wales is a case that illustrates van Berkel's point: one turns to the Cymmrodorion's proceedings, and to the Learned Society of Wales's programmes on History of Science and on Wales Studies.

8 See <www.clickonwales.org/2014/03/scholars-of-wales-stare-disaster-in-the-face> [accessed 2 February 2017]. Professor M. Wynn Thomas writes about Wales Studies in this volume of the *Transactions*.

9 *The History of Science in the Netherlands: Survey, Themes and References*, ed. by Klaas van Berkel, Albert van Helden and Lodewijk Palm (Leiden: Brill, 1999), p. 10.

Richard Price and his scientific circle

On some of his memorials, Richard Price is remembered as a Theologian, Philosopher, Mathematician; and as Philosopher, Preacher, Actuary, 'Cyfall Dynolryw'.[10] Most of his life and thought are to be found in his research papers, books, pamphlets, letters, and diary; his image is to be found in a portrait and in several satirical political cartoons of his last years. Scholars have added some memoirs, monographs, collected works, collected letters, and biographies. At the heart of Price scholarship are deep studies by D.O. Thomas and those of Price's biographer Paul Frame.[11]

I will not say much about the political and philosophical work for which he is most thoroughly studied, but consider him as a figure in the History of Science. How did he acquire his scientific interests? What science did he know? What problems did he tackle? What contributions did he make to the science of his day, either independently or in support of others? What influences surrounded him and his scientific network of friends? What happened to his science?

Answers to some of these questions are in Paul Frame's biography.[12] Price's early education in Wales and his later study at the Tenter Academy in London (1740–1744) under John Eames FRS (1686–1744) establishes his interest in science, which includes electricity, astronomy, and mathematics. In considering Price's scientific output, D. O. Thomas lists 42 publications which include books, journal papers, notes, and pamphlets. There are nine papers in the *Philosophical Transactions of the Royal Society*:

- two on probability theory (1764 and 1765);
- five on life assurance (1770, 1771,1774, 1775, and 1776);
- one on the Transit of Venus (1771);

and a short Postscript on light from bodies in combustion (1785) and a Letter on the longevity of women *v.* men (1786). I will focus on the subjects of probability theory and life assurance.

Price's scientific circle was wide and international. In London, through the Royal Society, he took his place in an elite scientific *milieu*. Of Price's scientific circle, four stand out for their intimacy, personal and scientific: Benjamin Franklin (1706–1790), Joseph Priestley (1733–1804), and Price's nephews William Morgan (1750–1833) and George Cadogan Morgan (1754–1798). All were engaged in investigations of the most fundamental phenomenon of the eighteenth and subsequent centuries: electricity. All were radicals.

In December 1765, after his publications on probability, Price became a Fellow of the Royal Society. Among eleven nominators were Franklin and John Canton (1718–1772), who was interested in electrostatics and close to Price, Franklin, and Priestley. Following his election, Price served on the Council of the Society.

10 At Newington Green Unitarian Church, London, and Carnegie House, Wyndham Street, Bridgend (formerly the Public Library), respectively.

11 D. O. Thomas, *The Honest Mind. The Thought and Work of Richard Price* (Oxford: Oxford University Press, 1977) and *The Correspondence of Richard Price*, ed. by W. B. Peach and D. O. Thomas, 3 vols (Cardiff: University of Wales Press, 1983–94); Frame, *Liberty's Apostle*.

12 Frame, *Liberty's Apostle*, chapters 3 and 4.

Benjamin Franklin is famous as an intellectual, diplomat, financial administrator, spy, and Founding Father of the United States of America. Franklin is honoured with many statues in prominent places, and many histories and biographies. Franklin's science was long neglected by historians and biographers and yet his science was the basis of his reputation and authority in his lifetime. Indeed, the colonist Franklin won the Royal Society's Copley Medal in 1753 and was elected FRS in 1756. Benjamin Franklin was in England between 1757 and 1762, and again from 1764 to 1775, during which period he met Price.

Franklin's basic idea of electricity as a fluid (1747) brought order into the subject, with its ideas about the conservation of electric charge and the positive-negative distinction. His *Experiments and Observations on Electricity* (1751) stand out.[13] Franklin's experiments on the nature of lightning and conductors (1752) had truly significant implications. Firstly, they showed that electrical phenomena occur in nature and on a large scale. Secondly, they demonstrated that the pursuit of basic science – curiosity – can have major practical applications.[14] Thirdly, the scientific understanding of lightning was a major contribution to the 'war' against superstition.[15]

Price met Joseph Priestley after Franklin. Price and Priestley are probably the two most prominent and controversial Dissenters of the late eighteenth century. They feature together in satirical cartoons along with the 3rd Earl of Stanhope, a gifted man of science.[16] Price brought Priestley to a meeting of the Royal Society and nominated him, along with Franklin and Canton, for the Fellowship in 1766. Price supported the younger Priestley in many ways, reading, commenting, and contributing to his *History and Present State of Electricity* (1767), and finding him a patron in Lord Shelburne at Bowood in 1772.

In a letter from Joseph Priestley to Richard Price, marked Warrington, 6 March 1766, we read this compliment to the network:

> I take it for granted, you have seen the letter I wrote, about a fortnight ago, to Dr Franklin. I desired he would show it to you, and Mr Canton. Writing upon a philosophical subject to any of you; I would have it considered as writing to you all.[17]

In addition to exchanges on technical matters, Price and Priestley had extensive discussions on the philosophical and religious aspects of science, such as the nature

13 In J. J. Thomson, *Recollections and Reflections* (London: G. Bell and Sons, 1936), the discoverer of the electron credits the fluid theory as a profoundly useful insight for the contemporary theory of electrons.

14 Sometimes attributed as Bacon's Thesis. The debate about this thesis is very much alive and one the Learned Society of Wales reignited in its publication by Sir John Cadogan, *Curiosity-driven 'Blue Sky' Research: A Threatened Vital Activity?* (Cardiff: Learned Society of Wales, 2014).

15 A full account of Franklin as a scientist is given by I. Bernard Cohen, *Benjamin Franklin's Science* (Cambridge, Mass: Harvard University Press, 1990).

16 As, indeed, was the 2nd Earl Stanhope (1714–1786) who resided at Chevening, fifteen miles from Tunbridge Wells, and conversed on probability with Bayes and de Moivre.

17 *The Correspondence of Richard Price*, I, p. 36.

of matter, the spirit, and free will. It was Joseph Priestley who gave the funeral oration for Price.[18]

Price's two nephews were active in science, both publishing on electricity. Of immediate interest is William Morgan's researches on the conduction of electricity in a vacuum, published in the *Philosophical Transactions* in 1785, communicated by Price.[19] Morgan's paper is clear and detailed, and compares well to modern standards of experimental reporting. The question is simply what happens to electrical current in a vacuum and, in modern terms, explores the combination of properties:

vacuum + gas + high voltage.

The apparatus he describes is an ancestor to what later developed into an important family of instruments, the discharge tubes. There are many subsequent tubes, including most notably the *Crookes Tube*. The experimental framework is of enormous importance on the road to understanding electricity and ultimately the atom. One early milestone is Faraday's extensive studies of electricity, contrasting Morgan's experiments and conclusions with those of Davy.[20] Morgan's electrical experiment of 1785 is noteworthy for another reason. The observations indicate that Morgan produced and observed plasma – the fourth state of matter – and what are called soft X-rays.[21]

William Morgan was well-established in the scientific community of his day, winning the Copley Medal in 1789 for his two papers on reversions and survivorships. He became a Fellow of the Royal Society in 1790, nominated by Henry Cavendish, Nevil Maskelyne, and, of course, Richard Price, among others. Trained by Richard Price, Morgan was Actuary of the Equitable Life Assurance Society from 1774 to 1830. His outstanding 56-year career is testimony to his high calibre and integrity.[22] Morgan also wrote a major book on pensions and insurance, about which subject a little more later.[23]

The younger nephew, George Cadogan Morgan (1754–1798), was also deeply interested in science and influenced by Price. While a dissenting minister and teacher in Norwich, he was involved in the controversy over the safety of

18 Joseph Priestley*, A Discourse on Occasion of the Death of Dr. Price: Delivered at Hackney, on Sunday, May 1, 1791* (London: J. Johnson, 1791).

19 William Morgan, 'Electrical Experiments Made in Order to Ascertain the Non-Conducting Power of a Perfect Vacuum', *Philosophical Transactions of the Royal Society of London*, 75 (1785), 272–78.

20 Michael Faraday, 'Experimental Researches in Electricity – Thirteenth Series', *Philosophical Transactions of the Royal Society of London*, 128 (1838), 125–68, on p. 154. For a general account of Faraday on electricity see chapter 8 in Joseph Agassi, *Faraday as a Natural Philosopher* (Chicago: Chicago University Press, 1971).

21 That is, low energy X-rays, e.g., electromagnetic waves with energy around 1keV.

22 The Price family's connection to the Equitable continued through William Morgan's son, Arthur Morgan FRS (1801–1870), who was Actuary of the Equitable 1830–1870.

23 William Morgan, *The Doctrine of Annuities and Assurances on Lives and Survivorships stated and explained. To which is added, An Essay on the present state of Population in England and Wales by R Price* (London: T. Cadell, 1779).

lightning conductors when Heckingham Hall of Industry, a workhouse, burned down in 1781.[24] Later, he wrote a paper containing experimental observations on the light emitted by combustion, electricity, and photoluminescence, which was communicated to the Royal Society by Price.[25] In London, he wrote two volumes of *Lectures on Electricity* (1794), building on the lectures he gave at the newly-founded dissenting academy, New College at Hackney, at which Price and Priestley also briefly taught. It was to George that Price left his scientific instruments.[26]

Richard Price and probability

Someone in Price's scientific circle who was to have a profound influence on his life and memory is Thomas Bayes (1701–1761). Bayes was a Dissenting Minister with mathematical interests, resident in Tunbridge Wells. He was an FRS and interested in the pure mathematical development of the calculus and the doctrine of chance, or probability theory as we now call it. Bayes was twenty years senior to Price, and was nearer to a generation of mathematicians contemporary with Sir Isaac Newton (1643–1727). Little is known of their social intercourse, but Bayes left Price £100 in his will and his family invited Price to examine Bayes's mathematical papers after his death. This was a difficult task but one that led to mathematical and philosophical treasure.

Over the course of two years, Richard Price prepared for publication some theorems about probability theory which he found among Bayes's papers. Price also wrote an explanation of their significance, and proved some new theorems. The papers Price created are landmarks in the history of probability and, as their legacy grows, in modern science.

The first paper was read by John Canton in December 1763 and appeared in the *Transactions of the Royal Society* in 1764.[27] The paper combines lightly edited transcriptions of Bayes's work with the new material by Price. Superficially, the structure and authorship seem to be as follows:

- Introduction (pp. 370–375, Price)
- Section I – Definitions and basic results (pp. 376–385, Bayes)
- Section II – Generic Example: 'Billiard' Table (pp. 385–392, Bayes)
- Scholium – Generalizations (pp. 392–399, Bayes and Price)
- Scholium – Generalizations (pp. 399–403, Price)

24 Simon Schaffer, 'Charged Atmospheres: Promethean Science and the Royal Society', in *Seeing Further: The Story of Science and the Royal Society*, ed. by Bill Bryson (London: Harper Press, 2010), 132–55.

25 George Cadogan Morgan, 'Observations on the Light of Bodies in a State of Combustion', *Philosophical Transactions of the Royal Society of London*, 75 (1785), 190–212.

26 For an appreciation of this less celebrated figure, see *Travels in Revolutionary France and a Journey Across America: George Cadogan Morgan and Richard Price Morgan*, ed. by Mary-Anne Constantine and Paul Frame (Cardiff: University of Wales Press, 2012).

27 Thomas Bayes and Richard Price, 'An Essay toward Solving a Problem in the Doctrine of Chances', *Philosophical Transactions of the Royal Society of London,* 53 (1763), 370–418. The volume for 1763 appeared in 1764.

- Appendix – Discussion of the use and significance (pp. 404–418, Price)

The paper contains Bayes's prerequisites in probability theory; a model problem to solve; and ten Propositions and three methods for calculating probabilities with error bounds. Notable theorems are the conditional probability Propositions 3 and 5, and the main Propositions 9 and 10.

The second paper appeared in 1765 and contains Price's transcriptions and new theorems sharpening the method of calculation with much improved error bounds.[28] Its structure is approximately:

- Introduction (pp. 296–297, Price)
- Rule 2 according to Bayes (pp. 298–310, Bayes)
- Rule 2 according to Price (pp. 310–324, Price)

The first paper is famous and is usually cited with an attribution to Bayes. But clearly the paper qualifies as being of joint authorship by any modern standard; if the approximation above is correct then Bayes's text in the *Essay* is 23 out of 48 pages, which is 48% of the published paper. In any case, Price's contributions are distinct and substantial, though his treatment in the immense scholarly literature referring to the paper does not reflect this.

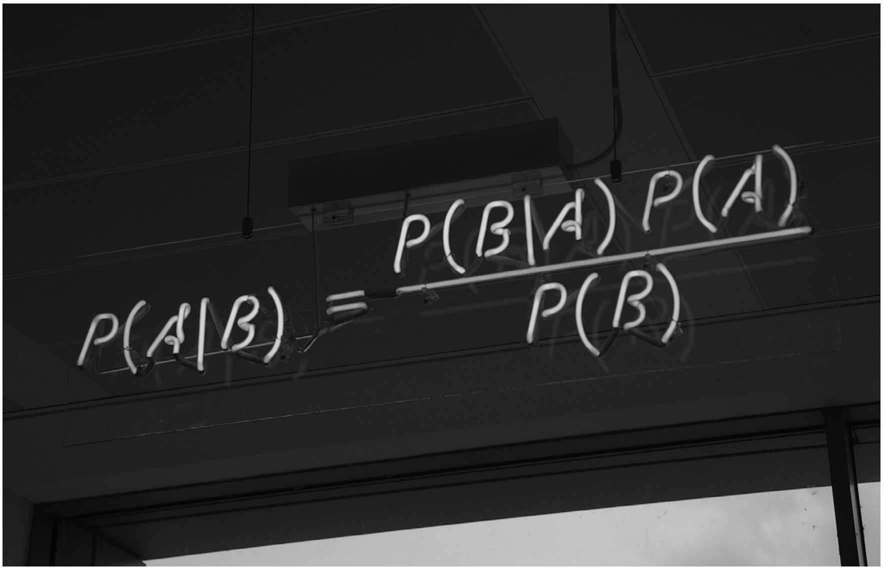
The papers are not easy reading. First, the notation is remote and comes from a period where Newtonian fluxions and geometrical methods were in use in Britain, and there were no standard notations for essential quantities (e.g., π, e^x and ${}^nC_k$). Second, modern readers of the paper have in mind the clean general forms of the theorems that began to emerge in the work of Pierre Laplace (from 1774), and which became elegant equations easily derived from twentieth-century axioms of probability. Third, at this early stage in the development of probability, the conceptual and technical basis of this subtlest of mathematical theories was far from clear and stable; the prerequisites section is difficult for this reason. In the centuries that have followed, there have been many reconstructions and commentaries on the two papers, which confirm the richness of the space of interpretations.[29]

The modern form of the Bayes-Price Theorem can be taught in school sixth-forms. In its simplest form, it is an instantly recognizable and celebrated formula. This is not the place to explain it, though we may admire it in lights (Fig. 1). Today, the mathematics is more abstract and simple but its philosophical interpretation is more complicated and controversial.

Indeed, probability theory has been plagued by controversies to do with what has come to be called *Bayesian Statistics* and *Bayesian Analysis*. The controversies arise from a nineteenth-century distinction between *objective* and *subjective* interpretations of probability that the early writers did not worry about. Roughly

28 Richard Price, 'A Demonstration of the Second Rule in the Essay toward the Solution of a Problem in the Doctrine of Chances', *Philosophical Transactions of the Royal Society of London*, 54 (1764), 296–325. The volume for 1764 appeared in 1765.

29 An excellent general history, with special attention to inverse probability, is Lorraine Daston, *Classical Probability in the Enlightenment* (Princeton, NJ: Princeton University Press, 1988). For mathematically able readers, there is Stephen M. Stigler, *The History of Statistics: The Measurement of Uncertainty Before 1900* (Cambridge, MA: Harvard University Press, 1986) and Anders Hald, *A History of Mathematical Statistics from 1750 to 1930* (New York: John Wiley, 1998).



[Fig. 1] *The modern Bayes-Price Theorem in lights*

speaking, objective probability confines the meaning of probabilities to physical data as evidence (such as observations of the frequencies of events); subjective probability introduces anthropomorphic ideas about observers' judgements and confidence: a probability denotes a numerical degree of belief based on evidence; the theorems determine how the probability is expected to be changed given new evidence. The modern term Bayesian refers to a subjective interpretation that is shaping modern data science. This subjective aspect of the original papers is pronounced in Price's sections, as we shall see shortly. Indeed, in a perceptive article Donald A Gillies has questioned whether Thomas Bayes was indeed a Bayesian in the modern sense.[30]

Bayesian Analysis has become commonplace, enabled by innovations in computer software and hardware. Since the 1990s, new developments in computer science, and a myriad of software applications, have come to depend on the legacy of Bayes-Price.[31] Why so much fuss, what are these papers about?

Inspired by games of chance, probability theory had been developed into a subject of mathematical research, most notably by Jacob Bernoulli (1654–1705)

30 Donald A. Gillies, 'Was Bayes a Bayesian?', *Historia Mathematica,* 14 (1987), 325–46.
31 An overview is given by Sharon Bertsch McGrayne, *The Theory That Would Not Die* (New Haven, CT: Yale University Press, 2012).

and Abraham de Moivre (1667–1754).[32] The theory had tackled problems of the form: given assumptions about a process or situation, calculate the probabilities that certain events may or may not happen. Crudely, the direction of the theory was

assumptions about a cause → calculations of probabilities of events.

But there was no theory for problems of the converse form, namely: given observational data about certain events that did happen or did not happen in the past, calculate probabilities of an event happening in the future. That is, the new theoretical direction needed was

data from observations of events → calculations of probabilities for their cause.

This form of problem was called *inverse probability*, or *converse probability* as Price sometimes termed it; it is better known as one of *statistical inference*.

I will return to focus on the original papers, hoping to give a clearer impression of their value.

Natural laws, miracles, and testimonies

Among Bayes's papers, Richard Price found a mathematical model of a generic statistical inference problem and its analysis. He edited, refined, and supplemented the work, and created the two papers that communicated the mathematics, established its originality and significance, and recognized its importance for science.

Bayes's generic mathematical problem that exemplifies the whole paper is this. Consider a rectangular table and throw down randomly a ball. Draw a vertical line L through the point at which the ball came to rest. The line L divides the table into two areas, one to the right of the line, one to the left. The problem is to calculate the probability of the position of the line lying between two given lines as error bounds. Suppose we mark the position of the line L on the bottom edge of the table by x and the position of the given error bounding lines by a and b. Then the problem is to calculate the probability that $a < x < b$ given only data about which of n balls randomly thrown lie to the left of the line L or to the right. Mathematically, this statistical inference problem is delightfully simple. The theorems solve this problem in a very general way.

The importance of the theory lies in its applications to observations in science of all kinds. Price writes:

> Every judicious person will be sensible that the problem now

32 De Moivre is especially influential in Price's science. A French immigrant in London, he was an exceptional mathematician and intimate with Newton. See David R. Bellhouse, *Abraham De Moivre: Setting the Stage for Classical Probability and Its Applications* (Boca Raton, FL: A. K. Peters/CRC Press, 2011).

> mentioned is by no means merely a curious speculation in the doctrine of chances, but necessary to be solved in order to [provide] a sure foundation for all our reasonings concerning past facts, and what is likely to be hereafter. […] But it is certain that we cannot determine, at least not to any nicety, in what degree repeated experiments confirm a conclusion, without the particular discussion of the before mentioned problem; which, therefore, is necessary to be considered by anyone who would give a clear account of the strength of analogical or inductive reasoning; concerning which, at present, we seem to know little more than that it does sometimes in fact convince us, and at other times not; and that, as it is the means of acquainting us with many truths, of which otherwise we must have been ignorant; so it is, in all probability, the source of many errors, which perhaps might in some measure be avoided, if the force that this sort of reasoning ought to have with us were more distinctly and clearly understood.[33]

For the converse problem 'shews us, with distinctness and precision, in every case of any particular order or recurrency of events, what reason there is to think that such recurrency or order is derived from stable causes or regulations in nature, and not from any irregularities of chance'.[34] And in the matter of the error bounds:

> And tho' in such cases the Data are not sufficient to discover the exact probability of an event, yet it is very agreeable to be able to find the limits between which it is reasonable to think it must lie, and also to be able to determine the precise degree of assent which is due to any conclusions or assertions relating to them.[35]

The inductive method of Newton, by which general laws arise from repeated empirical observations, was subject to philosophical dispute. David Hume (1711–1776), in his *Treatise of Human Nature* (which appeared in stages from 1739 to 1740), attempted to bring the scientific method to bear on the study of human nature. He questioned the very nature of the scientific method. He divided 'all the objects of human reason or enquiry' into two exclusive categories: relations between *ideas* and matters of *fact*. A fact is a proposition such that both it and its denial are fully conceivable, possible, and not self-contradictory. How do people arrive at their opinions concerning unobserved matters of fact?

Philosophers have returned again and again to the process of inferring a general law or principle from the observation of particular instances.[36] Induction is a

33 Bayes and Price, *Essay*, pp. 371–72.
34 Ibid., p. 374.
35 Ibid., p. 418.
36 The most original philosophical response is that of Karl Popper, whose model of scientific knowledge, based on conjectures that await refutations, avoids the need to think of induction let alone worry about it. See, e.g., Karl Popper, *Conjectures and Refutations* (London: Routledge and Kegan Paul, 1963).

classic problem of epistemology: on what basis can one justify inferring a general law or principle that says

all *P*s are *Q*s

from *finitely* many particular observed inferences that

all *n* observations confirm that *P*s are also *Q*s?

Hume denied that there was any philosophically sound basis for induction. However, it is clear from Price's contribution that here was indeed a logical basis for such practice. That the papers on inverse probability were focused on induction is clear from the titles Price gave them, recently discovered on offprints of the articles: what is commonly referred to as the *Essay* was originally entitled *A Method of Calculating the Exact Probability of All Conclusions founded on Induction*.[37]

Subsequently, in *Of Miracles,* Section X of his *Enquiries Concerning Human Understanding* (1748), Hume defines a miracle as something that contradicts laws of nature. Since factual knowledge is based on the evidence of what can be observed, how can one get evidence for a miracle? There is never enough evidence to outweigh the laws of nature and so enable a belief in miracles: 'That no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavours to establish'. This refutation of the possibility of miracles leads to what might be called the *Testimony Problem*: how can we evaluate the evidence of fallible witnesses?

Twenty years later, Richard Price addresses Hume's refutation in *On the Nature of Historical Evidence and Miracles*, which is the last of his *Four Dissertations* (1767). In Section II, he cites and discusses Hume at length, but what is remarkable is his use of the Bayes-Price paper to support his counter arguments. In a long footnote he gives a calculation that 'is enough to show how very inaccurately we are apt to speak and judge this subject, previously to calculation'.[38]

In summary, this is the calculation: suppose that there was testimony that a natural event – such as sunrise – had taken place 1,000,000 times consecutively with no exceptions. What is the probability that it will take place at the next opportunity? Price chooses to calculate that the chance (= probability) that the probability p of the event *not* happening satisfies $p > 1/1,600,000$. He shows it is 0.5353; and Price would say it was probable since it is greater than 0.5. From this he then calculates that the probability that the event will fail to occur in the next 1,000,000 occasions is 0.465. Thus, the familiar event failing to happen is far from unlikely. The point is that if a physical event has invariably occurred, and its occurrence qualifies as a natural law, then the event not happening will qualify as a miracle in Hume's

37 Stephen M. Stigler, 'The True Title of Bayes' Essay', *Statistical Science,* 28 (2013), 283–88. Price's second paper simply prefixed '*A Supplement to the Essay on'* to the offprint title of the first. Compare the title in notes 27 and 28 above.

38 I took this calculation from pp. 395–98 of the third edition (1772).

sense. But according to probability theory, such miracles may be possible and even far from improbable. Thus, in Price's reply to Hume's argument against Christian miracles we find the earliest known contribution to Bayesian Analysis.[39]

Let me note here that the Testimony Problem later attracted the attention of the computer pioneer Charles Babbage (1791–1871). In *On Hume's argument against miracles*, which is Chapter X of the *Ninth Bridgewater Treatise* (1838), Babbage gave a detailed general analysis of Hume's argument using probabilities on multiple witnesses and, in an appendix, showed that:

> If independent witnesses can be found, who speak truth more frequently than falsehood, it is always possible to assign a number of independent witnesses, the improbability of the falsehood of whose concurring testimony shall be greater than that of the improbability of the miracle itself.[40]

Richard Price and data

To Price, financial projects to create pensions and insurance were clearly designed to improve the lives of people and, in consequence, to improve society. The projects were self-evidently moral acts that met obvious needs. But were they practical? In the eighteenth century, they were commercially novel and vulnerable, having a high risk of failure – and failure would mean difficulties for beneficiaries. The problems start with creating and understanding financial models and their practical application. But what can be known about lives in the future? What is known of the lives of the past? For actuarial science to develop one needs technical developments: compound interest; probability theory; and statistical analysis of mortality data.

Price enters the primitive world of pensions and insurance by being asked for advice: first, about a scheme by lawyers to create a fund for widows of their profession; and second, in 1768, about the accounts of the Society for Equitable Assurances on Lives and Survivorships, remembered today as the Equitable Life Assurance Society. Price's comments proved influential, for the lawyers' scheme was abandoned as unsound, and the Equitable consulted him for the next fifteen years and acted upon his recommendations. Price received no payment for his work for the Equitable but he did receive gifts, such as scientific instruments valued at over £50. The Equitable, thanks to Price and Morgan, is immensely influential in the history of pensions and insurance.[41]

Price's work culminates in a *magnum opus*, his *Observations on Reversionary Payments, on Schemes for providing Annuities for Widows and Persons of Old*

39 Geoffrey Poitras, 'Richard Price, Miracles and the Origins of Bayesian Decision Theory', *The European Journal of the History of Economic Thought*, 20 (2013) 29–57.

40 *The Works of Charles Babbage*. *Volume 9*, ed. by Martin Campbell-Kelly (London: William Pickering, 1989), pp. 79–80.

41 Maurice Ogborn, *Equitable Assurances* (London: Gerorge Allen and Unwin, 1962).

Age; on the Method of Calculating the Values of Assurances on Lives; and on the National Debt. Published in 1771, the work ran to seven editions over forty years, three edited by his nephew William Morgan after Price's death. It was the standard work on pensions, insurance, and debt for a century, and for good reason. The book contained a rigorous, critical, and comprehensive account of all aspects of actuarial science.

The mathematics of designing annuities does not begin with Price. Some pioneers of probability theory addressed the application. De Moivre, for example, wrote the first textbook, *Annuities upon Lives* (1725).[42] But to apply the mathematics, good schemes and data must also be found and analysed. So, being an advisor to the Equitable involved Price in thinking about data: its collection, quality, analysis and use. Dominating Richard Price's work was the selection and analysis of data, the calculation of probabilities, and the creation of tables of survivorship and annuities, which are derived from *life tables*. A life table is a collection of data for evaluating and predicting life in a group or location. It gives mean probabilities that tell how long people live *on average* at every age. Such information is fundamental for financing pensions and insurance.[43] Such tables require thousands of calculations.

The origin of life tables is Edmund Halley's analysis of Caspar Neumann's demographic data for Breslau (today's Wroclaw, in Poland) for the years 1687–1691, and published in the *Philosophical Transactions* in 1693. Halley showed how to use the city's data about life and death, suitably analysed and displayed, to calculate how someone of age n might live for k years, how to determine life expectancy, and how to calculate the price of insurance for a single life. Edmund Halley (1656–1742) was a younger contemporary of Newton whose mathematical abilities were largely devoted to astronomy, a field of study rich in data and analytical problems. His name is celebrated by Halley's Comet. Halley's life and scientific work is varied and interesting, but, like many of his generation, it is shaped by Newton.[44] His 1693 paper is something of an isolated exercise but one in which he founded demography as a scientific subject. There are others that are active in the years before and after Halley.[45]

The construction and use of life tables is very difficult; in the eighteenth century it was, by modern standards, almost a hopeless task. Record keeping was poor, dispersed, and specific to the immediate needs of the record keepers. Defects were common as records were incomplete, far from uniform, and not centralized: there were poor church records, tax records (window, hearth), records on the consumption of alcohol, and so on. Then, as now, data needed careful evaluation, integration, and aggregation. Price addressed the problem rigorously, isolating concepts and assumptions. In the first three editions of his *Observations on Reversionary Payments*, he included a number of tables of probabilities of life from different

42 See Anders Hald, *A History of Probability and Statistics and Their Applications before 1750* (New York: John Wiley, 1990), chapter 25.

43 It also has value for government (e.g., for social provision in health) and can be applied to populations of living things quite generally.

44 Alan Cook, *Edmund Halley* (Oxford: Clarendon Press, 1998).

45 For an excellent account of the period, see C. G. Lewin, *Pensions and Insurance before 1800* (East Linton: Tuckwell Press, 2003).

places, especially at Northampton, based upon the Bills of All Saints Church for thirty-six years (1735–70). In the fourth edition, his table was revised to cover forty-six years (1735–80). This later table became famous, and later infamous, and is known as the *Northampton Table*. The fourth edition also had an analysis of Swedish data which, being more complete and consistent (noting gender, for example), better demonstrated the power of Price's methods.

The immense problems of establishing decent records is an important story in its own right.[46] Price ignited a controversy in seeking an answer to the question: has the population of the British Isles increased or decreased since the Glorious Revolution of 1688? The question was raised by the Reverend William Brakenridge in three papers in the *Philosophical Transactions* (1755–56). Price's interest has roots in his financial work and he wrote about the problem in various places, culminating in his *Essay on the Population of England and Wales*, which first appeared as an appendix to his nephew's *Doctrine* (see note 23 in this article) and later independently. Price's analysis showed that the population had declined substantially, a view that was provocative and, because the accuracy of the data was so poor, wrong. Because of Price's authority, the debate intensified and drew attention to the inadequacy of our records; it began to be resolved through the first census of 1801.

Conclusion

At the heart of the history of science is the history of what is known and how it changes. Changes arise from new theories, inventions, and data; from new problems and phenomena; and from new personal, commercial, and political interests. Scientific knowledge should be analysed historically *and* in terms of modern understanding, neither task being easy. Our narratives depend on today's views and tastes as they are expected to have an audience – in our case, people interested in history, science, or Wales.

Price's reputation lies in political and moral philosophy, and his contributions still resonate today, providing insights into political freedom, religious toleration, social security reform, finance and debt, and epistemology. An actor at the centre of eighteenth-century science, his contributions to the emerging field of electricity, and the ancient field of astronomy, were timely and modest. But his contributions to fledgling mathematical sciences are significant. In discovering, supplementing, and applying Thomas Bayes's unpublished work, he re-focused probability theory on the techniques and vast potential of statistical inference. In his work on pensions and insurance he developed rigorous methods for constructing data sets and computing probabilities. Today, these subjects are centre-stage in contemporary computing and data science.

In the ten years between the death of Thomas Bayes and the first appearance of his *Observations on Reversionary Payments*, Price in his spare time mastered

46 D. V. Glass, *Numbering the People* (Farnborough: D. C. Heath, 1973). See especially chapter 2.

and made lasting contributions to the mathematics of his day and its practical application. And what an application! His theories worked in practice and at scale, solved complex financial and social problems, and improved the lives of people and society. Price is rightly regarded as the most influential figure in the history of actuarial science.[47]

Meeting Richard Price, our sense of science grows.

[Fig. 2] *Smelling out a rat; or the atheistical-revolutionist disturbed in his midnight "calculations". James Gillray (1790)*

47 Craig Turnbull, *A History of British Actuarial Thought* (London: Palgrave Macmillan, 2017).