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### **Book chapter :**

Laramee, R., Chen, G., Jankun-Kelly, M., Zhang, E. & Thompson, D. (2009). *Bringing Topology-Based Flow Visualization to the Application Domain*. *Topology-Based Methods in Visualization II*, (pp. 161 Springer.

[http://dx.doi.org/10.1007/978-3-540-88606-8\\_12](http://dx.doi.org/10.1007/978-3-540-88606-8_12)

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# Bringing Topology-Based Flow Visualization to the Application Domain

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## Abstract

The visualization community is currently witnessing strong advances in topology-based flow visualization research. Numerous algorithms have been proposed since the introduction of this class of approaches in 1989. Yet despite the many advances in the field, topology-based flow visualization methods have, until now, failed to penetrate industry. Application domain experts are still, in general, not using topological analysis and visualization in daily practice. We present a range of state-of-the-art topology-based flow visualization methods such as vortex core line extraction, singularity and separatrix extraction, and periodic orbit extraction techniques, and apply them to real-world data sets. Applications include the visualization of engine simulation data such as in-cylinder flow, cooling jacket flow, as well as flow around a spinning missile. The novel application of periodic orbit extraction to the boundary surface of a cooling jacket is presented. Based on our experiences, we then describe what we believe needs to be done in order to bring topological flow visualization methods to industry-level software applications. We believe this discussion will inspire useful directions for future work.

**Keywords:** flow visualization, feature-based flow visualization, flow topology, applications

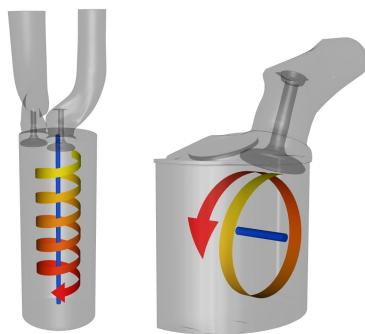
## 1.1 Introduction

Great progress has been made in the advancement of topology-based flow visualization methods since their introduction in 1989 [15]. Techniques for higher-order singularity extraction from vector fields have been introduced including a fast algorithm for 2D, steady-state data [43, 44]. Techniques for closed streamline extraction have been presented [52], including a grid-independent method [48]. The topology of unsteady flow can be extracted and visualized based on linear algebra [15], using streamline geometry [42], for 2D vector fields [50], and the tracking closed streamlines can be performed [53]. Several topology simplification algorithms have been written including multiresolution methods [8, 9], an area-based approach [10], topology-preservation-based compression of vector fields [35], and for

vector field design [54]. Surface-based topology [16] such as detection of separation and attachment lines has been investigated [24, 26], and a fast version of the aforementioned topology [49]. And many vortex core line and vortex core region algorithms have been implemented [13], like the well-known  $\lambda_2$  vortex core region-based method [19]. Levy et al. present an implementation that incorporates helicity [34]. Peikert and Roth describe and implementation that searches for local, parallel velocity and vorticity [39]. Roth and Peikert present an algorithm that extracts vortices of high-curvature [41]. The eigenvector method of Sujudi and Haines [46], and the swirl parameter method of Berdahl and Thompson [4] both employ the notion that a vortex core line occurs in a region of complex eigenvalues where the velocity is parallel to the associated real eigenvector. Applications of vortex core line extraction to aerodynamics is described by Kenwright and Haines [23, 25]. Vortex core line and region extraction techniques have also been developed for unsteady flow like the well-known predictor-corrector algorithm [1, 2] and an algorithm based on analysis of scalar values [3]. Reinders et al. present an application of tracking vortices trailing a tapered cylinder [40]. For a more complete overview of topology-based flow visualization research, we refer the readers to Laramee et al. [33].

Yet despite the many advances in topology-based analysis and visualization, this class of techniques is generally not used by domain experts in their daily routine. Functionality for the extraction and visualization of topology is not normally included in industry-level application software. The goal of this paper is to explore what needs to be done in order to bring topology-based methods to the application domain. We do so by drawing on our own experiences, namely, by applying topological methods to a collection of real-world data sets, identifying the insight they provide and at the same time, identifying the limitations of these approaches. We use topology-based extraction such as vortex core line extraction, singularity and separatrix extraction, and periodic orbit extraction techniques, in real engineering applications, including the visualization of engine simulation data such as in-cylinder flow, cooling jacket flow, as well as flow around a spinning missile. Our presentation includes the extraction of periodic orbits at the boundary surface of a cooling jacket—a novel application. We then describe what we believe needs to be done in order to bring topological flow visualization methods to industry-level software applications. We believe this discussion can play a role in steering future directions in the field.

We note that the current discussion fits in well with a larger trend in the visualization community. *Can visualization survive without customers?* [36] was the question posed by Bill Lorensen at the 2004 NIH/NSF workshop on visualization and research challenges [22]. Visualization research is not for the sake of visualization itself. In other words visualization is ultimately meant to help a user, i.e., someone normally outside the visualization community, gain insight into the problem they are trying to solve or the goal being sought after. Johnson called this problem “thinking about the science” [21]. Interdisciplinary collaboration can be very challenging. However, we do see signs of progress in this area. More quality, application-track papers have been published in recent years. We also note the emergence of the first Applied Visualization Conference (AppliedVis 2005) that took place in Asheville, North Carolina in April of 2005 (more information available at <http://www.appliedvis.org>). This topic was also the subject of recent panel discussions [14, 47] as well as a recent research paper [51].



**Fig. 1.1.** (left) The *swirl* motion of flow in the combustion chamber of a diesel engine. *Swirl* is used to describe circulation about the axis aligned with the valve cylinder. The intake ports at the top provide the tangential component of the flow necessary for swirl. The data set consists of 776,000 unstructured, adaptive resolution grid cells. (right) Some in-cylinder flows require a *tumble* motion flow pattern in order to mix fuel with oxygen. Tumble flow circulates around an axis perpendicular to the cylinder axis, orthogonal to the case of swirl motion.

## 1.2 Application: Simulation of In-Cylinder Flow

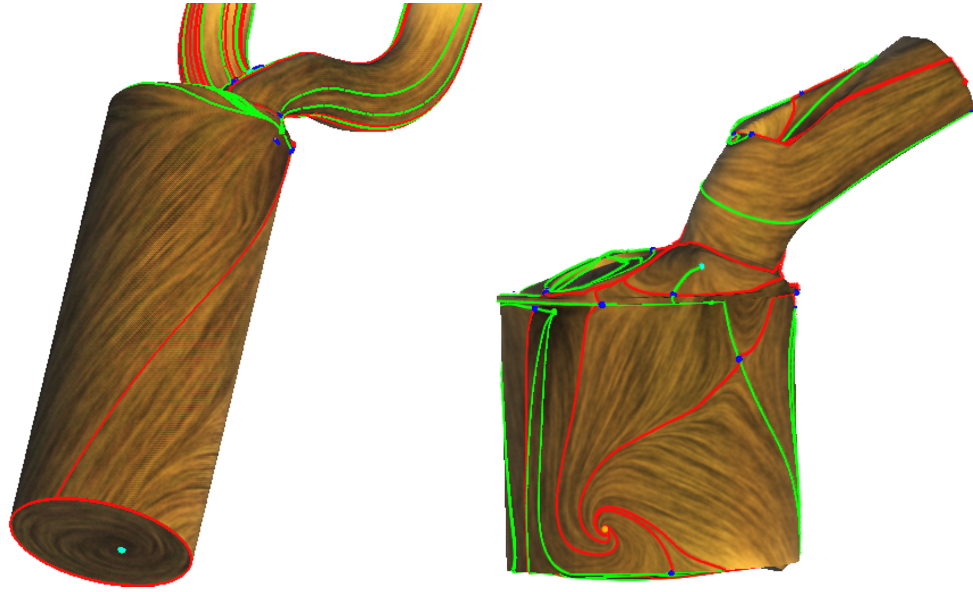
For flow entering and exiting a combustion chamber, the engineers responsible for the design try to create an ideal pattern of motion. The motion can be described as a swirling flow revolving around an imaginary, central axis residing inside the cylinder volume. One type of swirling motion, aptly called *swirl motion*, is depicted in Figure 1.1, left. The ideal swirl motion spirals around an axis aligned with the cylinder volume found at the center. Such an ideal is often strived for in diesel engines.

Another important pattern of flow is *tumble motion*, depicted in Figure 1.1, right. The axis of rotation in the tumble case is orthogonal to that of the swirl case. Also, the ideal motion is closer to a simple circle rather than a more spiral-like pattern. Since the axis of rotation is not aligned with the combustion chamber itself, this pattern of motion is more difficult to realize.

Achieving these ideal patterns of flow optimizes the mixture of oxygen and fuel during the ignition phase of the valve cycle. Optimal ignition leads to very desirable consequences associated with the combustion process including: more burnt fuel (less wasted fuel), lower emissions, and more output power.

### 1.2.1 Extraction and Visualization of Singularities and Separatrices

Extracting the singularities (or critical points) and related separatrices at the boundary of the geometry can provide valuable insight into the behavior of the flow directly, without the user having to search for the patterns of flow manually [15, 16, 17, 33]. Figure 1.2 shows the boundary topology, including singularities and separatrices for both cases of in-cylinder flow. In the case of the swirl motion associated with the diesel engine, the red separatrix at the boundary of the combustion chamber indicates a pattern of swirl motion consistent with the ideal shown in Figure 1.1, left [11, 32]. We can validate this to a certain extent by the addition of texture-based flow visualization, in this case using texture-advection approaches [30], to depict the characteristics of the entire flow at the boundary, not just the topological skeleton. In Figure 1.2 right, we see that the ideal pattern of tumble motion is not being realized. Instead of a single point-based recirculation zone directly in the middle of the combustion chamber, we see two dominant singularities: a saddle point in the upper, right-hand corner and a sink in the lower, left hand corner.



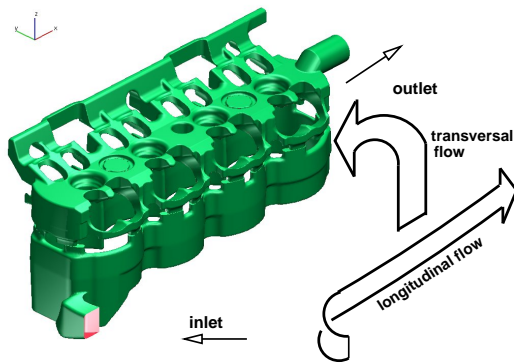
**Fig. 1.2.** (left) Swirl motion indicated by the helical separatrix at the boundary of the combustion chamber and (right) deviant tumble motion indicated by the boundary topology of the gas engine simulation. For the singularities, green = source, red = sink, orange = attracting focus, cyan = repelling focus, blue = saddle. Red separatrices end at an attractor (sink). Green separatrices end at a repeller (source).

### 1.2.2 Future Direction: Extraction of Arbitrary Flow Patterns

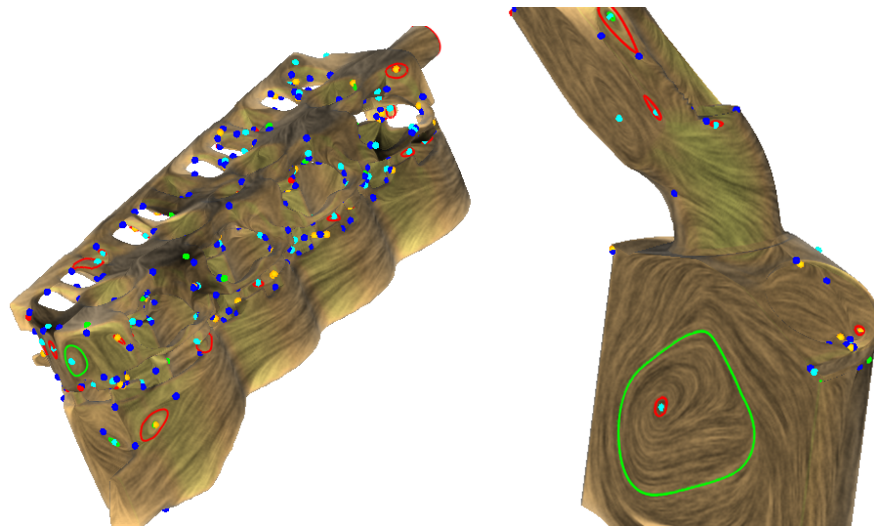
Based on our experience of trying to extract the most important features from in-cylinder flow [6, 11, 31, 32], we can make the following observation: Tools capable of extracting more arbitrary patterns of flow motion, in 2D, 2.5D, or 3D would be very helpful to the engineering analysis community. (By 2.5D we mean surfaces in 3D.) For example, in the case of swirl motion, the ability to extract a 3D helical pattern directly would be very useful. Ideally, the user could specify an arbitrarily shaped curve and search the vector field according to this set of user-specified geometry (and topology). This idea is inspired by the fact that most geometries from automotive engineering have an ideal pattern of flow that the engineers are trying to realize when designing their models [11, 27, 28, 31, 32].

### 1.3 Application: Heat Transfer

The job of a cooling jacket is to transfer heat away from the engine block of an automobile [31]. The cooling jacket has an extremely complex geometry. The model grid consists of over 1.5 million unstructured, adaptive resolution tetrahedra, hexahedra, pyramid, and prism volume elements, the volume of which differs by more than six orders of magnitude. There are two main components to the ideal pattern of flow through a cooling jacket: a *longitudinal* motion lengthwise along the geometry and a *transversal* motion from cylinder block to head and from the intake to the exhaust side. These two components are sketched in Figure 1.3. The location of the inlet and outlet are also indicated. Any flow that deviates from this ideal, essentially the most efficient volume-filling path from inlet to outlet, results in less transfer of heat away from the engine block.



**Fig. 1.3.** The major components of the flow through a cooling jacket include a longitudinal component, lengthwise along the geometry and a transversal component in the upward-and-over direction. The inlet and outlet of the cooling jacket are also indicated.



**Fig. 1.4.** (left) The flow topology, including 540 singularities and 140 periodic orbits extracted at the boundary surface of the cooling jacket. (right) Periodic orbits and singularities extracted at the boundary surface of the gas engine indicating tumble motion (green = source, red = sink, orange = attracting focus, cyan = repelling focus, blue = saddle)

### 1.3.1 Periodic Orbit Detection on Boundary Surfaces

Figure 1.4, left shows a novel application—the extraction of periodic orbits (also known as closed streamlines) at the boundary surface of the cooling jacket using the algorithm of Chen et al. [7]. The algorithm automatically extracts and visualizes closed streamlines, 140 total in this example. In this application, periodic orbits are very relevant because they indicate areas of flow recirculation. Recirculation zones are very important to the engineers studying the design of this engine component because they detract from the goal of transferring heat away from the engine block. Hence, one of the goals of the engineer is to minimize the number of recirculation zones. Rather than having to manually inspect the surface for recirculation, the user can now extract this circular pattern of flow (tumble) directly.

Figure 1.4, right shows another very useful application of the periodic extraction algorithm, namely, to the boundary surface of a gas engine. In this visualization, we can see a large green periodic orbit hinting at a recirculation zone that corresponds very well to the ideal tumble motion depicted in Figure 1.1, right.

### 1.3.2 Future Direction: Higher Dimensional Topology Simplification

The complexity of the result in Figure 1.4, left clearly motivates the need for automatic or semi-automatic simplification algorithms for boundary topology. Hundreds of topological elements, both singularities and periodic orbits complicate the visualization result possibly adding undesirable noise. A filtering operation based on parameters such as (1) the size of periodic orbits, (2) the distance between neighboring singularities, (3) an error threshold or (4) some other size/distance metric would be helpful to the engineer in order to filter out some of the smaller scale (or larger) scale singularities. A similar statement can be made for the case of 3D topology [31].

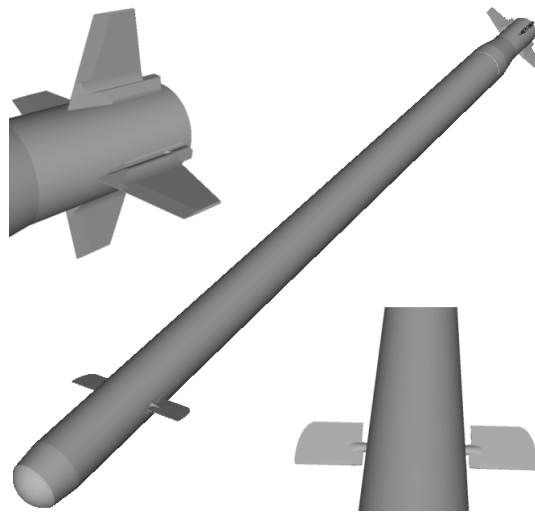
### 1.3.3 Future Direction: Further Development of Topological Methods for Unsteady Flow

Practitioners typically deal with unsteady flow. Natural phenomena are time-dependent. Yet, topology-based flow visualization methods are still not fully understood in the context of time-dependent flow. For example, the interpretation of a separatrix for unsteady flow remains unclear. Separatrices are curves that segment the flow into different regions of asymptotic behavior. In the case of unsteady flow pathlines appear to be a natural choice (as opposed to streamlines) for separating the flow into regions of similar asymptotic behavior. In 3D, pathsurfaces appear to be a natural choice but very little work on pathsurfaces has been presented. Streaklines may also be a natural choice for separating unsteady flow in 2D, but what about 3D? Are “streaksurfaces” a better choice? To our knowledge, no work on streaksurfaces has been done. What kind of separating behavior is most relevant in the case of unsteady flow? What is the best approach to segmenting the different regions of flow in the case of unsteady flow? Questions such as these are only starting to be addressed [45].

Another problem lies in periodic orbit detection. Consensus lacks on whether periodic orbit detection makes more sense in the context of steady versus unsteady flow. One can argue that in fact, periodic orbit visualization is misleading in the case of steady-state flow based on the argument that no such paths exist in reality. This is because truly steady-state flow does not exist if we define steady-state flow as instantaneous in time. On the other hand, periodic orbits are unlikely to be detected in unsteady flow because spatio-temporal behavior would have to remain identical over all cycles. Singular orbit detection may make more sense in the context of unsteady flow. Further development in this direction would also make topology-based methods more appealing to practitioners.

## 1.4 Application: Spinning Missile

The spinning missile with dithering canards is representative of the type of complex geometry routinely used in vehicle performance simulations. Several properties of the spinning missile geometry contribute to the complexity of the vortical flow surrounding it [5]. The missile is at a three degree angle with respect to the supersonic incident flow. Fixed, canted tail fins cause the missile to spin about its longitudinal axis at a rate of 8.75hz (Figure 1.5, upper left). The missile’s dithering canards provide pitch and yaw control by rotating about their attachment posts (Figure 1.5,



**Fig. 1.5.** Missile geometry: Canted tail fins (upper left) cause missile to spin about its longitudinal axis. Canards (lower right) rotate synchronously about axis passing through the missile body to provide pitch and yaw control.

lower right). To describe a single rotation of the missile, 360 time steps are employed. The canards complete several dither cycles during one missile rotation. The flow has some degree of periodicity, but due to the missile's nonzero angle of attack and spin, this period does not match the dither cycle. The simulation was performed on a mixed element mesh consisting of more than 35 million elements.

The engineers studying the missile were initially interested in whether the vortices coming off the canard tips impinged on the tail fins. Feature based vortex visualization was used to answer this question.

#### 1.4.1 Feature-Based Vortex Visualization

The vortex visualization method of Jankun-Kelly et al. [18] extracts vortex topology from computational fluid dynamics (CFD) data. Vortices can be topologically characterized by their core line and extent. The vortex core line is the curve about which streamlines swirl in a reference frame moving with the vortex. Please note this method uses **vortex topology**, not **vector field topology** (critical points, separatrices). This method exploits a technique to extract the vortex core lines from the line-type extrema of scalar fields and a novel k-means clustering algorithm to identify topologically complex vortical structures. The extent of a vortex is the boundary surface of the vortex core region [12]. Given core line and extent, additional vortex characteristics can be found. They include the sense of rotation, various measures of strength, and field values at the core.

This method has several strengths. Vortex detection is automatic rather than manual/interactive. Vortices can be extracted from practical engineering data that may be noisy, unstructured, and not resolved as well as we might like. Individual vortices of various strengths can be identified even in complex flows of multiple, interacting and merging vortices. Low level, large CFD data is distilled into compact, feature level vortex characteristic data such as core line, extent, and strength. These compact vortex detection results can be visualized interactively. All these properties of our method were requested by users.



### 1.4.2 Insight From Vortex Visualization

We have produced an animated visualization of the time varying vortical flow about the spinning missile. Several still images from the animation can be seen in Figure 1.6. Vortex core line and associated extent surface image pairs are shown from top to bottom, respectively. The color of each core line indicates rotation, clockwise or counterclockwise with respect to the local axial velocity. The extent surface is shaded by local tangential velocity, an indicator of vortex strength. This visualization allowed the engineers to answer their question: did canard tip vortices impinge on the tail fins and how strong were the vortices that did impinge. Other interesting vortex behavior was also observed. As the canards dithered, the strengths of their trailing vortices changed. This appears as a change in the shading of vortex extent. At certain roll angle / dither cycle combinations, each canard tip vortex changed its sense of rotation. This can be seen at time step 730 (Figure 1.6, third row from the top). The animation also revealed that vortices shed from the posts connecting the canards to the missile body traveled toward the tail fins. Engineers had originally been interested in canard tip vortices, but after seeing the vortex visualization animation, they decided that the post vortices merit further investigation. These insights were made possible by the topological feature extraction capabilities of this visualization method.

### 1.4.3 Future Direction: Noise Mitigation and Reduction

Noise is often present in the data we are given. We must work with the data we have, even when it is not quite ideal. This method is specifically designed to handle noisy data because doing so improves the quality of the visualization. More work needs to be done in the areas of noise reduction and mitigation. The two most pressing problems caused by noisy data are noisy core lines and occasional  $C^0$  core line discontinuity. For feature level applications, complete core lines are preferable to core line pieces. The vortex core line is needed to compute all other vortex characteristics. Less noise in the core line would make extent computation more robust. Therefore, improving core line quality would have a significant impact on the overall quality of the results.

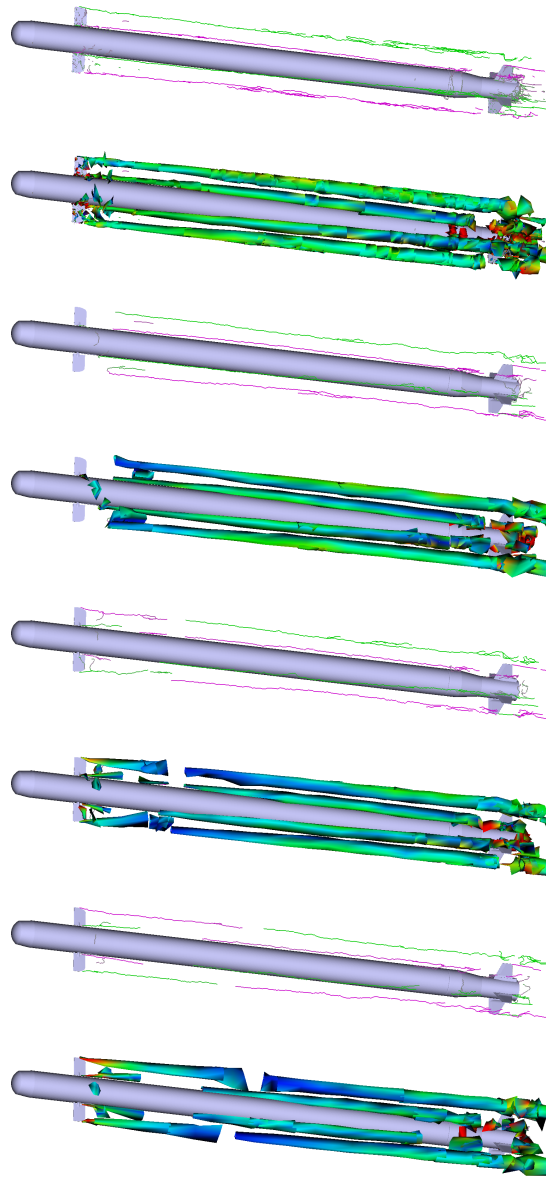
### 1.4.4 Future Direction: Improved Extent Computation

Several challenges need to be overcome in order to make extent computation more accurate and robust. Extent outliers, which appear as jagged spikes or indentations, are visually jarring inaccuracies. They can be removed by smoothing the entire extent, but this adversely affects extent accuracy. More work is needed to identify and locally repair extent outliers. A better extent model is needed. The one currently used works well for isolated vortices but has more difficulty with multiple, interacting vortices.

Further, the extent computation needs to be made Galilean invariant, which could be accomplished by a shift of reference frame to one traveling with the vortex if the translational motion of the vortex were known. It should be noted that, for nonstationary vortices, the fluid velocity at the vortex core will not be the velocity of the vortex core, thereby making Galilean invariance difficult to achieve. These enhancements would facilitate application of the feature based vortex visualization method to a wider range of data.

### 1.4.5 Future Direction: Feature Level Verification by Determining Whether a Detected Feature is a Vortex

Feature level verification is needed to automatically eliminate false positives, a task currently done manually. Feature level verification would reduce visual clutter and



**Fig. 1.6.** Vortex core lines and extent for spinning missile with dithering canards for time steps  $t=726, 728, 730,$  and  $732$  (top to bottom). Purple indicates a left handed rotation while green indicates a right handed rotation. Alternate images show vortex extent with surface colored by the local tangential velocity. The scale on the non-dimensional tangential velocity is blue (0) to red (0.189).

make visualization more meaningful and easier to interpret. All methods that detect vortices by searching for line-type local extrema suffer from false positives. This is because local extrema are a necessary but not a sufficient condition for the existence of vortices. Swirling flow must also be present. After detecting possible vortex core lines, Jiang et al. verified vortices by checking for the presence of streamlines that

made at least one complete revolution about the vortex core line [20]. The limitation of this vortex verification approach is that it is not Galilean invariant. Please note that the preceding vortex detection can be Galilean invariant.

#### **1.4.6 Future Direction: Vortex Tracking for Unsteady Flow**

This method could be further enhanced through the incorporation of vortex tracking. Vortex tracking would provide several benefits. Core line discontinuities, which should not be present, could be identified by comparing vortices at neighboring time steps. Vortex core velocity, needed for Galilean invariance, could be determined. Tracking could be another means of feature verification since vortices tend to persist over time, while some false positives are more transient. Tracking would permit rule mining, the discovery of rules governing vortical flow, leading to greater scientific insight [38].

### **1.5 Application Independent Directions**

Here we describe some general future directions for bringing topology-based visualization to the industry domain, independent of a specific application.

#### **1.5.1 Future Direction: More Accessible Implementations**

In general, topological extraction methods are complex and difficult to implement. For an example, see Section 1.8 (Appendix) which shows the pseudocode of the periodic orbit extraction algorithm [7]. We believe that the success of some algorithms, e.g., Marching Cubes [37] or simple particle tracing is a result of an accessible implementation. Graceful and easier implementations for topological analysis, extraction, and visualization would help in bringing this class of methods to the domain experts. Alternatively, if the visualization community provided engineers with source code or pre-compiled software, they would be more likely to use our methods than in the case of having to implement from scratch. However, perhaps some methods can only be algorithmically simplified to a certain extent before they become ineffective. In general, users would rather not re-implement methods. They prefer off the shelf software with an easy to use and intuitive UI.

#### **1.5.2 Future Direction: Faster Computation Time**

Clearly one aspect that would make topology-based flow visualization more attractive to practitioners is faster computation. In general, literature on feature-based flow visualization does not report performance times [33]. If performance times are not reported, there is usually a reason. Topology-based methods that are fast, or even interactive, would certainly enhance their attractiveness to those outside the visualization community.

### **1.6 Summary and Conclusions**

Despite the recent advances in topology-based flow visualization research, topological methods are still generally not found in commercial software packages. We have presented a range of topological analysis and visualization tools and their application to real-world problems. Included is a novel application—the extraction and visualization of periodic orbits at the boundary surface of a cooling jacket. Drawing on this experience, we presented a list of future work in this area necessary in order to bring topology-based flow visualization to the application domain. We have identified the following tasks:

1. Extraction of Arbitrary Flow Patterns: the ability to extract and visualize user-defined patterns from flow
2. Higher Dimensional Topology Simplification: methods that can either automatically or semi-automatically simplify topology on boundary surfaces and in 3D
3. More Accessible Implementations: implementations of extraction methods which are easier and more accessible to the engineers that must write them
4. Faster Performance: implementations that are interactive, or nearly interactive, would be ideal
5. Further Development of Topological Methods for Unsteady Flow: a greater understanding of topology-based methods in the context of unsteady flow is needed
6. Noise Reduction and Mitigation: the development of topology-based methods which are less sensitive to noisy data
7. Improved Extent Computation: accurate methods to handle outliers in the data
8. Feature Level Verification: automatic methods for the elimination of false positives
9. Vortex Core Tracking for Unsteady Flow: including a formal definition of a vortex
10. Improved Dissemination: a better transfer of knowledge from the visualization community is necessary.

Lack of communication between communities is also a problem. Currently there is quite a gap between the visualization research community and prospective users. In fact, other communities such as the engineering analysis community [29], are not even aware that a visualization community exists. Visualization scientists need to explain what tools and techniques are available and how they can be used to solve problems in science and engineering. Practitioners could also explain why they would like to visualize their data and what questions they're trying to answer. Closely related is the lack of inter-community knowledge transfer and a lack of educational literature.

We believe this discussion may play a role in steering future work in this field to the point where topological methods may even be included in industry-grade software. Certainly, a lot of work remains for topology-based methods to spread beyond the visualization community.

## 1.7 Acknowledgements

We would like to thank Ming Jiang, Raghu Machiraju, Christoph Garth, and Helwig Hauser for their valuable contributions. We also wish to thank Konstantin Mischaikow and Greg Turk for their valuable suggestions. We especially thank Harry Yeh in the Coastal and Ocean Engineering Department at Oregon State University for his very valuable user feedback on the paper.

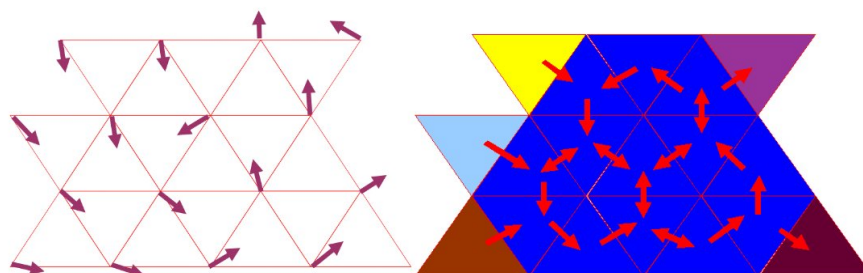
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**Fig. 1.7.** (An example for building a directed graph based on the input vector field defined on a triangular mesh. In the obtained directed graph, each node refers to a particular triangle, the direction of each directed edge are determined by the type of the edge. Based on the input vector field (left), we build the directed graph and compute the strongly connected components in the graph (right).

## 1.8 Appendix: Periodic Orbit Extraction Implementation

**Function:** extractPeriodicOrbit

**Input:** Mesh, vector field  $V$

**Output:** a set of periodic orbits

```
Directed_graph = buildDirectedGraph(Mesh, V); // see figure
SCC_list = Kosaraju-Sharir_algorithm(Directed_graph);
for i = 1 to number of SCC in SCC_list
    cur_SCC = SCC_list[i];
    if (probablycontainPeriodicOrbit(cur_SCC))
        a set of attachment points S = calculateAttachmentPoints(cur_SCC);
        a set of separation points A = calculateSeparationPoints(cur_SCC);
        // for finding attracting periodic orbits
        findPeriodicOrbit(V, A, cur_SCC);
        // for finding repelling periodic orbits
        findPeriodicOrbit(-V/* inverse of vector field of V */, S, cur_SCC);
    endfor
Return a set of detected periodic orbits
```

**Function:** findPeriodicOrbit

**Input:** Vector field  $V$ , set of points  $P\_list$  on the edges, SCC

```
for i = 1 to number of points in P_list
    cur_P = P_list[i];
    trace from cur_P.
    if (failToFindPeriodicOrbit(cur_SCC))
        continue;
    calculate the periodic orbit
    // This lets the current curve judge whether it is close to the detected
    // periodic orbit
    place dense markers on orbit
    add the detected periodic orbit to periodic orbit list
endfor
```

**Function:** failToFindPeriodicOrbit

**Input:** an SCC

**Output:** true or false



```
if tracing curve reaches a singularity
  Return false
if tracing curve leaves the cur_SCC
  Return false
if tracing curve reaches a previously detected periodic orbit
  Return false
Return true;
```

**Description:** Our periodic orbit detection method is inspired by Wischgoll and Scheuermann [52], in which they locate periodic orbits in a planar vector field by starting streamline tracing from a neighborhood of a singularity and keeping track of repeated cell cycles. While this method is capable of detecting many periodic orbits, it assumes that any periodic orbit can be approached by a singularity, which is not always true.

The basic idea of our approach comes from the observation that a periodic orbit must situate inside a region of flow recurrence, which corresponds to certain types of strongly-connected components in the domain. The contribution of our approach is to provide a way to find the starting point to start tracing for locating the cell cycle that can be converged to the desired cell cycle much quicker than previous methods. More details are described by Chen et al. [7].