Further Mathematics and the transition between school and university mathematics

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Abstract

This paper explores the motivations and barriers underlying student choice in Further Mathematics A-level and the implications of this choice on transition to mathematical university courses. The study used both questionnaires and individual interviews to access the opinions of undergraduate students. We identify several factors influencing student choice which include the attitudes of the students, their teachers and their parents, to both mathematics as a subject, and to Further Mathematics as a qualification. We show that studying Further Mathematics is beneficial to both mathematics and mathematics-related degrees at university and that many of the undergraduate students who had not chosen the qualification, felt they could have, and indeed now wished they had. However, we also show that Further Mathematics does not suitably prepare students for the rigorous approach to mathematics encountered in a mathematics degree. Our findings also suggest that studying through the Further Mathematics Support Programme, attending mathematics outreach events and studying mathematics extension papers may contribute to improved transition.

1. Introduction

Students need a strong foundation of mathematical skills to make a successful transition from school to studying science, technology, engineering and mathematics (STEM) disciplines at university. However, the dual nature of mathematics as both a subject in its own right, and a tool for use in other disciplines, creates difficulties in designing a suitable curriculum. The aim of increasing participation in STEM subjects has led to a need to broaden the appeal of mathematics courses at schools. In attempting this, the depth of the mathematical content of post-16 qualifications has been gradually replaced by a breadth of material, which has weakened the ability of these qualifications to suitably prepare students for studying mathematical subjects at university (Hoyles, Newman and Noss, 2001).

Given this background, it is not surprising that transition from school to undergraduate mathematics remains a focus in the ongoing reforms of mathematics education in England and Wales (see e.g., ALCAB, 2014). Moreover, it is now acknowledged that schools alone cannot resolve these issues and universities have been urged to put in place appropriate support mechanisms to ease student transition (Hodgen, McAlinden and Tomei, 2014).

The majority of students in England and Wales who apply to university have studied for GCE A-level qualifications. There are currently two mathematics qualifications available: 'Mathematics' and the more advanced 'Further Mathematics'. In this paper, we look at the specific role played by the Further Mathematics A-level qualification in the transition of students from school to both mathematics and non-mathematics STEM degrees.

2. Further Mathematics

At present Mathematics is the most popular A-level choice with 11% of all A-level students awarded the qualification in 2016. A recent study (Noyes and Adkins, 2016) identified that the growth in participation could be due to a broadening of the "pipeline" of students coming through the system thanks to the growing proportion of mathematics GCSEs awarded higher grades. However, in contrast, Further Mathematics remains one of the least popular options with only 1.8% of students studying the subject in 2016. This has been in part due

to difficulties experienced by schools in providing the Further Mathematics qualification (see, e.g., Tanner, Lyakhova and Neate (2016)).

To address these difficulties, the Further Mathematics Support Programme (FMSP) was established in 2003 in England and in 2010 in Wales. The FMSP provide an opportunity for students to study Further Mathematics (via online and face-to-face tuition) when it is not offered by their school/college. They also support provision in schools by offering professional development for teachers. While only a small proportion of all Further Mathematics students take their courses with the FMSP (e.g., 60 students are studying with the FMSP in 2016/17 in Wales which is approximately 12% of the number of A-level Further Mathematics qualifications awarded in Wales in 2016), the impact on the uptake of the subject in England and Wales has been significant. The proportion of A-level Mathematics students who were also awarded A-level Further Mathematics increased from 11% in 2003 to 17% in 2016 in England and from 7% in 2010 to 13.5% in 2016 in Wales. However, the FMSP tuition expects a lot from students in terms of workload and self-study which may indirectly contribute to narrowing the "pipeline" of Further Mathematics students. Thus, it has been argued that whenever possible students should be offered Further Mathematics as a fully timetabled option (see Tanner, Lyakhova and Neate (2016) for a discussion).

3. Methodology

In phase one of the study a survey was conducted of 377 undergraduate students from 7 universities across England and Wales. Students were either studying for a mathematics degree (241 students of whom 138 had studied Further Mathematics) or a non-mathematics STEM degree (136 students of whom 35 had studied Further Mathematics).
For the mathematics students, the universities were categorised based on their mean entry standard for mathematics degrees (taken from the Complete University Guide 2017) into three groups:

Group 1: mean entry greater than 480 UCAS points (equivalent to AAAA);
Group 2: mean entry between 340 and 480 UCAS points (AAB – AAAA);
Group 3: mean entry below 340 UCAS points (below AAB).

Of those studying for a mathematics degree, there were 54 participants in group 1 (50 of whom had studied Further Mathematics), 125 in group 2 (66 with Further Mathematics) and 62 in group 3 (22 with Further Mathematics). For reference, of the 61 universities in England and Wales classified for mathematics in the Complete University Guide 2017, 9 fell into Group 1, 24 into Group 2 and 28 into Group 3. Students who had studied for at least an AS level in Further Mathematics were classified as having participated in Further Mathematics. The survey contained a series of attitude statements on the choices made by the students at both A-level and university, and their feelings about their transition to university. These used a 5 point Likert scale from strongly agree to strongly disagree.

In phase two, 13 students from 8 universities who were studying for either a mathematics degree or a non-mathematics STEM degree were interviewed. The universities involved in phase 2 were not all involved in phase 1.

4. Results

Where statistical comparisons are made between groups, non-parametric tests have been used as the Likert scale data from the survey is ordinal. Mann-Whitney tests have been used for comparisons between two groups and Kruskal-Wallis tests for those between three groups.

The results given in this section only contain a selection of the overall findings of the study, which will be published elsewhere.

4.1. Mathematics Undergraduates

In terms of the decision to study for Further Mathematics, there are significant differences between those who had chosen to study Further Mathematics when compared with those who had not, with students who took Further Mathematics more likely to view themselves as gifted at mathematics, more likely to rate mathematics as their best subject, more likely to have had encouragement from teachers and parents to study mathematics, and more likely to view Further Mathematics as a route to university (see Table 1). This suggests that a student's confidence in their own mathematical abilities and the university they aspired to apply to, influenced their choice of Further Mathematics. It also reflects the importance of the attitudes of their parents and teachers. This reaffirms results from a study of school students (Tanner, Lyakhova and Neate, 2016).

Table 1. Selected significant differences in Mathematics degree students between those who had and had not studied Further Mathematics (Mann-Whitney, two tailed, α=0.05).

	Further Mathematics	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size	P
I am naturally gifted at mathematics.	Yes	26%	59%	12%	3%	1%	138	<0.001
	No	14%	47%	31%	9%	0%	103	
Mathematics is the subject I am best at.	Yes	60%	31%	6%	2%	2%	137	0.003
	No	42%	40%	14%	5%	0%	103	
Teacher(s) encouraged me to study AS/A-level mathematics.	Yes	60%	25%	9%	4%	1%	138	<0.001
	No	38%	36%	13%	8%	6%	103	
My parents thought it was important for me to study Further Mathematics.	Yes	12%	19%	51%	12%	7%	138	<0.001
	No	1%	11%	57%	23%	8%	101	
Having a qualification in Further Mathematics helps you to get into a better university.	Yes	21%	53%	14%	10%	2%	138	<0.001
	No	12%	33%	36%	18%	2%	101	

Of those who did not choose to study Further Mathematics, it was not a lack of ability that influenced the majority (only 5% felt they were not good enough), rather 56% wished to study other subjects and there was a perception among some students that Further Mathematics is harder and required more self-study than other subjects (see Table 2a).

Students who did study Further Mathematics felt that it had broadened their understanding of mathematics and had helped in the transition to university (see Table 2b). Indeed 72% of the students who had not studied Further Mathematics now wished that they had done so. There was no significant difference (Kruskal-Wallis, two tailed, α=0.05) across the 3 university groups in any of the questions in Table 2b, suggesting that Further Mathematics is beneficial for all mathematics undergraduates, not just those striving to attend the most selective universities. In interviews, students who had studied Further Mathematics were clear

that the advantage was in the additional topics covered, in particular matrices, complex numbers and vectors. However, 45% of all students have to work hard at mathematics "just to keep up", and there is no evidence that taking Further Mathematics affected this, with no significant difference between those who had studied Further Mathematics and those who had not (Mann-Whitney, two tailed, α=0.05, see Table 2 part c).

Table 2. Selected attitudes of mathematics undergraduate students.

Survey Statements	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size
a) Looking back on A-Level studies:						
Students who had not studied Further Mathematics:						
It was more important for me to study a range of subjects than to study Further Mathematics.	11%	45%	27%	17%	1%	103
I wasn't good enough to do Further Mathematics.	0%	5%	35%	46%	14%	102
There is a step up in difficulty between Mathematics and Further Mathematics.	12%	28%	56%	4%	0%	96
Further Mathematics requires more self-study than other subjects.	7%	22%	59%	9%	2%	96
b) Considering Transition to university studies:						
Students who had not studied Further Mathematics:						
I wish I had studied Further Mathematics.	36%	36%	18%	5%	5%	102
Students who had studied Further Mathematics:						
Studying Further Mathematics gave me a broader understanding of mathematics in general.	39%	50%	9%	2%	0%	135
Studying Further Mathematics made the transition to university level work easier.	46%	44%	8%	2%	1%	136
c) Studying at university:						
All Students:						
I have to work hard in mathematics just to keep up.	8%	37%	32%	21%	2%	240
I find it easier to memorise mathematical techniques than try to understand them.	8%	24%	27%	32%	10%	240

There was also no evidence that Further Mathematics developed the approach of students to mathematics as a subject, with no significant difference between the students who had and had not studied Further Mathematics in their opinions on using memorisation rather than trying to understand the material, (Mann-Whitney, two tailed, α=0.05). Here 32% of all students still found it easier to memorise the material. This was further confirmed by the interviews where several students commented on the new learning approach required by university for which they felt completely unprepared by A-levels. The students referred to the focus on the use of routine procedures and factual recall in both A-level Mathematics

and Further Mathematics while the more abstract and proof-based undergraduate mathematics was described as focussing on the application of conceptual knowledge to construct mathematical arguments. In particular, some of the interviewed students did not find anything in A-level Mathematics or Further Mathematics that helped prepare them for first year undergraduate courses in Analysis. Such courses were different from anything they had seen before, but they struggled to explain why and how.

In the interviews, the students who sat extension mathematics papers required by some selective universities, commented very strongly on the difference these made alongside attending mathematics enrichment events offered by universities. While Further Mathematics helped in terms of the extra content, it was the extension papers that exposed them to problem solving as is embedded into studying mathematics at university. In their opinion, learning as much mathematics beyond the syllabus as possible was the best preparation for university mathematics.

4.2. Non-Mathematics STEM Undergraduates

In terms of the original decision to study for Further Mathematics, there were significant differences in attitudes similar to mathematics undergraduates. Students who took Further Mathematics are more likely to view themselves as gifted at mathematics, more likely to rate mathematics as their best subject and more likely to have had encouragement from teachers and parents to study mathematics (see Table 3).

Table 3. Selected significant differences in non-Mathematics STEM undergraduate students between those who had and had not studied Further Mathematics (Mann-Whitney, two tailed, α=0.05).

Survey Statements	Further Mathematics	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size	P
I am naturally gifted at mathematics.	Yes	20%	66%	14%	0%	0%	35	<0.001
	No	4%	45%	36%	12%	4%	101	
Mathematics is the subject I am best at.	Yes	34%	26%	23%	17%	0%	35	0.016
	No	11%	34%	27%	22%	6%	100	
Teacher(s) encouraged me to study AS/A-level mathematics.	Yes	57%	23%	20%	0%	0%	35	0.011
	No	32%	40%	13%	9%	7%	101	
My parents thought it was important for me to study Further Mathematics.	Yes	11%	9%	54%	20%	6%	35	0.003
	No	0%	5%	47%	33%	15%	99	

Similar to mathematics undergraduates, a majority of those who chose not to study Further Mathematics did so to enable them to study other subjects. Again, there was a perception that Further Mathematics was harder (see Table 4a). Of those who had taken Further Mathematics, 91% felt it had broadened their understanding and 85% felt it had made their transition to university easier (Table 4b). Again, there was no significant difference (Mann-

Whitney, two tailed, α=0.05) in the answers in Table 4c between those who studied Further Mathematics and those who did not.

In contrast to the mathematics undergraduates, the emphasis from interviews of students studying degrees as varied as Computer Science, Earth Sciences, Economics and Engineering, was that studying Further Mathematics had been useful due to the fast pace and advanced nature of the mathematics they were expected to learn and rapidly apply in their degree course. Such an approach requires students to pick up techniques and methods quickly, without dwelling on understanding. As one student put it, such a module would "not teach you the maths that underpins it. You know what to do but do not know why", an approach similar to that often taken at A-level. This could explain why Further Mathematics helped with the mathematical content as well as enabling them to focus on the non-mathematical aspects of their course while also giving them the confidence to pick more mathematical optional modules.

Table 4. Selected attitudes of non-mathematics STEM undergraduate students.

Survey Statements	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size
a) Looking back on A-Level studies:						
Students who had not studied Further Mathematics:						
It was more important for me to study a range of subjects than to study Further Mathematics.	14%	47%	28%	9%	2%	99
I wasn't good enough to do Further Mathematics.	7%	20%	32%	30%	11%	100
There is a step up in difficulty between Mathematics and Further Mathematics.	13%	37%	47%	2%	0%	97
Further Mathematics requires more self-study than other subjects.	6%	27%	56%	11%	0%	97
b) Considering Transition to university studies:						
Students who had not studied Further Mathematics:						
I wish I had studied Further Mathematics.	12%	31%	23%	26%	9%	101
Students who had studied Further Mathematics:						
Studying Further Mathematics gave me a broader understanding of mathematics in general.	44%	47%	9%	0%	0%	34
Studying Further Mathematics made the transition to university level work easier.	50%	35%	15%	0%	0%	34
c) Studying at university:						
All Students:						
I have to work hard in mathematics just to keep up.	10%	39%	32%	19%	0%	136
I find it easier to memorise mathematical techniques than try to understand them.	10%	31%	21%	27%	10%	136

4.3. The Further Mathematics Support Programme and Student choice

In the survey sample, there were 173 students who had studied Further Mathematics, 18 of whom had studied through the FMSP. We found significant differences in the opinions of the FMSP students when compared to those that studied Further Mathematics in school (see Table 5), although the sample of FMSP students is small. The FMSP students are more likely to enjoy mathematics, more likely to think of mathematics as their best subject and more likely to have parental support. But the most striking finding was that students who use the FMSP also appear to have gained more from the experience as they are more likely to feel it broadened their understanding of mathematics in general.

In our opinion, the findings reflect the nature of the FMSP experience. The delivery model involves less traditional classroom teaching time, replacing this with independent study and online teaching. The interviewed students consistently reflected that the structure of the FMSP courses were closer to the university learning experience. While studying with the FMSP was harder, the students recognised the added value of the approach. However, to cope with such an approach students not only need to be good at mathematics but also need a supportive parental environment and the motivation to invest their own time and organise their own work.

Table 5. Selected significant differences in students between those who studied Further Mathematics using the FMSP and those who studied it in school. (Mann-Whitney, two tailed, α=0.05).

Questions	FMSP	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree	Sample Size	P
Studied Further Mathematics:								
I enjoy doing mathematics.	Yes	83%	11%	6%	0%	0%	18	0.008
	No	47%	48%	2%	2%	1%	154	
Mathematics is the subject I am best at.	Yes	89%	6%	6%	0%	0%	18	0.004
	No	51%	33%	10%	5%	1%	154	
My parents thought it was important for me to study Further Mathematics.	Yes	28%	22%	50%	0%	0%	18	0.006
	No	10%	16%	52%	15%	7%	155	
Studying Further Mathematics gave me a broader understanding of mathematics in general.	Yes	77%	24%	0%	0%	0%	17	0.001
	No	35%	53%	10%	2%	0%	152	

5. Discussion and Conclusions

Studying Further Mathematics helps the transition of students to a range of STEM degrees. In non-mathematics degrees, the extra content helps students to become better grounded in the mathematical theory before learning about its applications, and also gives them the confidence to continue studying more mathematics. For mathematics undergraduates, while there is an advantage in terms of the additional knowledge gained from studying Further

Mathematics, the interviews suggest that this does not fully prepare them for nature of mathematics degrees. There is evidence that the FMSP approach to teaching contributes to improved study skills but may also narrow the pipeline of the Further Mathematics students. As such, this approach provides a highly valuable experience but one which is not suitable for every student. We suggest that the study of Further Mathematics should be promoted to a much wider range of students beyond the best mathematicians. Moreover, while a balanced mathematics curriculum at post-16 may be difficult to achieve, schools and colleges should consider how to enrich the experiences of those students who intend to study mathematics at university by exposing them to mathematics "beyond the syllabus", including extension papers. Given that schools and colleges do not always have a capacity to do this, linking Higher Education mathematics departments with sixth forms may be beneficial.

Our findings suggest several avenues of further research. Firstly, we want to know why studying with the FMSP provides students with a broader understanding of mathematics. Is it the format of the FMSP courses, the different teaching styles or the students involved? Secondly, studying extension papers such as STEP was associated in the interviews with a change in attitudes to mathematics as a subject. But it is unclear how and why (although see the argument in Darlington (2015) on the benefits of extension papers). Do such courses prepare students better for studying courses like Analysis? Lastly, do university outreach events help with the transition to university? This was identified as important factor in interviews with the STEP students, but there is no obvious evidence why and how this helps.

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