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Calculable mass hierarchies and a light dilaton from gravity duals

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ABSTRACT

In the context of gauge/gravity dualities, we calculate the scalar and tensor mass spectrum of the boundary theory defined by a special 8-scalar sigma-model in five dimensions, the background solutions of which include the 1-parameter family dual to the baryonic branch of the Klebanov–Strassler field theory. This provides an example of a strongly-coupled, multi-scale system that yields a parametrically light mass for one of the composite scalar particles: the dilaton. We briefly discuss the implications of these findings towards identifying a satisfactory solution to both the big and little hierarchy problems of the electro-weak theory.

1. Introduction

Many extensions of the Standard Model (SM) of particle physics are motivated by the (big) hierarchy problem. New dynamics and symmetries stabilize the electroweak scale, leading to the expectation that new particles should appear just above it. But such particles have not been detected experimentally in direct nor indirect searches. The little hierarchy between the mass of the Higgs and the new particles demands an explanation.

QCD dynamically explains the insensitivity to high-energy scales of the pion decay constant. New strong dynamics might replicate such success in the electro-weak theory. However, besides the calculability limitations of a strongly-coupled theory, the discovery of the Higgs particle [1] exacerbates the little hierarchy problem in such scenario, as one would have expected a proliferation of bound states to appear above the electroweak scale.

Fig. 1 provides a pictorial representation of the little hierarchy problem, by showing the SM mass spectrum, the current range of bounds from direct searches for exotica from the ATLAS collaboration, and the mass spectrum of a generic, hypothetic strongly-coupled new theory that evades them. Current bounds range from 570 GeV (Higgs triplet) [2] to 6.58 TeV (Kaluza–Klein graviton) [3], and (coarse-grained over the details) this range represents the LHC reach. The spectrum is model dependent, but consists of infinitely many bound states of all spins. If the Higgs scalar and the new physics have a common strong-coupling origin, and if the lack of evidence for new physics is confirmed, the anomalous suppression of the mass of the Higgs particle must also arise dynamically.

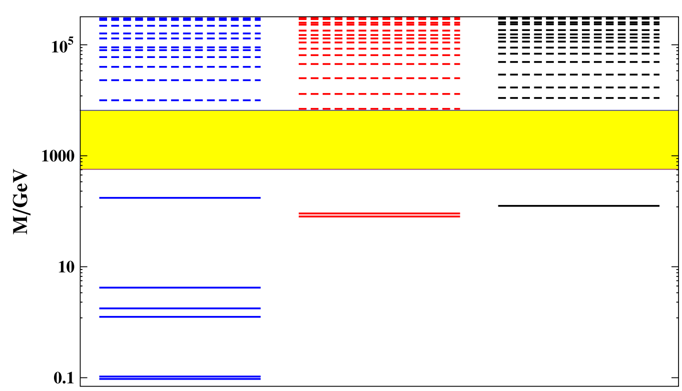

Fig. 1. The mass spectrum of SM particles (continuous lines) and of a generic strongly-coupled new theory (dashed lines) with new states heavy enough to evade the bounds from LHC direct searches (shaded region) [2,3]. Fermions are rendered in blue, vectors in red and scalars in black. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

To make strongly-coupled models viable, it is imperative to find an example of a strongly-coupled, four-dimensional theory, no matter what the microscopic origin, that exhibits one scalar state parametrically lighter than the plethora of bound states. This possibility arose long ago within walking technicolor [4], and has been discussed at length in many contexts since [5–9], including Randall–Sundrum models stabilized à la Goldberger–Wise [10–12], suggesting that the gravity dual of a theory with a moduli space

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Table 1
The field content, in terms of chiral superfields, and its classical symmetries [15]. $SU(M) \times SU(M+N)$ is the gauge group ($M = kN$). An additional Z_2 symmetry exchanges $A \leftrightarrow B$ and conjugates the gauge fields.

	$SU(M)$	$SU(M+N)$	$SU(2)_A$	$SU(2)_B$	$U(1)_B$	$U(1)_R$
A_α	M	$\overline{M+N}$	2	1	+1	+1/2
B_α	$\overline{M}$	$M+N$	1	2	−1	+1/2

might provide a concrete realisation of a system in which enhanced condensates and hierarchies of scales emerge. The underlying dynamics is approximately scale invariant, and condensates induce spontaneous symmetry breaking, yielding a light scalar particle in the spectrum: the dilaton.

In this paper, we provide a calculable example of a strongly-coupled theory that realizes this scenario, although it does not implement electro-weak symmetry breaking. Calculability is provided by the regular background in dual (super-)gravity [13,14]: the baryonic branch of the Klebanov–Strassler (KS) system [15, 16]. We compute the spectrum via the gauge-invariant fluctuations of the background in its 5-dimensional sigma-model description [17–20]. We refer to [21] for technical details. We report the results, discuss their origin, potential applications, and limitations.

2. The baryonic branch of KS: field theory

The four-dimensional $\mathcal{N}=1$ supersymmetric theory is discussed for example in [22–26]. It has gauge group $SU(M) \times SU(M+N)$, with $M = kN$ (for k integer), and bifundamental matter fields (see Table 1).

The superpotential contains the nearly-marginal[1]:

$$W = h\mathrm{Tr}\left[A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right] . \tag{1}$$

The moduli space of the theory contains a baryonic branch. Following [23,25], we illustrate some of its properties with tree-level arguments. For a fixed choice of $k = q$, the F-term equations are solved by $B_i = 0$. We define

$$\Phi_1 = \begin{pmatrix} \sqrt{q} & 0 & \cdots & 0 & 0 & 0 \\ 0 & \sqrt{q-1} & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \sqrt{2} & 0 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 \end{pmatrix} , \tag{2}$$

$$\Phi_2 = \begin{pmatrix} 0 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & \sqrt{q-1} & 0 \\ 0 & 0 & \cdots & 0 & 0 & \sqrt{q} \end{pmatrix} , \tag{3}$$

where each element represents a block proportional to the identity matrix $\mathbb{I}_N$. The D-term equations are

$$0 = -g_q \sum_i \mathrm{Tr}_{q+1}\left[A_i^\dagger T_q^A A_i + B_i T_q^A B_i^\dagger\right] , \tag{4}$$

$$0 = -g_{q+1} \sum_i \mathrm{Tr}_q\left[A_i T_{q+1}^A A_i^\dagger + B_i^\dagger T_{q+1}^A B_i\right] , \tag{5}$$

where the labels q and $q+1$ refer to the $SU(qN)$ and $SU((q+1)N)$ groups, respectively, g_j are the gauge couplings, and T_j^A the generators. Taking $A_i = c\ \Phi_i$:

$$\mathrm{Tr}\left[T_q^A\left(A_1 A_1^\dagger + A_2 A_2^\dagger\right)\right] = (q+1)|c|^2 \mathrm{Tr}\, T_q^A = 0\,. \tag{6}$$

Another classical branch has $A_i \leftrightarrow B_i$. The operator

$$\mathcal{U} = \frac{1}{q(q+1)N}\mathrm{Tr}\left[A_i^\dagger A_i - B_i B_i^\dagger\right] , \tag{7}$$

normalized so that for $A_i = c\Phi_i$ and $B_i = 0$ one has $\mathcal{U} = |c|^2$, is the order parameter of Z_2 symmetry breaking, and of the Higgsing $SU(qN) \times SU((q+1)N) \to SU(N)$.

The matrices Φ_i obey the $SU(2)$ algebra [25], and indeed the perturbative calculation of the spectrum of gauge bosons yields $M^2 = g^2|c|^2\lambda_{\ell,\pm}$ (for $g_q = g_{q+1} \equiv g$) where

$$\lambda_{\ell,\pm} = q + \frac{1}{2} \pm \sqrt{\left(q+\frac{1}{2}\right)^2 - \ell(\ell+1)}\,, \tag{8}$$

where $\ell = 0, 1, \cdots, q-1$ [25], and where the eigenvalues have multiplicity $(2\ell+1)N^2$ for $\ell \neq 0$ and $N^2 - 1$ for $\ell = 0$. In addition there are $(2q+1)N^2$ states with mass $M^2 = g^2|c|^2 q$. The $N^2 - 1$ massless vectors represent the unbroken gauge group $SU(N)$. The unbroken $SU(N)$ theory with adjoint matter field content can be obtained by twisted compactification on a 2-sphere of the CVMN six dimensional field theory [28], the degeneracies exactly match, while the numerical values of the masses agree for $\ell \ll q$ [29]. This theory has a dynamical (confinement) scale Λ.

The perturbative approach provides a lot of insight [25], but leaves many open questions.

- The two gauge couplings and the coupling h are not independent, but non-perturbatively related [22].
- The couplings run, because of the presence of the anomaly. The RG flow is best described in terms of a cascade of Seiberg dualities that progressively reduce the group as in

$$\begin{aligned} SU(kN) \times SU((k+1)N) &\to SU(kN) \times SU((k-1)N) \\ &\to SU((k-2)N) \times SU((k-1)N) \\ &\to \cdots . \end{aligned} \tag{9}$$

- The cascade stops at $k = q$ because of the Higgsing due to $\mathcal{U}$, but the constant c should be determined non-perturbatively.
- Supersymmetry allows to infer that the gaugino condensate forms, breaking $Z_{2N} \to Z_2$ at scale Λ, and to classify the quantum moduli space [23], but not to calculate the spectrum of bound states.
- The Kähler part of the supersymmetric action is not protected by non-renormalization theorems, hence the whole spectrum requires non-perturbative treatment.
- The constant c should be linked with q. We assume in the following that the position along the (quantum) baryonic branch be characterized by a non-perturbatively defined α, such that $\alpha \to -\infty$ corresponds to $\mathcal{U} = 0$, and $\alpha \to +\infty$ to $q \to +\infty$.
- There are two dynamical scales: Λ is the scale of explicit symmetry breaking given by dimensional transmutation of the scale anomaly (beta functions), but $\mathcal{U}$ is unconstrained, defines a scale that can be taken to be larger than Λ, and breaks scale invariance spontaneously, suggesting the presence of a dilaton in the spectrum, if the latter effect is larger than the former.

[1] The $SU(M) \times SU(M)$ theory is a CFT [27], as for $N = 0$ there is no anomalous breaking of $U(1)_R \to Z_{2N}$. The CFT can be obtained as the low-energy IR fixed point reached by a mass deformation of the $\mathcal{N}=2$ supersymmetric gauge theory, itself obtained by Z_2-orbifold of the $\mathcal{N}=4$ theory with gauge group $SU(2M)$. At the IR fixed point the anomalous dimension is non-perturbative, and hence W in Eq. (1) is not irrelevant.

For all of these reasons, we need a non-perturbative description of the theory at strong coupling, which is provided (at large N) by the known gravity dual [16].

3. The baryonic branch of KS: gravity

The baryonic branch is described in gravity by a family of type-IIB supergravity backgrounds [16] (see also [30–32]) within the PT ansatz [33], characterized by a compact five-dimensional manifold with the symmetries of $T^{1,1}$ [35], and a non-compact five-dimensional space with metric ansatz:

$$ds_5^2 = e^{2A(r)} ds_{1,3}^2 + dr^2 \,. \tag{10}$$

The space can be foliated along r in Minkowski slices, related by a conformal factor e^{2A} dependent only on r, so that the radial direction is interpreted in field-theory terms as a renormalization scale.

The general problem of finding solutions and studying their fluctuations can be conveniently formulated in terms of a truncation to a five-dimensional sigma-model with 8 scalars Φ^a coupled to gravity, and Lagrangian

$$\mathcal{L} = \frac{R}{4} - \frac{1}{2} G_{ab} g^{MN} \partial_M \Phi^a \partial_N \Phi^b - V(\Phi^a) \,, \tag{11}$$

where R is the five-dimensional Ricci scalar, G_{ab} is the sigma-model metric, g_{MN} is the metric, and $V(\Phi^a)$ is the potential. The detailed form of potential and kinetic terms can be found elsewhere [17]. The full lift to 10 dimensions is known [33].[2]

The background solution can be found by using Eq. (10) and the assumption that the scalars have non-trivial profiles $\Phi^a = \bar{\Phi}^a(r)$, and looking for non-singular (in 10 dimensions) solutions of the system of coupled differential equations [16].

The baryonic branch solutions are characterized by two scales r_0 and $\bar{r}$, the separation of which is controlled by the integration constant α [21]. When $r > \bar{r}$, the background is approximated by the KS solution, and captures the cascade of Seiberg dualities for $k > q$. For $r < \bar{r}$ the background approaches the CVMN solution [28]: the quiver gauge theory is higgsed to the $SU(N)$ one. As $r \to r_0$, the space ends: the theory confines and the gauginos condense. The gravity background captures all the non-perturbative features expected from field theory.

In computing the spectrum, we restrict our attention to tensor and scalar modes by fluctuating the five-dimensional sigma-model around the background solutions. We employ the gauge-invariant formalism of [17–19]. The tensor $\mathfrak{e}^{\mu}{}_{\nu}$ is the transverse and traceless part of the fluctuations of the four-dimensional metric. The gauge-invariant scalars $\mathfrak{a}^a$ are written in terms of the fluctuations φ^a of the bulk scalars and h of the trace of the four-dimensional part of the metric:

$$\mathfrak{a}^a = \varphi^a - \frac{\partial_r \bar{\Phi}^a}{6 \partial_r A} h \,. \tag{12}$$

The tensors obey the linearized differential equations

$$0 = \left[\partial_r^2 + 4 \partial_r A \partial_r + e^{-2A} m^2 \right] \mathfrak{e}^{\mu}{}_{\nu} \,, \tag{13}$$

where m is the four-dimensional mass, and for the scalars

$$0 = \left[\mathcal{D}_r^2 + 4 \partial_r A \mathcal{D}_r + e^{-2A} m^2 \right] \mathfrak{a}^a - \left[V^a{}_{|c} - \mathcal{R}^a{}_{bcd} \partial_r \bar{\Phi}^b \partial_r \bar{\Phi}^d + \frac{4(\partial_r \bar{\Phi}^a V^b + V^a \partial_r \bar{\Phi}^b) G_{bc}}{3 \partial_r A} + \frac{16 V \partial_r \bar{\Phi}^a \partial_r \bar{\Phi}^b G_{bc}}{9 (\partial_r A)^2} \right] \mathfrak{a}^c \,. \tag{14}$$

In order to interpret the eigenstates as states in the dual theory, we impose the boundary conditions:

$$\partial_r \mathfrak{e}^{\mu}{}_{\nu} \Big|_{r=r_i} = 0 \,, \tag{15}$$

and [20]

$$\frac{2 e^{2A} \partial_r \bar{\Phi}^a}{3 m^2 \partial_r A} \left[\partial_r \bar{\Phi}^b \mathcal{D}_r - \frac{4 V \partial_r \bar{\Phi}^b}{3 \partial_r A} - V^b \right] \mathfrak{a}_b + \mathfrak{a}^a \Big|_{r_i} = 0 , \tag{16}$$

where $r_i = r_I, r_U$ are cutoffs, acting as regulators.

The two regulators have no physical meaning: they are needed in the numerical calculation for practical reasons, but the physical results are obtained by extrapolating to $r_I \to r_0$ and $r_U \to +\infty$ [21]. Here we report only the final physical results, contained in Fig. 2, where we show the spectrum of scalar and tensor modes as a function of the parameter α characterizing the position along the baryonic branch. We are interested only in ratios of masses, to facilitate the comparisons we chose the normalization so that for all values of α the next-to-lightest state agrees with the lightest state of the CVMN spectrum [18].

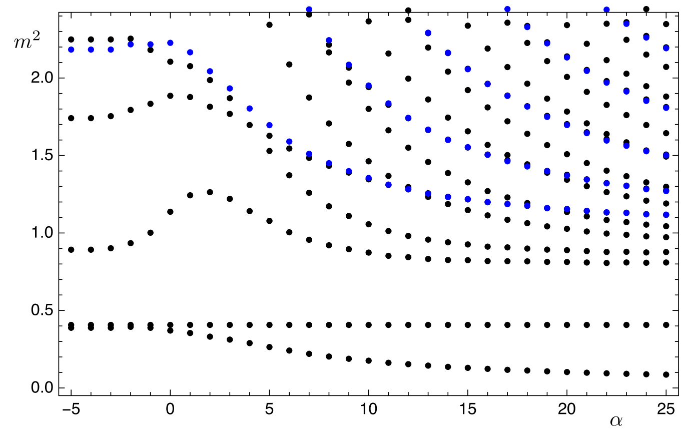

Fig. 2. The mass spectrum m^2 of scalar (black) and tensor (blue) states. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The main results that emerge are the following.

- Scalar and tensor spectra agree with the KS case for small α [17,36], and show evidence of deconstruction at the non-perturbative level: at large α, part of the spectrum consists of a dense sequence of states, above the continuum thresholds of the CVMN case ($\alpha \to +\infty$) [17,18].
- The spectrum of scalars contains one state the mass of which is suppressed going to large α. In the limit $\alpha \to +\infty$, we expect this state to become massless and decouple, as suggested by Fig. 2 and by the fact that this state is absent in the spectrum of the CVMN background, in which case it does not correspond to a normalizable, massive mode [21].

The latter is the main element of novelty of this paper.

[2] The five-dimensional system we study is obtained by imposing a set of constraints on the consistent truncation on $T^{1,1}$ in [34]. In particular, the reduction to eight scalars plus gravity is obtained by imposing a non-linear constraint that is in general not integrable. We treat the resulting sigma-model as a five-dimensional system on its own, and study its fluctuations. To study the full lift to type-IIB supergravity one would need to extend the five-dimensional sigma model along the lines of [34], to include additional scalar fields as well as vectors.

4. Discussion

All the qualitative expectations emerging from the study of the field theory are confirmed quantitatively by the gravity dual, including the fact that the spectrum of glueballs along the baryonic branch deconstructs a compact manifold, and interpolates between the known spectra of the KS and CVMN backgrounds.

In addition, we find a new result not accessible using field theory methods: the spectrum of scalars contains one parametrically light state, the mass of which is suppressed moving far from the origin of the baryonic branch. We interpret it as a dilaton, on the basis of the fact that all the solutions considered here are dual to the same field theory, with scale Λ controlled by the explicit breaking of scale invariance, but differ by the tunable choice of the vacuum value of the operator $\mathcal{U}$.

The dynamical scale Λ is controlled by explicit symmetry breaking (the scale anomaly), and hence is natural. There emerges in addition a tunable hierarchy between the mass of one isolated scalar (the dilaton) and the typical scale of the other states. This is hence an example of a strongly-coupled theory that naturally provides both a big and a little hierarchy of scales, thanks to the role of scale invariance and to the hierarchy between the scales of its spontaneous and explicit breaking.

A complete and rigorous understanding of the field theory requires extending the formalism for treating the fluctuations to adapt it to a more general truncation including vectors in the sigma model [34]. We leave this task for future work. Furthermore, the theory is supersymmetric, and the gravity calculation captures only the large-N limit. The latter provides a technical advantage, as it allows to perform a conceptually simple (although numerically challenging) calculation. The dilaton is parametrically light because the theory admits a classical moduli space along a flat direction that is lifted only by controllable quantum effects (the running of the couplings), with the quantum moduli space still non-trivial [23], and hence there are condensates that are parametrically larger than the scale introduced by the explicit breaking of scale invariance due to the anomaly. Whether such phenomena arise in non-supersymmetric theories is an open problem.

The results of this paper support the expectation that in a strongly-coupled theory in which condensates are enhanced, the mass spectrum of bound states (including the lightest scalar) would reproduce the qualitative features in Fig. 1. A phenomenologically viable solution of the hierarchy problem(s) of the electroweak theory would require to implement electro-weak symmetry and its breaking. One possible way to achieve this, within the specific context of the model studied here, might follow the lines of the Sakai–Sugimoto model [37], by embedding in the background extended objects [38,39] that implement the $SU(2)\times SU(2)$ global symmetry of the SM Higgs sector, and its breaking. The gauging of $SU(2)_L\times U(1)_Y$ must be reinstated via holographic renormalization, by including boundary-localised terms that cancel the divergence of the gauge boson wave function, and hence retain a finite gauge coupling in the limit in which the UV cutoff is taken to infinity. While finding the embedding is technically challenging [40], given the great potential of this or alternative model-building approaches, we think this line of reasoning deserves further future study.

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