Experimental Realization of a Thermal Squeezed State of Levitated Optomechanics

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We experimentally squeeze the thermal motional state of an optically levitated nanosphere by fast switching between two trapping frequencies. The measured phase-space distribution of the center of mass of our particle shows the typical shape of a squeezed thermal state, from which we infer up to 2.7 dB of squeezing along one motional direction. In these experiments the average thermal occupancy is high and, even after squeezing, the motional state remains in the remit of classical statistical mechanics. Nevertheless, we argue that the manipulation scheme described here could be used to achieve squeezing in the quantum regime if preceded by cooling of the levitated mechanical oscillator. Additionally, a higher degree of squeezing could, in principle, be achieved by repeating the frequency-switching protocol multiple times.

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While squeezing a quantum state of light [1] has a long history of experiments, the squeezing of a massive mechanical harmonic oscillator has not seen many experimental realizations so far. The first demonstration of squeezing in a classical mechanical oscillator was by Rugar and Grütter [2]. Squeezing of classical motional states in electromechanical devices by parametric amplification and weak measurements has subsequently been proposed [3], and it has been experimentally demonstrated in an optomechanical system [4]. Schemes relying on sinusoidal modulation of the spring constant have also been proposed and discussed by numerous authors [5–8]. In optomechanical cavities, Genoni *et al.* suggested that squeezing below the ground-state fluctuations (quantum squeezing, for brevity) may be attainable via continuous measurements and feedback [9]. Quantum squeezing of a high-frequency mechanical oscillator has only been experimentally demonstrated very recently, in a microwave optomechanical device [10,11]. Also only very recently, a hybrid photonic-phononic waveguide device has shown the correlation properties of optomechanical two-mode squeezing [12]. Another interesting method of generating squeezing, of relevance to this Letter, relies on nonadiabatic shifts of the mechanical frequency. Such a method was initially discussed in relation to light fields [13,14]. Similar ideas, utilizing impulse kicks on a mechanical oscillator, have been recently discussed [15,16]. In this Letter we report the first experimental demonstration of mechanical squeezing via nonadiabatic frequency shifts, thus realizing a useful tool to manipulate the state of a levitated optomechanical system.

Theory.—In what follows we shall present a quantum mechanical treatment of our squeezing protocol, in anticipation of future experiments that may achieve quantum squeezing. Because of the linearity of the Heisenberg equations of our system, it should be pointed out that formally identical results may be obtained through classical statistical mechanics [17]. We consider a nanosphere of mass m trapped in a harmonic potential. Along the z axis, we can manipulate the system by switching between two Hamiltonians $\hat{H}_1, \hat{H}_2$, where $\hat{H}_j = (\hat{p}^2/2m) + \frac{1}{2}m\omega_j^2\hat{z}^2$, $\hat{z}, \hat{p}$ denoting the z components of the canonical position and momentum operators, and the trapping frequency may assume either of two distinct values: ω_1 or ω_2. (In our experiment, we adopt $\omega_2 < \omega_1$.) As we shall see shortly, our squeezing protocol relies on the rapid (i.e., *nonadiabatic*) switching between the two Hamiltonians [13,14]. It is instructive to write down the annihilation operators—say, $\hat{a}$ and $\hat{b}$—corresponding to the two trap frequencies ($\hbar = 1$):

$$\hat{a} = \sqrt{\frac{m\omega_1}{2}}\left(\hat{z} + \frac{i\hat{p}}{m\omega_1}\right), \quad \hat{b} = \sqrt{\frac{m\omega_2}{2}}\left(\hat{z} + \frac{i\hat{p}}{m\omega_2}\right). \quad (1)$$

Through simple algebra, one may notice that $\hat{a}$ and $\hat{b}$ are related by a squeezing transformation of the form $\hat{b} = \cosh(r)\hat{a} - \sinh(r)\hat{a}^\dagger$, with $r \equiv \frac{1}{2}\log(\omega_2/\omega_1)$ being the squeezing parameter. We may exploit the mathematical relationship between modes $\hat{a}$ and $\hat{b}$ to generate mechanical squeezing as follows. Let the particle be initially prepared in an arbitrary state (in our experiment, this will be a thermal state of $\hat{H}_1$). At time $t = 0$, we suddenly change the trapping frequency from ω_1 to ω_2 such that the Hamiltonian becomes $\hat{H}_2$. We then let the system evolve until a time $t = \tau$ (the *squeezing pulse duration*) before rapidly switching back to the Hamiltonian $\hat{H}_1$. In the Heisenberg picture, this amounts to a simple harmonic evolution $\hat{b} \to \hat{b}e^{-i\omega_2\tau}$ for the operator $\hat{b}$. In terms of the quadratures $\hat{X} = (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$, $\hat{P} = -i(\hat{a} - \hat{a}^\dagger)/\sqrt{2}$, however, the transformation is nontrivial: $(\hat{X}, \hat{P})^\intercal \to M(\hat{X}, \hat{P})^\intercal$, where the matrix

$$M = \begin{pmatrix} \cos(\omega_2\tau) & e^{2r}\sin(\omega_2\tau) \\ -e^{-2r}\sin(\omega_2\tau) & \cos(\omega_2\tau) \end{pmatrix} \quad (2)$$

embodies a combination of rotation and squeezing in the phase space of mode $\hat{a}$. Note that, in general, the squeezed quadrature will be a linear combination of $\hat{X}$ and $\hat{P}$. The associated squeezing parameter $\lambda(\tau)$ is encoded in the singular values of M and can be found as follows. Since $\det(MM^\intercal) = 1$, we can parametrize the eigenvalues of $MM^\intercal$ as $(\mu, 1/\mu)$ for some parameter where $\mu > 0$. Note that $\sqrt{\mu}$ quantifies the deformation of the *standard deviations* of the rotated quadratures. The mechanical squeezing parameter thus reads (in decibels)

$$\lambda(\tau) = 10|\log_{10}(\sqrt{\mu})|. \quad (3)$$

The analytical expression for $\lambda(\tau)$ is unwieldy if τ is left generic. It is, however, readily verified that maximum squeezing can be obtained by setting $\omega_2\tau = (\pi/2)$, in which case $\lambda_{\max} = 10\log_{10}(\omega_1/\omega_2)$.

Experiments.—We trap a silica nanosphere of radius 32 nm (± 5 nm) and mass 3.1×10^{-19} kg ($\pm 1.4 \times 10^{-19}$ kg) in an optical dipole trap. The size of the particle is evaluated from fitting a Lorentzian to the power spectral density of the signal, as described in Ref. [19] and shown in Fig. 3(b); from this, the mass is obtained as well. We use a 1550 nm laser, directed into a parabolic mirror which focuses the light to a diffraction limited spot, where the particle is trapped. Experiments are performed in a vacuum chamber at a pressure of 1×10^{-1} mbar. In this regime, the damping of the particle motion by random collisions with background gas is linear in the pressure $p_{\rm gas}$, and the related damping coefficient can be approximated by

$$\Gamma \approx 15.8\frac{r^2 p_{\rm gas}}{m v_{\rm gas}}, \quad (4)$$

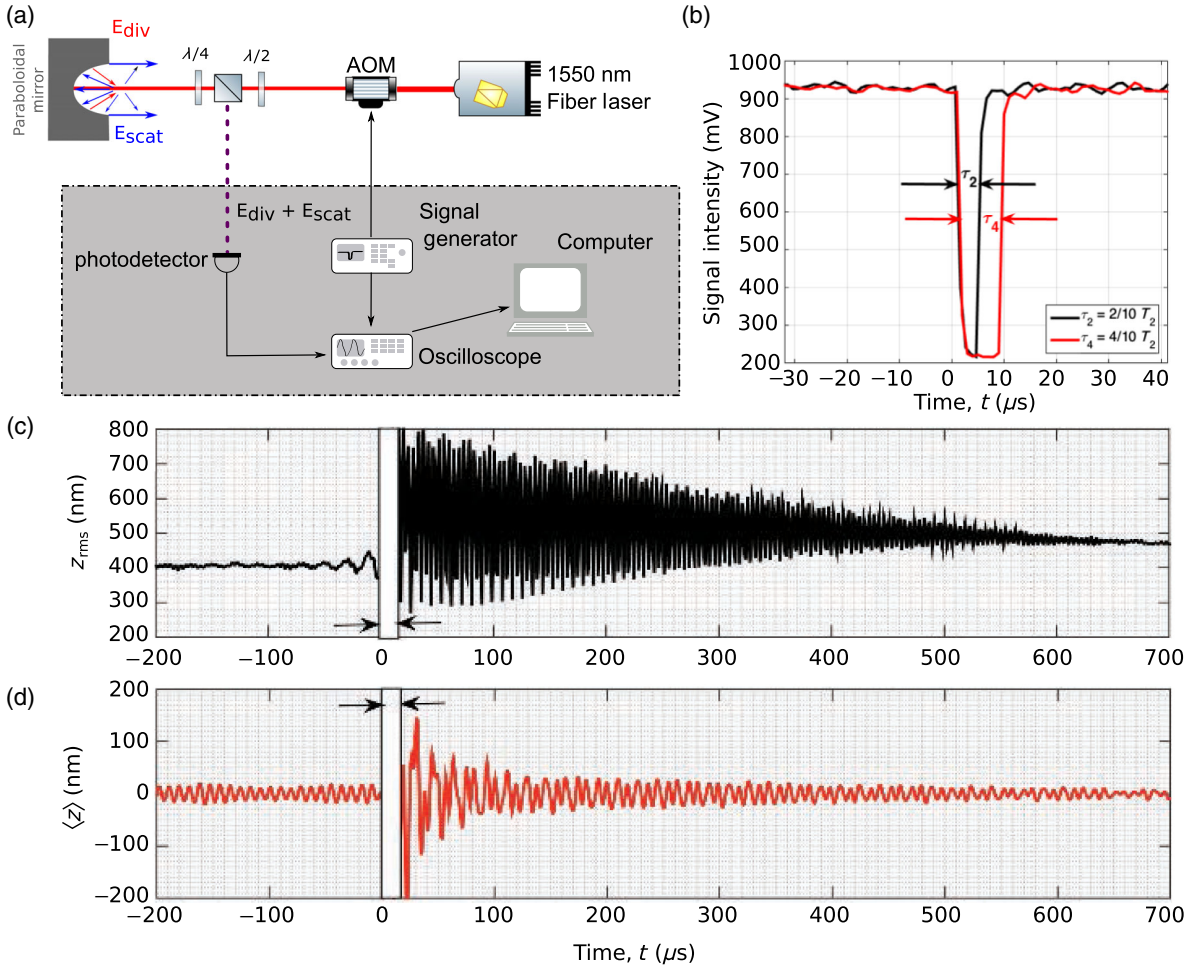

FIG. 1. Experimental implementation of squeezing levitated optomechanics. (a) Schematic of the squeezing setup. The paraboloidal trapping system interferes the $E_{\rm div}$ divergence field and the $E_{\rm scat}$ Rayleigh scattered field from the trapped particle. The grey region is for homodyne detection, as well as pulse application. (b) Negative square pulse for squeezing generation, as seen by the photodetector. Two different pulse durations are shown, τ_2 and τ_4. (c) Root-mean-square position as a function of time, $z_{\rm rms}(t) = \sqrt{\langle (z - \langle z \rangle)^2 \rangle}$, obtained from 1500 pulse sequences applied to the same particle. Oscillations for $t < 0$ are due to bandpass filtering. (d) Time dependence of the mean position $\langle z \rangle$ (the center of the thermal distribution). This quantity also shows oscillations at ω_1 after the squeezing pulse.

with m and r being the radius and the mass of the nanosphere, respectively, and $v_{\mathrm{gas}} = \sqrt{3k_BT/m_{\mathrm{gas}}}$ the mean thermal velocity of the background gas of mass m_{gas} [20]. We evaluate $\Gamma = 2\pi \times 227$ Hz ($\pm 2\pi \times 9$ Hz) for this experiment, while the main uncertainty in mass comes from the pressure measurement ($\sim$15%).

As shown in Fig. 1(a), the position of the single nanosphere is measured using an optical homodyne method. More details about the particle trapping and detection can be found elsewhere [19].

A short squeezing pulse of duration τ is applied by switching between two different trapping laser powers, P_1 and P_2 [see Fig. 1(b)], using a free space acousto-optical modulator (AOM). The trapping frequency is given by $\omega = \sqrt{k_0/m}$, where $k_0 = 8\alpha P/(c\pi\epsilon_0 w_f^4)$ for motion in the z direction, with α being the polarizability of the particle, c the speed of light, ϵ_0 the electric field constant, and w_f the waist of the laser beam at the focal point. The laser power can be modulated by changing the voltage applied to the AOM; we switch between trap frequencies $\omega_1 = 2\pi \times 112$ kHz and $\omega_2 = 2\pi \times 49.3$ kHz. The time scale of the switch is determined by the AOM bandwidth, which is more than 1 MHz and therefore much larger than both trap frequencies. Hence, we model the switch as instantaneous. Here, evidently, $\omega_2 < \omega_1$. The condition $\omega_2 > \omega_1$ may also be used, and it would result in squeezing of a different quadrature. Experimentally, we found it more practical to employ the tighter trapping potential (corresponding to ω_1) most of the time, so as to minimize the probability of losing the particle.

The same signal generator which is used to generate the squeezing pulse triggers an oscilloscope to record a time trace with a duration of 1 s. The same single pulse sequence is repeated 1500 times for the same trapped particle, while allowing for 1 s between the pulses to restore the initial thermal state. The recorded time traces initially include signals from the x, y, and z motional degrees of freedom. However, the pulse scheme is only optimized for a single motional frequency, namely, the one in the z direction which is perpendicular to the mirror surface. This is primarily because the z motion is predominant in our detection signal. We filter the signal around the ω_z frequency peak to extract the impact of the pulses on the z motion alone. The root mean square (rms) of the position of the particle z_{rms} is used to analyze the state of motion; see Fig. 1(c). The entire experiment takes over 10 h, during which time drifts in laser power (and hence in trap frequency) may occur. Thus ω_2, while known in principle, is taken as a free parameter in the fitting model.

Results.—The Fourier transform of the oscillation in the mean position $\langle z \rangle$—i.e., the motion of the center of the thermal distribution—shows prevalence by the frequency ω_1 before and after the pulse; see Fig. 1(a) and the Supplemental Material [17]. We can thus infer that our pulse imparts a small phase-space displacement to the particle. To correct for this, we subtract the average displacement from the data, while we try to account for the remaining effects and experimental imperfections through an effective dephasing model (see the Supplemental Material [17]).

Initially, z_{rms} is constant as the phase of the oscillation is random between the 1500 individual pulse experiments. After the squeezing pulse, the motion shows damped phase-coherent oscillations. The rms oscillation decays within about 680–690 μs, which gives a rate of thermalization to the temperature of the background gas molecules between $2\pi \times 230$ and $2\pi \times 234$ Hz. This is in good agreement with the value for Γ estimated via the Lorentzian fit.

We are operating in the classical regime in that we observe quadrature variances that are several orders of magnitude larger than those in the quantum ground state. Therefore, we may estimate the particle's momentum by simply taking the time differential of the position measurement. In passing, we note that, in the quantum regime, our continuous measurement process would require a more

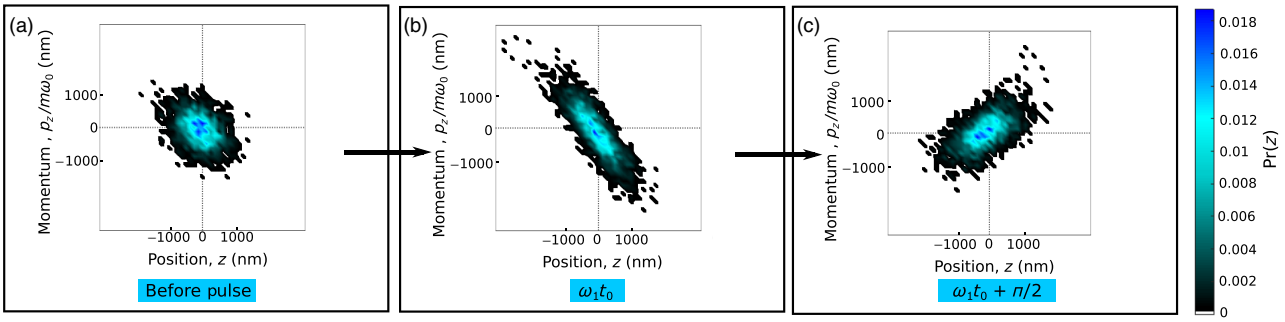

FIG. 2. Experimentally measured phase-space distributions of the mechanical state before and after the squeezing pulse. The average displacement of the state has been subtracted [see Fig. 1(d)]. (a)–(c) Density plots of the phase-space distributions for z motion, at three different times, for a pulse duration $\tau = 3/10T_2$. (a) State of the particle motion before the pulse is applied. The former is well approximated by a Gaussian distribution, as is typical for a thermal state. (b) Phase-space distribution shortly after the pulse has been applied (at the time t_0): note how it presents clear signatures of squeezing. (c) Phase-space distribution at the time $t_0 + \frac{1}{4}T_1$. The squeezed state rotates in phase space while squeezing degrades with time, mainly due to background gas collisions that tend to restore the initial thermal distribution.

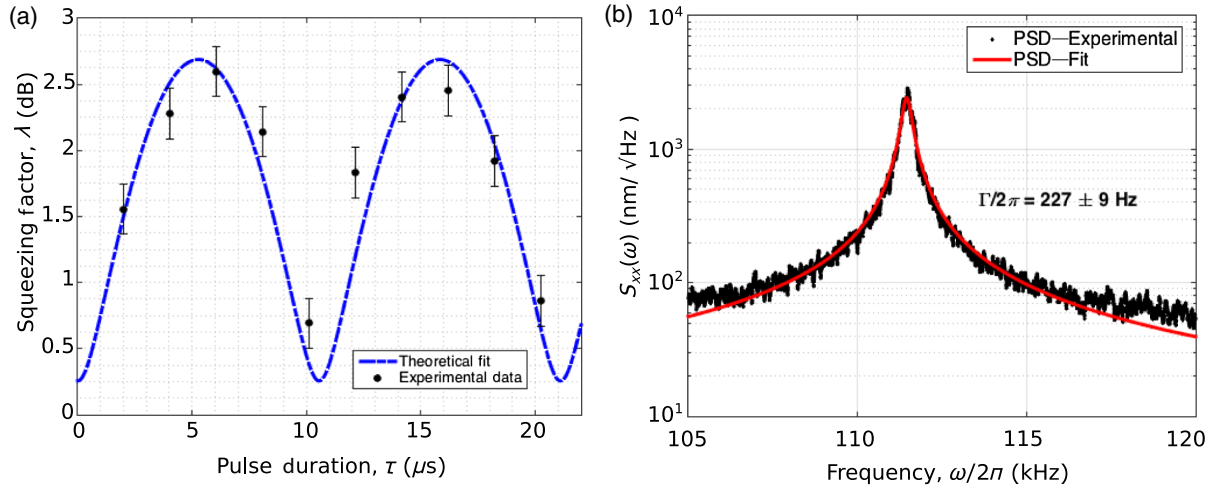

FIG. 3. Quantitative analysis of the squeezing effect. (a) Squeezing factor λ as a function of pulse duration τ (measured in microseconds). This is extracted by comparing the minor axes of the phase-space ellipses (see, e.g., Fig. 2) before and after the pulse. The theoretical fit to the data (the blue line) has been done according to Eq. (10) in the Supplemental Material [17]. (b) Lorentzian fit to the power spectral density (PSD) of the z motion. This is used to extract the radius and the mass of the particle, as well as the collisional damping rate Γ according to Eq. (9) in the Supplemental Material [17].

rigorous treatment [21]. Applying the described strategy to our data set, we generate the phase-space distribution of the trapped particle motion. Figure 2(a) shows the distribution of the system before the pulse is applied. Such an initial distribution is nearly Gaussian, and its small asymmetry can be attributed to the nonlinear response of the position measurement at large oscillation amplitudes [22].

Immediately after the pulse, we observe the typical features associated with squeezing. It is evident that the applied pulse deforms the phase-space distribution of the particle, which then displays the typical oblong shape of a squeezed state; see Fig. 2(b). Following the pulse, the distribution rotates in phase space according to the harmonic oscillator evolution—see, for instance, Fig. 2(c). During such an evolution, the distribution progressively relaxes back towards a thermal one; we attribute this to thermalization via collisions with the background gas.

The measured degree of squeezing as a function of pulse duration is shown in Fig. 3(a). While the theoretical prediction for the squeezing parameter $\lambda(\tau)$ agrees qualitatively with the experimental results, the largest squeezing factor we achieve experimentally is 2.7 dB, lower than the expected $\lambda_{\max} \simeq 3.56$ dB. We can obtain a reasonable fit of the data by assuming that the squeezing pulse is affected by some phase noise whose strength is τ independent [17], and which we assume is associated with the abrupt voltage changes. In a nutshell, this amounts to rescaling $\langle \hat{b}^2 \rangle \to \eta \langle \hat{b}^2 \rangle$ at the end of the squeezing operation, where $0 \leq \eta \leq 1$ quantifies the residual "phase coherence" [17]. For the best fit, as shown in Fig. 3(a), we obtain $\omega_2 = 2\pi \times 47.9$ kHz (± 1.55 kHz) and $\eta = 0.73$ (± 0.10). We have assumed that other sources of noise (e.g., thermalization with the background gas) can be neglected during the pulse, i.e., that the motion of the particle at short time scales during the squeezing operation is affected predominantly by such phase noise.

Conclusion.—The demonstrated squeezing technique could be used for enhanced sensing and metrology based on levitated optomechanics, such as for force sensing applications [23] and nonequilibrium dynamics studies [22]. Truly quantum squeezing may be approached by precooling the motional state [24–26]. Center of mass motion temperatures of trapped nanoparticles of below 1 mK have been experimentally demonstrated [19,20,27] via parametric feedback, while alternative methods include quantum measurement techniques [21,28], which have been successfully applied to membrane and cantilever optomechanical devices [29,30]. Future work will include the investigation of multiple pulses to increase the achievable levels of noise reduction [31], and of methods for probing the nonclassicality of mechanical oscillators.

Finally, we would like to comment on our measurement scheme, which relies on a continuous monitoring of the particle's position. At first, this might appear to be undesirable from the future perspective of approaching the quantum regime, due to the well-known disturbance induced by the quantum measurement process. Yet, it was recently shown that, if correctly accounted for, continuous monitoring may in fact improve the achievable mechanical squeezing [21].

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