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# The two-loop $n$ -point all-plus helicity amplitude

David C. Dunbar, Guy R. Jehu and Warren B. Perkins

*College of Science,  
Swansea University,  
Swansea, SA2 8PP, UK  
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## Abstract

We propose a compact analytic expression for the polylogarithmic part of the  $n$ -point two-loop all-plus helicity amplitude in gauge theory.

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## I. INTRODUCTION

Computing perturbative scattering amplitudes is a key challenge in quantum field theory both for comparing theories with experiment and for understanding the symmetries and consistency of theories. Explicit analytic expressions for scattering amplitudes have proved particularly useful in understanding the behaviour and symmetries of the underlying theory. Calculating amplitudes from their singular structures is an extremely powerful tool which has a long history [1] and has seen tremendous development based on techniques including unitarity [2, 3] and on-shell recursion [4].

Recently the leading in color component of the two-loop all-plus five-point amplitude has been computed in QCD [5, 6] using  $d$ -dimensional unitarity techniques. Subsequently this amplitude was presented in a very elegant and compact form [7]. In this form the amplitude consists of a piece driven by the infra-red (IR) and ultra-violet (UV) singular structure of the amplitude and a “remainder” piece.

In ref. [8] it was demonstrated how this form can be generated using a combination of four-dimensional unitarity and (augmented) recursion which provides an understanding of the simplicity of the amplitude. In this article we propose an expression for the polylogarithms in the leading-in-color part of the  $n$ -point all-plus amplitude. This proposal is based on collinear limits and unitarity.

Using the conventions of ref. [7] the leading in color component of the amplitude can be expressed,

$$\begin{aligned} \mathcal{A}_n(1^+, 2^+, \dots, n^+) |_{\text{leading color}} &= g^{n-2} \sum_{L \geq 1} (g^2 N c_\Gamma)^L \\ &\times \sum_{\sigma \in S_n/Z_n} \text{tr}(T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} \dots T^{a_{\sigma(n)}}) A_n^{(L)}(\sigma(1)^+, \sigma(2)^+, \dots, \sigma(n)^+) \end{aligned} \quad (1.1)$$

and the result we present is for the color-stripped two-loop amplitude  $A_n^{(2)}(1^+, 2^+, \dots, n^+)$ .<sup>1</sup>

The IR and UV behaviour of this amplitude is very-well specified [9] and it can be split into singular terms plus a finite remainder function,  $F_n^{(2)}$ , which is to be determined:

$$A_n^{(2)} = A_n^{(1)} I_n^{(1)} + F_n^{(2)} + \mathcal{O}(\epsilon), \quad (1.2)$$

where

$$I_n^{(1)} = \left[ - \sum_{i=1}^n \frac{1}{\epsilon^2} \left( \frac{\mu^2}{-s_{i,i+1}} \right)^\epsilon + \frac{n\pi^2}{12} \right]. \quad (1.3)$$

Since this one-loop amplitude is finite there are no  $\epsilon^{-1}$  terms [9]. In this equation  $A_n^{(1)}$  is the all- $\epsilon$  form of the one-loop amplitude [10, 11]. Although the one-loop amplitude is rational to  $\mathcal{O}(\epsilon^0)$ , the all- $\epsilon$  expression contains polylogarithms which, when combined with the  $\epsilon^{-2}$  factor generate finite polylogarithms in the two-loop amplitude.  $F_n^{(2)}$  contains further polylogarithmic terms,  $P_n^{(2)}$  and rational terms  $R_n^{(2)}$ :

$$F_n^{(2)} = P_n^{(2)} + R_n^{(2)}. \quad (1.4)$$

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<sup>1</sup> The factor  $c_\Gamma$  is defined as  $\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)/\Gamma(1-2\epsilon)/(4\pi)^{2-\epsilon}$ .  $S_n/Z_n$  are the cyclically-distinguishable permutations of the  $n$ -legs and  $T^{a_i}$  are the color-matrices of  $SU(N)$ .

The ansatz for  $P_n^{(2)}$  is in terms of the functions

$$F^{2m}[S, T, K_2^2, K_4^2] = \text{Li}_2\left[1 - \frac{K_2^2}{S}\right] + \text{Li}_2\left[1 - \frac{K_2^2}{T}\right] + \text{Li}_2\left[1 - \frac{K_4^2}{S}\right] \\ + \text{Li}_2\left[1 - \frac{K_4^2}{T}\right] - \text{Li}_2\left[1 - \frac{K_2^2 K_4^2}{ST}\right] + \text{Log}^2(S/T)/2. \quad (1.5)$$

As the notation suggests, these are related to the one-loop box integral functions with two massive (and two massless) legs. The function corresponds to a box integral function where the momenta  $K_2$  and  $K_4$  are non-null and  $S = (k_1 + K_2)^2$  and  $T = (K_2 + k_3)^2$ . This combination of dilogarithms can be regarded as either a) the  $D = 4$  integrals truncated to remove the IR divergent terms or b) the  $D = 8$  integrals [12, 13]. The function is smooth in the limit  $K_2^2 \rightarrow 0$ ,

$$F^{2m}[S, T, 0, K_4^2] = \text{Li}_2\left[1 - \frac{K_4^2}{S}\right] + \text{Li}_2\left[1 - \frac{K_4^2}{T}\right] + \text{Log}^2(S/T)/2 + \frac{\pi^2}{6} \quad (1.6)$$

which is the one-mass function,  $F^{1m}[S, T, K_4^2]$ .

$P_n^{(2)}$  is obtained by summing over all possible  $F^{2m}$  including the degenerate cases when  $K_2$  corresponds to a single leg and  $F^{2m}$  reduces to  $F^{1m}$ . Leg  $K_4$  must have at least two external legs. Defining the kinematic invariants  $t_i^{[r]}$

$$t_i^{[r]} = (k_i + k_{i+1} + \dots + k_{i+r-1})^2 \quad (1.7)$$

then

$$F_{n:r,i}^{2m} = F^{2m}[t_{i-1}^{[r+1]}, t_i^{[r+1]}, t_i^{[r]}, t_{i+r+1}^{[n-r-2]}] \quad (1.8)$$

which is shown diagrammatically in fig. 1. The case  $r = 1$  is the one-mass case  $F_{n:i}^{1m} \equiv F_{n:1,i}^{2m}$ .

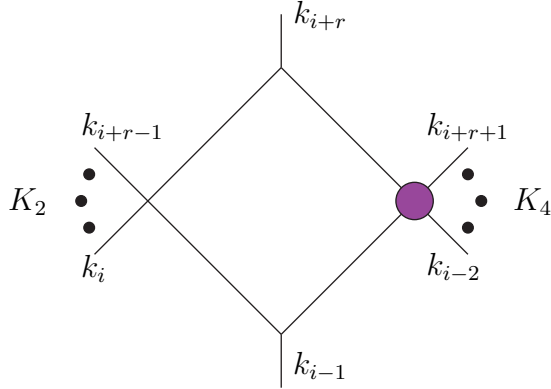


FIG. 1: Diagrammatic representation of the functions  $F_{n:r,i}^{2m}$ . The functions are symmetric between  $K_2$  and  $K_4$  but their coefficients are not. The summation is over all such functions including the case when  $K_2$  is a single leg ( $r = 1$ ) but leg  $K_4$  must contain at least two legs (indicated by a solid disc).

The ansatz for  $P_n^{(2)}$  is

$$P_n^{(2)} = \frac{i}{6 \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \cdots \langle n1 \rangle} \sum_{i=1}^n \sum_{r=1}^{n-4} c_{r,i} F_{n:r,i}^{2m} \quad (1.9)$$

where the coefficient is

$$c_{r,i} = \left( \sum_{a<b<c<d \in K_4} \text{tr}_-[abcd] - \sum_{a<b<c \in K_4} \text{tr}_-[abcK_4] + \sum_{a<b \in K_4} \frac{\langle i-1|K_4baK_4|i+r \rangle}{\langle i-1i+r \rangle} \right) \quad (1.10)$$

where

$$\text{tr}_-[abcd] \equiv \langle ab \rangle [bc] \langle cd \rangle [da] , \quad (1.11)$$

$K_4$  is the set  $\{i+r+1, \dots, i-2\}$  with a cyclic definition of indices and inequality refers to ordering within the set.

Note that  $P_n^{(2)}$  has transcendentality two (w.r.t polylogarithms) as do one-loop amplitudes although the full amplitude,  $A_n^{(2)}$ , has higher transcendentality. For  $n=4$ ,  $P_4^{(2)}=0$  and the remainder function is purely rational [14].

## II. COLLINEAR LIMITS

We consider the collinear limit of the amplitude as an important consistency test and to illustrate some key features. The collinear limit occurs when adjacent momenta  $k_a$  and  $k_{a+1}$  become collinear,

$$k_a \longrightarrow z \times K, \quad k_{a+1} \longrightarrow (1-z) \times K = \bar{z}K . \quad (2.1)$$

In this limit, amplitudes factorise as

$$A_n^{(L)}(\dots, k_a^h, k_{a+1}^{h'}, \dots) \longrightarrow \sum_{L_s, h''} S_{-h''}^{hh', (L_s)} \times A_{n-1}^{(L-L_s)}(\dots, K^{h''}, \dots) , \quad (2.2)$$

where  $S_{-h''}^{hh', (L_s)}$  are the various splitting functions. For the all-plus amplitude the tree amplitude vanishes for both choices of  $h''$  and

$$A_n^{(2)}(\dots, k_a^+, k_{a+1}^+ \dots) \longrightarrow S_-^{++, \text{tree}} \times A_{n-1}^{(2)}(\dots, K^+, \dots) + \sum_{h=\pm} S_{-h}^{++, (1)} \times A_{n-1}^{(1)}(\dots, K^h, \dots) . \quad (2.3)$$

The term with  $h=-$  is purely rational and is irrelevant for the polylogarithmic term.

The first important proposal is that we use an all- $\epsilon$  form of the one-loop amplitude to ensure that *to all orders in  $\epsilon$* ,

$$A_n^{(1)} \longrightarrow S_-^{++, \text{tree}} \times A_{n-1}^{(1)} . \quad (2.4)$$

With this, we only need the order  $\epsilon^0$  form of the one-loop amplitude to check the collinear limit.

In the collinear limit,

$$I_n^{(1)} \longrightarrow I_{n-1}^{(1)} + r_-^{++} + \Delta , \quad (2.5)$$

where [2]

$$r_-^{++} = -\frac{1}{\epsilon^2} \left( \frac{\mu^2}{z\bar{z}(-s_{a,a+1})} \right)^\epsilon + 2 \ln z \ln \bar{z} + \frac{1}{3} z\bar{z} - \frac{\pi^2}{6} \quad (2.6)$$

and

$$\Delta = \log\left(\frac{-s_{a,a+1}}{\mu^2}\right) \log(z\bar{z}) - \log\left(\frac{-s_{a-1,a}}{\mu^2}\right) \log(z) - \log\left(\frac{-s_{a+1,a+2}}{\mu^2}\right) \log(\bar{z}) - \log(z) \log(\bar{z}) - \frac{1}{3} z\bar{z} + \frac{\pi^2}{4}. \quad (2.7)$$

The combination  $S_-^{++,\text{tree}} r_-^{++}$  is the one-loop splitting function. Consequently,

$$\begin{aligned} A_n^{(1)} \times I_n^{(1)} &\longrightarrow S_-^{++,\text{tree}} A_{n-1}^{(1)} \left( I_{n-1}^{(1)} + r_-^{++} + \Delta \right) \\ &= S_-^{++,\text{tree}} \left( A_{n-1}^{(1)} I_{n-1}^{(1)} \right) + \left( S_-^{++,\text{tree}} r_-^{++} \right) A_{n-1}^{(1)} + S_-^{++,\text{tree}} A_{n-1}^{(1)} \Delta. \\ &= S_-^{++,\text{tree}} \left( A_{n-1}^{(1)} I_{n-1}^{(1)} \right) + S_-^{++,(1)} A_{n-1}^{(1)} + S_-^{++,\text{tree}} A_{n-1}^{(1)} \Delta. \end{aligned} \quad (2.8)$$

In the last term,  $S_-^{++,\text{tree}} A_{n-1}^{(1)} \Delta$ , we need only keep the one-loop amplitude to order  $\epsilon^0$ . Consequently we require

$$P_n^{(2)} \longrightarrow S_-^{++,\text{tree}} P_{n-1}^{(2)} - S_-^{++,\text{tree}} A_{n-1}^{(1)} \Delta' \quad (2.9)$$

where  $\Delta'$  is the non-rational part of  $\Delta$  of eq. (2.7).

The overall pre-factor of

$$\begin{aligned} \frac{i}{6 \langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \cdots \langle n1 \rangle} &\longrightarrow \frac{1}{z\bar{z} \langle aa+1 \rangle} \times \frac{i}{6 \langle 12 \rangle \cdots \langle a-1K \rangle \langle Ka+2 \rangle \cdots \langle n1 \rangle} \\ &= S_-^{++,\text{tree}} \times \frac{i}{6 \langle 12 \rangle \cdots \langle a-1K \rangle \langle Ka+2 \rangle \cdots \langle n1 \rangle} \end{aligned} \quad (2.10)$$

which is the tree splitting function times the descendant  $n-1$ -point pre-factor. Consequently we require

$$\sum_{i=1}^n \sum_{r=1}^{n-4} c_{r,i} F_{n:r,i}^{2m} \longrightarrow \sum_{i=1}^{n-1} \sum_{r=1}^{n-5} c_{r,i} F_{n-1:r,i}^{2m} - \bar{A}_{n-1}^{(1)} \Delta' \quad (2.11)$$

where  $\bar{A}_{n-1}^{(1)}$  is the one-loop amplitude divided by the prefactor.

Each term in  $P_{n-1}^{(2)}$  where leg  $K$  is within either  $K_2$  or  $K_4$  arises directly from a single term in  $P_n^{(2)}$  with legs  $a$  and  $a+1$  both in  $K_2$  or  $K_4$  respectively. In these cases the functions have a smooth limit:

$$\begin{aligned} F_{n:r,i}^{2m} &\longrightarrow F_{n-1:r-1,i}^{2m} && a, a+1 \in K_2 \\ F_{n:r,i}^{2m} &\longrightarrow F_{n-1:r,i}^{2m} && a, a+1 \in K_4 \end{aligned} \quad (2.12)$$

while the corresponding coefficients behave as

$$\begin{aligned} c_{r,i} &\longrightarrow c_{r-1,i} && a, a+1 \in K_2 \\ c_{r,i} &\longrightarrow c_{r,i} && a, a+1 \in K_4, r \neq n-4 \end{aligned} \quad (2.13)$$

When  $a$  and  $a + 1$  are the only two legs within  $K_4$  (i.e.  $r = n - 4$ ),

$$c_{n-4,a+3} = \frac{\langle a-1 | K_4(a+1) a K_4 | a+2 \rangle}{\langle a-1 a+2 \rangle} = \frac{\langle a-1 a \rangle [a a+1]^2 \langle a a+1 \rangle \langle a+1 a+2 \rangle}{\langle a-1 a+2 \rangle} \quad (2.14)$$

which vanishes in the collinear limit and there is no contribution from this function. This configuration is illustrated in the first part of fig. 4.

When one of the massless legs is  $a$  or  $a + 1$  we use the identity

$$F_{n:r+1,a+1}^{2m} + F_{n:r,a+2}^{2m} \longrightarrow F_{n-1:r,a+2}^{2m} \quad (2.15)$$

which is shown diagrammatically on fig. 2. This identity follows from Abel's identity. There is also the corresponding identity when one of the  $F^{2m}$  has  $K_2$  null,

$$F_{n:2:a+1}^{2m} + F_{n:a+2}^{1m} \longrightarrow F_{n-1:1;a+2}^{1m} . \quad (2.16)$$

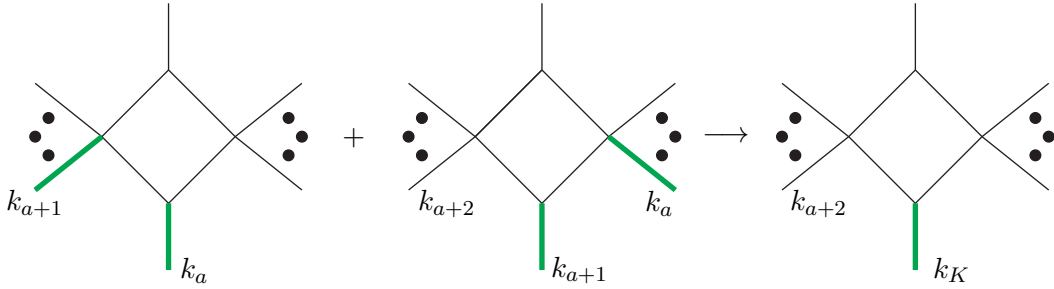


FIG. 2: Pictorial representation of the identity amongst the  $F$ -functions.

The functional identities in eq. (2.15) and eq. (2.16) are very similar to those that appear in the collinear limit of the one-loop ‘‘Maximally-Helicity-Violating’’ (MHV) amplitude in  $\mathcal{N} = 4$  although in that case the identities are for the untruncated box integrals. The coefficients of the functions shown in fig. 2, both descend to the appropriate coefficient in the collinear limit

$$c_{r+1,a+1} , c_{r,a+2} \longrightarrow c_{r,a+2} \quad (2.17)$$

and consequently

$$c_{r+1,a+1} F_{n:r+1,a+1}^{2m} + c_{r,a+2} F_{n:r,a+2}^{2m} \longrightarrow c_{r,a+2} F_{n-1:r,a+2}^{2m} . \quad (2.18)$$

This identifies where all the  $F^{2m}$  terms of the  $n - 1$  point amplitude arise from in the collinear limit.

There are a few limiting cases in the  $n$ -point we must consider. The functions shown in fig. 4 have vanishing coefficients and do not contribute. The two functions represented in fig. 3 satisfy

$$F_{n:a}^{1m} + F_{n:a+1}^{1m} \longrightarrow -\Delta' \quad (2.19)$$

and have coefficients satisfying

$$\frac{ic_{1,a}}{6 \langle 1 2 \rangle \cdots \langle n 1 \rangle} , \frac{ic_{1,a+1}}{6 \langle 1 2 \rangle \cdots \langle n 1 \rangle} \longrightarrow S_-^{++,\text{tree}} A_{n-1}^{(1)} \quad (2.20)$$

so that

$$\frac{i}{6 \langle 12 \rangle \cdots \langle n1 \rangle} (c_{1,a} F_{n:a}^{1m} + c_{1,a+1} F_{n:a+1}^{1m}) \longrightarrow -S_-^{++,\text{tree}} A_{n-1}^{(1)} \Delta' \quad (2.21)$$

as required by eq. (2.9).

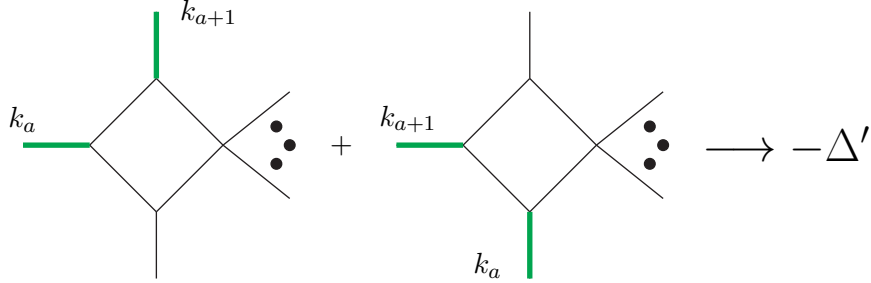


FIG. 3: The collinear limit of these two  $F$ -functions.

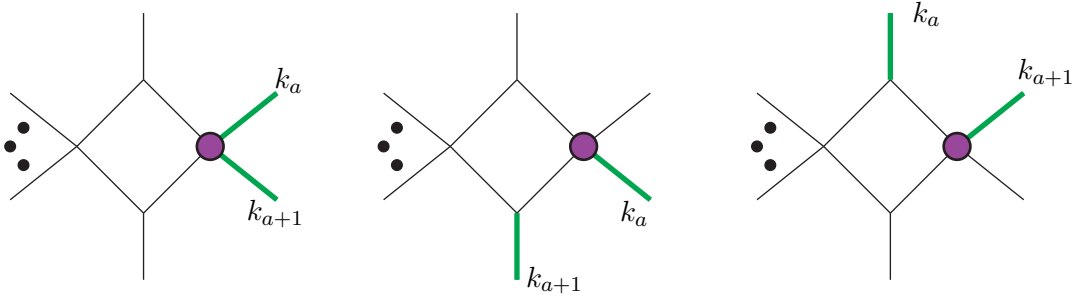


FIG. 4: Functions whose coefficients vanish in the collinear limit

Consequently, the ansatz has the correct collinear limit up to rational terms.

### III. UNITARITY CHECKS

This section is rather a reverse of how the ansatz was formed. We can use unitarity, subject to certain assumptions, to generate the functions in the remainder function. This is possible using four-dimensional unitarity where the cuts are evaluated in four dimensions rather than  $4 - 2\epsilon$  dimensions. This process in principle will miss certain functions, specifically purely rational terms and terms that are sub-leading in  $\epsilon$ . As discussed above, we assume that the sub-leading in  $\epsilon$  terms can be deduced from IR consistency allowing us to generate an ansatz for the polylogarithmic part of  $F_n^{(2)}$ .

Using four dimensional unitarity, the one-loop all-plus amplitude has no cuts and may be regarded as a vertex. With this philosophy, the coefficient of the one-loop functions can be determined by one's favourite technique. We use quadruple cuts [12] to determine the box coefficients and canonical forms [15] for the triangle and bubble functions.



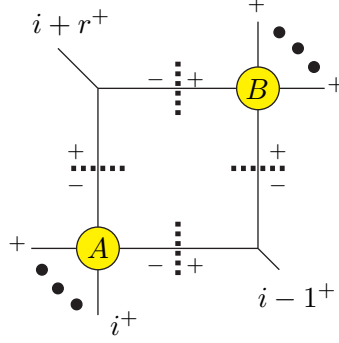


FIG. 5: The non-vanishing quadruple cut.  $A$  is a MHV tree amplitude whereas  $B$  is a one-loop all-plus amplitude.

When all external legs are of positive helicity the non-vanishing quadruple cut shown in fig. 5 is

$$M_3((i-1)^+, -l_2^+, l_1^-) \times M_{r+2}^{\text{tree}}(i^+, \dots, (i+r-1)^+, -l_3^-, l_2^-) \times \\ M_3((i+r)^+, -l_4^-, l_3^+) \times M_{n-r}^{(1)}((i+r+1)^+, \dots, (i-1)^+, -l_1^+, l_4^+) \quad (3.1)$$

where the tree amplitude may be a three-point amplitude ( $r=1$ ) but the one-loop amplitude must have at least two external legs. Using the  $n$ -point all-plus one-loop amplitude [10],

$$A^{(1)}(1^+, 2^+, \dots, n^+) = -\frac{i}{3} \sum_{1 \leq k_1 < k_2 < k_3 < k_4 \leq n} \frac{\langle k_1 k_2 \rangle [k_2 k_3] \langle k_3 k_4 \rangle [k_4 k_1]}{\langle 1 2 \rangle \langle 2 3 \rangle \dots \langle n 1 \rangle} + O(\epsilon), \quad (3.2)$$

we obtain

$$\frac{[i-1|K_4|i+r][i+r|K_4|i-1]}{\langle 1 2 \rangle \langle 2 3 \rangle \langle 3 4 \rangle \dots \langle n 1 \rangle} \\ \times \left( \sum_{a < b < c < d \in K_4} \text{tr}_-[abcd] - \sum_{a < b < c \in K_4} \text{tr}_-[abcK_4] + \sum_{a < b \in K_4} \frac{\langle i-1|K_4baK_4|i+r \rangle}{\langle i-1 i+r \rangle} \right). \quad (3.3)$$

This is the coefficient of the one-loop box function  $I_4^{2m}$ . This integral function satisfies

$$2(ST - K_2^2 K_4^2) I_4^{2m} = \left( -\frac{(-S/\mu^2)^{-\epsilon}}{\epsilon^2} - \frac{(-T/\mu^2)^{-\epsilon}}{\epsilon^2} + \frac{(-K_2^2/\mu^2)^{-\epsilon}}{\epsilon^2} + \frac{(-K_4^2/\mu^2)^{-\epsilon}}{\epsilon^2} \right) + F^{2m}[S, T, K_2^2, K_4^2] \quad (3.4)$$

where  $[i-1|K_4|i+r][i+r|K_4|i-1] = ST - K_2^2 K_4^2$ . This contains IR-infinite terms,  $(-s/\mu^2)^{-\epsilon}/\epsilon^2$ , together with  $F^{2m}$ .

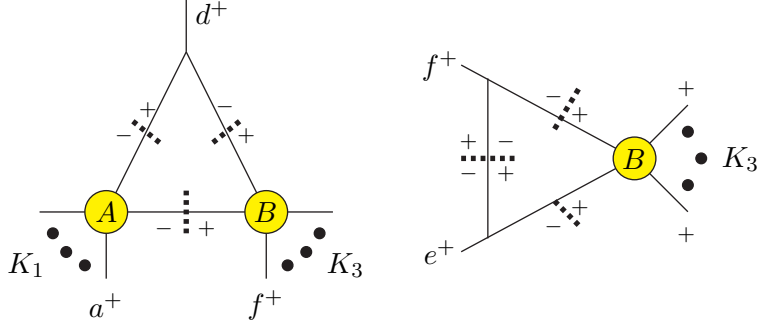


FIG. 6: The non-vanishing triple cuts. The one-mass triangle has two non-vanishing configuration.  $A$  and  $B$  are as in fig. 5.

There are also triangle integral functions whose coefficient may be determined from triple cuts [13, 16–18]. The non-vanishing triple cuts as shown in fig. 6. These can be evaluated using canonical forms [15]. The coefficient of the two mass triangle shown in fig. 6 is

$$\frac{\langle f a \rangle [d|K_3|d]}{\langle 1 2 \rangle \langle 2 3 \rangle \cdots \langle n 1 \rangle \langle a d \rangle \langle f d \rangle} \sum_{i < j \in K_3} \langle d|K_3 i j K_3|d \rangle \quad (3.5)$$

and that of the one-mass triangle is

$$\frac{s_{ef} \langle d e \rangle \langle e f \rangle \sum_{i < j \in K_3} [e i] \langle i j \rangle [j e]}{\langle 1 2 \rangle \langle 2 3 \rangle \cdots \langle n 1 \rangle \langle d f \rangle} + \frac{s_{ef} \langle e f \rangle \langle a f \rangle \sum_{i < j \in K_3} [f i] \langle i j \rangle [j f]}{\langle 1 2 \rangle \langle 2 3 \rangle \cdots \langle n 1 \rangle \langle e a \rangle}. \quad (3.6)$$

The integral functions are

$$I_3^{2m}(K_1^2, K_3^2) = \frac{1}{\epsilon^2} \frac{(-K_1^2/\mu^2)^{-\epsilon} - (-K_3^2/\mu^2)^{-\epsilon}}{(-K_1^2) - (-K_3^2)} \quad (3.7)$$

and

$$I_3^{1m}(K_3^2) = \frac{1}{\epsilon^2} (-K_3^2/\mu^2)^{-1-\epsilon}. \quad (3.8)$$

Each IR divergent term, such as  $(-S/\mu^2)^{-\epsilon}/\epsilon^2$ , occurs in both triangle and the box functions. When we sum over the box and triangle contributions we obtain an overall coefficient of  $A_n^{(1),\epsilon^0}(1^+, 2^+, \dots, n^+)$ ,

$$\left( \sum C_i I_{4,i}^{2m} + \sum C_i I_{3,i}^{2m} + \sum C_i I_{3,i}^{1m} \right) \Big|_{IR} = A_n^{(1),\epsilon^0}(1^+, 2^+, \dots, n^+) \times \sum_{i=1}^n \frac{1}{\epsilon^2} \left( \frac{\mu^2}{-s_{i,i+1}} \right)^\epsilon, \quad (3.9)$$

where  $A_5^{(1),\epsilon^0}(1^+, 2^+, \dots, n^+)$  is the order  $\epsilon^0$  truncation of the one-loop amplitude<sup>2</sup>. A key

<sup>2</sup> This has been checked on random kinematic points for  $n \leq 25$ .

step is to promote the coefficient of these terms to be the all- $\epsilon$  form of the one-loop amplitude which then gives the correct singular structure of the amplitude.

Note that in principle the amplitude could contain bubble functions. These can be obtained from two-particle cuts. However, the two-particle cuts are  $\mathcal{O}(\ell^{-1})$ , indicating that the bubble functions have vanishing coefficients and are thus absent. This is consistent with the known IR and UV singular structure of these particular amplitudes.

#### IV. RATIONAL TERMS

The rational terms in the amplitude are of course very important. After identifying the non-rational part the rational part may be obtained by recursion. This was illustrated for the five point case in [8]. Recursion is however fairly complicated because the rational terms contain double poles which means we require subleading information about the amplitude.

While this has proved possible for some amplitudes [19–22], we have not yet been able to obtain the rational part of the  $n$ -point amplitude in closed form.

#### V. CONCLUSIONS

We have proposed an explicit compact expression for the polylogarithms of the all-plus two-loop  $n$ -point amplitude which has:

- a) the correct IR and UV structure
- b) the correct collinear limits
- c) the correct four dimensional cuts

A key element is that the most complex polylogarithms are contained in the leading singular terms. The expression for the remaining polylogarithms is constructed from simple building blocks which are combinations of dilogarithms corresponding to simple one-loop box integrals. Whether there is an underlying symmetry reason for the simplicity (such as the link between the one-loop all-plus amplitude and the amplitudes of self-dual Yang-Mills [23, 24]) remains to be seen. We hope that the simplicity of the result will inspire attempts to produce compact analytical expressions for further multi-loop gauge theory amplitudes.

#### VI. ACKNOWLEDGEMENTS

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