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Topology-Based Flow Visualization, The State of the Art

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Abstract

Flow visualization research has made rapid advances in recent years, especially in the area of topology-based flow visualization. The ever increasing size of scientific data sets favors algorithms that are capable of extracting important subsets of the data, leaving the scientist with a more manageable representation that may be visualized interactively. Extracting the topology of a flow achieves the goal of obtaining a compact representation of a vector or tensor field while simultaneously retaining its most important features. We present the state of the art in topology-based flow visualization techniques. We outline numerous topology-based algorithms categorized according to the type and dimensionality of data on which they operate and according to the goal-oriented nature of each method. Topology tracking algorithms are also discussed. The result serves as a useful introduction and overview to research literature concerned with the study of topology-based flow visualization.

Keywords: flow visualization, feature-based flow visualization, flow topology, state of the art report

1.1 Introduction

Research in topology-based flow visualization is making rapid advances. Helman and Hesselink introduced the visualization community to the notion of flow topology in 1989 [21, 23]. Classical flow oriented topology research is based on the detection and classification of critical points in the vector field, as shown in Figure 1.2. What makes topology-based methods attractive is their ability to represent very large data sets in a concise and compact manner. Unlike other flow visualization approaches

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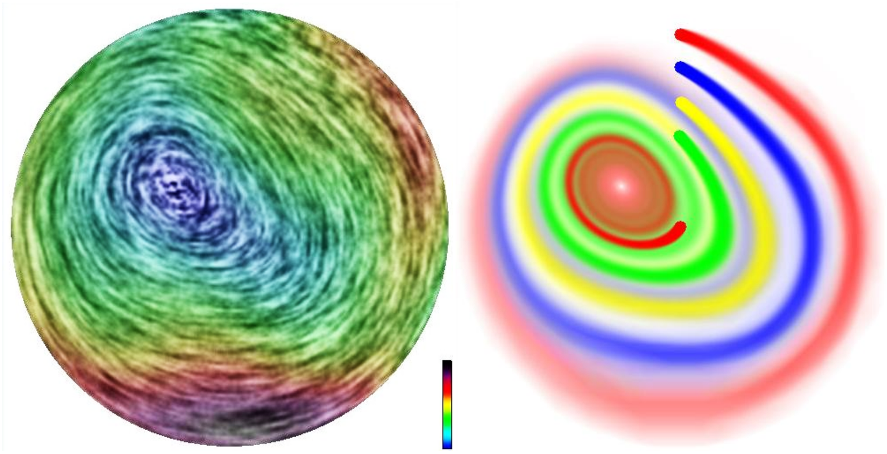

Fig. 1.1. Visualization of flow around a critical point using texture advection and dye injection [35]. In contrast to these methods, topology-based methods extract and visualize critical points directly.

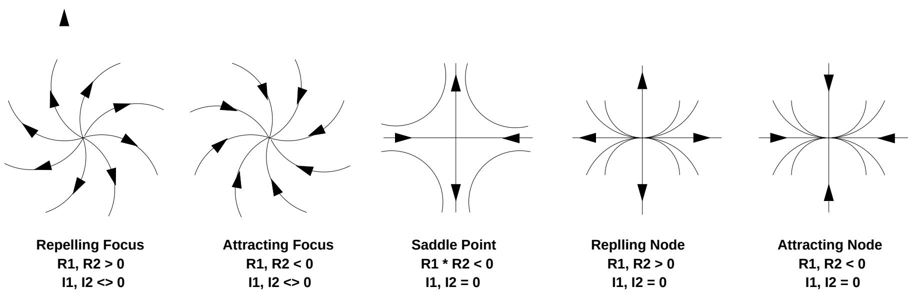

Fig. 1.2. Vector field topology: critical points are usually classified by the eigenvalues of the Jacobian [21]. R represents the real components and I the imaginary components of the Jacobian.

(Figure 1.1), critical points of a data set are extracted and the relationships between those points are depicted accordingly. We refer the reader to Abraham and Shaw for an introduction to topological analysis [1].

Topology-based research in flow visualization has come a long way since 1989 –the progress of which we will describe in Section 1.2. Yet, despite the many advances, there are still many unanswered questions in the field of topology-based research. There are still topic areas completely untouched by researchers at the time of this writing, e.g., vector and tensor field topology simplification in three-dimensions, for both steady and time-dependent (or unsteady) data.

Here, we summarize the progress that has been made up to this point in the field. We introduce a novel classification of topology-based methods in flow visualization based on topology extraction and simplification of vector and tensor fields (Section 1.2). The classification points out clearly those areas rich in previous work and some areas which still remained unaddressed by the visualization community.

1.2 Topology-Based Methods in Flow Visualization, The State of the Art

In this section, we review the current state of the art in topology-based methods in flow visualization. We start off with a description of our classification before describing the algorithms themselves. Our overview relates different research results with one another and highlights relative advantages and disadvantages of each approach.

1.2.1 Classification

		Topology Extraction			Topology Simplification	
Scalar		Static	Dynamic		Static	Dynamic
Data	2D	[19]		2D	[5] [6]	
	2.5D	[44] [26] [45]	[7]	2.5D	[13] [16] [61]	
	3D	[40] [62]		3D	[17]	
Vector		**Steady**	**Unsteady**		Steady	Unsteady
Field	**2D**	[55] [56] [87] [73]	[21] [53] [80] [89]	2D	[8] [10] [9] [38]	
Data			[83] [68] [71] [72]		[77] [82] [76] [64]	
					[67] [66]	
	2.5D	[22] [30] [32] [75]		2.5D		
		[84]				
	3D	[37]$_v$ [27]$_v$ [59]$_v$ [29]$_v$	[2]$_v$ [3]$_v$ [49]$_v$ [4]$_v$	3D	[85]	
		[31]$_v$ [51]$_v$ [18]$_v$ [46]$_v$				
		[28]$_v$ [41]$_v$ [58]$_v$ [54]$_v$				
		[23] [39] [88] [43]	[15]			
		[42] [70] [41] [14]				
		[74] [60] [86] [33]				
Tensor		Steady	Unsteady		Steady	Unsteady
Field	2D	[91]	[11] [79]	2D	[78] [82] [81]	
Data	2.5D			2.5D		
	3D	[24] [90] [92] [93]		3D		
		[25]				

Table 1.1. An overview and classification of topology-based methods in visualization. Research is divided up into topology extraction and topology simplification literature. Methodology is further classified according to scalar vs. vector vs. tensor field data analysis. Finally, a sub-classification is made based on data dimensionality, both spatial and temporal. References are listed in chronological order within each spatio-temporal dimensionality. In Section 1.2. we focus on the research with **bold** emphasis–topological analysis of vector field data. References subscripted with a $_v$ denote research related to vortex core extraction.

Table 1.1 illustrates our classification of topology-based methods in visualization. At the broadest level of classification, we have divided up the literature into work that focuses on either *extraction* of topological features, i.e., topological analysis or *simplification* of a given topology. Conceptually, simplification can be thought of as an extension of extraction. We separate the literature focused on simplification because much of it is dedicated to simplification of an *a priori* topology, especially in the area of flow visualization–the focus of this overview. We have further divided up the literature into *vector* and *tensor* field analysis. Each sub-classification is then further classified based on the spatial and temporal dimensionality of the vector or tensor field data to which the respective algorithm is applied. The topology research on scalar data is divided into *static* and *dynamic* cases rather than steady and unsteady in order to be more general. Dynamic analysis of scalar data sets can also include a transformation from one static surface to another surface [7]. Within a single spatio-temporal dimension, references are listed in chronological order. Our overview focuses on those categories with **bold** emphasis, namely, topological analysis and extraction of vector field data. The focus on vector field analysis was chosen in order to limit the scope of the review. The topics of scalar and tensor field topology can be covered in future state of the art reviews. Note that within the category of **3D, vector field extraction**, literature which focuses on vortex core extraction is denoted with subscript $(_v)$. We now describe the literature in increasing order of dimensionality, grouped together by topic. Another overview is given by Scheuermann and Tricoche [57].

Although the topology of *scalar fields* serves as a third category of research, our review of the literature does not focus on the topological analysis of scalar fields [19, 26, 62] which includes the extraction of features such as ridge and valley lines and extremal features. Our survey of scalar topology analysis is also not exhaustive, but supplies the reader references for further reading. Here we briefly mention some research in the field. Monga et al. [44] compute ridge lines on isointensity surfaces in 3D volume data and use them for data registration and automatic atlas generation. Interrante et al. [26] use ridge and valley lines in order to perceptually, enhance the visualization of multiple, transparent surfaces in 3D. Szymczak and Vanderlyde describe an algorithm that extracts topologically simple isosurfaces [61]. Morse theory has been applied to extract the topology of arbitrary surfaces by Ni et al. [45].

1.2.2 Topology Extraction of Vector Field Data

2D, Steady

Extraction of Higher-Order Critical Points: Most critical point detection algorithms are based on piecewise linear or bilinear approximation. These methods do not properly represent local topology if nonlinear behavior is present. Scheuermann et al. [55, 56] choose a polynomial approximation in areas with nonlinear behavior and apply a suitable visualization–streamlines seeded at the critical points with additional annotations.

Extraction of Closed Streamlines: Wischgoll and Scheuermann [87] present an algorithm for detecting closed streamlines in planar flows. Closed streamlines are of interest because they may indicate regions of recirculating flow. It is based on monitoring streamlines as they enter, exit, and re-enter cells of the vector field domain. We urge the reader to use caution when interpreting the visualization results. This is because an spatial dimension inherent to the applied domain has been left out of the analysis.

The first approach to detecting closed streamlines in planar flow was based on monitoring polygon-based entrance and exit events of a streamline during integration [87], This approach is extended to time-dependent flows by Wischgoll et al. [89]. At each time step, closed streamlines are extracted. Afterwards, a time-dependent correspondence between individual streamlines is computed. Theisel et al. [73] present an alternative approach to computing closed streamlines. A 2D vector field is transformed into a 3D vector field. This can be done by representing time as a third spatial dimension. Then streamsurfaces are seeded in the 3D domain. Finally, closed streamlines are detected by intersecting streamsurfaces. The difference to previous work is that this approach avoids mesh-based dependency, e.g., examining and testing individual mesh polygons.

Vector Field Design: Theisel presents a novel method that allows the user to design higher order vector fields of arbitrary topology [64]. The technique is based on control polygons that let the user specify the characteristics of critical points. This enables a mechanism by which to test topology extraction algorithms. The result can also be used for compression purposes. We note that this research does not fit cleanly into our classification partially because it spans more than one area.

2D, Unsteady

Detection and Classification of Critical Points: Helman and Hesselink introduced the visualization community to flow topology [21]. Their analysis included the detection, classification, and visualization of critical points in planar flows (Figure 1.3). They applied their algorithms to both steady-state and unsteady flow. They represent time as a third spatial dimension for the case of time-dependent, planar flow.

Vortex Detection Based on Streamline Geometry: Sadarjoen and Post [53] present two methods for detecting vortex structures in 2D vector fields. They are both based on an analysis of streamline geometry. The first method uses local cumulations of curvature that may indicate a group of vortices in very close proximity to one another. The second method looks at the curvature of a single streamline and computes a winding angle–a metric of geometric curvature. One advantage of this technique is that it detects weak vortices because it does not depend on velocity magnitude at a single point. A disadvantage, however, is the large number of streamlines that must be seeded and computed in order to maintain complete coverage of the flow.

Detection of Topological Transitions: A novel topology-based method for the visualization of time-dependent 2D flows is given by Tricoche et al. [80]. Extending the work of Helman and Hesselink [21, 23], they identify and visualize topological

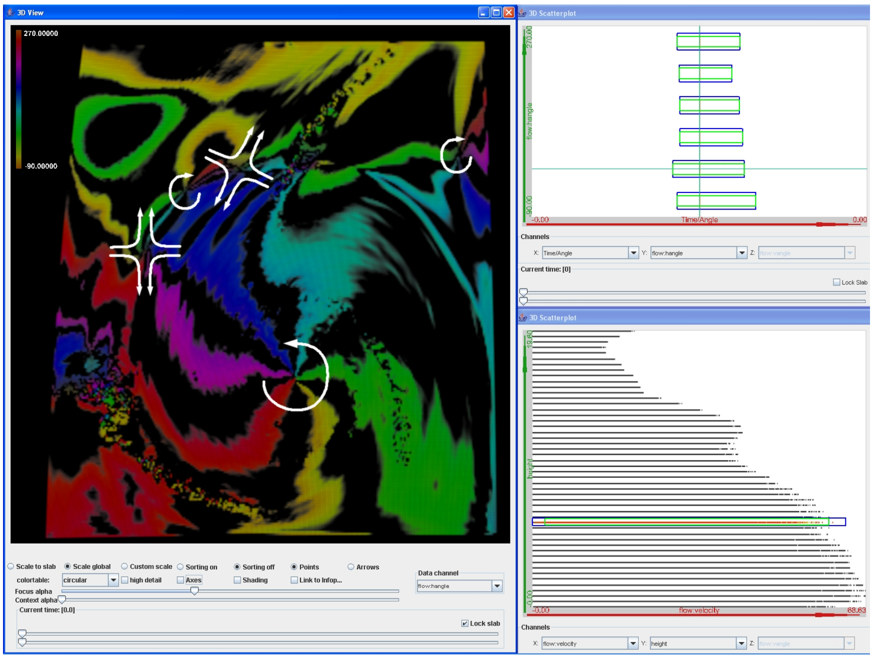

Fig. 1.3. Here, the topology of a hurricane is visualized and depicted with the SimVis system [12].

transitions–the qualitative change of topology structure from one stable state to another over time. Three types of transitions are investigated: (1) a Hopf-like transition–a transition of a singular point from an attracting focus (i.e. sink) to a repelling focus (i.e. a source), (2) a fold-like transition–the pairwise annihilation or creation of a saddle and a source or sink, (3) a basin transition–the case when two saddle points start independent of one another, join briefly, and again separate. Again we caution the reader when interpreting these results. A spatial dimension inherent to the original domain has been omitted from the analysis.

Critical Point Tracking: Theisel and Seidel introduce an alternative critical point tracking method for 2D, unsteady flow based on streamlines [68]. The temporal dimension of the planar flow is represented as a third spatial dimension and streamlines are traced along critical points as they evolve. This space-time representation is called a feature flow field. In addition to visualizing the path of critical points over time, events such as fold bifurcations are visualized.

Streamline and Pathline Oriented Topology: Topological methods often segment vector fields using curves based on streamlines, e.g., separatrices or streamsurfaces such as separating streamsurfaces. In addition to streamline oriented topology, Theisel et al. [71, 72] also consider pathline oriented topology. In the study of streamline oriented topology, they propose new approaches to detect bifurcations like saddle connections and cyclic fold bifurcations. Saddle connections are bifurcations that appear when two separatrices originating from saddle points coincide. A cyclic fold bifurcation is the case of when two closed streamlines collapse and disappear. The also propose a novel approach to detect and track closed streamlines in 2D, time-dependent vector fields. In the study of pathline oriented topology, they segment the vector field into regions where pathlines show attracting, repelling, or saddle-like behavior.

Vector Field Comparison: Although it does not fit cleanly into our classification, we briefly mention a closely related topic–vector field comparison. Theisel et al. [65] introduce a topology-based metric by which vector fields can be compared or related to one another. Preliminary approaches based on comparison metrics (i.e., distance measures) were based on local deviations of direction and magnitude of flow vectors [20, 63]. These previous distance functions yield a fast comparison of vector fields, but do not take into account any structural information. Levin et al. [36] introduce the first topology-based approach to vector field metrics with the Earth Mover's Distance (EMD [52]), a technique from image retrieval. The limitations of this algorithm are that: (1) it's critical point coupling strategy does not consider the location of critical points in the vector fields and (2) all critical points are compared to one another which can lead to a worst case complexity of $o(n!)$ where n is the number of critical points. To overcome these critical point coupling limitations, Theisel et al. [65] introduce a comparison metric that uses feature flow fields [68].

2.5D, Steady

Separation and Attachment Lines: Separation and attachment lines correspond to loci where flow leaves or converges at a surface. Prior to Kenwright [30], the only algorithm that could automatically detect separation and attachment lines was presented by Helman and Hesselink [22]. Previous approaches were generally based on observations. Helman and Hesselink's technique is based on vector field topology. Their algorithm detects closed separation lines, that is, lines that begin at a saddle or node and end at another saddle or node. Kenwright's algorithm also detects open separation, i.e., lines that do not always start or end at critical points in the vector field. This algorithm is based on phase plane analysis.

Kenwright et al. [32] expand the work of Kenwright [30] by introducing another algorithm, the parallel vector algorithm, for detecting open separation and attachment lines. The parallel vectors algorithm is based on the observation that one of the eigenvector directions was always parallel to the local streamlines in regions where streamlines asymptotically converged. The advantage of this approach is that it provides a local test that may be performed at any point in the vector field. Kenwright et al. show that the parallel vectors algorithm is slightly superior to their previous algorithm (called the phase plane algorithm), however, it is more difficult to implement. The phase plane algorithm uses self-contained analysis within each triangle, making it well suited for unstructured meshes. The parallel vector algorithm requires calculation of vector gradients on irregular triangulations. But for curvilinear meshes, the parallel vector algorithm is best because vector gradients can be calculated using central differences. The parallel vector algorithm also resolves the line discontinuity problem associated with the phase plane algorithm.

Tricoche et al. [75] propose a method for the detection of separation of attachment lines in 2D flows defined over arbitrary surfaces in 3D. They build primarily on the work of Kenwright and Haimes [30, 32] by improving performance. They do so by using both local flow properties and global structural information such that feature searching and extraction is fast and accurate.

Boundary Switch Connectors: Weinkauf et al. [84] extend the work of Theisel et al. [70] with the introduction of *boundary switch connectors*, a topological element that complements saddle connectors. Theisel et al. [70] considered separation surfaces emanating from saddle points only. Weinkauf et al. [84] extend this work to include separating surfaces starting from boundary switch curves. The intersection of separating surfaces emanating from boundary switch curves results in boundary switch connectors.

3D, Steady

Vortex Core Line Extraction: Sujudi and Haimes [59] present a line-based vortex core extraction algorithm that locates points that satisfy the following two criteria: (1) the velocity gradient tensor contains complex eigenvalues and (2) the velocity in the plane perpendicular to the real eigenvector is zero. The individual points are then connected to form the vortex core line. The disadvantage here is that it is not always possible to form a continuous line. This problem is addressed by Haimes and Kenwright [18] who present adapt the algorithm to be face-based rather than cell-based.

Vortices can cause many undesirable effects for aircraft, such as reduced lift and noise. They can lead to structural fatigue and even premature airframe failure in severe cases. Kenwright and Haimes [29, 31] applied the eigenvector method of Sujudi and Haimes to flow analysis around an aircraft.

Roth and Peikert build on the work of Sujudi and Haimes [59] by introducing a higher-order method for vortex core line extraction. While the eigenvector method of Sujudi and Haimes [59] is correct for linear vector fields, it fails to detect curved vortex core lines, especially in the case of turbomachinery data sets. Roth and Peikert demonstrated this limitation previously [50]. Their method overcomes the previous limitations stemming from the use of a linear vector field for vortex core line extraction by introducing higher-order derivatives that can be used to detect bent vortex cores.

This vortex core line extraction algorithm is later formulated at a higher level of abstraction, namely as a parallel vectors operator by Peikert and Roth [46]. The basic idea behind the parallel vectors approach is to derive two vector fields from a given 3D vector field such that vortex core lines are locations where the two derived vector fields are parallel.

Some vortex core extraction methods, like that from Jeong and Hussain [27], can be described as Galilean invariant, i.e., they are invariant when a constant vector field is added. This is because their computation uses only derivatives of the vector field. Many vortex core line extraction algorithms are Galilean variant because they depend on a certain reference [2, 3, 46, 59]. Sahner et al. [54] present an approach to extracting vortex core lines that is Galilean invariant, i.e., the result does not depend on the frame of reference. The extracted features remain unchanged when adding a constant vector field. They do so by considering ridge or valley lines of Galilean invariant vortex region quantities.

Vortex Core Region Extraction: A general problem with vortex core line extraction algorithms is their computational complexity and that they may generate more than one vortex core line within a vortex core region. Mahrous et al. [41] present a vortex core region detection based on Sperner's lemma–adapting a notion from combinatorial topology. The approach analyzes the behavior of a vector field based on the vectors found at the boundaries of each grid cell. Velocity vectors exhibit characteristic patterns in the neighborhood of a vortex. The algorithm searchers for these patterns.

In our overview, we focus on vortex core line extraction rather than vortex core region extraction. Thus the method of Jeong and Hussain, known as the λ_2 method [27] is not described in detail here (Stegmaier and Ertl present a GPU-based implementation of the λ_2 method [58]). Similarly, we do not focus on vortex core extraction based on isosurface extraction in a scalar field [37]. A more general overview of vortex analysis from a feature-based flow visualization point of view is given by Post et al. [47].

Separating Surfaces: Helman and Hesselink build on their previous work [21] and extract surface topology and separating surfaces of flow in 3D [23]. A surface topology skeleton is extracted and visualized by projecting the 3D vector field in the neighborhood of the surface onto the plane tangent to the body and applying a 2D detection algorithm. They also compute streamsurfaces which separate 3D vector fields into disparate regions of flow. Included is a description of how these streamsurfaces are tessellated in an efficient manner. They also uses icons such as arrows and disks to display critical points in 3D.

Mahrous et al. [41, 42] present an algorithm for efficient computation of separatrices in 3D vector fields. They present methods that accelerate the extraction of separatrices. Enhancements are made to reduce the number of sample streamlines and their length. Streamlines are seeded in a more meaningful and a efficient matter rather than using a brute-force approach of seeding streamlines at all cell locations. Texture advection is applied to stream surfaces by Laramee et al [34].

Dynamical Systems: Löffelmann and Gröller [39] visualize the topology of dynamical systems. Dynamical systems provide a mathematical model comprised of a set of state variables whose goal is to characterize real world phenomena, e.g., a stock market, a chemical reaction, or a food chain. Their visualization couples characteristic streamlines emanating from fixed points in the domain with a thread of streamlets. The characteristic streamlines play the role of seed points for a thread of streamlets. The large number of streamlets provide more information about the behavior of the dynamical system in the neighborhood its characteristic trajectories. Thus a trade-off between domain coverage and perceptibility is realized in 3D.

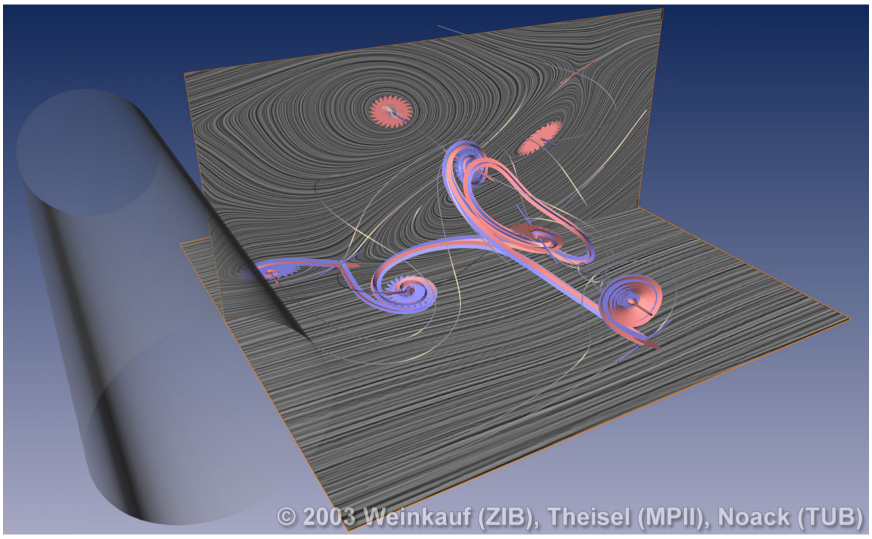

Fig. 1.4. Visualization of flow past a circular cylinder using critical points and saddle connectors [70]. *Image courtesy of H. Theisel et al.*

Detection of Closed Streamlines: Wischgoll and Scheuermann [88] extend their previous work [87] of detecting closed streamlines to 3D vector fields. The algorithm is based on preventing infinite cycling during streamline integration. **Saddle Connectors:** Theisel et al. [70] introduce a new topological element of vector fields called a *saddle connector*. A saddle connector is a streamline that joins two saddle points in a vector field (Figure 1.4). A saddle connector is found essentially by computing the intersection of the separation surfaces of two saddle points. These topological structures achieve a visually sparser, more compact topological representation of the vector field, thus avoiding the visual complexity associated with showing too many separating streamsurfaces.

Hybrid Visualization and Vortex Breakdown: Tricoche et al. [74] use a combination of 3D volume rendering of a vector field's scalar fields with vector field topology projected onto a moving cutting plane. The goal is to gain insight into the behavior of vortex breakdowns with this novel hybrid visualization (Figure 1.5).

Critical Point Modeling and Classification: Weinkauf et al. [85] extend the work of Theisel [64] for designing vector fields. In particular they: (1) model 3D vector fields of arbitrary topology. Previously, only first order points and the index of higher order critical points were considered [43], (2) introduce a complete classification of 3D critical points and (3) adapt the notion of saddle connectors in order to model the intersection curves of separation surfaces. Thus, the problem of modeling a vector field is reduced to the problem of modeling the topological skeleton using control polygons.

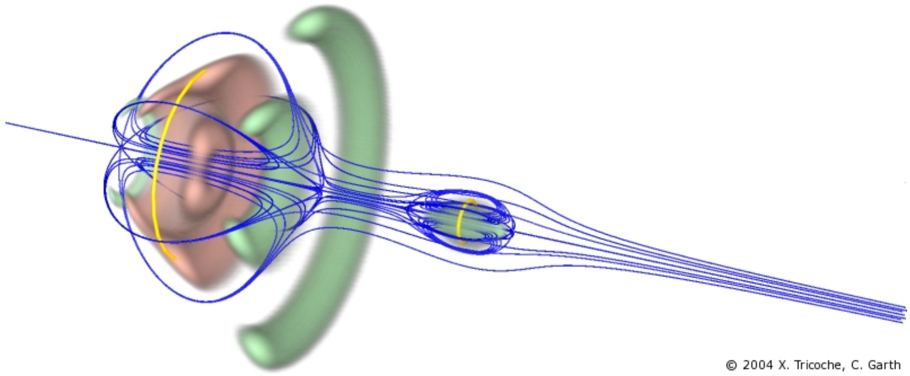

Fig. 1.5. The visualization of a vortex breakdown bubble. Flow topology is depicted with stagnation points in red, singularity paths in yellow and streamlines in blue [74]. *Image courtesy of X. Tricoche et al.*

Weinkauf et al. [86] extend the work of Tricoche et al. [77] to 3D. They introduce an extraction and classification scheme for higher order critical points in 3D. The approach is based on enclosing a critical point, or a cluster of critical points by a bounding surface. The properties of the vector field at the boundary surface are then examined in detail, i.e., subsets of the surface are divided up into inflow, outflow, hyperbolic, and elliptic regions of flow. The classification of critical points in 3D is then determined by the corresponding regions on the bounding surface. The simplified structure of the flow within the bounded regions is then visualized with an appropriate icon(s).

Applications of Topology-Based Flow Visualization: Sun et al. [60] apply a topological analysis to visualize the power flow through a C-shaped nano-aperture. Such an aperture may be very effective at power transmission with applications including data storage, particle manipulation, and nano-scale photonic devices. Their topological analysis of this data set results in a heightened understanding of the critical factors affecting power transmission of these apertures including: polarization effects, efficiency, the size of interaction regions, resonant transmissions, and more.

Laramee et al. [33] apply topology-based flow visualization methods in order to gain insight into the behavior of flow through a cooling jacket. This application is discussed in more detail in a following chapter. Other applications of topology-based flow visualization are discussed by Garth et al. [15] and Tricoche et al. [74].

3D, Unsteady

Vortex Core Line Extraction and Tracking: Banks and Singer [2, 3] developed an algorithm for vortex tube reconstruction based on the assumption that a vortex core is a vorticity line–a streamline in the vorticity field. and pressure is a minimum in the core. The algorithm consists of four basic steps: (1) compute the vorticity

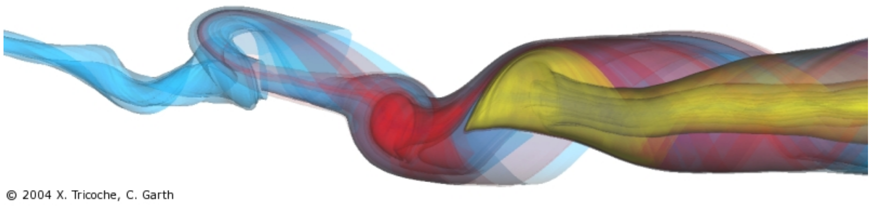

Fig. 1.6. The visualization of vortex breakdown using transparent separation surfaces originating at stagnation (saddle) points [15]. *Image courtesy of C. Garth et al.*

along a vortex core line (seeded based on threshold vorticity magnitude and pressure), (2) predict the next point along the core line by stepping in the vorticity vector's direction, (3) compute the vorticity at the new predicted point, and (4) update (or correct) the point to the location of minimum pressure in the plane perpendicular to the core.

Reinders et al. [49] present an application which detects and tracks vortex tubes in flow past a tapered cylinder. First, they apply the winding-angle method [53] is used to detect the vortices on a number of horizontal slices. Second, the 3D vortex tubes are constructed from the 2D vortices by applying a spatial feature tracking procedure based on attributes of the vortices [48]. The same feature tracking algorithm is then applied in the temporal domain for vortex core tracking.

Theisel et al. [69] describe a novel method to extract parallel vectors [46] based on the use of feature flow fields [68]. They derive appropriate vector fields such that vortex core lines appear as streamlines (in the feature flow fields). Thus, the extraction of vortex core lines is reduced to a well-known streamline integration computation. They also introduce a novel classification of transitions (or events) associated with time-dependent vortex core lines as well as the methodology used in tracking core lines. The classification includes: (1) saddle transitions, (2) closed collapse transitions, (3) and inflow and outflow boundary transitions.

Singularity Tracking and Vortex Breakdown: Garth et al. [15] present a method to efficiently track singularities in 3D, unsteady flow. The method also applies to data defined on unstructured grids. Conceptually, it is an extension of the work of Tricoche et al. [83]. The concept of a singularity index is discussed and extended from the well known 2D case to the more complex 3D domain. The results are particularly insightful for the study of vortex breakdown. Occurrences are vortex breakdown (or bursting) are correlated with local extrema in physical quantities and visualized with corresponding views from information visualization (Figure 1.6).

1.2.3 Discussion and Future Prospects

Table 1.1 clearly illustrates those areas with a heavy concentration of topology-based research, e.g., 3D steady-state, and those areas with little to no work. In fact, Table 1.1 highlights areas that remain untouched up to this point in time, e.g., topology simplification in 3D tensor fields. Other areas still requiring research work include:

- Interactive techniques to support topology extraction and tracking: At present, topology-based techniques are, in general, still slower relative to traditional flow visualization techniques such as particle tracing or texture-advection methods.
- Extraction and analysis of new types of topological structures: Surely, not all important topological structures have been clearly identified and studied.
- Integration of topology-based methods with other flow visualization techniques such as texture advection: A topological skeleton by itself, sometimes leaves out other important properties of the flow such as downward and upstream direction.
- The practical application of topological methods outside the visualization community: Still, much work remains to be done in the application of topology-based flow visualization to data sets from industry or some application domain area in order to demonstrate their utility in a convincing manner.
- More theoretical development to support cognition of results: Topological analysis still leaves open questions with respect to interpretation of the results. For example, how do we interpret pathline-oriented topology? More theory may be needed to aid such cognition.

Thus, the field of topology-based methods in visualization is still rich in unsolved problems.

However, there may be reasons why so much of the spatio-temporal domain in our classification remains virtually unexplored in the research literature. Reasons may include high levels of complexity and applicability to real-world problem domains. We discuss possible reasons for this in a later chapter.

1.3 Acknowledgments

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