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Paper:

Data-driven and hybrid coastal morphological prediction methods for mesoscale forecasting

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Dominic E. Reeve [a,*], Harshinie Karunarathna [a], Shunqi Pan [b], Jose M. Horrillo-Caraballo [a], Grzegorz Różyński [c], Roshanka Ranasinghe [d]

[a] College of Engineering, Swansea University, Wales, UK
[b] School of Engineering, Cardiff University, Wales, UK
[c] Institute of Hydro-Engineering, Polish Academy of Sciences, Gdańsk, Poland
[d] UNESCO-IHE/Delft University of Technology, The Netherlands

A R T I C L E   I N F O

A B S T R A C T

It is now common for coastal planning to anticipate changes anywhere from 70 to 100 years into the future. The process models developed and used for scheme design or for large-scale oceanography are currently inadequate for this task. This has prompted the development of a plethora of alternative methods. Some, such as reduced complexity or hybrid models simplify the governing equations retaining processes that are considered to govern observed morphological behaviour. The computational cost of these models is low and they have proven effective in exploring morphodynamic trends and improving our understanding of mesoscale behaviour. One drawback is that there is no generally agreed set of principles on which to make the simplifying assumptions and predictions can vary considerably between models. An alternative approach is data-driven techniques that are based entirely on analysis and extrapolation of observations. Here, we discuss the application of some of the better known and emerging methods in this category to argue that with the increasing availability of observations from coastal monitoring programmes and the development of more sophisticated statistical analysis techniques data-driven models provide a valuable addition to the armoury of methods available for mesoscale prediction. The continuation of established monitoring programmes is paramount, and those that provide contemporaneous records of the driving forces and the shoreline response are the most valuable in this regard. In the second part of the paper we discuss some recent research that combining some of the hybrid techniques with data analysis methods in order to synthesise a more consistent means of predicting mesoscale coastal morphological evolution. While encouraging in certain applications a universally applicable approach has yet to be found. The route to linking different model types is highlighted as a major challenge and requires further research to establish its viability. We argue that key elements of a successful solution will need to account for dependencies between driving parameters, (such as wave height and tide level), and be able to predict step changes in the configuration of coastal systems.

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1. Introduction

Planning and development on our shorelines is increasingly undertaken within the framework of structured shoreline management plans that require the consideration of morphological change over a window of up to 100 years into the future. Methods to perform this have been scarce and predictions have been made on an ad hoc, case by case, basis. The absence of a consistent predictive framework has provided the motivation to develop morphological models that can provide useful mesoscale (of the order of 10^1 to 10^2 km length scales and 10^1 to 10^2 year timescales) estimates of coastal morphological change. The hurdles to developing such models are significant. The deterministic process models that have proved useful for predicting short-term storm response encounter difficulties when applied to mesoscale problems. Not only does the increased number of time steps required lead to an unacceptable accumulation of numerical errors, as well as huge increases in computational time, but the approach has difficulties in reliably reproducing some of the broad morphological tendencies observed in practice.

In this paper we discuss some of the methods that have been attempted to make mesoscale predictions of coastal change and then propose an alternative approach, based on using information contained in the growing amount of observational evidence gathered in coastal monitoring programmes. This approach has gained the epithet 'data-driven' modelling and is demonstrated through a number of applications to study sites from around the world. We also provide a demonstration of how data-driven methods can be combined with reduced

complexity models as an example of how a good observational database can be combined with elements of physical understanding to form the basis of prediction. Accordingly, we argue for the importance of maintaining and extending long records of good quality observations, such as those gathered at Duck in the USA, Lubiatowo in Poland and the Channel Coastal Observatory in the UK, and the intention to expand the observational network to cover a wider range of shoreline types and exposures. Further, methods that combine the observational evidence with elements of our understanding of the key physical processes seem to show some promise, and go some way towards addressing the criticism that purely data-driven methods are based entirely on historical measurements.

One question that arises naturally from scale categorisation is whether forecasting methods can (or should) be developed at each scale, or whether the detailed process knowledge at small scales should simply be extended into the description of larger scale processes, at consequent computational cost. Section 2 provides some background to the different types of methods that have been developed for predicting mesoscale coastal morphology. Section 3 presents a range of data-driven techniques together with a selection of applications to specific sites. The merging of data-driven and mechanistic approaches in hybrid models is discussed in Section 4. The paper concludes with Section 5.

2. Background

Our coastlines change as a consequence of the aggregation of forces due to winds, waves and tides and the consequent movement of sediments. The net result of continual sediment transport is an alteration of the shape or morphology of the shoreline. De Vriend et al. (1993) concluded over 20 years ago that understanding this process was one of the most challenging issues confronting coastal engineers and managers. Despite improvements in process understanding, computational power and monitoring techniques, our understanding of coastal morphodynamics remains limited. Unfortunately there are practical limitations to simply expanding the process models that have worked quite well for short-term prediction to the mesoscale problem. These arise partly through the potential for errors associated with the numerical approximation of derivatives to accumulate and eventually dominate the procedure, rendering the solution useless (e.g. Lilly, 1965). There are theoretical arguments that suggest there may be an inherent uncertainty or limit of predictability in the equations used for coastal process modelling due to their strong nonlinearity. In practical terms this means that the computations for very similar starting conditions will at some point diverge to very different, but equally valid, solutions. This type of behaviour has been termed "chaos" after the pioneering work of Lorenz (1963) into dynamical systems. The consequence of apparently deterministic equations of motion supporting chaos is that even if our set of morphological prediction equations are solved perfectly, we cannot be sure that our predictions will be perfect because of the limited accuracy of the initial conditions. Both Baas (2002) and Southgate et al. (2003) discuss a number of methods developed in the discipline of nonlinear processes and explain how these can be applied in the context of coastal engineering. To date, it has not been established whether the equations for morphodynamic evolution support chaos or not. From a pragmatic point of view, it seems only prudent to assume that uncertainties in initial conditions, measurement errors and numerical errors are likely to limit the period over which useful predictions can be obtained through deterministic process modelling. Some progress has been made using the 'brute force' approach of running process models over areas of several square kilometres and for periods up to 5 years or so as reported by Lesser (2009). The primary computational constraint is that the hydrodynamic and morphodynamic time scales are quite different. The currently preferred technique for addressing this is the morphodynamic acceleration method, in which the bed level changes that occur during a single hydrodynamic time step are multiplied by a factor so that the morphodynamic updating step does

not have to be computed for each short hydrodynamic step. Detailed discussion of the technique in a range of applications can be found in Lesser et al. (2004), Jones et al. (2007), van der Wegen and Roelvink (2008), Roelvink et al. (2009), Ranasinghe et al. (2011) and Dissanayake et al. (2012) amongst others. Thence, there is good motivation to find alternative means to make mesoscale predictions that are required to inform coastal planning.

Coastal management, design of coastal structures, and flood risk all depend upon our understanding of how the shoreline changes, especially at decadal and longer timescales. Engineered structures such as groynes, artificial headlands and detached breakwaters are used as means to control the movement of sediment on the beach or near the coast. Despite such measures, changes in the prevailing conditions can lead to dramatic variations in coastal morphology and hence flood and erosion risks. Gaining insight into the physical processes that govern mesoscale morphological evolution (French et al., in this issue; van Maanen et al., in this issue), is crucial for the successful design of coastal defence systems and formulation of shoreline management strategy. From a forecasting perspective, various classes of approach are possible:

a) Process-based modelling: These models include explicit representations of physical processes to describe the complex interaction between waves, tides, sediment transport, coastal defence structures and the resulting morphological and shoreline changes. This approach can be successful for short-term forecasting, such as single or multiple storm events, often at a limited spatial scale associated with specific engineering schemes. It becomes less feasible for longer term simulations and larger domains for the reasons already mentioned above. Nevertheless, process-based modelling is capable of providing valuable insights into complex processes, thus improving the level of understanding of those processes, as demonstrated in the reviews by de Vriend et al. (1993), Nicholson et al. (1997), Roelvink (2006), Pan et al. (2010) and others.

b) Data-driven modelling: This relatively new class of approach is discussed in detail in Section 3, so only a brief outline is given here. In essence, data-driven models use measurements of past conditions at a site, together with sophisticated statistical techniques, to identify patterns of behaviour that are then extrapolated into the future to form a forecast.

c) Hybrid Modelling: This covers models in which simplifications to the governing equations or the forcing, or both, are made in order to make mesoscale forecasting tractable. Such approaches have also been termed 'reduced complexity methods' or 'behaviour-oriented' models. This class also includes approaches that combine two or more modelling concepts to form hybrids, such as combining empirical equilibrium models with wave sequences to introduce an evolutionary element (e.g. Yates et al., 2009; Splinter et al., 2014). In some situations information from complex process models is used to provide parameterised data to simpler models, such as one-line or N-line models, in order to retain the primary coastal processes. An example of this type of approach includes the use of Unibest-CL+ with parameterizations derived from a number of simulations using the model suite Delft3D reported by van Koningsveld et al. (2005). Huthnance et al. (2008) provide a brief survey of hybrid models developed for estuary and coastal inlet morphology prediction with a range of complexity, including the Analytical Emulator described by Manning (2007), the Hybrid Regime model of HR Wallingford (2006), the SandTrack approach of Soulsby et al. (2007), ASMITA described by Wang (2005), and the inverse techniques of Karunarathna et al. (2008), that have demonstrated applications with encouraging results.

d) Probabilistic modelling: This class of modelling is used to quantify uncertainties and is included as a separate approach because the input data and output quantities are distinct from those of deterministic models. Specifically, descriptions of the probability distribution functions and correlation properties of the drivers are required as

input, and the output is (sample) statistics of the dependent variables. Many probabilistic models are essentially deterministic models run many times over to create a Monte Carlo simulation, in order to assess the levels of uncertainty in the predictions. For example, Monte Carlo simulations have been presented by Vrijling and Meijer (1992) for a port development, by Dong and Chen (1999) to investigate the effect of storms on an open coast, by Lee et al. (2002) for forecasting cliff erosion, Wang and Reeve (2010) to assess the performance of a detached breakwater scheme and Callaghan et al. (2013) to estimate storm erosion. The output from these models can provide an indication of the uncertainty in the predictions of deterministic models but are difficult to validate in the sense that there are rarely sufficient measurements to verify that the statistics of the model output correspond to the statistics of the measurements. To address this deficiency other probabilistic approaches have been developed to provide an independent check of Monte Carlo results. These are quite complex and have been developed for 1-line model conditions only to date. In one of the first examples, Reeve and Spivack (2004) provided analytical expressions for the first two moments of beach position in the case of a beach nourishment. Subsequently Dong and Wu (2013) formulated a Fokker–Planck equation for the probability distribution of shoreline position, and Reeve et al. (2014) presented a closed-form analytical solution for the mean beach position near a groyne, subject to random wave attack.

3. Data-driven methods

Most models are driven by data through initial or boundary conditions so the term 'data-driven' might at first seem rather all-encompassing. However, the term 'data-driven method' refers to techniques that rely solely on the analysis of measurements, without invoking knowledge of physical processes. Attribution of particular behaviours found through the analysis to particular physical processes is through inference. At its most basic, a data-driven method involves analysing a sequence of measurements of forcing variables or coastal state indicators in order to find evidence of trends, cycles or other smoothly varying modes of change. The term 'data driven model' is best reserved for the process of making a forecast of future coastal state formulated around an extrapolation of the patterns found from analysis. Changes in morphological regime are unlikely to be captured unless the past records also contain such episodes. Much of the literature covers the application of methods to finding patterns in measurements and interpreting these in terms of physical processes. There is less work reported on the use of data-driven methods for forecasting.

The origins of data-driven modelling are difficult to pinpoint but evolved from the application of prognostic analysis techniques and a recognition that many coastal datasets seemed to exhibit coherent patterns of temporal behaviour that could be extrapolated to form a prediction. The extent of signal extraction has improved as the sophistication of statistical methods has increased. One of the most basic problems encountered in coastal data analysis is that measurements usually provide poor resolution in time and are often intermittent, thereby making interpolation very error-prone. Spatial sampling is often much better with surveying along fixed profiles at regular intervals becoming a norm in monitoring schemes. Analysis methods based on Fourier analysis (in time) are therefore difficult to use and alternatives have been sought, with Empirical Orthogonal Function analysis being one of the most widely used. Early analyses with Empirical Orthogonal Functions, (EOF), such as those of Winant et al. (1975) and Aranuvachapun and Johnson (1978), relied mostly on the records of beach profile measurements. In contrast, the later studies of Wijnberg and Terwindt (1995) and Reeve et al. (2001) extended analyses to nearshore and offshore morphology respectively. Notable exceptions are the analyses of

Aubrey and Emery (1983), Solow (1987) and Ding et al. (2001) of data from long-established national tide-gauge networks, used for tidal harmonic decomposition and surge analysis.

The link between data-driven methods and process understanding is not a one-way relationship. For example, we might well expect from dynamical considerations that a beach subjected to seasonally changing wave conditions would show a seasonal signature in its response. This deduction can be tested through data-driven analysis such as that reported by Haxel and Holman (2004) who used EOF analysis to isolate seasonal patterns in beach alignment. Both Haxel and Holman (2004) and Thomas et al. (2010) were also able to identify longer interannual patterns in beach plan shape which were attributed to the El Niño–La Niña atmospheric oscillation, something that could be tested in process modelling.

National beach monitoring programmes, such as those run by the US Army Corps at Duck, Carolina, the Polish Academy of Sciences Lubiatowo Coastal Research Station, and the New Forest District Council's Channel Coastal Observatory in the UK, (New Forest District Council, 2014), have spurred a rapid expansion in the type and sophistication of statistical methods that have been used for analysis. For example, Singular Spectrum Analysis (SSA) was employed to trace forced and self-organised components of shoreline change (Różyński et al., 2001), Principal Oscillation Patterns were used to derive a data-driven model of changes in nearshore bathymetry by Różyński and Jansen (2002) and Canonical Correlation Analysis (CCA) was employed by Różyński (2003) to study interactions between bars on the multi-barred beach at Lubiatowo while Larson et al. (2000) used it to analyse the links between beach profile changes and wave conditions at Duck, North Carolina.

Hsu et al. (1994) described one of the first studies to use data-driven methods for forecasting beach levels. They combined beach level measurements with an indicator of wave conditions, (Irribarren number), in an EOF analysis so as to forecast beach levels on the basis of the prevailing wave conditions. The technique was moderately successful but did not find widespread use. CCA has one advantage over the other methods in that it explicitly establishes linked patterns of behaviour between two variables, thereby opening the possibility of forecasting one variable on the basis of the other. This means it is ideally suited to morphological prediction where beach response is strongly dependent upon hydrodynamic conditions. The thinking behind this is thus: our predictive capability for waves and tides is generally greater than it is for coastal morphology; if we can establish a link between the hydrodynamics and the beach morphology then this, together with hydrodynamic forecasts, might be used as an effective predictor of coastal morphology. Horrillo-Caraballo and Reeve (2008, 2010) demonstrated how CCA could be employed to analyse beach profile and wave measurements at a sandy beach at Duck, North Carolina and a mixed sand/gravel beach at Milford-on-Sea on the south coast of the UK. Forecasts of beach profiles at both sites, based on the correlations between waves and beach profiles, had a quality useful for planning purposes over a period of about a decade. In a parallel, but mathematically quite similar, development artificial neural networks have also found application in coastal sciences. Very often presented as a method in which an artificial 'brain' is first 'trained' on a range of input and output data and then used to make forecasts using new input data, this method is in essence a multiple regression technique. One of the earliest applications in coastal engineering was by van Gent and van den Boogaard (2001), who used measurements obtained in laboratory experiments to create a neural network to predict forces on vertical structures. A similar process was used to create a wave overtopping calculator for coastal structure design in the EurOtop Manual (2007), and many subsequent applications for engineering design purposes. The use of neural networks for coastal morphology was discussed by Southgate et al. (2003), and Pape et al. (2007, 2010) have described the application of a variety of neural net algorithms to the problem of predicting the movement of a nearshore bar over the period of several years. The

core assumption of data-driven models is that information on the evolution of a system can be extracted from signals that are manifestations of that evolution.

In the following subsection a description of the Lubiatowo coastal station and dataset is provided as this forms much of the basis of the discussion of the different data-driven methods that follows. In the subsequent subsections the different analysis techniques are described briefly, following which the process of applying this sequence of data-driven methods to the data is illustrated to demonstrate how the individual methods can be applied and what extra information becomes available at each stage.

3.1. Lubiatowo coastal station

The IBWPAN Lubiatowo Coastal Research Station is located on the shores of the southern Baltic Sea, near Gdansk, Poland (Fig. 1). The beach is sandy and approximately north facing. Tides are extremely small (a few centimetres) and most water level variations arise as a result of surges due to the passage of low pressure atmospheric depressions, and local wind and wave set up.

The coastal station was constructed in the 1970s and consisted of 8 measuring towers from which measurements including waves, water levels and seabed elevations were made, (Fig. 2 left panel). Storm damage and economic constraints have meant that the most offshore towers have not been replaced so that now there are two towers remaining (Fig. 2 right panel).

Measurements performed and archived at the station include:

- Routine monthly records of beach topography and shoreline configuration along 27 geodetically referenced cross-shore profiles, since 1983 (Fig. 3);
- Routine annual/bi-annual records of nearshore topography along the same profiles; cross-shore range ca. 1000 m;

- Records of wave height and wave driven currents within the surf zone (wave gauges, current metres) during field experiments;
- Records of deepwater wave height, period and direction by a waverider — during field campaigns;
- Records of wind speed and direction as supplementary measurements.

The Lubiatowo site provides a very good example of a beach with multiple bars. Its minimal tides mean that this process can be discounted as a major factor when trying to understand the evolution of the shoreline. On beaches with multiple bars the outer bars control the evolution of inner bars by altering hydrodynamic regimes, through wave breaking and energy dissipation that affect inner bars. During moderate storms they are much less active (waves may steepen but do not break), so almost all energy dissipation occurs over inner bars. Thus, apparently simple non-tidal, longshore uniform beaches with multiple bars are highly nonlinear and therefore difficult for classical process based modelling. That is why statistical techniques are a useful option to further our understanding of such cases.

Fig. 4 shows the nearshore seabed evolution over a few years. The inner bars are fairly stable features; they are well-defined in the time mean, whereas the outer region of the seawardmost bar is much less homogeneous in its behaviour.

3.2. Methods based on signal covariance structures

Data-driven techniques based on the covariance structure of analysed signals often assume that all information about statistical properties of those signals is contained by that structure. This is equivalent to the assumption that all random variables of the signal are normal (Gaussian). However approximate this assumption may be, it is usually exact enough to be accepted. The most important consequence of normality is that if random variables, centred about their mean values,

Fig. 1. Location of IBW PAN Coastal Research Station at Lubiatowo.

Fig. 2. (Left) 1974–8 measuring towers operational, erected for joint COMECON experiments in 1974 and 1976; (Right) 2008–2 towers remaining.

are orthogonal, then they are uncorrelated and, by virtue of their normality, independent. Methods that fall into this category include Empirical Orthogonal Functions, Complex Principal Component Analysis, Canonical Correlation Analysis, Singular Spectrum Analysis and Multichannel singular spectrum analysis. In the following subsections these analysis techniques are described briefly, following which the process of applying this sequence of data-driven methods to the data is illustrated to demonstrate how the individual methods can be applied and what extra information becomes available at each stage.

3.3. Signal covariance methods

3.3.1. Theoretical background

The common idea of all techniques investigating signal covariance structure is based on breaking down the covariance matrix into eigenvectors, which are orthonormal. Particular variants of this approach are related to whether the analysis is done in the spatial or temporal domain or in both. When the covariance matrix is computed in the spatial domain, the associated method is termed the Empirical Orthogonal Function or Principal Component Analysis. Its formalism is simple and well known. Let us represent the measured seabed depths, centred about their mean depths as h_{xt}, where x ranges between 1 and n_x and t between 1 and n_t. The terms of the symmetric covariance matrix A^{EOF} can then be expressed as:

$$a_{i,j} = \frac{1}{n_x n_t} \sum_{t=1}^{n_t} h_{i,t} h_{j,t}. \tag{1}$$

The matrix A^{EOF} possesses a set of positive eigenvalues λ_n and a set of corresponding eigenfunctions e_{nx}, defined by the matrix equation:

$$A^{EOF} e_n = \lambda_n e_n. \tag{2}$$

We can directly see from this equation that the sum of all eigenvalues is equal to the trace of the matrix, i.e. the global signal variance. Each of the n eigenvalues represents a portion of that variance and they are usually rearranged in decreasing order of magnitude. The magnitude of the ordered eigenvalues typically decreases quite rapidly and the number of eigenvalues that are significant can be determined using an empirical 'rule of thumb' proposed by North et al. (1982). Due to the orthonormality of eigenvectors, the principal components c_{nt} are obtained from:

$$c_{nt} = \sum_{x=1}^{n_x} h_{xt} e_{nx} \tag{3}$$
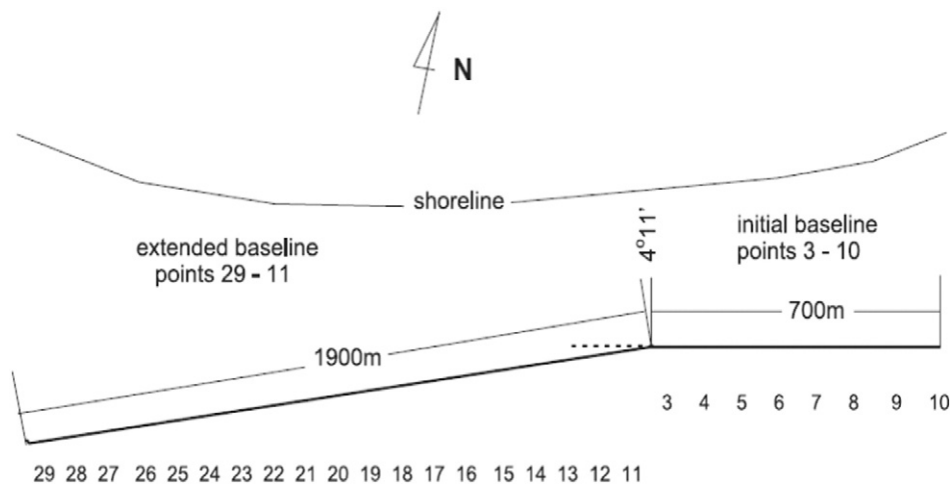
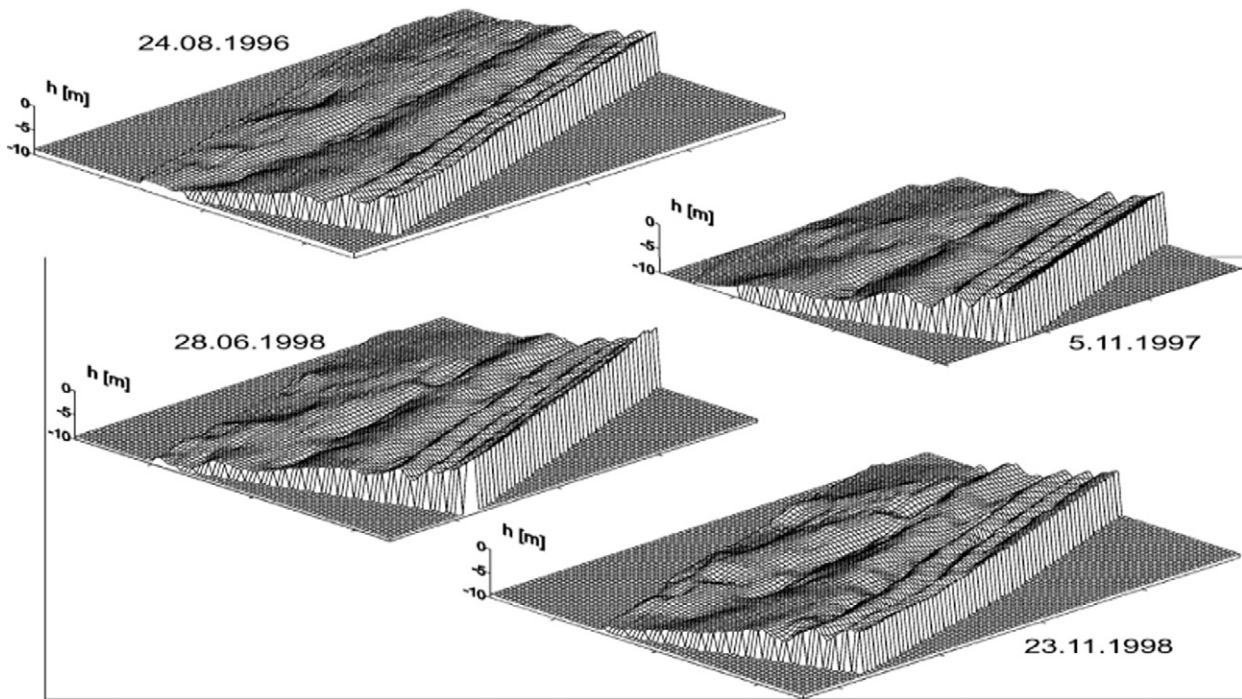**Fig. 3.** Location of survey profiles along the baseline at Lubiatowo.

Fig. 4. Snapshots of the multiple bar alignment at Lubiatowo.

The principle components encapsulate the time variation in the measurements. The key merit of EOF decomposition is the separation of the temporal and spatial patterns within the signal. The spatial patterns (eigenvectors) are usually plotted in the spatial domain. Since the eigenvectors are scaled to unit length regions where they have relatively large amplitude indicate areas of significance of a particular eigenvector. The principal components can reveal trends, oscillations and other types of behaviour. Studied jointly with the associated eigenvectors these can assist in interpreting the patterns within the signal.

Given that we might expect, from consideration of the physical processes, a linkage between two variables, it is reasonable to ask whether there are methods that could test the strength of such a linkage. One such technique is Canonical Correlation Analysis (CCA). It was introduced by Hotelling (1935) and used in climatology (Glahn, 1968). Larson et al. (2000) were the first to apply the method in a coastal engineering context. The method consists in analysing two data sets in form of vector time series $Y_{t,y}$ and $Z_{t,z}$ where t indicates observations in time of spatial variations y and z. The number of observations must be the same for each data set but the number of spatial points do not need to be equal. Subtracting mean values for each spatial location centres the data sets. Finally, we construct linear combinations of Y and Z respectively to define new variables that are maximally correlated. Additionally, a scaling is introduced such that these new variables have unit variances and are orthogonal. The CCA method then proceeds by finding, through matrix algebra, the (linear regression) relationship between the predictor field Y and predictand Z. Linear regression methods
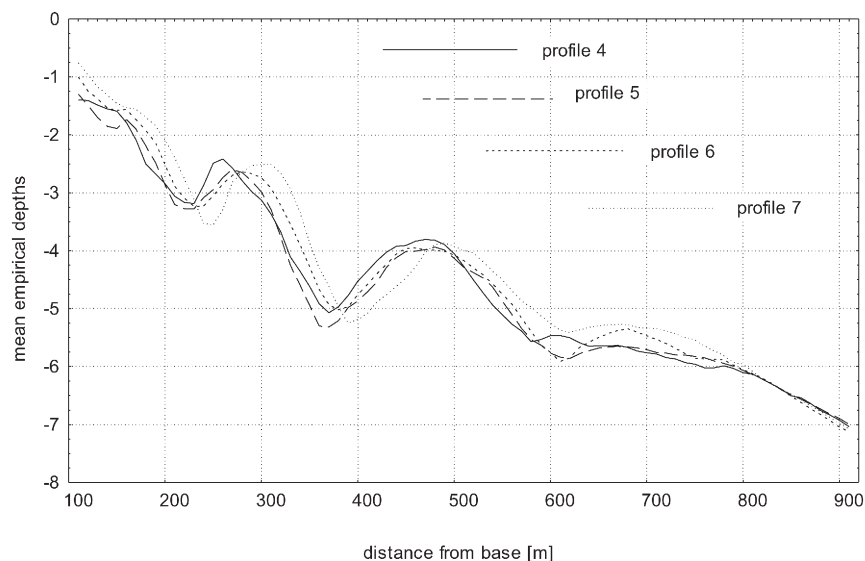

Fig. 5. Mean profiles 4, 5, 6 and 7, Lubiatowo, Poland 1987–1998, (from Różyński and Jansen, 2002).

have been used in coastal engineering in order to investigate dependencies and establish empirical relationships between variables (e.g., Larson and Kraus, 1995; Hedges, 2001).

In practice, the performance of regression analysis methods is subject to the characteristics of the observations, the level of noise in the data, and the choice of regression function and technique. In using regression for prediction it should always be borne in mind that the regression relationship is based on past measurements, so that if the system being measured undergoes a major change, or a new driver suddenly becomes important, predictions based on historic data may no longer be reliable, and may give misleading results (Freedman, 2005).

3.3.2. Application of EOF and Canonical Correlation Analysis

An example of an application of the EOF method is presented below, for four neighbouring cross-shore profiles, (profiles 4, 5, 6 and 7) at Lubiatowo. The seabed is usually characterised by 4 alongshore bars, but during the measurements the number of crests varied between 3 and 6. Fig. 5 presents the average profiles, which are very much uniform alongshore. The spatial structures of the three most important eigenvectors, associated with the greatest eigenvalues of the covariance matrix and their respective principal components are plotted in Fig. 6.

The first three EOFs explain 65% of the total variance (1st EOF: 34%, 2nd EOF: 19% and 3rd EOF: 12%). The 1st EOF, is very consistent for all four profiles; it displays similar amplitudes of its extremes all over the study area, highlighting high alongshore uniformity of the Lubiatowo beach. The extremes coincide with average positions of crests and troughs (see Figs. 5 and 6). This implies that the 1st EOF describes vertical oscillations of the whole system, which are expected to be a bit more significant in the area of outer bars. The 2nd EOF, is also fairly well alongshore uniform and is most pronounced for the area of inner bars 1 and 2. The most substantial changes, related to the 3rd EOF occur for the area stretching from the offshore slope of bar 2 up to the trough between bar 3 and 4. It may therefore describe interactions between inner bars 1 and 2 and outer ones 3 and 4 in the form of cycles with the period of approximately 12 years. The 3rd EOF shows dissimilarities from one profile to the next, indicating that interactions amongst inner and outer bars are not always uniform in the alongshore direction. The presence of independent oscillations in the inner bars suggests that the inner and outer bars may form separate sub-systems in their long-term evolution.

When using CCA the choice of predictor and predictand requires some care, as noted by Różyński (2003). Some of the results of a CCA analysis are shown in Fig. 7. The standard deviations of measurements are drawn as a thick solid line at the top. We can see they have minima over bar crests and troughs (the latter except for the trough between bars 3 and 4), whereas the maxima are associated with onshore and offshore bar slopes. Such a pattern indicates that crests and troughs are fairly firmly fixed. Firm positions of crests are quite surprising at first glance, but together with less surprising firm positions of troughs they imply that bar oscillations occur in fairly narrow bounds, because it looks unlikely for crests to travel far enough to take positions of troughs and vice versa. However, bar oscillations are strong enough to cause very pronounced changes in depth over bar slopes. In other words, the slope nearer a crest at one time becomes the slope nearer a trough at some other time, which can only occur in connection with dramatic changes in depth. It should be remembered though, the bars do not oscillate as rigid structures and the records of bed topography only reflect joint effects of movements of each and every sediment grain.

Since the EOF analysis suggested the existence of the sub-system of inner and outer bars, an initial choice was to select part or the whole outer subsystem as a predictor and the rest as a predictand, so the first analysis was done for bar 4 as the predictor (610–900 m) and bars 1, 2 and 3 as the predictand (100 to 600 m), in order to assess the dependence of inner bars on the outermost bar. The standard deviations of predictions are plotted in Fig. 7 as a dense dotted line. This line shows that roughly speaking prediction errors are proportional to standard deviations of measurements, so positions of crests and troughs are predicted with better accuracy than onshore and offshore bar slopes.

The 2nd analysis took positions of bars 1 and 2 as predictand (100 – 400 m) and bar 3 as predictor (410–620 m). The dense dashed line of prediction standard deviations is very similar to the previous one. The results hardly change when the outer bars 3 and 4 are used as a predictor (410 to 900 m). Standard deviations of predictions are shown as a dotted line and are again very close to previous results. It can be therefore concluded that predictands and predictors must share a common feature, so that practically the same results are obtained for various
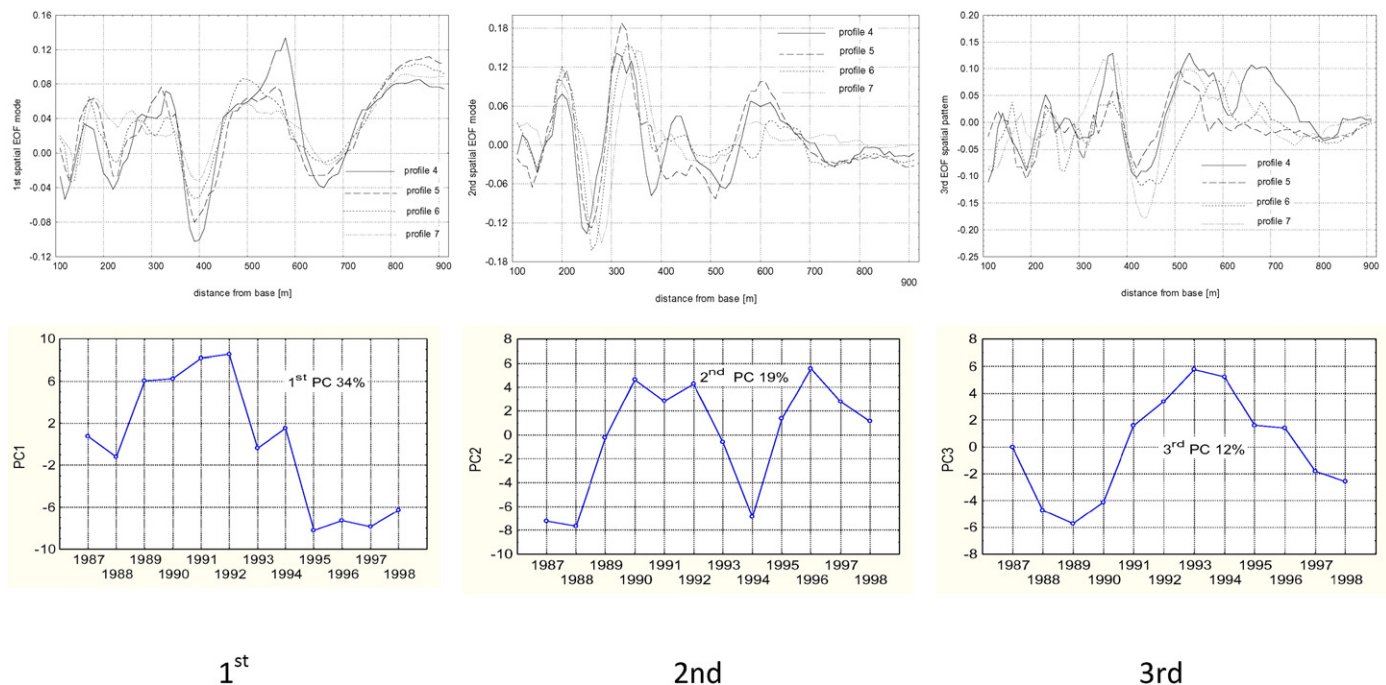

Fig. 6. EOF eigenvectors (top) and principal components (bottom) at Lubiatowo, Poland, 1987–1998, (from Różyński and Jansen, 2002).
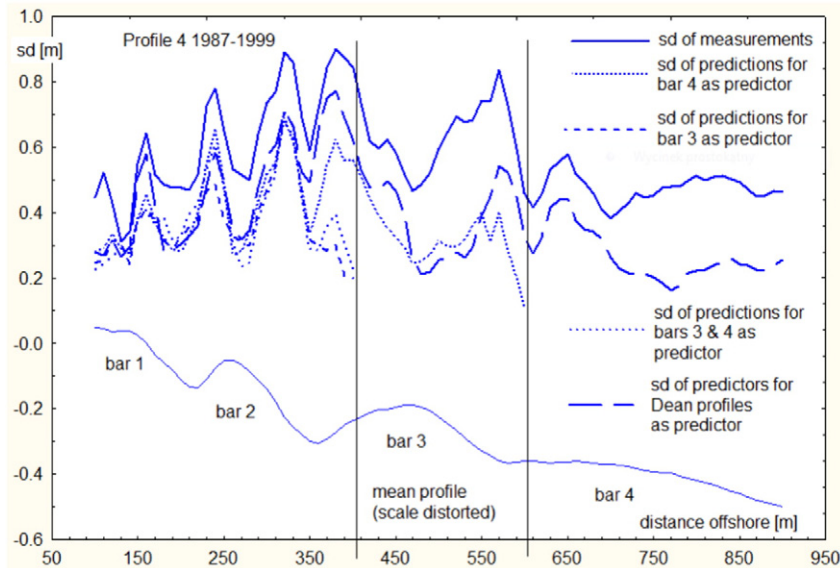
Fig. 7. CCA analysis of outer and inner bar interactions at Lubiatowo, Poland (Różyński 2003).

combinations of predictor and predictand. They also point out the upper threshold (roughly 60%) of the variability of inner bars that corresponds to the variability of outer bars.

While helpful, such results do not relate directly to empirical understanding. One means of injecting such knowledge into the CCA approach is to try to link the observed seabed changes to the seabed equilibrium profile, expressed by Dean's coefficients derived upon least square fit to the measurements. These define equilibrium profiles, which may serve as the predictor to measurements treated as the predictand, over the entire stretch 100–900 m. The CCA analysis performed for such a pair of predictor–predictand illustrates to what extent equilibrium profiles control different portions of beach profiles. The line of standard deviations of predictions using the equilibrium profiles is plotted as intermittent line in Fig. 7 and matches both the patterns of previous analyses as well as the line of standard deviations of measurements. Różyński (2003) found that the average prediction error over the entire profile is very similar to results found with the previous predictors. Thus from the analysis over the entire line 100–900 m with the Dean profiles as predictors the outcome of partial CCA computations over the profile subsets can be deduced very accurately.

The above results demonstrate inner bars 1 and 2 depend much less on equilibrium profiles than the outermost bar 4. Since changes in equilibrium profiles depict vertical profile variability, it may therefore be concluded that the outermost bar is dominated by vertical fluctuations, whereas for inner bars vertical movements play less crucial role. It confirms the EOF analysis, where the 1st pattern, expounding vertical oscillations of the whole seabed, was most pronounced in the vicinity of the outermost bar. This conclusion also illustrates the synergistic effect that using two methods for the same data set and obtaining consistent results may have.

It might be considered that while using the measurement of seabed elevation in one location to predict the location of the seabed in another is helpful to support physical explanations of the observed behaviour it doesn't help much in terms of practical prediction, (if you are measuring the seabed level at one point it is probably quicker and cheaper to go and measure the elevation at another nearby point while you have kit and crew mobilised). The CCA method is ideally suited for use in a forecasting role to predict one variable on the basis of another one. This works best when there is a physically plausible link between the two series such as wave energy and shoreline position. While the CCA makes no use of any physical process understanding, a user can improve its performance by using informed judgement in the choice of predictor

and predictand. One possible approach is as follows: given a sequence of beach profile measurements and wave conditions we split this into two sections; the first section will be used for the CCA analysis to compute the regression matrix, while the second section will be used to assess 'forecasts' made on the basis of the regression matrix; in order to make the number of beach profiles (approximately monthly) and wave measurements (approximately 3 hourly) the same some aggregation method is required for the waves. Larson et al. (2000) suggested that as any of the measured profile changes were likely to be the result of the wave conditions between this and the previous profile survey this aggregating could be achieved by creating the probability density function, (pdf), of wave heights of the waves in the interval; forecasts could be made on a monthly basis by using the pdf of the wave heights between the starting point and a month hence, and then estimating the corresponding beach profile using this pdf combined with the regression matrix. Larson et al. (2000) investigated the performance of this approach for different wave parameters as well as inshore and offshore wave conditions. Horrillo-Caraballo and Reeve (2008) extended Larson et al.'s (2000) study with additional measurements from Duck and investigated how the choices made in describing the wave climate could influence the quality of predictions made on the basis of the CCA results. They found the best performance was achieved using an empirical description of wave heights and suggested that resampling techniques could be useful in quantifying the forecast uncertainties. In a subsequent investigation Horrillo-Caraballo and Reeve (2010) applied the same technique to measurements taken at a shingle beach in Milford-on-Sea, UK. Results were as good, if not better, than for the Duck site, demonstrating that the CCA can find patterns of linked behaviour in two distinct datasets, irrespective of the details of the underlying physical processes, which are quite different on sandy and shingle beaches. Fig. 8 shows an example of the forecasts made at Milford over a 3 month period using the CCA regression together with offshore wave measurements. The root mean square error in the forecast profile is of the order of 0.3 m which, for a 3 month forecast compares favourably with the quality of forecast obtained from process-based models over the same time frame.

The right hand panels (Fig. 8) show the distribution of the error across the profile, which reaches a maximum across the intertidal area, a feature also found for predictions at Duck. This reflects the fact that both beaches are micro-tidal and therefore their response will reflect the influences of tides. This effect is absent from the CCA regression and is likely to be one of the main sources of the discrepancies. These
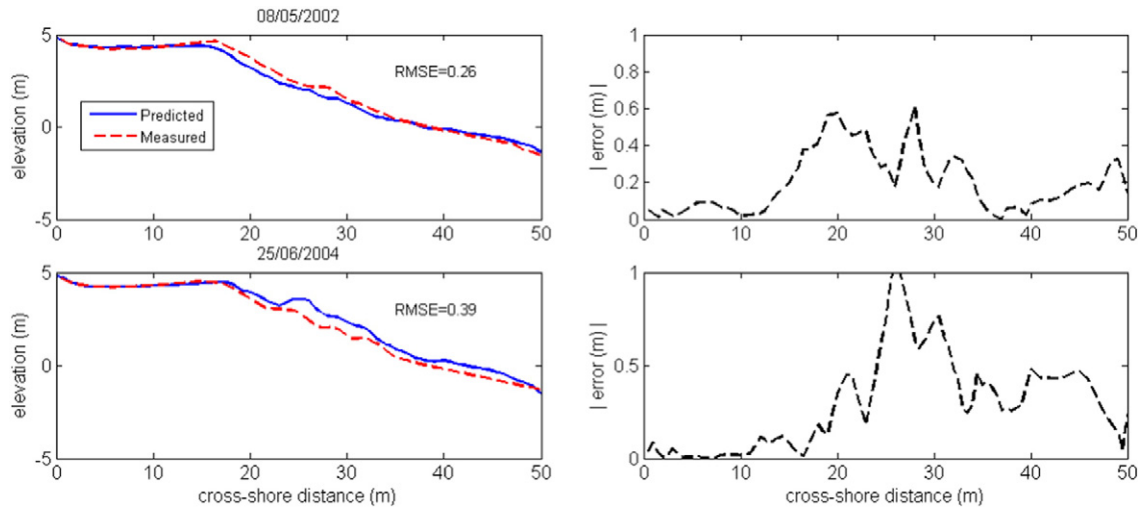
Fig. 8. Comparison of hindcast profiles and actual for May 2002 and June 2004 (left panel). Cross-shore distribution of error (right hand panel). Hindcasts were made over a period of 6 months — the gap between successive profile measurements.

results also indicate that any model of the mesoscale evolution of these beaches should account for both tides and waves for best results. The method is not limited to beach profiles and may also be applied to other coastal indicators such as beach plan shape. De Alegría-Arzaburu et al. (2010) describe an application of CCA to derive forcing–response relations between the wave climate and shoreline position on a macrotidal gravel barrier located in the southwest of the U.K. that is known to rotate in response to variations in the prevailing wave direction. The link between wave direction and beach plan shape orientation was isolated from the measurements purely from the statistical analysis without any direct physical process knowledge. More recently the CCA method was used to relate offshore wave conditions to beach plan shape at three morphologically distinct areas along the North Kent coast; one in an open exposure, one sheltered behind a breakwater and subject to diffracted waves, and one bounded by shore normal groynes. Using CCA Reeve and Horrillo-Caraballo (2014) showed that the beach shape in each of the three locations could be forecast to a useful degree of accuracy from the same offshore wave conditions, but with different regression coefficients. This result demonstrates that if there is a link in the mutual patterns of behaviour between two datasets, the CCA will find them irrespective of the details of the physical processes and irrespective of their relative geographical locations. The significance of such results is that it provides a means of guiding decisions on the density and location of wave buoys for coastal management. On the basis of the locations studied so far this suggests that a network of offshore rather than inshore buoys might be a more economical means of routine monitoring of coastal wave conditions.

Returning again to the multiple bars at Lubiatowo, it seems we are able to understand the bathymetric variations to a moderate degree within linear methods, but this leaves a significant component unexplained. One possibility, mentioned in the Introduction, is that the complicated observed behaviour is a manifestation of a chaotic system. That is, a system that is governed by deterministic equations but, nevertheless, can exhibit seemingly unpredictable behaviour. There is a large literature on this subject but for our purposes we are interested in the associated time series analysis methods. The germane point from our perspective is that while it is now straightforward to analyse and identify chaotic behaviour from the solution of a set of nonlinear equations known to show chaotic behaviour, given only the output and an analysis that confirms chaotic behaviour, it is still virtually impossible to deduce what the governing equations were, or indeed how many governing equations there are. A technique called Singular Spectrum Analysis (SSA), can be used to assist in separating noise from underlying irregular but smooth changes in a signal. It can also give some insight into the

dimension of the problem, that is, how many governing equations there might be producing the observed behaviour. Broomhead and King (1986) showed how the method, based on deep mathematical results, could be applied to the output of the Lorenz equations. Vautard et al. (1992) provide a useful summary of SSA and its applications to adaptive filtering and noise reduction. The method is based on analysing the lagged covariance matrix of the a time series x_i, where $1 \leq i \leq n_t$. To perform the analysis the user needs to define the number M_L which is the *window length* or *embedding dimension* and its value needs to be chosen with care. It is, in effect, your estimate of the dimension of the system producing the output being analysed. The covariance matrix is symmetric, so its eigenvalues λ_k are all positive, and the corresponding eigenvectors are orthogonal. The eigenvectors form the time-invariant part of the SSA decomposition, whereas the variability of a given system is contained in principal components (PCs).

The PCs are time series of length $n_t - M_L$ and are orthogonal to each other as M_L consecutive elements of the original time series are needed to compute one term of every PC. In one of the first applications of the SSA method to coastal morphodynamics Różyński et al. (2001) analysed 16 years of shoreline position measurements (1983–1999), sampled at equal 100 m intervals upon a monthly basis along a 2.8 km shoreline segment at Lubiatowo. No systematic behaviour could be deduced from the observations, but the SSA results suggested that the shoreline exhibits standing wave behaviour with periods lasting several decades found in the western part of the coastal segment, approximately 16 years in the middle part of the segment, and ~8 years in the central-eastern part of the segment. A more sophisticated version of the SSA, multi-channel SSA (MSSA), in which covariances in both time and space are computed, was subsequently applied to the same dataset by Różyński (2005). Some of the results of this analysis are shown in Fig. 9. The reconstructed components shown in Fig. 9 are analogous to the eigenfunctions of EOF analysis.

Fig. 9 (left panels) shows the 1st reconstructed component. Western profiles are shown on the left (lines 17 to 29), eastern on the right (lines 11–16 and 03–10). They feature a long shoreline standing wave with a period of several decades. The amplitude of that wave on the eastern sector is much less pronounced, so it had not been detected by the ordinary SSA method. Fig. 9 (right panels) demonstrates trajectories of the 2nd MSSA reconstructed components. They can also be interpreted as standing waves with a period of 7–8 years, with nodes at profiles 27–26 and 16–15. The antinodes can be observed at profiles 22–19 and 03–06, and a corresponding wavelength of 1000 to 1300 m can be deduced. Since this pattern was not as strong as the 1st reconstructed component in the western sector, it had not been detected by the
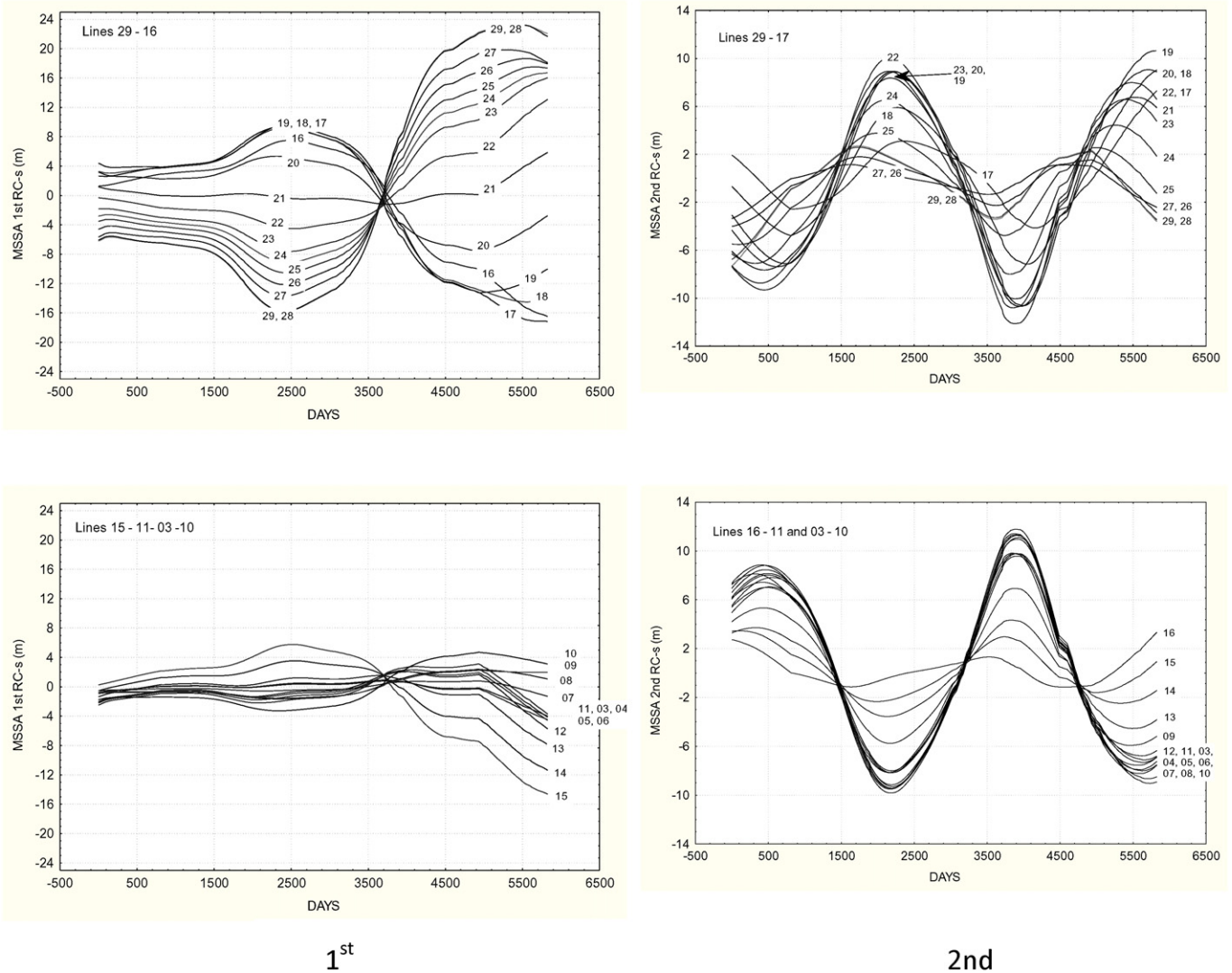
Fig. 9. Reconstructed components (1st — Left; 2nd — Right) using MSSA, Lubiatowo 1983–1999, (Różyński, 2005).

ordinary SSA method either. Thus, the MSSA had confirmed the detection of the same shoreline standing waves as the SSA and found some extra patterns as well.

As another example of how findings from data-driven analyses can then initiate further research on the dynamics the detection of this 8 year oscillation prompted extensive studies into the coupling of beach response with a similar periodic component of the North Atlantic Oscillation (NAO) and its winter index (NAOWI) (Różyński, 2010). It was found that this coupling is transmitted by the wave climate during winters.

In concluding this section we summarise the methods based on the analysis of covariance structures within measurements as:

1. Always begin with fairly simple methods like EOF or SSA, where covariances in either space or time are considered;
2. When the amount of data is sufficient, and the results of Step 1 are inconclusive, a more advanced tool (eg. MSSA, CPCA) may be able to extract just a few dominant patterns;
3. When the coastal system is complicated by other additional processes, for example tides or stratification, there may be 2D spatial patterns or migrating seabed forms that may require advanced tools (CPCA, MSSA);
4. Beach nourishment schemes are usually frequently monitored, so they can be a perfect subject for data-driven analyses;

5. When hydrodynamic information is available, every effort should be made to use this in the analysis or interpretation.

3.4. Other methods

We conclude our discussion of data-driven methods with a section on two of the more recent techniques which perhaps have more in common with Fourier decomposition than covariance methods. Namely, wavelets and empirical mode decomposition (EMD). Both methods have been developed to isolate underlying features of a time series that may be obscured by high frequency noise, but have yet to be used in a predictive scheme for coastal morphology.

Wavelets are conceptually similar to Fourier decomposition, the main difference being that the functions used in the expansion, the wavelets, are non-zero only in a finite interval, rather than extending to infinity. Just like Fourier analysis there are continuous and discrete versions; the continuous version being more appropriate for theoretical developments while the discrete version being more suitable for practical application to time series of measurements taken at discrete points in time or space. Further, Fourier transforms implicitly assume periodicity of the analysed signal, and hence stationarity of the studied signal. Although they can provide useful information about a signal, it is frequently not enough to characterise signals whose frequency content changes over time. Hence, Fourier analysis is not an ideal tool for

studying signals that are non-stationary. By contrast, due to the localisation property wavelet transforms enable us to obtain orthonormal expansions of a signal that is intermittent and nonstationary.

The discrete wavelet transform, (DWT), has been used widely in studies where there is a need to examine intermittent or variable phenomena in long data series, such that by Mallat (1989) in signal analysis, by Kumar and Foufoula-Georgiou (1997) in geophysics, by Iyama and Kuwamura (1999) on earthquake analysis and by Smallwood (1999) for shock analysis. One attractive property of wavelets is time (or space)–scale localisation so that the signal variance at a particular scale is related to a particular time (location). In contrast to Fourier analysis, which uses harmonic sine and cosine functions, wavelet analysis is based on a 'mother wavelet' function. Some examples of wavelet functions that have been employed are shown in Fig. 10. One useful analogy for thinking about the difference between Fourier and wavelet analysis is to imagine the result of analysing a piece of piano music with both. Fourier analysis tells you how much of each note was played, whereas wavelet analysis will tell you what notes were played and when. Some of the first applications of wavelet analysis in a coastal engineering context were the studies by Short and Trembanis (2004) who analysed beach levels with continuous wavelets, and Różyński and Reeve (2005) who analysed water level and current records over the period of a storm using DWT. Li et al. (2005) developed an adapted maximal overlap wavelet transform to investigate the multiscale variability of beach levels at Duck, while Esteves et al. (2006) applied DWT to investigate seasonal and interannual patterns of shoreline changes in Rio Grande do Sul, Southern Brazil. The application of wavelet analysis to identify elements of predictive morphological models that required improvement was suggested by French (2010). One variant of DWT, wavelet packet transforms which yields improved resolution of the scale intervals of the variability than DWT, were applied by Reeve et al. (2007) to the Duck beach profile dataset. In much the same way that Rozynski's MSSA analysis of the Lubiatowo site confirmed and extended the earlier SSA analysis so Reeve et al.'s (2007) wavelet packet analysis of the Duck site confirmed their DWT results but also lead to the finding that over 25% of the temporal variance in beach levels was attributable to interannual periods (16 to 22 months).

Pruszak et al. (2012) examined extended records of shoreline and dune foot positions at Lubiatowo, covering 25 years of observations between 1983 and 2008.

Using the coif5 wavelet function they found that the shoreline standing wave, detected as the 1st reconstructed component by the MSSA study, is also imprinted in the variations of dune foot, (see Fig. 11, top panel). Moreover, they were able to assess its length $L \approx 3700$ m and period $T \approx 30$ years as well as the amplitudes which, unsurprisingly, were more pronounced for the shoreline wave. A comparison of the outputs of the MSSA and DWT analyses (Figs. 9 and 11), demonstrates clearly that both MSSA and DWT methods captured the most important

morphological patterns, but the greater number of records analysed in the DWT analysis allowed for a more precise description of its parameters.

Empirical mode decomposition was introduced by Huang et al. (1998) as an adaptive method suitable for analysing all types of signal, containing nonlinear and non-stationary components. In common with other decomposition methods discussed above, EMD expresses a signal as the sum of functions termed Intrinsic Mode Functions, (IMFs). The IMFs are defined by the zero crossings of the signal and represent an oscillation embedded in the data. The signal being analysed must fulfil three rather general constraints: (1) the signal has at least two extrema: one maximum and one minimum; (2) the characteristic time scale is defined by the time lapse between the extrema; (3) if the signal has no extrema but contains inflexion points, it can be differentiated once or more times to reveal the extrema and the final results can be obtained by integration(s) of the components.

The decomposition of a signal into the IMFs is called the sifting process. In this, oscillations are removed successively from the original signal, creating a sequence of functions that iterate towards a signal that meets the conditions for an IMF. The first time through this procedure yields one IMF which contains high frequency content of the original signal. This IMF is then removed from the original signal and the process is repeated to find the next IMF. A sequence of IMFs can thus be computed from a single signal. The process is completed by imposing a halting criterion, such as when the magnitude of the standard deviation computed from two consecutive sifting results falls below a specified value.

The EMD method has been applied to beach profiles at Lubiatowo. Fig. 12 top left shows an example record of profile 4 from 1987 together with its IMFs. The purpose of this decomposition is illustrated in Fig. 13. The ultimate goal of the study was to assess the rates of wave energy dissipation in fully developed wave breaking regime to identify the areas of erosion and accumulation. To do so, the linear trend and low-frequency components were recombined to arrive at a smooth but monotonic trend. From this short term departures of wave energy dissipation rates from the constant rate valid for the Dean profile could be estimated. As a result, Różyński and Lin (2015) demonstrated that not only could the EMD method be used to extract smooth monotonic profiles reminiscent of equilibrium profiles but it was also possible to incorporate the hydrodynamic information to identify areas of the cross-shore profile prone to erosion and accumulation during storms.

4. Hybrid models

As we have seen in the last section purely statistical methods are data hungry and do not make use of physical understanding (except indirectly through the choice of quantities being analysed). On the other hand, process models have difficulties with mesoscale forecasting for the reasons already mentioned in Section 1. There are also arguments
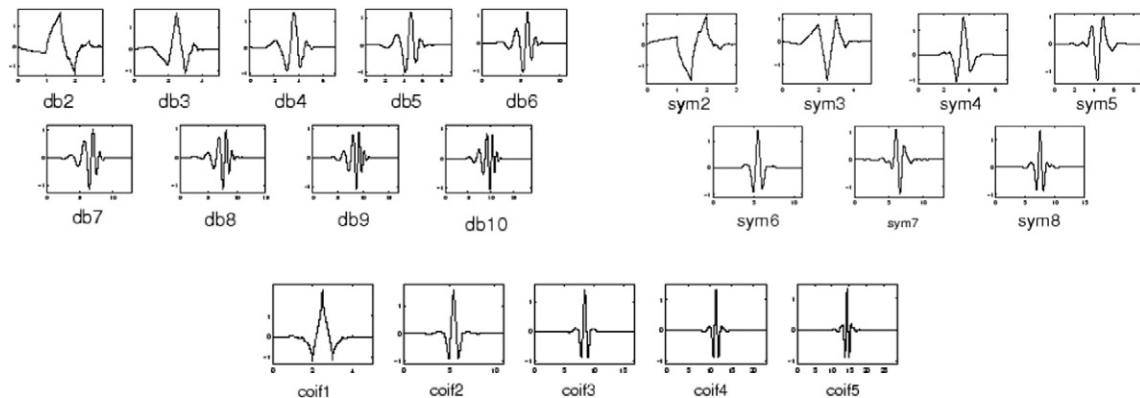

Fig. 10. Orthogonal wavelets: near anti-symmetric dbN (top left), near symmetric symN (top right) and near symmetric coifN (bottom centre).
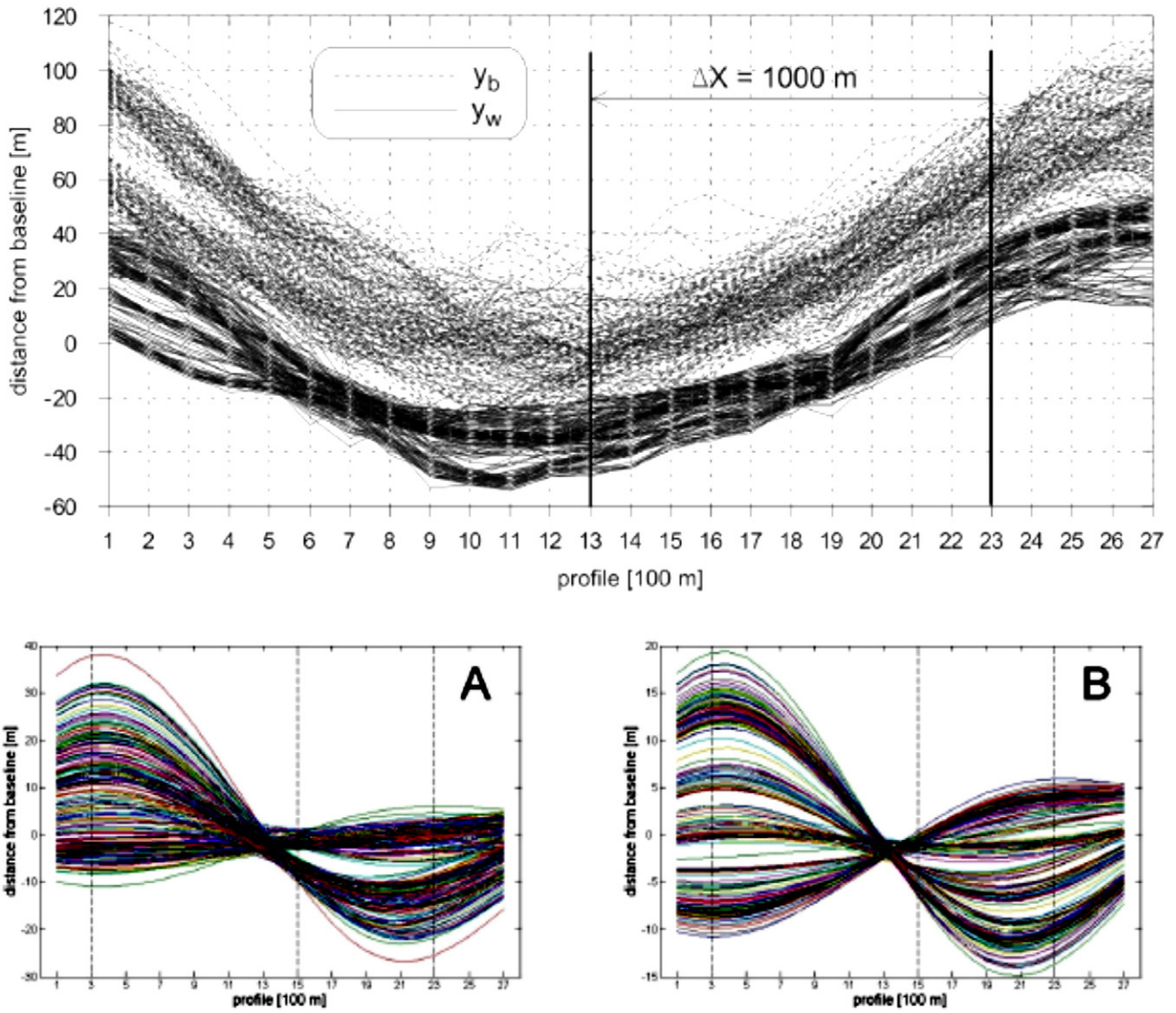
Fig. 11. Extended (1983–2008) measurements of shoreline (y_b) and dune foot (y_w) positions, Lubiatowo, Poland (top panel). Shoreline (bottom left) and dune foot (bottom right) standing wave fixed by the DWT method using coif5 wavelet function (bottom panels) (Pruszak et al., 2012).
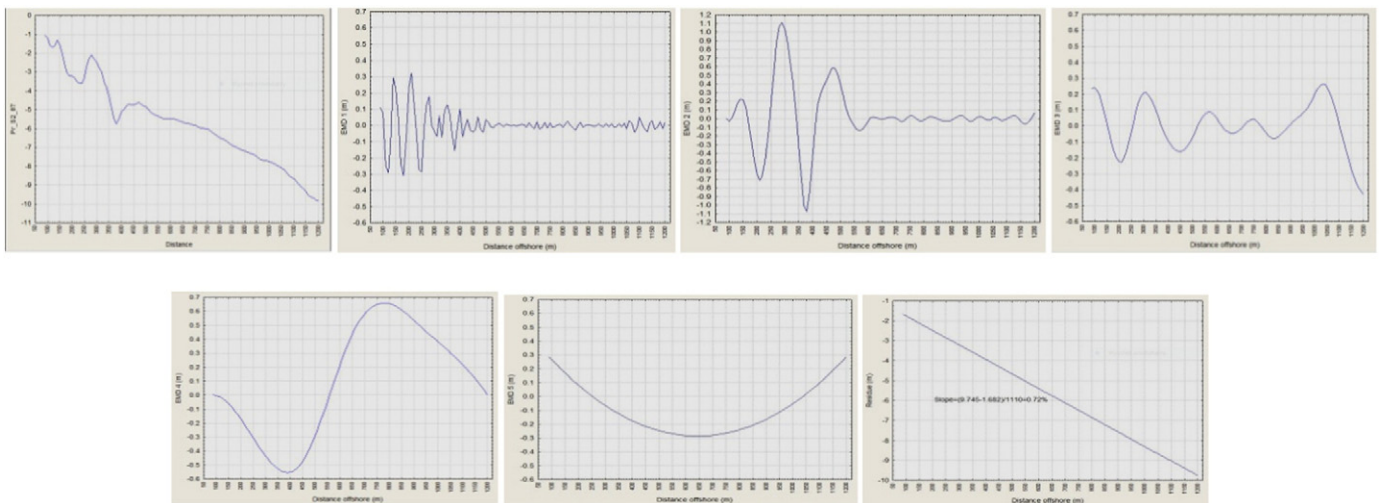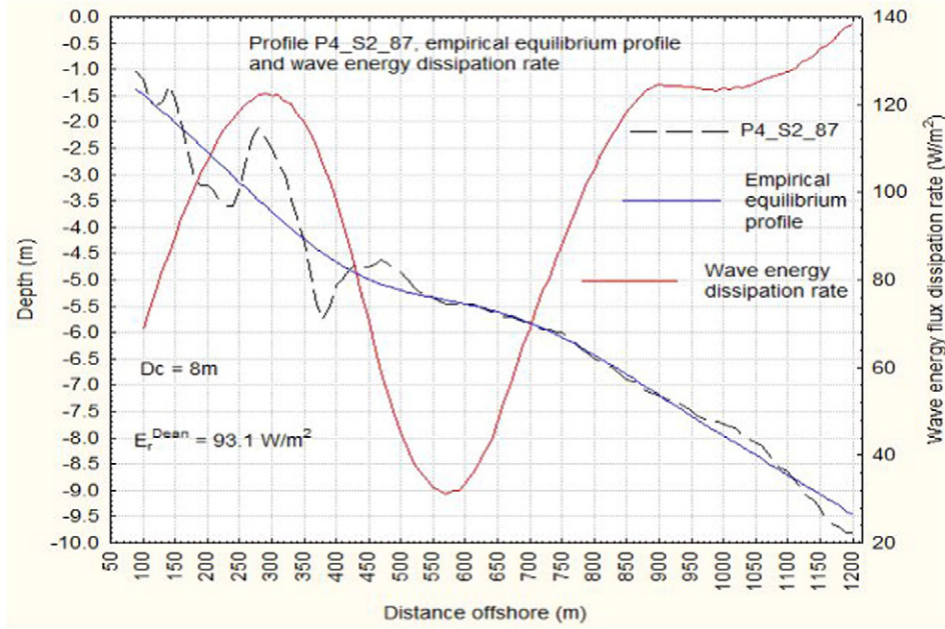

Fig. 12. Decomposition of cross-shore profile with multiple bars with EMD method: surveyed profile (top left), IMFs (short oscillations top centre left, bars top centre right, low frequency modes top right and bottom left and centre, trend bottom right).

Fig. 13. Raw profile (intermittent line), EMD monotonic trend (solid line) and wave energy dissipation rates (red line) evaluated using monotonic trend: rates greater/lower than 93 W/m^2 indicate erosion/deposition areas, seabed beyond D_c = 8 m should not develop wave breaking regimes, Różyński and Lin (2015).

that a scale-based approach to modelling would be a more successful way forward, and these chime with the wavelet type of analysis that is very much based on the concept of scale rather than correlation. Such arguments generally advance along the lines that morphological models developed upon our understanding of processes at small (human) scale do not contain all the necessary ingredients to encapsulate observed meso- and macro-scale variations. Murray (2007) provides an eloquent exposition of these qualitative arguments. They are supported by observations of shoreline evolution, Murray et al. (2015), and the difficulties experienced by process models in simulating observed mesoscale behaviours. This leaves us facing something of a dichotomy: Is it possible to scale process models up to mesoscale or is there a fundamental difficulty with this approach? The deterministic reductionist approach would argue that we should be able to predict at all scales if all the relevant processes are included in the model. By comparing model output with observations the performance of the model can be tested and the absence of relevant processes in the model gauged. If there are discrepancies then further experiments/observations are required to find and identify the missing process. The development of beach profile modelling provides an interesting lesson in this regard. In the 1980s and 1990s many beach profile models were developed for predicting beach response to storms, such as: HR Wallingford's COSMOS model, Nairn and Southgate (1993); UNIBEST-TC of Delft Hydraulics, Reniers et al. (1995); CROSMOR2000 from the University of Utrech, Van Rijn and Wijnberg (1996); and many others. Roelvink and Broker (1993) and Van Rijn et al. (2003) noted that while very good performance was achieved by these models when compared against laboratory experiments of beach response to storm waves, these same models were not able to reproduce the observed post-storm beach recovery. The development of the XBeach code (Roelvink et al., 2009) sparked renewed interest in profile modelling. Subsequent development of the code, such as that reported by Jamal et al. (2014), have included the slightly longer time scale process of infiltration, (amongst other processes), to reproduce post-storm beach building. The point being that to identify a missing process from observations that has only a slightly longer time scale than a wave period, and to build this into a process model, has taken several decades.

For mesoscale modelling there is also a practical problem in that as time and space scales get larger so do the monitoring requirements. There is another, more undecidable, problem in that the system is non-linear; nonlinear systems can exhibit chaos and emergent behaviour, which is predictable if you know the governing equations but if not, as discussed earlier not even methods such as SSA can determine the equations solely from observations. As a result a different class of model has emerged; termed variously 'reduced complexity', 'behaviour-oriented' or 'hybrid'. These terms encapsulate the idea behind the approach to create a model that distils the essence of mesoscale evolution without the difficulties associated with process models nor the data requirements of data-driven methods. It is fair to highlight a distinction between hybrid models and the other forms of model; hybrid models have been very much focussed on developing understanding as opposed to creating accurate forecasts.

In hybrid models, elements of two or more model types are combined. Some elements of the physics are eliminated in order to reduce computational costs and simplify the dynamics on the assumption that the broad scale morphological changes will be captured. They use simplified governing equations that exhibit the behaviour of the application. Good examples of such models in a coastal context are: the empirical equilibrium beach profile model of Dean (1977) and beach plan shape model of Hsu and Evans (1989); 1- or N-line shoreline evolution models pioneered by Pelnard-Considere (1956) and subsequent developments due to Hanson and Kraus (1989), Hanson et al. (2003), Dabees and Kamphuis (1998); the shoreline evolution model of Karunarathna and Reeve (2013); the cross-shore profile evolution models of Stive and de Vriend (1995) and Niedoroda et al. (1995); the estuary and tidal inlet models proposed by Stive et al. (1998), Karunarathna et al. (2008) and Spearman (2011); and the equilibrium shoreline models described by Yates et al. (2009) and Splinter et al. (2014). Hybrid models have been developed on the basis of physical intuition, hypothesis and mathematical considerations. By their very nature they tend to be process or scenario specific and, as yet, there is no well-defined or standardised methodology of deriving them from first principles. They will normally take as a starting point either a guiding physical principle (such as conservation of mass), or a specific physical process (such as transport or diffusion). An early example of such a model is due to

Cowell et al. (1995) which relies only on sea level change and changes in geometry to predict shoreface translation, rather than hydraulic forcing.

4.1. Diffusion models

One of the most widely used type of hybrid models is based on diffusion-type equations. Indeed the model proposed by Pelnard-Considere (1956) is such an equation for describing the evolution of the beach plan shape on the basis of laboratory experiments. In order to account for cross-shore sediment transport, 1-line models have been extended to track two or more contours along the beach. This group of models is known as multi-line models or 'N-line' models, and have been developed successively by Perlin and Dean (1983), Dabees and Kamphuis (1998), Hanson and Larson (2000) and Shibutani et al. (2009). In contrast to 1-line models, N-line models simulate changes in cross-shore beach profile as well as beach plan form. However, they require detailed information on both cross-shore sediment transport rates and cross-shore distribution of alongshore sediment transport rates. This is often implemented as the rate of cross-shore transport being based on the principle that any profile deviation from the equilibrium profile will result in cross-shore exchange of sediment between contours as in the formulations proposed by Steetzel and de Vroeg (1999) and Hanson et al. (2003). While a diffusion type equation for longshore transport can be derived from physical principles, and the assumption of small wave angles, the same is not true for models of cross-shore transport. Nevertheless, one and two dimensional advection–diffusion type formulations have been used in dynamical beach evolution models, with appropriate initial and boundary conditions. In these models, sea bottom bathymetry change is assumed to take place as a result of the collective effects of diffusive and non-diffusive processes.

Diffusion has the effect of smoothing irregularities in the sea bed. However, smoothing is not the only morphological response of a seabed and other morphological changes have to be included in some way. A mathematically convenient way to achieve this is by including a source function in the equation which is an aggregation of changes driven by physical processes other than diffusion. The natural question that arises is of course how the source function should be specified. One way to investigate this is to take observations at two distinct times and determine the source function that is required by predicting the earlier observation to the second observation with the simplified model. This process amounts to what is termed an inverse problem; one is using observations to determine the parameter and function values in an equation. Many of the simplified morphological evolution equations take the form of diffusion-type equations and some progress has been made recently in developing methods to solve inverse problems of this nature. In fact, because diffusion-type models have been proposed for beach plan shape, beach profile and beach area problems the methods have a level of generality. If the source term does in fact represent the aggregation of coherent morphological processes it may be possible to hypothesise particular causal physical mechanisms on the basis of the shape of the source function in different settings. If this is indeed the case then it provides a natural way in which to extend the simplified model by including the newly identified process. In any case, if the inversion is carried out with sequential observations it is possible to build up a time sequence of source terms and analyse these for patterns of behaviour which could be extrapolated into the future to allow predictions of morphological evolution.

Here we discuss the method as applied to beach plan shape and profiles to illustrate how the inversion method can be used to understand mesoscale morphological changes. For beach plan shape the idea is to use the 1-line model as the governing equation, with observations of the beach position to determine the corresponding values of the diffusion coefficient and forcing function. The first such approach with this method was Reeve and Fleming (1997) who used a constant diffusion coefficient and a specific form of forcing function. These restrictions were subsequently relaxed in developments of the method by Spivack

and Reeve (1999, 2000). Based on this initial work and extending the form of 1-line equation as proposed by Larson et al. (1997), Karunarathna and Reeve (2013) developed an advection–diffusion model to simulate beach plan shape evolution relative to a fixed line of reference:

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial x}\left(K(x,t)\frac{\partial y}{\partial x}\right) + S(x,t) \qquad (4)$$

Eq. (4) describes the variation of shoreline position $y(x,t)$ defined relative to a fixed reference line at longshore location x at time t. $K(x,t)$ is interpreted as the space- and time-dependent diffusion coefficient which relates the response of the shoreline to the incoming wave field through longshore transport. $S(x,t)$ is a space- and time-dependent source function which describes all processes that contribute to shoreline change other than longshore transport by incident waves, (including tides, wave induced currents and anthropogenic impacts).

Solving Eq. (4) to find K and S simultaneously, given a sequence of observations of beach plan shape is a challenging mathematical problem. Karunarathna and Reeve (2013) used an approximate, two step approach to determine K and S for Colwyn Bay Beach in the UK, from a series of historic shoreline surveys and wave measurements. (Shorelines were measured every 6 months from 2001 to 2006 and waves for the same duration, at five locations along the beach, were derived from hindcasting). This site was selected because Colwyn Bay beach has a clearly defined longshore extent and exhibits substantial morphodynamic variability; shoreline recession of 0.69 m/year has been observed at unprotected parts of the beach. There are also good records of waves and beach positions. In addition there are some rock groynes on the beach that provide the potential for interrupting longshore drift and causing some cross-shore transport; a signature that ought to be picked up in the source function.

The time mean diffusion coefficient was computed using the equation:

$$K(x,t) = \frac{2Q_0}{D_c}. \qquad (5)$$

The depth of closure, D_c, was determined using the formula due to Hallermeier (1981) and Q_0 was computed from the time history of wave conditions and the CERC equation due to US Army Corps of Engineers (1984) to generate a time sequence of $K(x,t)$ along the beach. The time mean diffusion coefficient $\overline{K}(x)$ was obtained by time averaging $K(x,t)$ over intervals between successive beach surveys. The source function was determined as the inverse solution to Eq. (4).

The resulting longshore variation of the time mean diffusion coefficient for Colwyn Bay is shown in Fig. 14.

The distribution of K shows that the lefthand side of the beach is less mobile than the righthand side. It could reflect a grading in sediment size or wave exposure. The sediment shows little sign of alongshore
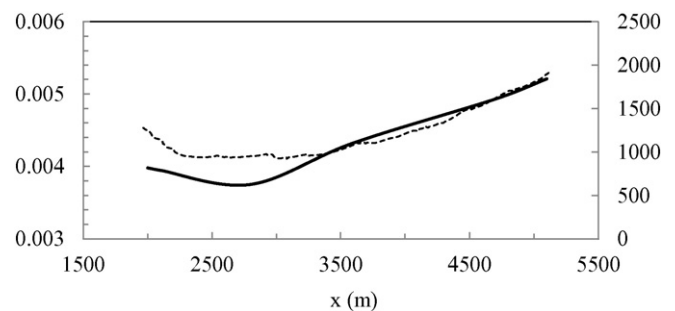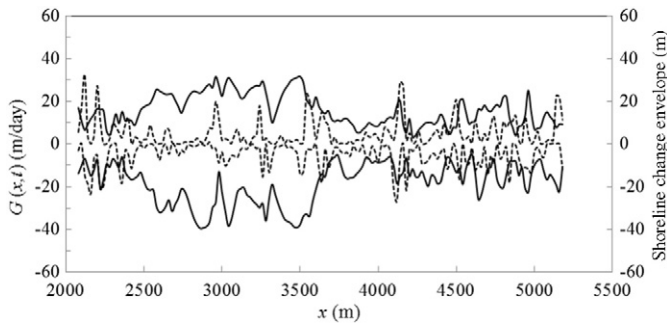
Fig. 14. Longshore variation of mean diffusion coefficient along Colwyn Bay (dark line) and the mean shoreline (dotted line), (Karunarathna and Reeve, 2013). Left axis shows diffusion coefficient and the right axis the shoreline position relative to a reference line.

Fig. 15. Spatial distribution of time varying source function envelope (dotted lines), measured shoreline change envelope (dark line) (Karunarathna and Reeve, 2013).

grading, but an analysis of the nearshore wave climate confirms that the righthand end of the beach is exposed to larger and slightly more oblique waves.

Fig. 15 shows the envelope of all source functions recovered from historic shoreline measurements along the beach. The envelope of shoreline change, determined directly from measured shoreline surveys, is also shown in the figure. The two envelopes are distinctly different. The source function relates to the residual of cross-shore wave transport processes. Its largest values coincide with the region of the beach which has rock groynes. Therefore, the difference between the two envelopes provides an estimate of the alongshore transport contribution to shoreline change.

The combination of a hybrid morphodynamic shoreline model, data, and an inverse technique has provided an approximate means of distinguishing between the contributions of cross-shore and alongshore transport to shoreline change. Further, the mean diffusion coefficient can be related to the longshore variations in wave climate, thus giving some tangible physical interpretation to the somewhat abstract quantities K and S.

Stive et al. (1991) and Niedoroda et al. (1995) proposed an advection–diffusion model for predicting cross-shore beach profile change, of the form given in Eq. (6):

$$\frac{\partial h(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(K(x)\frac{\partial h(x,t)}{\partial x}\right) + S(x,t). \tag{6}$$

The cross-shore position of the profile is described as a function of profile depth where beach profile evolution is assumed to take place as a result of the collective effects of diffusive and non-diffusive processes. In Eq. (6), K is a cross-shore varying diffusion coefficient and S is a time and space dependent source function. The specification of the diffusion coefficient and source term was discussed as being likely to be site dependent but little practical guidance on how these should be determined in practice was given. Karunarathna et al. (2009) demonstrated how the method developed by Spivack and Reeve (2000) could be used to retrieve the diffusion coefficient and source term from a sequence of historic beach profile measurements, taking K and S as a sum of time varying and time averaged components as in a Reynolds expansion. Assuming that there are no drastic changes to the morphodynamic forcing in future, the sequence of K and S can be analysed for patterns which can be extrapolated to formulate suitable future values to be used in forecasts made with Eq. (6). Karunarathna et al. (2011) established a correlation between incident wave conditions and the source function, using CCA, which was then used to predict future source terms. The predicted source functions were then used, with the mean distribution of K calculated previously, to determine future changes in beach profiles at three monthly intervals. A comparison of measured and predicted beach profile change using the above method is shown in Fig. 16 (lefthand panels), and the absolute errors in Fig. 16 (righthand panels).

A comparison by eye suggests that the error in forecasts using this method is less than the errors obtained by forecasting evolution directly from the profile data and a CCA regression. A quantitative assessment of the comparative performance is the subject of current research. Karunarathna et al. (2012) also noted that the long term mean profile was well-represented by Dean's equilibrium profile and, on the basis that the Milford-on-Sea mean profile represents an equilibrium, showed that the mean diffusion coefficient should follow an $x^{1/3}$ distribution. The computed coefficient did indeed follow this distribution to a good degree, demonstrating a clear link between the heuristically defined diffusion process and the sediment grain size.

Despite the dynamical simplicity of hybrid modelling approaches, an encouraging level of performance has been found by different researchers using a variety of techniques. The absence of detailed exactitude in such models may be troubling to reductionists, and the authors have some empathy with this. However, given that many researchers and practitioners are at ease using equilibrium-type models for development of conceptual arguments and design, there is perhaps a case for revisiting what metrics are most appropriate for gauging the performance of mesoscale models as opposed to those for small-scale models. This is reinforced by the fact that hybrid models are data-dependent and thus site- or scenario-specific and as a result, the accuracy of model predictions will be dependent on the accuracy and resolution of the data as well as model assumptions.

4.2. Other models

In addition to the approaches described above, there are a number of other techniques that have been developed and applied to investigate coastal morphodynamics. Amongst these are: stochastic methods that treat the coast as a stochastic variable, driven by random forcing, for example the Monte Carlo methods proposed by Vrijling and Meijer (1992), Lee et al. (2002), Wang and Reeve (2010) and Callaghan et al. (2013); models that investigate the nature of shoreline instabilities due to the interaction of waves and shoreline as described by Ashton et al. (2001) and Idier et al. (2011); network or systems models that attempt to describe the behaviour of the coast as a system of linked elements like those of Baas (2002), Reeve and Karunarathna (2009) and French and Burningham (2011). The last of these types of models is discussed in detail by Payo et al. (in this issue).

One issue potentially of relevance to mesoscale coastal change that has gained prominence in recent years is that of wave chronology and its significance, or otherwise, to morphological evolution. The importance of the order in which storm episodes occur, or storm sequencing or 'chronology', was raised by Southgate (1995) and investigated on a more formal basis with Monte Carlo simulation by Dong and Chen (1999). They concluded that chronology effects could be significant in the short to medium term but became less significant over longer periods. Indeed, for periods of storm groups, in a 1-line model study of beach response near a groyne Reeve (2006) showed that while using time-averaged wave conditions yielded the same net transport as the corresponding time varying conditions, the final beach configuration was dependent on the sequence of storms. Walton and Dean (2011) showed analytically that the final planform shape of the shoreline may depend on the wave sequence despite wave conditions being spatially uniform. More recently, Reeve et al. (2014) demonstrated analytically that for the simple case of waves varying randomly in angle only, the timescale of changes in beach position are directly proportional to the timescale of changes in wave direction. Taking a storm time scale of one week this means that we would expect any chronology effects in the beach position to be negligible after a period of a few time scales, say a fortnight to a month. In an analysis of pocket beaches Turki et al. (2012) found that to explain the observed behaviour of several pocket beaches near Barcelona over the period of years it was necessary to explicitly include an element of beach memory of antecedent wave conditions in their hybrid model. Suffice to say this is an active area of
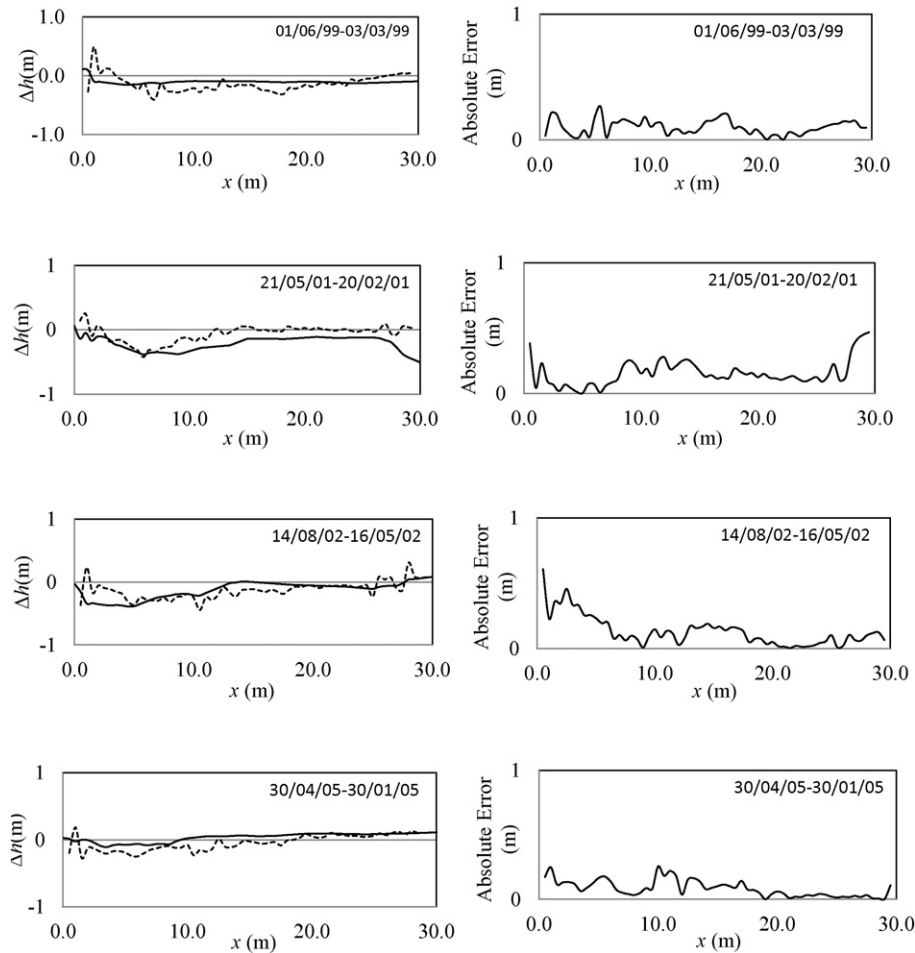
Fig. 16. Comparison of measured (dark line) and computed (broken line) cross-shore profile change by the diffusion model [left panel] and absolute errors between the two [right panel] (Karunarathna et al. 2011).

research, and that current understanding indicates that the direct impact of chronology effects is likely to be at the smaller scale end of mesoscale, although this will depend on the structure of correlation between the different driving variables.

This brings us back to the statistical description of morphology and its forcing. This section concludes with a discussion of how some ideas from turbulence could be applied to hybrid modelling. For the purposes of this discussion we consider the 1-line equation for the evolution of a beach in response to waves under the small angle assumption. This is a simple morphological model but illustrates the stochastic approach which can also be applied to more complicated descriptions. If the shoreline position and wave conditions are considered to be random variables, with $y = <y> + y'$ and $K = <K> + K'$ where $< >$ denotes an ensemble average and 'denotes the fluctuation about the ensemble average then we may write the ensemble average of the 1-line equation as:

$$\frac{\partial <y>}{\partial t} = <K> \frac{\partial^2 <y>}{\partial x^2} + \left\langle K' \frac{\partial^2 y'}{\partial x^2} \right\rangle \quad (7)$$

where $<y'> = <K'> = 0$, $\ll y \gg = <y>$ and $\ll K \gg = <K>$. This assumption is the basis of 'Reynolds Averaging' in fluid mechanics, and is used to develop theories of turbulence. An important aspect of this assumption is that the fluctuations average to zero over the chosen averaging period. That is, the overall motion has a separation of scales of change that means such an averaging can be performed unambiguously. Assuming that this is the case it is clear from Eq. (7) that the ensemble

average shoreline position depends on the ensemble averaged wave forcing, $<K>$, as might be expected, but also on the correlations between the fluctuations in wave forcing and the second derivative of the beach plan shape; in essence a form of 'morphodynamic turbulence'.

To solve Eq. (7) it is necessary to specify the last term on the right hand side in some way. This is not straightforward and is analogous to the 'turbulence closure problem' in fluid mechanics. One approximation would be to assume the correlation term is negligible; another would be to parameterise it in terms of the ensemble quantities, for example by replacing it with a term proportional to $<y>$ or its spatial gradient. Yet another approach, open to us in this age of computational modelling, is to use Monte Carlo simulation. Thus, if wave conditions are considered random variables driving a stochastic shoreline response, then by repeating simulations of shoreline evolution with multiple but independent realisations of wave conditions a corresponding set of realisations of likely shoreline responses can be generated. Ensemble statistics of shoreline position can be calculated from the realisations directly, without the need to specify the turbulence term in Eq. (7).

This might seem like a winning strategy which neatly bypasses the problems of 'turbulence closure'. It can certainly be a powerful method but it comes with drawbacks. For instance, irrespective of the number of realisations generated, these can only form a small subset of all possible outcomes. That is, the ensemble averages calculated from Monte Carlo simulation are sample statistics, calculated from the sample comprising the set of generated realisations, and can only ever be an approximation to the true solution. Further, Monte Carlo simulations can only ever be as good as the accuracy of the statistical characteristics of the input variables. If the statistics of the input variables do not accurately represent

the statistics of the variables in nature, both in terms of distribution and correlation properties, then the resulting outputs will be unreliable. Unfortunately, good statistics requires very extensive observations against which to fit statistical distributions. One such example is the Dutch coast and Li et al. (2014) describe a recent modelling investigation that incorporates dependencies between variables based on observed data.

5. Discussion and concluding remarks

Data-driven methods constitute a rigorous means of identifying trends, cycles and other patterns of behaviour. When used for prediction, by extrapolating such patterns, they are unlikely to be able to predict changes in coastal system state or configuration unless such changes are captured in the measurement record used for the pattern analysis. It may also have occurred to the reader that the hybrid morphological prediction methods described in the preceding sections really constitute a wide range of tools, rather than a seamless and integrated system of models ready to be used by the practitioner. In the view of the authors this is indeed the case; reflects the current state of understanding and is perhaps indicative of the fact that we are at a crossroads where, to make improvements in process modelling necessary for mesoscale forecasting, we need to develop a better understanding of the processes that dominate mesoscale evolution of coastal morphology. This understanding may come from direct modification of process models through, for example, experimentation with different parameterisation schemes or time compression algorithms, or through establishing empirical relationships with statistical techniques or through the development of hybrid models designed to capture mesoscale features of morphological evolution. Whichever one, or combination of these, transpires to be successful it will require more measurements; specifically, the continuation of current coastal monitoring programmes and perhaps the initiation of new programmes to extend monitoring to coastal types and exposures not represented by the current programmes. Long-term monitoring will underpin all forecasting methods, model development and validation; requiring significant and continued investment. Developments in monitoring technology such as video, LIDAR, and UAV systems, mean that much larger areas can now be monitored quickly.

The uncertainties in modelling the medium and long term that arise from the accumulation of rounding errors, the potential nonlinear sensitivity to initial condition, and limitations of our understanding of the key processes, mean that at least for the present, we must accept that predictions with such techniques must be uncertain. These uncertainties can be estimated through statistical methods such as Monte Carlo simulation. This would appear to be one possible area for further research that has the potential to provide bounds on the uncertainty of predictions and also to provide insight into which elements of the problem contribute most to the overall uncertainty. The importance of long term measurements and their role in guiding the development of our understanding cannot be understated. These provide an extremely valuable resource for investigating the statistical nature of the quantities we are trying to forecast. Furthermore, the longer the record the greater the chance of observing infrequent erosion events, breaching of barriers or dunes and inlet closures that can all lead to radical changes in the morphodynamics of the shoreline. Most valuable of all are datasets that contain contemporaneous records of the driving forces and the shoreline response, since these facilitate cross-correlation analysis as well as testing process model predictions.

From a practical perspective medium to long term forecasts of coastal evolution are being demanded for coastal management. In such situations an answer of some form has to be provided and the selection of forecasting approach depends very often on the availability and quality of measurements at a site. This is likely to remain the case for the foreseeable future, while the existing monitoring programmes continue to build the database of measurements to an extent that allows some of the uncertainties to be reduced or eliminated and our understanding

to improve. As such, having the flexibility of a variety of methods to make forecasts seems both apposite and persuasive.

Finally, the prospect of linking some of the many different approaches is appealing and would help to develop a system-wide capability for forecasting coastal change. Linking data-driven models with hydraulic quantities has always been difficult and challenging, due to the fact that contemporaneous measurements are very rare. However, some progress is being made in this regard, for instance, combining CCA with simple wave transformation calculations as proposed by Valsamidis et al. (2013), or combining EMD and wave transformation to investigate details of the impact of wave breaking over a multi-barred beach as proposed by Różyński and Lin (2015). The route to linking different model types, (data-driven with hybrid, hybrid with process-based and so on), is not clear at this moment and requires further research to establish its viability; which if demonstrated, will require the elucidation of practical methods of implementation.

To conclude, the quantity, quality and duration of observations are the vital factors when considering options for mesoscale morphological predictions and when assessing the accuracy of the model output. It is important to bear in mind the limitations of any model. Data-driven methods can play an important role by providing forecasts that are site specific but independent of any particular process-based modelling package. Process models provide a flexible methodology that is not site specific, but do require intensive computational resources and can be fraught with difficulties associated with numerical stability, accumulation of rounding errors and sensitivity to small changes in boundary conditions. Hybrid models often provide the optimum means of forecasting given constraints on observational records and physical process understanding at all but the best monitored sites. However, the outputs from such models can be frustratingly devoid of detail. The increasing availability of more high quality measurements provides the promise of improved forecasts, better modelling methodologies, reduced uncertainties and a more complete understanding of the mechanisms driving mesoscale coastal morphodynamics.

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