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Beach memory and ensemble prediction of shoreline evolution near a groyne

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ABSTRACT

In this paper we address the question of estimating the average position of a beach and its inherent variability about this mean. It is demonstrated how, even in a much simplified situation, the ensemble average of beach plan shape involves cross-correlation of the beach position and wave conditions. This renders the governing equations inimical to analytical treatment. A new analytical expression for the mean beach plan shape and its variation are derived for the case of a single groyne exposed to waves varying in direction only. This demonstrates that 'beach memory' is directly related to the autocorrelation of wave direction. For more general conditions a semi-analytical expression for the ensemble average of the shoreline position is derived. This solution is estimated with site specific wave conditions using Monte Carlo simulations. The characteristics of the solution are investigated and it is demonstrated that, for this case at least, the terms involving the wave direction are virtually uncorrelated with the terms that do not. It is concluded that, in an ensemble sense, the morphodynamic impact of wave direction is decoupled from that due to wave height and period.

1. Introduction

Changes in the configuration of our shorelines occur as a response to waves, tides and sediment movement. Understanding and predicting longshore sediment transport driven by varying wave and tidal conditions is one of the most important issues confronting coastal engineers and managers. Coastal management, design of coastal structures, environmental impact assessment and flood risk assessment all depend upon our understanding of how the shoreline changes and will react to interventions. It is the transport of sediment, driven by wave energy at the shoreline, which is most altered by shoreline management strategies that typically involve some combination of coastal structures and beach nourishment. Tides are driven by gravitational forces and can be predicted to a very good degree of accuracy by and large. In contrast, waves are driven by surface winds which are notoriously unpredictable and often considered to be randomly varying about underlying seasonal or annual trends. Shorelines respond strongly to the incident wave conditions and therefore can also exhibit large natural variations. This can make it difficult to reconcile instantaneous observations of the shoreline with the expected long term trends. A good example would be an unexpectedly strong shoreline response to the construction of a beach control structure. The shoreline manager has to answer the question 'Is the shoreline response simply a variation from the anticipated behaviour due to unusual conditions, or does it signify an unintended impact due to an imperfect understanding of the physical processes?'. The management response to each case will be very different and have very contrasting costs and impacts on the neighbouring shoreline infrastructure and communities. Understanding how fluctuations in wave conditions transfer to variations in shoreline position is extremely important in informing our response to situations where the behaviour of a beach is observed to diverge from that expected.

To some extent Monte Carlo simulation can assist in this. If wave conditions are considered random variables driving a stochastic shoreline response, then by repeating simulations of shoreline evolution with multiple but independent realisations of wave conditions a corresponding set of realisations of likely shoreline responses can be generated. Ensemble statistics of shoreline position can be calculated from the realisations directly such as described by Vrijling and Meijer (1992) for port applications and Wang and Reeve (2010) for a detached breakwater scheme.

An aspect of Monte Carlo simulation that can cause difficulties is the accuracy of the statistical characteristics of the input variables. If these do not accurately represent the statistics of the variables in nature, both in terms of distribution and correlation properties, then the resulting outputs will be unreliable. Verifying the results of Monte Carlo simulation can also be problematic because Nature only gives us one realisation. In this situation it is helpful to invoke the principle of ergodicity. That is, the assumption that the ensemble average is identical to a time average taken over a suitably long period. Ergodicity is not guaranteed, but is often assumed, so that we can compare time averages

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of observed quantities with the ensemble averages generated from numerical models. This avoids difficulties arising from the accumulation of numerical errors as well as numerical instabilities that can be encountered in simulations over long periods of time.

An alternative to Monte Carlo simulation is to evaluate ensemble averaged quantities directly. One approach is to develop a Fokker–Planck evolution equation for the probability distribution function of beach position as described by Dong and Wu (2013). Another approach is to formulate equations for the evolution of particular statistical moments of beach position. Neither approach is particularly straightforward, even for idealised cases. In this paper, we develop a formal moment solution for the case of a groyne on an initially straight beach, subject to random wave conditions. The solution is analytically intractable in the general case, but can be estimated through Monte Carlo simulation. Here, we use Monte Carlo simulation to generate 600 realisations to establish sample statistics. The Monte Carlo realisations are also used to investigate the correlation properties of different terms in the formal solution. As a result, inferences can be made about the relative impacts of wave direction and height on the statistics of the beach response.

When the angle of the shoreline is small with respect to the wave fronts breaking at the shore, a linear partial differential equation governing shoreline change can be derived from the continuity equation and the longshore transport equation. In the simplest case, wave conditions are taken to be uniform in space and constant in time. This approach was pioneered by Pelnard-Considère (1956) who proposed an equation to forecast changes in coastline position near groynes and which has become known as 'the one-line model'. This model describes the evolution of the distance of a single height contour from a reference line, y, as a function of longshore distance, x, and time, t:

$$\frac{\partial y}{\partial t} = K\frac{\partial^2 y}{\partial x^2} \tag{1}$$

where (when using the CERC sediment transport formula),

$$K = \frac{\kappa\rho_w H_b^2 c_{gb}}{8D(\rho_s - \rho_w)(1-p)}.$$

H_b is the breaking wave height, c_{gb} is the group velocity of the waves at breaking, α_b is the wave angle at breaking, ρ_w is the density of sea water, ρ_s is the density of the sediment, p is the sediment porosity, D is the height of the active profile (usually taken as the sum of the berm height and the depth of closure), κ is the 'coastal constant' whose value is set according to the sediment size distribution. Eq. (1) can be solved for numerous combinations of fixed boundary and initial conditions, see eg. Crank (1956) and Carslaw and Jaeger (1959). While Pelnard-Considère proposed Eq. (1) on the basis of observations from laboratory experiments of accumulation of sediment near a groyne, the concept was subsequently extended to describe: sand bypassing a groyne by Bakker (1969); the spreading of beach nourishments by Walton and Chiu (1979); and the impacts of sediment supply to beaches from rivers by Wind (1990). These studies considered constant, uniform wave conditions. One of the first analytical studies to relax the constraint of constant wave conditions was Larson et al. (1997) who provided solutions for, amongst many examples, the case of a groyne compartment with sinusoidal time variation of the wave angle. Subsequently, solutions for arbitrarily varying wave conditions for the case of a single groyne were presented by Reeve (2006), and for a groyne compartment by Zacharioudaki and Reeve (2008).

While this body of research has been successful in broadening the range of cases that can be described with analytical solutions they still rely on the assumption of small angles. In practical applications the 'small angle approximation' is usually relaxed and a time marching numerical solution procedure is adopted to solve the continuity, sediment transport and wave angle equations simultaneously. Modelling suites that include elements of wave prediction, nearshore wave transformation, modifications to allow for wave diffraction, longshore variations in wave angle and height, and variations in beach slope have been described by Hanson and Kraus (1989) and Dabees and Kamphuis (1999). The general application of analytical solutions based on the assumptions of small angles, constant and uniform wave conditions, negligible diffraction effects to real life cases cannot be justified a priori. Nevertheless, their robustness is demonstrated by their continuing use for: numerical code testing; quick estimation and pedagogy. They also have the advantages over more complete but complex models of avoiding the cumulative effect of rounding errors and providing an efficient means of estimating shoreline evolution without the need for iteration.

The importance of the order in which storm episodes occur, or storm sequencing, was discussed by Southgate (1995) and investigated further with numerical Monte Carlo simulation by Dong and Chen (1999, 2001) who concluded that chronology effects could be significant in the short to medium term but became less significant over longer periods of a few years. In an analysis of beach response near a groyne Reeve (2006) noted that while using the time average wave conditions yielded the same net transport as the corresponding time varying conditions, the final beach configuration was dependent on the sequence of storms. Walton and Dean (2011) used an analytical method based on the Heaviside technique to demonstrate that the final planform shape of the shoreline may depend on the wave sequence despite wave conditions being spatially uniform. Further, Valsamidis et al. (2013) found that results computed with this technique had a strong dependence upon the sampling period, demonstrating that short-term correlation in wave conditions has an influence on the short-term beach response. In an analysis of pocket beaches Turki et al. (2012) proposed a beach evolution model that explicitly included an element of beach memory of antecedent wave conditions to explain the observed behaviour of several pocket beaches near Barcelona. This observed tendency can be explained in the context of 1-line theory.

This paper is organised as follows: Section 2 provides the background to the deterministic analytical solution and its adaptation to derive ensemble averaged solutions; in Section 3 an exact analytical solution for the ensemble average beach position is presented for a simplified case; in Section 4 the study site and wave conditions used for the Monte Carlo simulation are introduced; results are described in Section 5 and a discussion and conclusions are provided in Section 6.

2. Mathematical background

If, in Eq. (1), we treat the shoreline position and wave conditions as random variables, with $y = \langle y\rangle + y'$ and $K = \langle K\rangle + K'$ where $\langle\,\rangle$ denotes an ensemble average and $'$ denotes the fluctuation about the ensemble average then we may write the ensemble average of Eq. (1) as:

$$\frac{\partial\langle y\rangle}{\partial t} = \langle K\rangle\frac{\partial^2\langle y\rangle}{\partial x^2} + \left\langle K'\frac{\partial^2 y'}{\partial x^2}\right\rangle \tag{2}$$

where it is understood that $\langle y'\rangle = \langle K'\rangle = 0$, $\langle\langle y\rangle\rangle = \langle y\rangle$ and $\langle\langle K\rangle\rangle = \langle K\rangle$.

Eq. (2) shows that the time evolution of the ensemble average shoreline configuration depends not only on the ensemble averaged wave forcing, $\langle K\rangle$, but also on the correlations between the fluctuations in wave forcing and the second derivative of the beach plan shape; in essence a form of 'morphodynamic turbulence'. It is quantifying this second term that is difficult and which is analogous to the 'turbulence closure problem' in fluid mechanics. An obvious approximation is to neglect the correlation term, setting it equal to zero, or to parameterise it in terms of the ensemble quantities (eg. replacing it by a term proportional to <y> or its spatial gradient).

Neither of these approximations is entirely satisfactory, so alternatives have been sought. A probabilistic theory which accounts for temporal correlation in the wave climate was developed by Reeve and Spivack (2004) who presented solutions for the moments of the shoreline position for the case of a bell-shaped nourishment on an otherwise straight beach. The presence of random fluctuations in the diffusion coefficient was found to accelerate the dispersion of nourishment in comparison to the case where there were no fluctuations.

Consider the solution of Eq. (1) for the case of an impermeable groyne located at $x = 0$. The boundary conditions in this instance are $\partial y / \partial x = h(t)$ at $x = 0$ and $y \to 0$ as $x \to \pm\infty$. For an impermeable groyne there is zero transport across the groyne, so $h(t) = \tan(\alpha_b(t))$, where $\alpha_b(t)$ is the wave angle at breaking and the second boundary condition corresponds to the condition that there is an undisturbed beach far from the groyne. The wave conditions are taken to be a random function of time, so that the diffusion coefficient, K, is also a random function of time. Assuming an initially straight beach ($y = 0$ at $t = 0$), Reeve (2006) showed that the solution may be written as,

$$y(x,t) = -\frac{1}{\sqrt{\pi}}\int_0^t \left(\int_w^t K(u)du\right)^{-1/2} \exp\left\{-\left(\frac{x^2}{4\int_w^t K(u)du}\right)\right\} K(w)h(w))\,dw \tag{3}$$

with w being a dummy variable of integration running from time 0 to arbitrary time t. Although this is an analytical solution its evaluation for any realistic sequence of wave conditions requires numerical integration. Hence we term this a semi-analytical solution in what follows. Given a sequence of wave conditions (H_b, T and $\alpha_b(t)$) it is possible to construct a corresponding sequence of $K(t)$, and $h(\alpha_b(t)) = \tan(\alpha_b(t))$, and then evaluate the solution at any particular time using numerical integration.

The closed-form nature of this solution provides two options to determine the statistics of the beach position. These are:

1. Use the solution to generate 'Monte Carlo' simulations of shoreline evolution, from which statistics can be computed, and
2. Derive the ensemble solution directly from the semi-analytical solution.

Both procedures require long sequences of wave data. In the first case, the route to the Monte-Carlo solution is straightforward once we have established a suitable wave climate and means to create statistically accurate realisations of this. This step is not always straightforward. For example, Vrijling and Meijer (1992) and Dong and Chen (1999) noted that it is often necessary to make some assumptions about the statistics of the waves which restrict the application of this method to more general situations. In the second case, the data are used to specify the distribution function and correlation function of the waves.

We now consider the second option. The semi-analytical solution given in Eq. (3) yields the temporal evolution of the shoreline position in the vicinity of a groyne. If we ensemble average this equation over all possible sequences of wave conditions the first moment (the mean of the solution) will be written as

$$\langle y\rangle = -\frac{1}{\sqrt{\pi}}\int_0^t \left\langle \left(\int_w^t K(u)du\right)^{-1/2} e^{\left(-\left(\frac{x^2}{4\int_w^t K(u)du}\right)\right)} K(w)h((w))) \right\rangle dw \tag{4}$$

where $\langle f\rangle$ denotes the ensemble average of the stochastic function f and is the triple integral $\int_0^\infty\int_0^\infty\int_0^{2\pi} fp(\alpha_b, T, H)d\alpha_b dTdH$ over all possible values and combinations of wave direction, period and height, and $p(\alpha_b, T, H)$ is their joint distribution function. This is difficult to treat analytically because it involves a cross-correlation of three terms. Furthermore, we require the joint distribution function of wave height, period and direction. We return to this problem in Section 4 but first consider an analytical solution to Eq. (4) for a simpler case. Finally, it is helpful to reprise some of the main the assumptions in deriving Eq. (3):

(i) the small angle assumption, that the angle between wave crests and the shoreline is small and that the angle between the local shoreline and the datum line is small
(ii) the effects of diffraction are ignored.

Note that (i) doesn't exclude large changes in shoreline position, only that the angle of the shoreline does not vary greatly in doing so. As an example, an accumulation of 100 m updrift of a groyne will lead to a change in beach angle of 0.1 rad if the beach realigns over a frontage of 1 km. Sharp changes in beach angle, particularly those associated with localised wave angle change due to diffraction, cannot be modelled.

3. Analytical ensemble mean solution

In what follows we drop the 'b' suffix on the wave angle. For simplicity's sake we consider α small, so that $tan(\alpha) \approx \alpha$. Further, consider wave conditions that are constant in height and period and vary randomly only in direction. To solve Eq. (4) the probability distribution of the wave angle is required. Here, we write $\alpha = \langle\alpha\rangle + \alpha * \delta$ where $\langle\alpha\rangle$ is a constant mean value of α, δ is a random variable with zero mean and unit variance (so $\mathrm{Var}(\alpha) = \alpha^{*2}$), density function $p_\delta(\delta)$ and autocorrelation function $\rho(\xi)$, where ξ is the time lag. Now, the ensemble average of an arbitrary function of a stochastic variable θ, $f(\theta)$, can be written as

$$\langle f\rangle = \int_{\theta_{\min}}^{\theta_{\max}} fp_\theta(\theta)d\theta. \tag{5}$$

Hence, the ensemble average solution for the beach position may be written as

$$\langle y(x,t)\rangle = \left\langle -\alpha\sqrt{\frac{4Kt}{\pi}}\left[e^{-\frac{x^2}{4Kt}} - \frac{x\sqrt{\pi}}{\sqrt{4Kt}}erfc\left(\frac{x}{\sqrt{4Kt}}\right)\right]\right\rangle \equiv \langle -\alpha G(x,t)\rangle. \tag{6}$$

In Eq. (6) it should be noted that the function $G(x,t)$, although a function of time, is not randomly varying. Hence,

$$\begin{aligned}\langle y(x,t)\rangle &= -G(x,t)\int_0^{2\pi} \alpha p_\alpha(\alpha)d\alpha \\ &= -G(x,t)\langle\alpha\rangle.\end{aligned} \tag{7}$$

That is, in this very constrained situation, the ensemble average shoreline may be found by substituting the mean wave direction in the deterministic solution.

The variance of the shoreline position is defined by:

$$\langle y(x,t) - \langle y(x,t)\rangle\rangle^2 = \left\langle y(x,t_1,t_2)^2\right\rangle - \langle y(x,t)\rangle^2 \tag{8}$$

which requires the second moment of the shoreline to be determined (see eg. Papoulis, 1987). The second moment function, with respect to time, is defined as:

$$\begin{aligned}\langle y(x,t_1).y(x,t_2)\rangle &= G^2(x,t)\langle\alpha(t_1)\alpha(t_2)\rangle\\ &= G^2(x,t)\big\langle\left(\langle\alpha(t_1)\rangle+\alpha^*\delta(t_1)\right)\left(\langle\alpha(t_2)\rangle+\alpha^*\delta(t_1)\right)\big\rangle\\ &= G^2(x,t)\left(\langle\alpha(t_1)\rangle\langle\alpha(t_2)\rangle+\alpha^{*2}\langle\delta(t_1)\delta(t_2)\rangle\right)\\ &= G^2(x,t)\left(\langle\alpha(t_1)\rangle\langle\alpha(t_2)\rangle+\alpha^{*2}\rho(\xi)\right)\end{aligned} \tag{9}$$

where $\xi = t_2 - t_1$, and the last line follows from the definition of the autocorrelation function. Substituting Eqs. (7) and (9) into Eq. (8) gives:

$$\langle y(x,t)-\langle y(x,t)\rangle\rangle^2 = G^2(x,t)\alpha^{*2}\rho(\xi). \tag{10}$$

The variance of the beach position at any alongshore point, x, and time t can be obtained from Eq. (10) by setting $t_1 = t_2 = t$, or $\xi = 0$. More generally, Eq. (10) gives us the autocorrelation function of the beach plan shape over time. For forms of autocorrelation function that are often used to describe physical processes (negative exponential or Gaussian forms), a common trait is that $\rho \to 0$ as $\xi \to \infty$. The characteristic timescale over which the autocorrelation function falls from a maximum to a small fraction is termed the 'correlation period'. In this case the correlation period will be of the same order as the typical storm duration. That is, the beach plan shape will, on average, show some similarity to itself over small periods of time, but that this similarity is expected to vanish as the period of time increases. The beach plan shape will thus exhibit some 'memory' of antecedent conditions dependent upon the autocorrelation properties of the wave angle. Put another way, sequences of 'storms' are anticipated to lead to chronology effects in beach plan shape over periods several times the duration of the storm grouping. Eq. (10) also indicates that the greatest variance in shoreline position is expected to be close to the groyne; this variance decreases as one moves away from the groyne; and the 'memory' of the beach shape is dependent on the autocorrelation of the incoming wave conditions. This provides the theoretical underpinning, in a special case, for the computational findings of Southgate (1995) and Dong and Chen (1999).

It is also interesting to compare these findings with the results of Reeve and Spivack (2004) who found, in the case of small angle waves impinging on a nourishment scheme, that any beach memory arose from the correlation in the diffusion coefficient which, in the majority of transport formulations, does not depend on wave direction. In the current case, the presence of a structure promotes the importance of wave direction in its vicinity, despite the small wave angle assumption.

In the above, 'storms' have been defined in terms of their direction only in order to make the analysis tractable. In reality, storms will be characterised by changes in wave period and wave height too. The influence of these additional parameters is the focus of the remainder of the paper.

4. Case study

The site chosen for our study is Aberystwyth, a town on the Welsh coast of the United Kingdom. Offshore wave conditions covering a period of four years have been furnished by Royal Haskoning and Aberystwyth City Council and correspond to hourly offshore hindcast wave heights, wave periods and directions at a point in a water depth of 32 m. Fig. 1 shows the location of the bay and the offshore point at which waves were hindcast.

The purpose of the case site is not to attempt to simulate in detail the beach at Aberystwyth, but rather: to employ the type and quality of data that might be used in the design; to transform this wave data to a shallow water, thereby allowing for changes in wave statistics that might be expected in real applications; to drive a semi-analytical model of the beach response near a groyne with time varying but uniform wave conditions and thereby investigate the statistics of the beach changes. This procedure includes the main wave transformation and beach response processes but specifically excludes the local tidal variations and localised diffraction effects that might lead to sharp changes in beach orientation very close to the groyne. In order to construct a continuous time

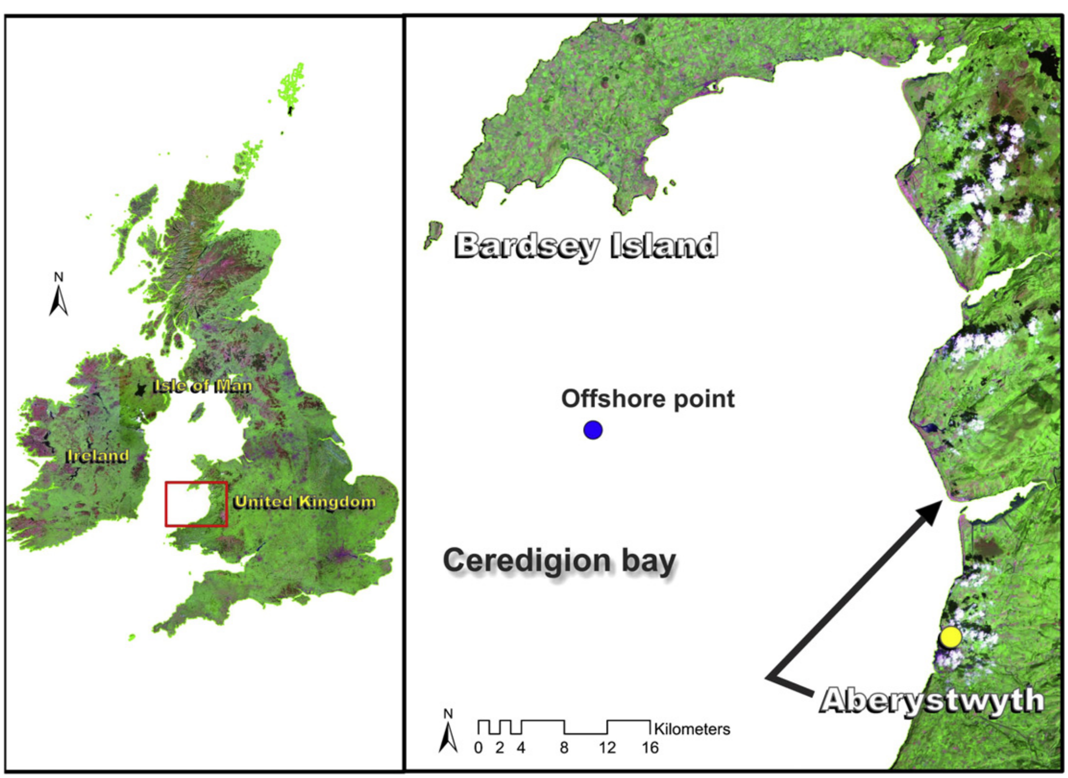

Fig. 1. Location of the case study and offshore wave climate point.

series of nearshore wave climate, the sequence of hourly offshore wave conditions were transformed inshore to a fixed water depth contour (2 m) using the wave transformation model SWAN (see Booij et al., 1999; Ris et al., 1999). In this process the mean water level (+2 m relative to the local datum), has been used for the SWAN computations. The choice of inshore location is arbitrary but is informed by (a) the need to use a contour that will evolve under the local wave conditions and (b) the largest inshore wave height in the transformed series is approximately equal to 1.6 m, which is close to the water depth times the breaker index of 0.78. A further assumption is required. Random waves will break at different points but the beach model requires the broken wave angle at the particular contour. This issue occurs whether you have an analytical or numerical model. The assumption is made that waves that have not broken by the 2 m depth contour will have comparatively little impact on the sediment transport and can be disregarded, while for waves that have already broken any change in direction that occurs between the breaker point and the chosen depth contour will be small. Bathymetric data was obtained from Admiralty charts, and interpolated onto a regular grid using kriging. The left panel in Fig. 2 shows the resulting bathymetry and the right panel shows a sample output illustrating the propagation of wave into the bay from westnorthwest.

A time series of wave conditions on the 2 m depth contour at Aberystwyth were constructed from the results extracted from the SWAN model output. Fig. 3 shows probability density functions (PDFs) for wave height (middle panel) and period (right panel), as well as the wave rose for the wave directions (left panel).

It is clear from this figure that the PDF of wave heights does not resemble a Rayleigh distribution, i.e. does not have a bell shape; a result of wave breaking. In contrast, the PDF for wave periods has an approximately bell-shape distribution.

5. Generation of realisations

In order to estimate ensemble averaged quantities from Monte Carlo simulation numerous wave sequences are required. The four years of hindcast waves are considered a single realisation of wave conditions. This can be used to generate more sequences with similar statistical properties, in order to generate an ensemble of solutions. Here, we employed the method proposed by Walton and Borgman (1990), further elucidated in Borgman and Sheffner (1991), to generate wave sequences with the same statistical properties as the input data set. The method consists of a piecewise, month-by-month, multivariate, stationary simulation approach, which preserves the marginal distributions and the first and second order moment properties that describe the intercorrelations of the data sequences. Seasonal changes are imposed by simulating each month separately based on the information of the original time series for each month and then a square-root interpolation scheme is carried out in order to force a smooth transition in the time series and intercorrelations from month to month. The procedure uses as a basis an empirical normal score transformation which maintains the first-order multivariate and higher order univariate moments of the data.

We have used this technique to generate 600 sequences, each with a length of 4 years, of wave height, period and direction which preserve the statistical properties of the original time series shown in Fig. 3. The marginal densities of wave height and period, as well as the wave rose, were computed for all the wave sequences, to check the statistical properties of the synthetic data. Fig. 4 presents an example of these results, where the wave rose and marginal densities of 5 synthetic time series are illustrated. Left panels show the wave rose and the middle and right panels show the probability density functions for wave heights and periods respectively. Comparing these plots against those obtained for the original data (Fig. 3), it is evident that the technique preserves the distributional properties reasonably well.

As an additional check on the simulated sequences, a comparison of the autocorrelation function of the detrended diffusion coefficient K was performed. Fig. 5 presents the results of this comparison at different time scales, from one year to four days. It shows that for all of these scales a reasonable reproduction of the temporal auto-correlation function is achieved. The annual cycle is captured as well as the e-folding time at shorter lags; although the shape of the autocorrelation function at very short lags (<10 h) is slightly more bell-like in the synthetic data.

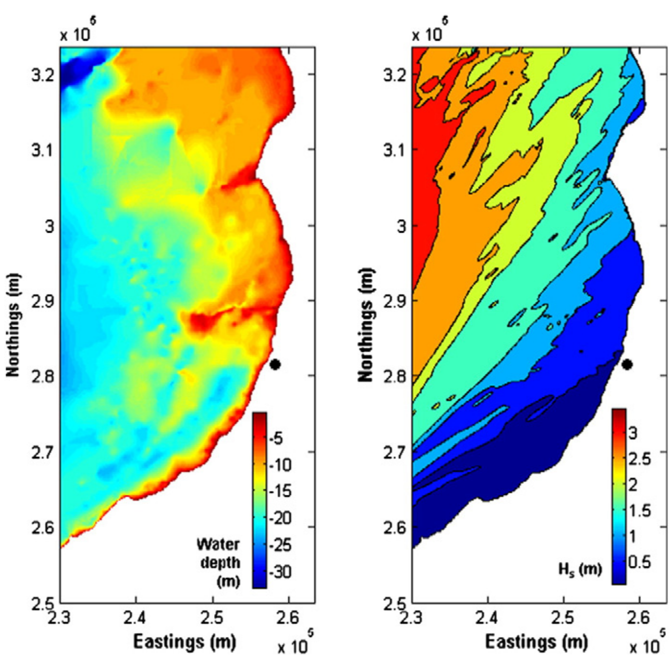

Fig. 2. Left panel—bathymetry for Ceredigion Bay in Wales, UK. Right panel—wave transformation sample output for waves approaching from WNW (the black dot represents the position of Aberystwyth).

6. Ensemble average of the semi-analytical solution

6.1. Hypothesis

We take as a hypothesis that, when wave height and period are allowed to vary in time as well as wave direction, the contributions to the beach response from wave direction are largely uncorrelated with wave height and wave period. If this is the case then the expression for the ensemble average beach position given in Eq. (4) will be well approximated by the expression in Eq. (11). That is, the ensemble average of the product of the terms in the integrand can be approximated as the product of the ensemble averages of the individual terms, thus:

$$\langle y\rangle \approx -\frac{1}{\sqrt{\pi}}\int_0^t \left\langle \left(\int_w^t K(u)du\right)^{-1/2} \exp\left\{-\left(\frac{x^2}{4\int_w^t K(u)du}\right)\right\}\right\rangle \langle K(w)\rangle\langle h(w)\rangle\, dw. \tag{11}$$

Equivalently, this is a statement that the cross-correlations between the terms are negligible. We introduce some terminology here to allow some abbreviation in the discussion. Eq. (4) can be written as

$$\langle y\rangle = -\frac{1}{\sqrt{\pi}}\int_0^t \langle T_1 \cdot T_2 \cdot T_3\rangle dw \tag{12}$$

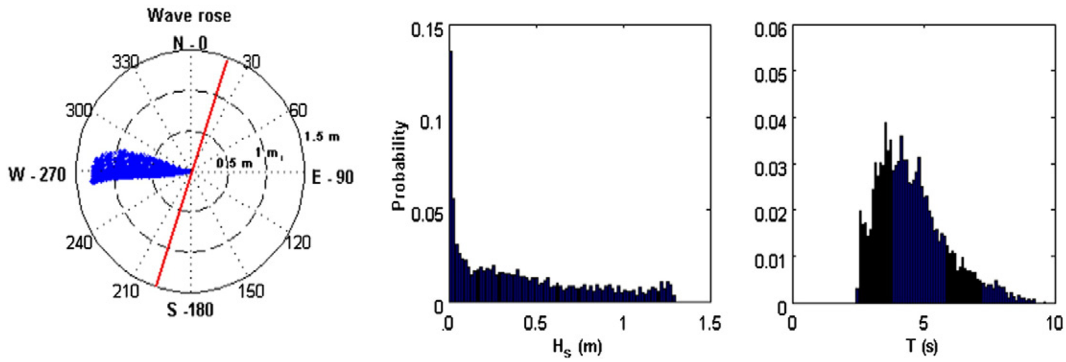

Fig. 3. Left panel—wave rose for transformed waves at the 2 m depth contour. Middle and right panels—corresponding probability density functions for wave height and period respectively.

where

$$T_1 = \left(\int_w^t K(u)du\right)^{-1/2} \exp\left\{-\left(\frac{x^2}{4\int_w^t K(u)du}\right)\right\}$$
$$T_2 = K(w)$$
$$T_3 = h(w)$$

and the hypothesis is that

$$\langle y\rangle \approx -\frac{1}{\sqrt{\pi}}\int_0^t \langle T_1\rangle\langle T_2\rangle\langle T_3\rangle dw. \quad (13)$$

6.2. Ensemble averages of key terms

For the purpose of testing the hypothesis we consider the situation where there is an impermeable groyne situated at $x = 0$ m on an

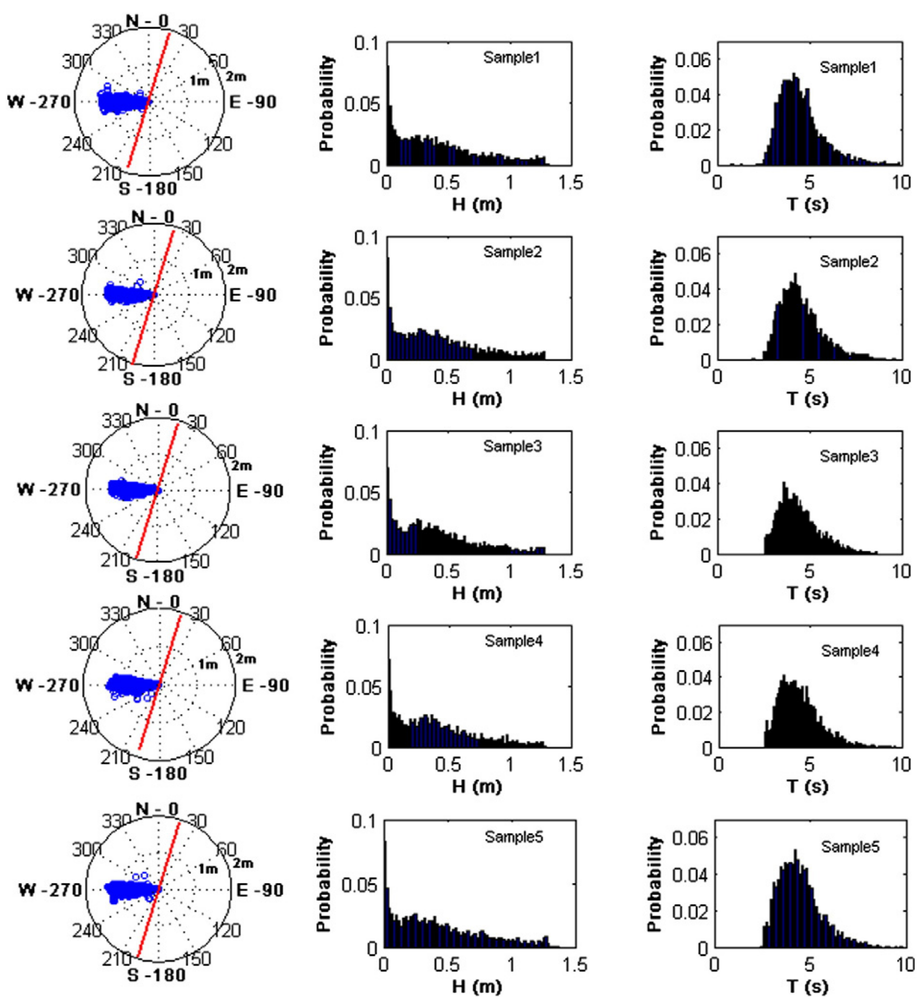

Fig. 4. Wave rose and probability distribution functions for wave angles, wave heights and periods for 5 samples of simulated data. Left panel—wave rose for directions; middle and right panels—probability distribution functions for wave height and period.

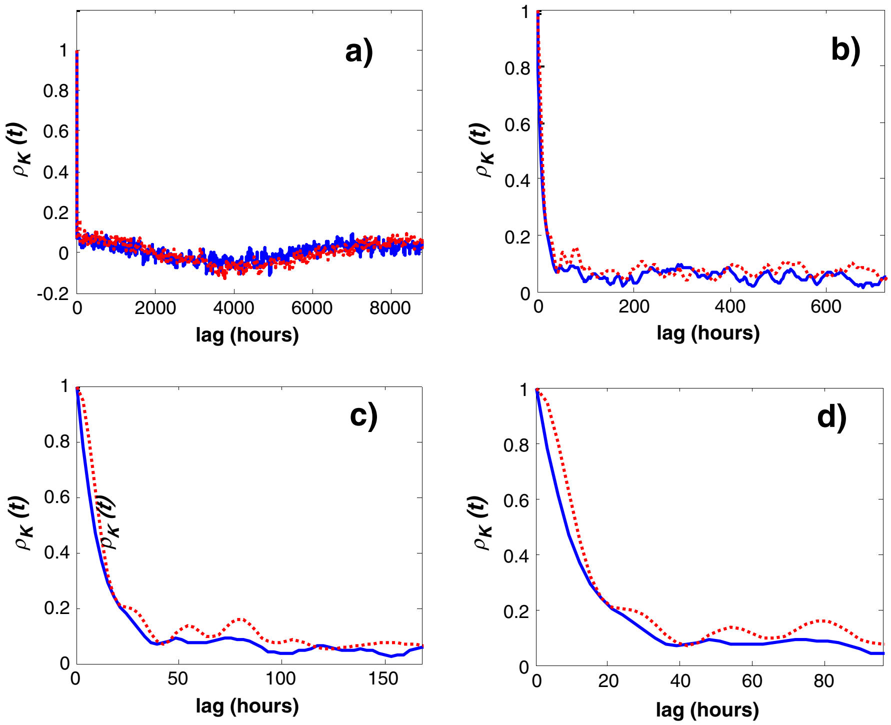

Fig. 5. Autocorrelation function of the detrended time series of K: (a) over ~1 year, (b) ~1 month, (c) ~1 week, and (d) 4 days; (solid line—calculated directly with the nearshore transformed time series; dotted line—calculated from the synthetic time-series using the Borgman & Sheffner technique).

otherwise straight beach coinciding with the line $y = 0$. For each realisation of wave conditions the beach evolution over a period of four year is computed using Eq. (3). In addition, for each realisation the individual terms T_1, T_2 and T_3 are stored for later processing.

One of the issues arising in Monte Carlo simulation is how to determine a suitable number of realisations from which to compute ensemble statistics. While there is some guidance from statistical sampling theory on the least number, there is not much guidance as to an upper limit on the number of realisations required that should be generated. We start by examining the behaviour of the ensemble averages of each of the terms T_1 and T_2 and T_3. This will provide information on the variability of each of these in the solution, as well as the convergence or otherwise of their ensemble average with respect to the number of realisations.

Fig. 6 is a colour-scale plot of the exponential term in T_1 over the first two years. The top panel shows this term for the first realisation, and

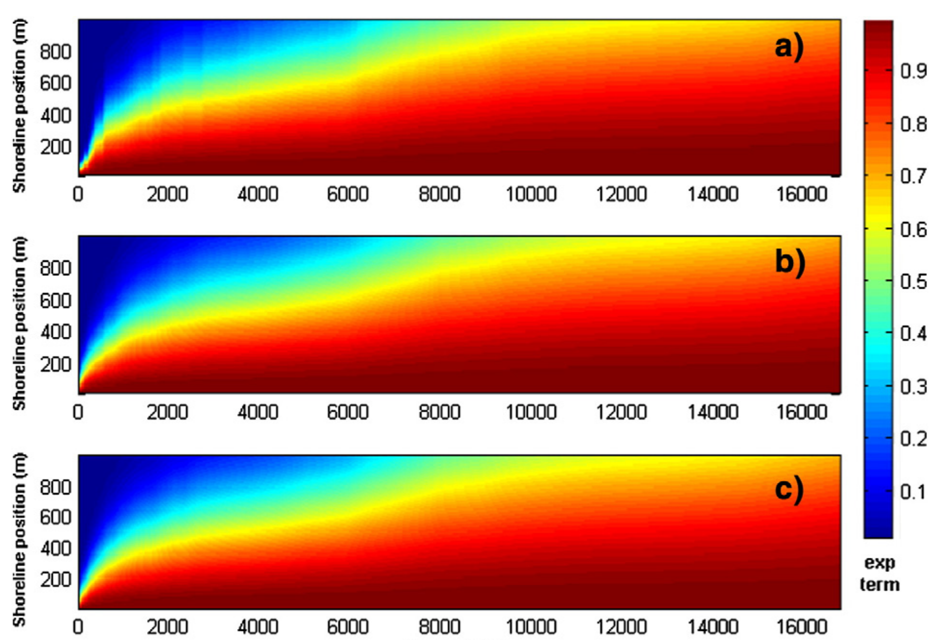

Fig. 6. (a) T_1 for one realisation; (b) ensemble average of T_1 over 10 realisations and (c) ensemble average of T_1 over 50 realisations.

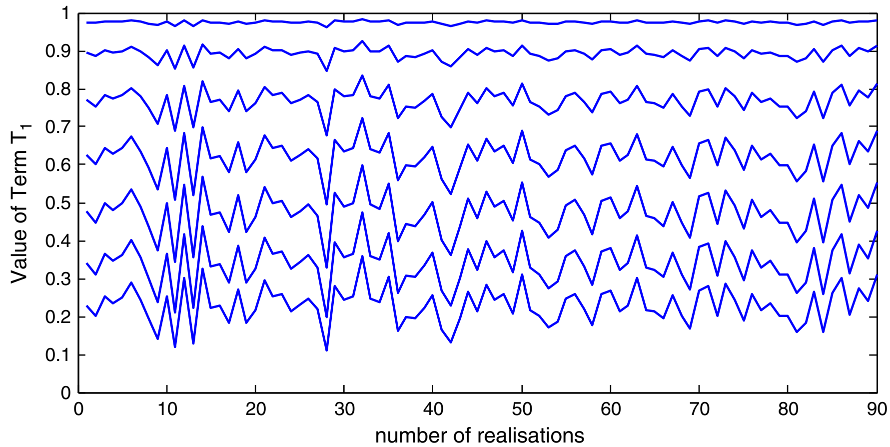

Fig. 7. Variability of the exponential term at $x = 100$ m (top line), 200 m (second from top), 500 m, 600 m, 700 m, 800 m and 950 m (lowest line), along the groyne after 1500 h for 90 separate realisations.

thus no averaging is involved; whereas the second and third panels show the ensemble averages computed over 10 and 50 realisations respectively.

From the examination of this figure, two features are clear. First, the general trend of this term is shown even for the single realisation result (top panel). Second, it is also evident that relatively few realisations are needed in order to compute the ensemble average of this term to a reasonable degree of accuracy; this is illustrated in the small differences between the middle and bottom panels of this figure. Fig. 7 shows the values of T_1 at several different points along the shoreline as a function of realisation number after 1500 h have elapsed. The selected positions are at 100, 200, 500, 600, 700, 800 and 950 m. No trend is expected as the realisation number increases as the realisations should be independent. However, when following each line the range in values with the realisation number gives an immediate indication of the variability in the value of T_1 at a particular position along the shoreline. A feature

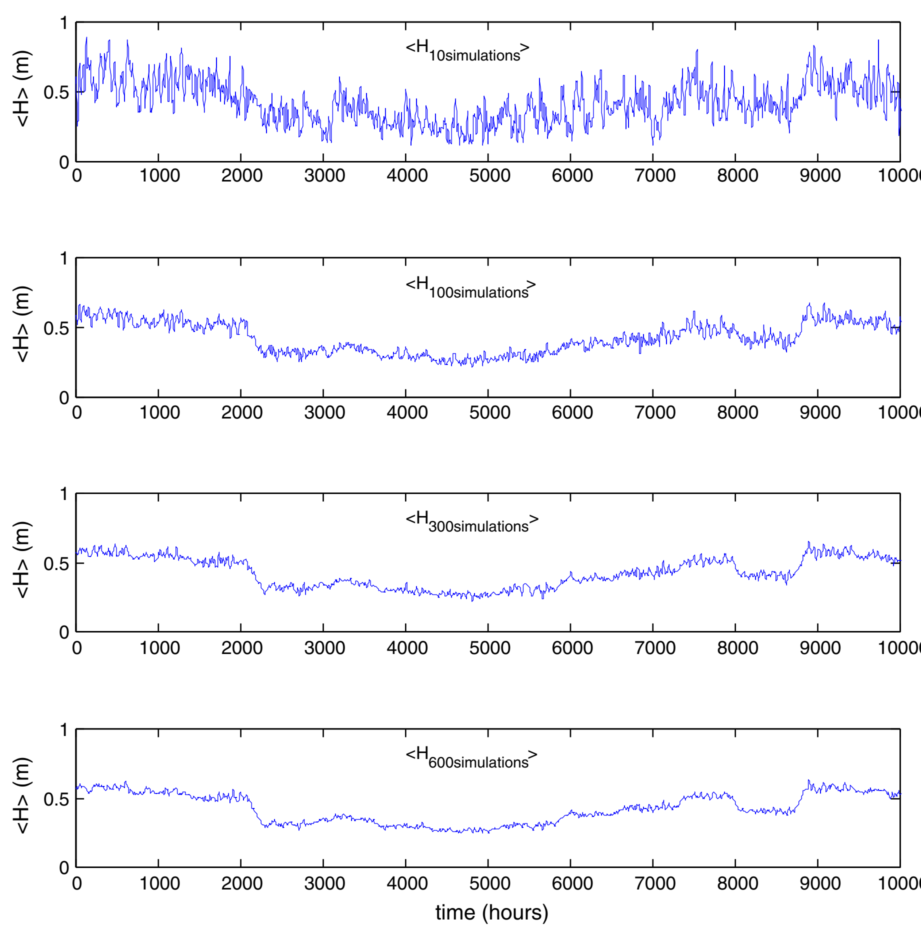

Fig. 8. Ensemble averages of H with a different number of realisations.

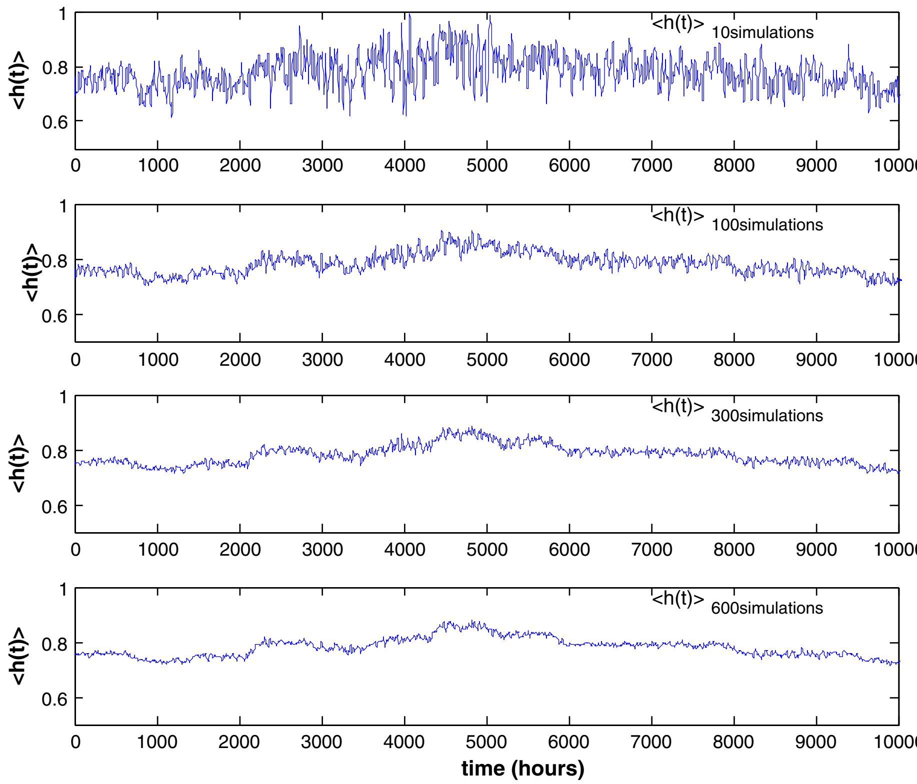

Fig. 9. Ensemble averages of $h(t)$ over 10,000 h computed with 10, 100, 300 and 600 realisations.

that is evident from the figure is that the range in value of T_1 is much larger in the vicinity of the groyne.

The second term, T_2, is the diffusion coefficient K, which is a function of the wave climate (H and T, and indirectly θ through the refraction processes). The trend in K is very similar to that of wave height due to the nature of the CERC transport formula. Sequences of the significant wave height have been determined for all the time series. Ensemble averages determined from 10, 100, 300 and 600 realisations are shown in Fig. 8 over a time period of 10,000 h ($\approx$14 months). The seasonal trend of the wave climate is evident in the bottom two panels, which correspond to the averages with 300 and 600 simulations. It is clear that oscillations shown in the top panel are smoothed as the number of realisations used to compute the ensemble average increases, which is as expected. Moreover, the general trend that is observed in the significant wave heights is achieved when the ensemble average is evaluated with 100 realisations.

The third term T_3, is a function of the wave angle. Ensemble averages of T_3 are illustrated in Fig. 9, which shows that a long term trend is identifiable in the ensemble average for 100 simulations and results using 300 and 600 realisations appear to be similar. However, the difference between seasons is not great.

6.3. Test of the hypothesis

In the previous sections the preparation of realisations of the full solution (Eq. (3)) and corresponding realisations of separate terms of the solution has been described. The hypothesis (Eq. (13)) can now be investigated by comparing the ensemble average of the full solution and the ensemble average of the approximate solution. Fig. 10 shows, in the top panel, the result of the ensemble average of the integrand $<T_1T_2T_3>$, calculated with 600 realisations. The bottom panel is the corresponding ensemble average, again using 600 realisations, calculated from the ensemble averages of each of the terms separately, that is, $<T_1><T_2><T_3>$.

To obtain the solution of the shoreline position requires an evaluation of the integral over time of the functions shown in Fig. 10, which

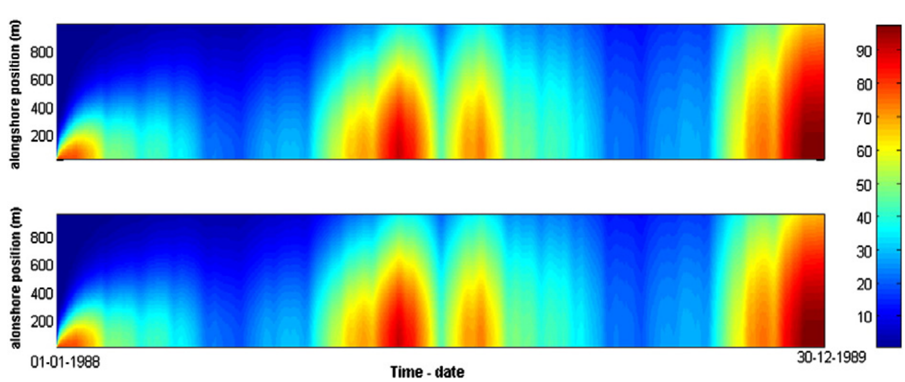

Fig. 10. The quantity $<T_1T_2T_3>$ (top panel); the quantity $<T_1><T_2><T_3>$ (bottom panel).

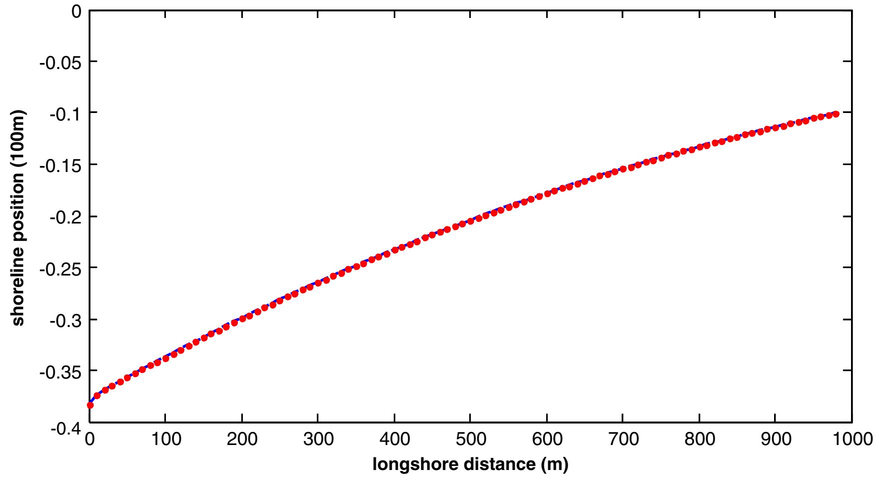

Fig. 11. Ensemble average of the solution estimated with Eq. (12) at the end of the four year period (solid line); ensemble average of the solution estimated with Eq. (13) at the end of the four year period (dotted line). Note: y-axis units are in units of 100 s of metres.

is performed numerically. Fig. 11 displays the mean shoreline position at the end of four years. This demonstrates that the results are very close, although there is a small underestimate of the mean shoreline position using the approximate form of the solution.

This result suggests that an assumption of no correlation between the terms in the integrand can provide a very good approximation to the full solution. A potential side benefit is that the ensemble averaging can be performed separately for each term in the integrand, thereby simplifying the calculation procedure. This also provides some opportunity to develop analytical solutions for idealised cases that could provide an independent check on Monte Carlo solutions obtained using numerical procedures. Further, as one of the terms, T_3, depends solely on wave direction it may be inferred that the lack of correlation arises largely due to the decoupling between wave direction on the one hand and the wave height and period on the other. That is, any temporal similarity in the wave conditions in terms of their height and/or period is not matched by like behaviour in wave direction.

7. Conclusions and discussion

The question of how wave chronology can affect beach shape, sometimes cast as whether our shorelines exhibit 'beach memory', has received continuing discussion in the literature. In this paper we have addressed this question from the perspective of treating the shoreline as a random variable, driven by random waves. Expressions for the ensemble average beach position and variance have been derived for a highly constrained situation in which waves vary with direction only. The solutions exhibit several features observed in the field or in numerical studies such as the large variability of beach position near groynes and temporal correlation in beach shape over the period of storm groups which reduces over longer periods.

To relax some of the restrictions of the analytical solution a semi-analytical solution for shoreline evolution under wave conditions in which wave height, wave period and wave direction all vary in time has been used as the basis for investigating the direct evaluation of the evolution of the ensemble average shoreline near a groyne. Site-specific wave conditions have been used to generate statistically similar wave sequences in order to create an ensemble of forecasts of beach evolution near a groyne over a four year period. The ensemble of forecasts has been used to investigate the contribution of different terms to the overall ensemble average beach plan shape. Terms involving the wave direction that arise principally from the presence of the groyne have negligible correlation with terms dependent on wave height and period. The lack of correlation suggests a statistical decoupling between the two sets of wave parameters. One explanation for this is the short autocorrelation period of wave direction.

Other constraints of the analytical solution, such as the treatment of diffraction, the small angle assumption and the influence of tides remain outstanding and efforts to remove the associated assumptions are the subject of continuing research.

Finally, the site chosen in this study is on an open coast, but sheltered from distant swell. For sites with a stronger temporal correlation in wave direction, such as those with restricted fetch directions or those dominated by swell, the level of decoupling would be expected to be less. For situations similar to the open coast site used in this study, the decoupling of the terms in the semi-analytical solution suggests that analytical solutions for ensemble quantities in a range of simplified situations can be developed in order to provide checks for Monte Carlo numerical models.

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