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A MODEL OF VOID FORMATION

YASMIN FRIEDMANN AND TSVI PIRAN

Racah Institute, The Hebrew University, Jerusalem, 91904, Israel; yasmin@merger.fiz.huji.ac.il; tsvi@nikki.fiz.huji.ac.il

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ABSTRACT

We introduce a simple model for the formation of voids. In this model the underdensity of galaxies in voids is the product of two factors. The first arises from a gravitational expansion of the negative density perturbation. The second is due to biasing: galaxies are less likely to form in an underdense region. One feature of the model is an upper cutoff in void sizes. We calculate the volume-filling factor of characteristic voids for different cold dark matter (CDM) models and find that our formation model points to Λ CDM models as preferred models of the power spectrum. A natural consequence of our model is that the underdensity of the dark matter inside voids is smaller than the galaxy underdensity.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

Visual inspection of redshift surveys has revealed (De Lapparent, Geller, & Huchra 1986; Kirshner et al. 1987; Geller & Huchra 1989; Da Costa et al. 1995; Geller et al. 1997; Shectman et al. 1996) that a large fraction of the universe is made of “voids,” regions in which the typical galaxy density is significantly lower than the mean galaxy density. Most galaxies tend to be found in two-dimensional sheets that encompass these voids. Using the VOID-FINDER algorithm, an automated algorithm that detects voids in three-dimensional surveys and measures their volume, El-Ad & Piran determined the sizes and depths of voids in several surveys: the Southern Sky Redshift Survey 2 (SSRS2), *IRAS*, and Optical Redshift Survey (ORS) (El-Ad & Piran 1997, 2000; El-Ad, Piran, & Da Costa 1997). They found that the void distribution is remarkably stable and that different surveys that encompass the same regions in the sky see the same voids. They also found that: (1) The voids occupy $\approx 50\%$ of the volume. (2) Void radii are in the range $13\text{--}30 h^{-1}$ Mpc. There appears to be an upper cutoff to the sizes of the voids. This upper limit does not depend on the properties of a particular survey or on the effective depth. This upper cutoff is also seen in visual inspection of the deepest survey existing today, the Las Campanas Redshift Survey (LCRS; Shectman et al. 1996), whose effective depth is $100 h^{-1}$ Mpc. (3) The density contrast of galaxies in voids is in the range $[-0.70, -0.95]$.

In current surveys, the sizes of observed structures are not much smaller than the effective depth of the survey. Therefore, only about a dozen voids have been identified so far. Hence, there are not enough data to produce a good statistics on the distribution of the voids’ sizes and depths. This situation will change with the new generation of automated redshift surveys, the Two-Degree Field (2dF; Lahav 1996) and the Sloan Digital Sky Survey (SDSS; Loveday 1996). With these surveys we will be able to identify dozens of voids and to quantify their features. With this situation in mind, we are beginning to set the ground here for analysis of these properties and for a comparison of the observations with a simple model for void formation.

The existence of significant inhomogeneities on the scale of tens of Mpc should be an important clue to the formation of large-scale structure, and can be useful in exploring the power spectrum on this scale range. Indeed, positive fluctuations on scales of 10 Mpc (clusters and superclusters)

provide a powerful tool for exploring the power spectrum (see Bahcall & Fan 1998). However, voids have not been used as yet, for several reasons. First, the present-day sky surveys are not comprehensive enough to allow a full quantitative assessment of void sizes and of the distribution function of void sizes. Second, there is no simple theory for the formation of voids.

A theory for the formation of voids should explain the physical mechanisms that operate in the formation of the voids. It should be able to explain qualitatively the appearance of the apparent upper cutoff on their sizes. Using this theory, one could compare the properties of voids (more specifically, voids sizes and filling factor) that arise from different primordial perturbation spectra with the observations. Another goal of such a model is to predict the underdensity of the dark matter within the voids and to provide us with a prediction of the effective biasing factor within the voids.

Blumenthal et al. (1992), Dubinski et al. (1993), and Piran (1997) considered a purely gravitational scenario for the formation of voids. Their model is based on the assumption that light traces matter on the scale of voids. In this case, the observed underdensity in the galaxy distribution corresponds to a comparable underdensity in the dark matter. According to this model, the observed voids today are primordial negative perturbations that grew gravitationally and reached shell-crossing today. Shell-crossing happens when the radius of the perturbation has grown by a factor of 1.7, corresponding to a density contrast of -0.8 . However, at this stage the perturbation is highly nonlinear (the corresponding linear amplitude would have been 2.7). Such a large amplitude requires too much power on the scale of voids and is inconsistent with the number density of clusters and superclusters on slightly lower scales.

In this paper we suggest a new approach that can be used as a formalism for analyzing voids. We present a simple intuitive model that describes the formation of voids as due to gravitational growth and biasing. We relax the assumption that light traces matter on these large scales and claim that the observed underdensity in the galaxy density is a product of two factors. The first arises from a simple gravitational expansion of the negative density perturbation. The second factor arises due to biasing: galaxies are less likely to form in an underdense region (see, e.g., Zel'dovich & Shandarin 1982). We consider spherical underdensities. This is,

quite generally, a good approximation, since negative density perturbations become more and more spherical as they evolve (Icke 1984; Lin, Mestel, & Shu 1965). To estimate the biasing factor, we use a simple peak biasing formalism developed by David & Blumenthal (1992) for the calculation of biasing in clusters.

Our model explains why voids appear in a relatively small range of sizes and in particular why there is an upper limit to the sizes of the voids. We use it to calculate the expected sizes and volume-filling factor of voids in different cosmological models, and we compare our results with current observations. The comparison is made to a simple interpretation of the data: voids occupy 50% of the volume, their radii are in the range of 13–30 h^{-1} Mpc, and the typical underdensity in the galaxy distribution is taken to be -0.8 . This is a simplified picture and should be modified in future when data from new surveys are available and when we have a more refined model. Finally, we use our model to calculate the expected dark matter underdensity within the voids.

We find that cosmological models that agree with other constraints on the power spectrum can in general produce voids with properties as described above, even if not as many as those observed. In particular, we find, in spite of the crudeness of the model and the uncertainties in present-day data, that flat Λ CDM with a current density parameter $0.25 < \Omega_0 < 0.35$ is the most preferable model, in agreement with other observations.

The paper is organized as follows. In § 2 we present the details of the model and a general calculation of the underdensity of galaxies inside voids. In § 3 we calculate the relative volume of the universe in the form of voids in universes characterized by different cosmological parameters and cold dark matter (CDM) power spectra. We discuss the implication of our results in § 4.

2. THE MODEL

We begin by calculating the dynamics of a negative density perturbation in a general cosmology. Our goal is to calculate η , the ratio of the comoving size of the perturbation to its initial comoving size, in terms of δ_L , the linear amplitude of the perturbation. The factor $\eta^{-3} - 1$ is the underdensity due to gravitational growth. Because it is expressed in terms of δ_L , it can be calculated directly from the linear power spectrum once the relevant scale is chosen. Then we turn to calculate the underdensity of galaxies in a larger scale negative density perturbation.

2.1. Gravitational Growth of Voids

During the linear phase, perturbations grow in amplitude but not in comoving size. As the perturbations become nonlinear, their comoving radius begins to grow. To find η we solve the differential equation that governs the evolution of a spherical shell surrounding a negative density region. At some initial time, t_i (at a redshift z_i), the shell is expanding at the same rate as the background (that is, we have an initial density perturbation). The initial small (negative) density contrast is δ_i , and the initial radius of the shell is R_i . The background evolution is characterized by the present values of the Hubble constant H_0 , the density parameter Ω_0 , and the cosmological constant Λ . For convenience, we define $\lambda_0 = \Lambda/3H_0^2$. As long as there is no shell-crossing, the mass inside the shell remains constant and energy conservation yields a differential equation for R , the shell's radius (Lahav

et al. 1991):

$$\dot{R}^2 = H_0^2[-\Omega_0(1+z_i)^3 R_i^2 \delta_i - (\Omega_0 + \lambda_0 - 1)(1+z_i)^2 R_i^2 + \Omega_0(1+z_i)^3 R_i^3(1+\delta_i)/R + \lambda_0 R^2]. \quad (1)$$

We combine this equation with the equation for the background's redshift:

$$\frac{1}{1+z} \frac{d(1+z)}{dt} = -H_0 P(z), \quad (2)$$

where

$$P(z) = \Omega_0(1+z)^3 - (\Omega_0 + \lambda_0 - 1)(1+z)^2 + \lambda_0, \quad (3)$$

to obtain an equation for dR/dz . We solve this equation numerically and obtain the radius R as a function of the redshift. Since the comoving radius increases as $1/(1+z)$, we find that $\eta = [R(z)(1+z)]/[R_i(1+z_i)]$.

The growing solution of the linear perturbation equation is given by Heath (1977):

$$\delta_L^+ = C P^{1/2}(z) \int_z^\infty \frac{(u+1)du}{P(u)^{3/2}}. \quad (4)$$

The constant C depends on the initial conditions. Since we have considered earlier an initial density perturbation, δ_i , the corresponding initial density contrast of the growing mode is $3\delta_i/5$.

We can now obtain the growth factor as a function of δ_L (we drop the superscript “+” hereafter). Figure 1 depicts η as a function of δ_L for three cases: an Einstein–de Sitter universe ($\Omega_0 = 1$, $h = 0.5$, $\lambda_0 = 0$), an open low-density universe without a cosmological constant ($\Omega_0 = 0.37$, $\lambda_0 = 0$), and a flat low-density model with cosmological constant ($\Omega_0 = 0.37$, $\lambda_0 = 0.63$). The function $\eta(\delta_L)$ is practically independent of the cosmological parameters, and the different curves overlap each other. For very small values of δ_L , when the perturbations are still linear, the growth factor η is very close to 1. This is expected, since in the linear theory perturbations grow in amplitude only. Here η increases only as the perturbation becomes nonlinear.

2.2. Biased Galaxy Formation in Voids

We turn now to the statistical determination of the underdensity of galaxies within voids. Following David &

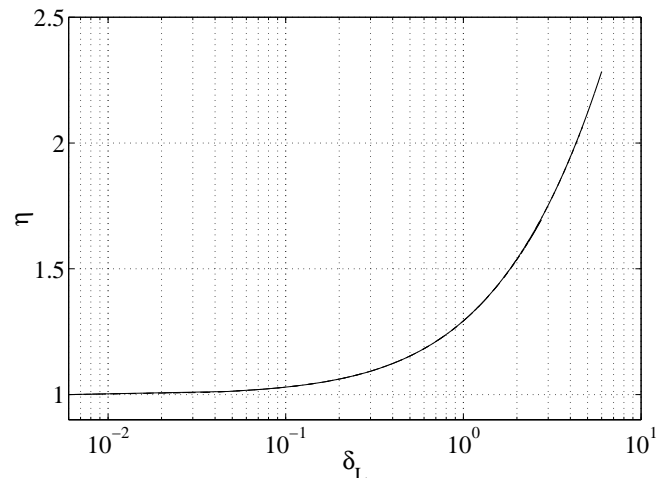


FIG. 1.—Growth factor, η , of a spherical perturbation as a function of the corresponding linear amplitude for three cases: an $\Omega_0 < 1$ open universe, $\Omega_0 + \lambda_0 = 1$, and a flat universe with $\lambda_0 = 0$.

Blumenthal (1992), we consider a simple model in which galaxies form in peaks that exceed a global galaxy formation threshold. We define the “efficiency” of galaxy formation in some volume V , ϵ^V , as the fractional volume of V that is contained in galaxies,

$$\epsilon^V = \frac{V_{\text{gal}}}{V_{\text{tot}}} \quad (5)$$

To determine ϵ^V , we use the conditional probability $f(v_g, v_v)$ of finding a galaxy-size fluctuation with a relative overdensity $v_g = \delta_g/\sigma_g$ within a void-size fluctuation with a relative underdensity $v_v = \delta_v/\sigma_v$. Here σ_g and σ_v are the rms mass fluctuations filtered on galaxy and void scales, respectively. The scale of a galaxy, R_g , is related to its mass, M_g (including the dark matter), through $M_g = (4\pi/3)R_g^3\langle\rho\rangle$. Then

$$f(v_g, v_v) = \frac{1}{2\pi\sqrt{1-r^2}} \times \exp\left[-\frac{1}{2\sqrt{1-r^2}}(v_g^2 + v_v^2 + 2rv_g v_v)\right], \quad (6)$$

where r is the correlation coefficient between the two scales, given by $\sigma_{gv}^2/\sigma_g\sigma_v$, where

$$\sigma_{gv}^2 = \frac{1}{(2\pi)^3} \int d^3k |\delta_{\mathbf{k}}|^2 W(kR_g)W(kR_v), \quad (7)$$

and $W(kR)$ is a window function. We choose a top-hat window function,

$$W(kR) = \frac{3(\sin kR - kR \cos kR)}{(kR)^3}. \quad (8)$$

We use a single typical galaxy mass of $M_g = 1.2 \times 10^{12} M_\odot (M/L)/(100 M_\odot/L_\odot)$, which is the median of the galaxy luminosity function. This is clearly an approximation, and possibly the crudest one we make in this work. The scale related to this mass, R_g , varies according to the cosmological parameters of the model. Assuming that only the peaks that exceed a global threshold v_{th} become luminous galaxies, the efficiency of galaxy formation in voids of a given radius R and with a given v_v is

$$\epsilon^{\text{void}}(v_v, R) = \frac{\int_{v_{\text{th}}}^{\infty} f(v_g, v_v) dv_g}{\int_{-\infty}^{\infty} f(v_g, v_v) dv_g} = \frac{1}{2} \operatorname{erfc}\left[\frac{v_{\text{th}} + rv_v}{\sqrt{2(1-r^2)}}\right]. \quad (9)$$

The galaxy formation threshold, v_{th} , is calculated using the global efficiency of galaxy formation,

$$\epsilon^{\text{bg}} = \epsilon^{\text{void}}(R_v = \infty) = \frac{1}{2} \operatorname{erfc}\frac{v_{\text{th}}}{\sqrt{2}}. \quad (10)$$

Empirically, one possible way of determining the fraction of mass residing in galaxies is to divide the mass-to-light ratio of a typical galaxy by the mass-to-light ratio of the universe. Following Bahcall, Lubin, & Dorman (1995), we take the M/L ratio of the universe to be $1350\Omega_0 h$, and that of a typical galaxy to be $100h$. Now we have

$$\frac{1}{2} \operatorname{erfc}\frac{v_{\text{th}}}{\sqrt{2}} = \frac{(M/L)_{\text{gal}}}{1350\Omega_0 h}. \quad (11)$$

Using equation (11) we can determine, for any given Ω_0 , the global galaxy formation threshold.

2.3. The Combined Underdensity

The current underdensity of galaxies in voids, δ_{gal} , is

$$1 + \delta_{\text{gal}} = \frac{\rho_{\text{gal}}^{\text{void}}}{\rho_{\text{gal}}^{\text{bg}}} = \frac{\epsilon^{\text{void}}}{\epsilon^{\text{bg}}\eta^3} \quad (12)$$

where $\rho_{\text{gal}}^{\text{void}}$ is the density of galaxies in the void, and $\rho_{\text{gal}}^{\text{bg}}$ is their average density in the background universe. The second equality holds once the growth factor of the void is taken into account, and all the galaxies are taken to be of the same typical scale ($\approx 1 h^{-1}$ Mpc).

3. THE VOID CONTENT OF THE UNIVERSE IN DIFFERENT CDM MODELS

Given the model described above, we now calculate the expected sizes and the volume-filling factors of voids in different cosmological models. We also calculate the dark matter underdensity in voids in these models. Our aim is to find the dependency of the filling factor on cosmological parameters. A second goal is to predict the dark matter underdensity in voids and through this to learn the biasing between dark and luminous matter on these large scales.

We consider first the SCDM model ($\Omega_0 = 1$, $h_0 = 0.5$, $\Lambda = 0$, $n = 1$, $\Omega_b = 0.0125 h^{-2}$). It is already established that this is not a valid model of the universe; it does not agree simultaneously with *COBE* and with cluster abundance data. However, because of the simplicity of the SCDM model, we use it as a tool to demonstrate how the void content of the universe changes with the normalization of the power spectrum. We use the transfer function calculated by Bardeen et al. (1986) as the shape of the dark matter power spectrum. For the normalization we consider two possibilities: *COBE* normalization as calculated by Bunn & White (1997) ($\sigma_8 = 1.27$) and cluster abundance normalization as given by Pen (1998) ($\sigma_8 = 0.53$).

We present contour lines of constant galaxy underdensity as a function of the radius of the voids today, R , and the relative underdensity in the dark matter, v . Figure 2 depicts several contour lines for the cluster-normalized SCDM model. If we look at constant v , we find that there are

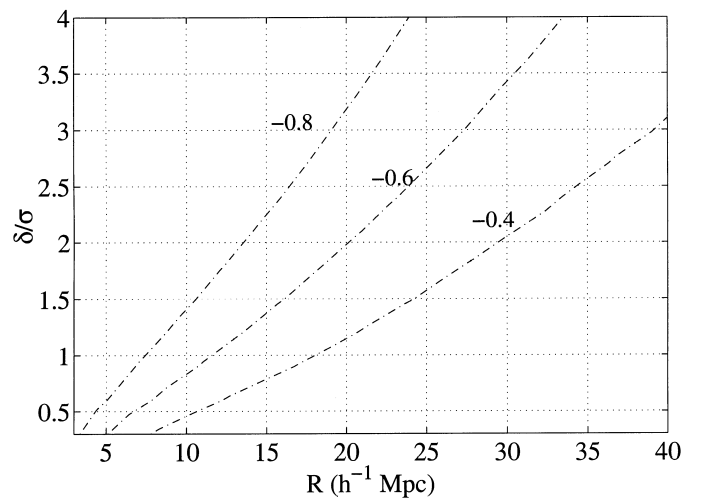


FIG. 2.—Contours of constant δ_{gal} displayed as a function of the radius of the voids today, R , and the relative underdensity of the dark matter, v , in a standard CDM model that is cluster-normalized. We can see that the typical voids ($R = 20 h^{-1}$ Mpc $\delta_{\text{gal}} = -0.8$) are 3σ objects, i.e., they are produced very rarely.

relatively more galaxies in larger voids. This is due in part to the statistical properties of the fluctuations and in part to the gravitational expansion of the underdensities. At larger scales the amplitude of the perturbations is smaller, so to form a galaxy in a larger underdensity we need galaxy-size perturbations of smaller amplitude. These will be more abundant because the distribution function of the fluctuations is a Gaussian. Thus, there will be more galaxies in larger voids, and the relative underdensity of the galaxies will decrease. The gravitational expansion factor does not compensate for this; in fact, it becomes less important because δ_L of the underdensities decreases as R grows (see Fig. 1). If we look at voids of constant radius, we see that there are relatively fewer galaxies at larger ν . The contribution to this behavior is also twofold. Negative perturbations of higher ν correspond to deeper voids; in such voids we need galaxy-size perturbations of larger amplitude to form galaxies. These are less abundant and therefore the relative underdensity of the galaxies is larger in deeper voids. To this we add the fact that negative perturbations of higher ν are of higher δ_L , and for these the gravitational factor is larger. Thus, the volume of the void will grow and the relative underdensity in the galaxy distribution will be even greater.

An important feature to notice in this figure is that for a given underdensity of the galaxy distribution inside a void, larger voids are produced exponentially more rarely, since they require large and hence extremely rare initial perturbations. Thus, there is a sharp upper limit to the sizes of voids.

To compare with observations, we calculate the filling factor of the voids. We calculate the fraction of the universe that is composed of spherical and isolated negative density perturbations that are large enough and deep enough to produce voids of radii 13–30 h^{-1} Mpc. The number of spherical inhomogeneities of a radius R and an amplitude in the range $[\delta, \delta + d\delta]$ inside the horizon is

$$N(R, \delta)d\delta = \frac{c^3 H_0^{-3}}{R^3} \frac{1}{(2\pi\sigma_R^2)^{1/2}} e^{-\delta^2/2\sigma_R^2} d\delta. \quad (13)$$

Clearly, the isolated spherical approximation would break down at low ν values, and it might be violated around the lower limits ($\nu \sim 1.5$) of our integration. We expect it to hold at higher values. The total volume of the corresponding voids is $(\eta R)^3 N(R, \delta)$, and the relative volume is

$$f(R, \delta)d\delta = \frac{\eta^3}{(2\pi\sigma_R^2)^{1/2}} e^{-\delta^2/2\sigma_R^2} d\delta. \quad (14)$$

The voids with radii 13–30 h^{-1} Mpc correspond to initial fluctuations of sizes of about 10–25 h^{-1} Mpc, depending on the model. Thus, to obtain the overall filling factor, denoted by f , we integrate equation (14) along the contour of $\delta_{\text{gal}} = -0.8$ in the appropriate range. This method of counting might be complicated by the possibility of overcounting: a void of a certain radius and amplitude might be counted again as a void of larger radius and smaller amplitude. This cannot happen if the underdensity δ increases with R , since the larger void would be deeper. We therefore checked the behavior of δ , and found that it increases monotonically with R .

We have carried out this calculation for two CDM models with different normalizations. The contour lines of $\delta_{\text{gal}} = -0.8$ of the two models are presented in Figure 3.

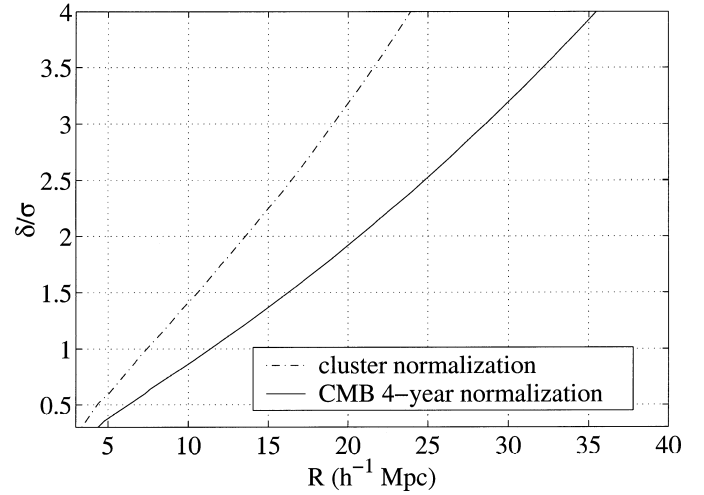


FIG. 3.—Two contours of $\delta_{\text{gal}} = -0.8$: one for cluster-abundance-normalized SCDM (*dashed line*), the other for *COBE*-normalized SCDM. The effect of the different normalizations is very obvious: when the power on the scale of voids is higher (*COBE*), the observed voids are much more frequent.

The difference between the two models is very pronounced: the model with more power on the scale of the voids (*COBE* normalized) yields more voids. This can be explained as follows: when the power on the scale of the voids is larger, the amplitude needed to produce the voids that we see today is reached by fluctuations with lower ν , which are therefore more frequent. In models with less power on void scales, the same amplitude of underdensities requires higher ν values and are therefore less frequent. This is reflected in the calculated values of the filling factors of $f = 33\%$ for the *COBE*-normalized model, and only 11% for the cluster-normalized model.

However, neither of these CDM models is an acceptable model of the universe. We now estimate the void content of the universe in the context of power spectra that are compatible with observations. We consider open CDM and flat Λ CDM models. The transfer function used is as above (Bardeen et al. 1986). The normalization is according to the 4 yr *COBE* DMR experiment, as calculated by Bunn & White (1997). To determine the models' parameters, we first set Ω_0 , then choose a tilt such that the model is also cluster normalized. This is done by calculating σ_8 and finding a tilt such that the normalization condition given by Pen (1998),

$$\sigma_8 = (0.53 \pm 0.05)\Omega_0^{-0.45} \quad (15)$$

for open models and

$$\sigma_8 = (0.53 \pm 0.05)\Omega_0^{-0.53} \quad (16)$$

for flat models, is satisfied. We take Hubble's constant to be $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$, in agreement with recent results from the *Hubble Space Telescope* (*HST*) Key project ($H_0 = 71 \pm 6 \text{ km s}^{-1} \text{ Mpc}^{-1}$; see Mould et al. 2000) and measurements of the time delay between multiple images of gravitational lens systems ($H_0 = 69^{+13}_{-19} \text{ km s}^{-1} \text{ Mpc}^{-1}$; see Biggs et al. 1999); $\Omega_b h^{-2} = 0.015$ in all models. The models are described in Table 1. Columns (1) and (2) of the table give Ω_0 and n . In column (3) we list σ_8 , the amplitude of mass fluctuations in spheres of radius 8 h^{-1} Mpc. All σ_8 are within the ranges allowed by the cluster normalization. As another check on the validity of our models, we show that

TABLE 1
LIST OF THE MODELS CONSIDERED

Model (1)	Ω_0 (2)	n (3)	σ_8 (4)	Γ (5)	f (%) (6)	δ_{DM} (7)
Open CDM						
1	0.3	1.3	0.92	0.18	19	-0.56
2	0.35	1.17	0.85	0.21	18	-0.53
3	0.4	1.07	0.81	0.24	18	-0.52
4	0.45	0.98	0.76	0.27	18	-0.49
Λ CDM (flat)						
5	0.2	1.2	1.2	0.11	31	-0.60
6	0.25	1.1	1.11	0.15	29	-0.56
7	0.3	1	0.95	0.18	25	-0.53
8	0.35	0.96	0.93	0.21	25	-0.52
9	0.4	0.91	0.86	0.24	24	-0.50
10	0.45	0.88	0.83	0.27	22	-0.49

the shape parameter Γ of each of the models is within the limits ($0.15 < \Gamma < 0.3$) allowed by measurements of the angular correlation function from the Automated Plate Measuring (APM) galaxy survey (Efstathiou, Bond, & White 1992). The values of Γ are listed in column (4). Finally, column (5) gives the calculated filling factor, and column (6) lists the calculated dark matter underdensity.

As before, we present the results as contour lines of constant δ_{gal} as a function of the radius of the voids today, R , and as a function of their relative underdensity in the dark matter, v . The contour lines for the open and Λ CDM models are presented in Figures 4 and 5, respectively. We note first that all the models show a common behavior, which was also manifested in the SCDM models: larger voids of $\delta_{\text{gal}} = -0.8$ are produced exponentially more rarely. The sharp upper limit to the sizes of voids exists in all CDM models.

Figure 4 describes voids of $\delta_{\text{gal}} = -0.8$ in open CDM models. It is clear that the void distribution does not depend strongly on Ω_0 . The filling factor is almost constant,

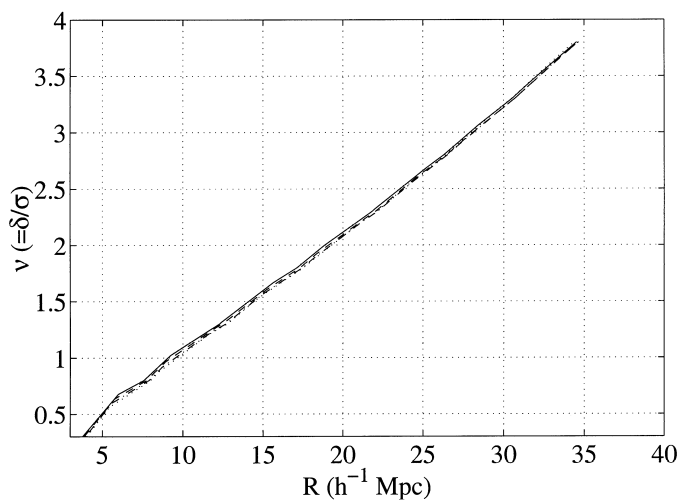


FIG. 4.—Contours of $\delta_{\text{gal}} = -0.8$ in the four open CDM models checked, displayed as a function of the radius of the voids today, R , and relative underdensity, v . It is clear that in open models, the distribution of the voids is not a function of Ω_0 . This can be explained by the fact that the power spectra for these models are almost the same on all scales.

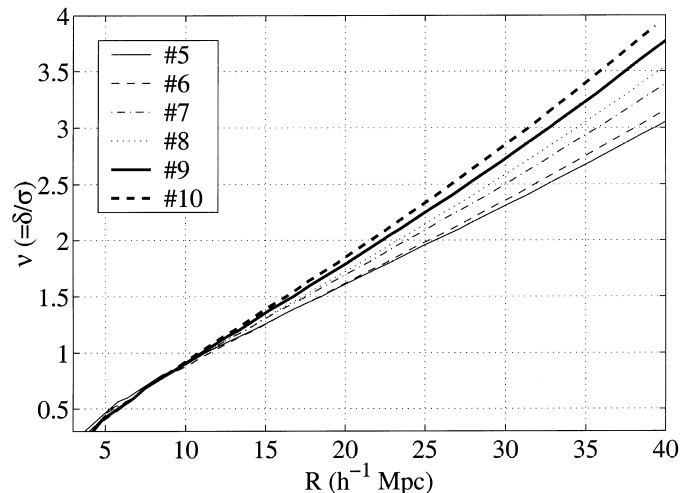


FIG. 5.—Contours of $\delta_{\text{gal}}(v, R) = -0.8$ in the six Λ CDM models as a function of the radius of the voids today, R , and of the relative underdensity of the void, v . Here we see a stronger dependence on Ω_0 , and we also see that voids are more frequent in these models than in the open models.

having values of 18%–19%. Figure 5 describes the same voids in flat Λ CDM models. Here there is a stronger dependence of the void distribution function on Ω_0 , and the filling factor is larger than in the open models. It is in the range 22%–31%, decreasing with Ω_0 .

We have also calculated the expected underdensity of the dark matter. This underdensity is given simply by $\delta_{\text{DM}} = \eta^{-3} - 1$. It is listed, for $20 h^{-1}$ Mpc voids in the different models, in column (6) of Table 1. The underdensities are in the range $[-0.5, -0.6]$ for all the models. These typical values are a factor of 1.3–1.6 smaller than the galaxy underdensity, indicating this factor as the biasing between galaxies and dark matter perturbations on the $20 h^{-1}$ Mpc scale within the voids.

4. DISCUSSION

We have presented here a model for the formation of voids. In this model voids arise from initial negative density perturbations. Such underdensities grow in comoving volume, and this growth increases the underdensities of both the galaxies and the dark matter within the voids. The galaxy underdensity is enhanced further because positive galaxy-size perturbations are less frequent within negative void-size perturbations. This mechanism inhibits the formation of galaxies within the voids. In our model, both mechanisms contribute comparable factors to the overall galaxy underdensity.

We use the model to investigate the void content of the universe for different power spectra that are in agreement with *COBE* and cluster abundance data. Qualitatively, we found in all the cosmological models we tested that the probability of finding voids of a certain δ_{gal} falls exponentially with the radius. This behavior may explain the observed upper limit of the radii of voids.

In order to quantitatively test our model, we have calculated the filling factor of the observed voids ($R_v \in [13-30] h^{-1}$ Mpc, $\delta_{\text{gal}} = -0.8$). We find that in all the models that we considered, the observed voids fill only half the expected volume. However, there is a clear trend toward higher filling factors in Λ CDM models, where the relevant voids appear

more frequently and fill a larger fraction of the universe. We also found that in the open models that we have tried, since the power spectra were very similar, the distribution of the void sizes and the filling factor did not change with Ω_0 . However, in Λ CDM models, as Ω_0 grows the relevant voids become less frequent and the void content of the universe decreases. This can be explained by the fact that as we increase Ω_0 , the amplitude of fluctuations on the scale of voids is decreased. The most preferable models are the Λ CDM models with $0.25 < \Omega_0 < 0.35$; these comply with all the constraints and have the highest void-filling factors. Still, even these values fall short of the observations by a factor of ≈ 1.4 .

We suspect that the small filling factor is due in part to the oversimplified model of galaxy formation that we have used. A more realistic model should allow for a range of galaxy masses and a more elaborate biasing mechanism between the dark matter and galaxies. This will be the next step toward a more reliable model. Note also that, as already mentioned, another important assumption of our model is spherically symmetric isolated evolution. We have assumed that the underdensities are spherical and isolated when calculating the gravitational growth and the filling factor, ignoring possible mergers between neighboring voids and the influence of positive overdensities on nearby underdensities. Void mergers might lead to the disappearance of smaller voids with deeper underdensities alongside the appearance of larger asymmetric voids. Positive nearby overdensities could exert forces on matter inside underdensities and increase their growth rate. Both effects could increase the filling factor of voids.

Finally, we have computed the underdensity of dark matter in typical voids of radius $20 h^{-1}$ Mpc. While the dark matter is influenced only by the gravitational expansion of the negative density perturbations, the number of galaxies is also influenced by the biasing factor. For this reason, we have $|\delta_{\text{DM}}| < |\delta_{\text{gal}}|$. The expected dark matter underdensities that we find are about a factor of 1.3–1.6 lower than the underdensities of the galaxy density. These values should be regarded only as an upper limit to the real underdensity expected in nature. Since real voids are more frequent, they must correspond to lower ν values, and their

gravitational growth factor would be smaller. This will result in a less negative dark matter density contrast. This prediction should be compared with estimates of the dark matter density in voids from N -body simulations and with future measurements of the dark matter underdensity within the voids.

It will be interesting to apply our model to account for the evolution of void sizes and abundances as a function of redshift. We suspect that in critical-density universes, the evolution of voids will be stronger than in low-density universes; we have shown in § 2 that the growth of the radius of the void depends only on the linear amplitude of the perturbation and not on cosmological parameters. Thus, in a universe with $\Omega_0 = 1$, where the linear amplitude grows like the scale factor, the radius of the void will grow constantly. However, in models where matter ceases to dominate, such as open models that become curvature-dominated at small z or flat models with a cosmological constant that begins to dominate at late times, the linear amplitude reaches a constant value and stops growing. In such cases, the comoving radius of the voids will also stop growing at late times. Thus, we could use the model to predict the change in comoving radius of voids as a function of z in different cosmological models, and by comparing to the next generation of deep sky surveys discriminate between low- and critical-density models (for example, in critical-density universes, older voids will be smaller in radius, and the galaxy underdensity in them will also be smaller).

Upcoming sky surveys, such as the Sloan Digital Sky Survey, will increase the available galaxy distribution data by several orders of magnitude. In particular, such surveys will include more voids, and we hope enough voids to obtain the distribution and evolution of the void sizes. That would allow us to compare our model with observations in a more accurate way and constrain Ω_0 and other cosmological parameters using the void distribution.

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